



7th Galileo Xu-Guangqui meeting  
Pescara — July 11, 2025



# Exploiting galaxy clusters as powerful cosmological probes through strong gravitational lensing

Giorgia Di Rosa

PhD student @ University of Ferrara

M. Lombardi, P. Bergamini, C. Grillo, P. Rosati, L. Bazzanini,  
A. Acebron, M. Meneghetti, D. Abriola, S. Schuldt, M.  
Fogliardi, G. Angora, A. Mercurio, E. Venzella et al.



# GRAVITATIONAL LENSING

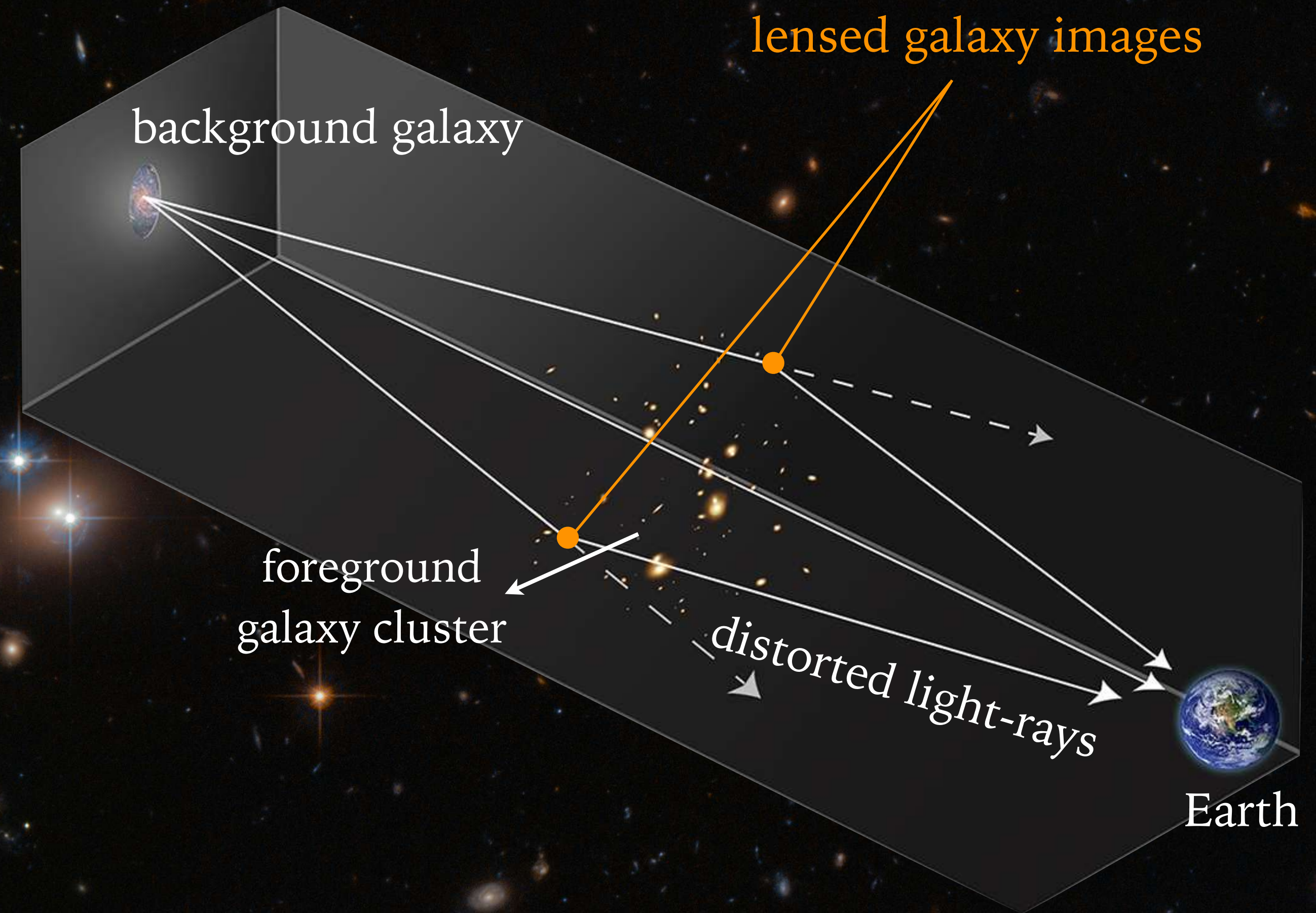
One of the most powerful means to investigate how matter is distributed within different scales of the Universe

**Strong Lensing (SL)** occurs if:

- source, lens and observer are closely aligned
- the lens is sufficiently dense (supercritical) → galaxies and central region of **galaxy clusters**

Effects:

- production of distinct and resolvable multiple images of the same background source
- amplification, magnification and distortion of the multiple images
- **time delays** between multiple images





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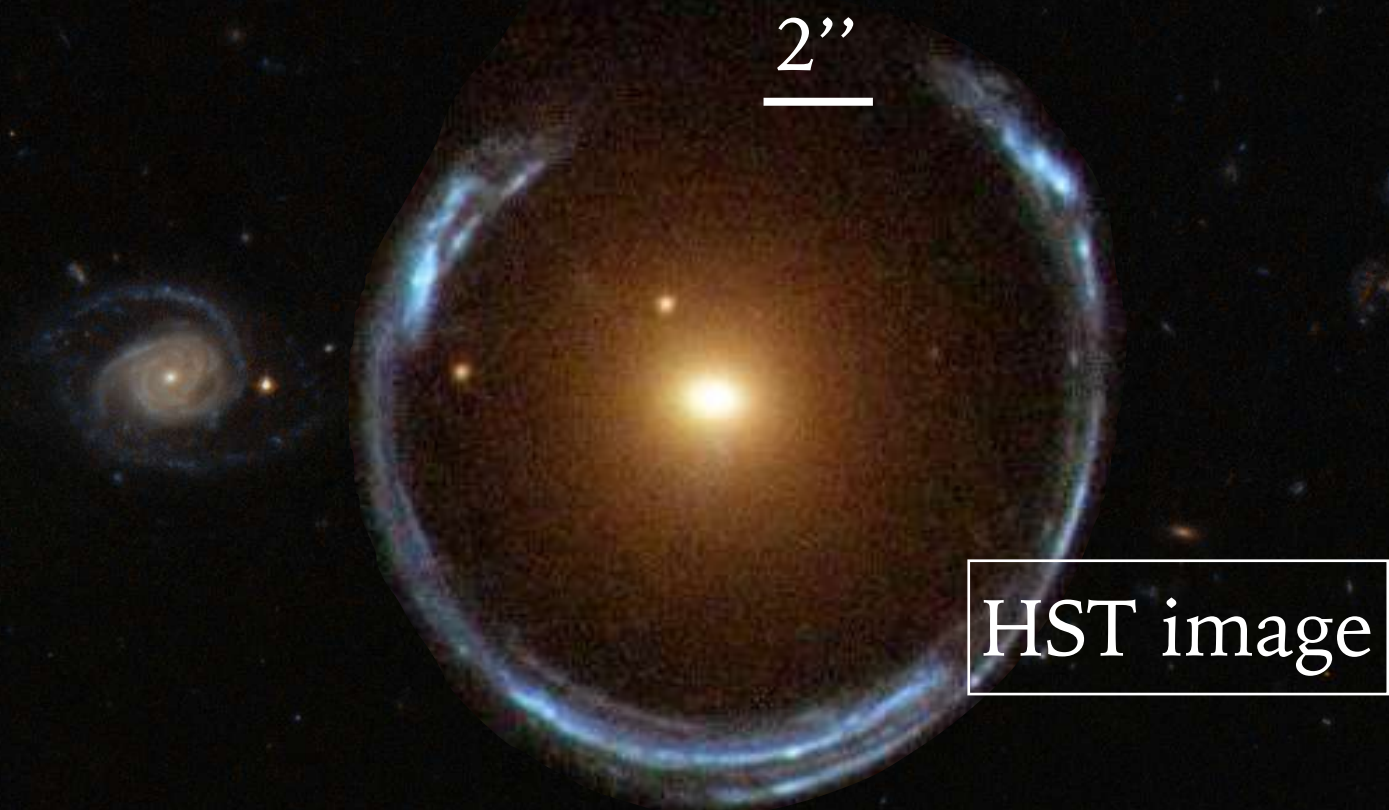
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Einstein ring



Einstein cross



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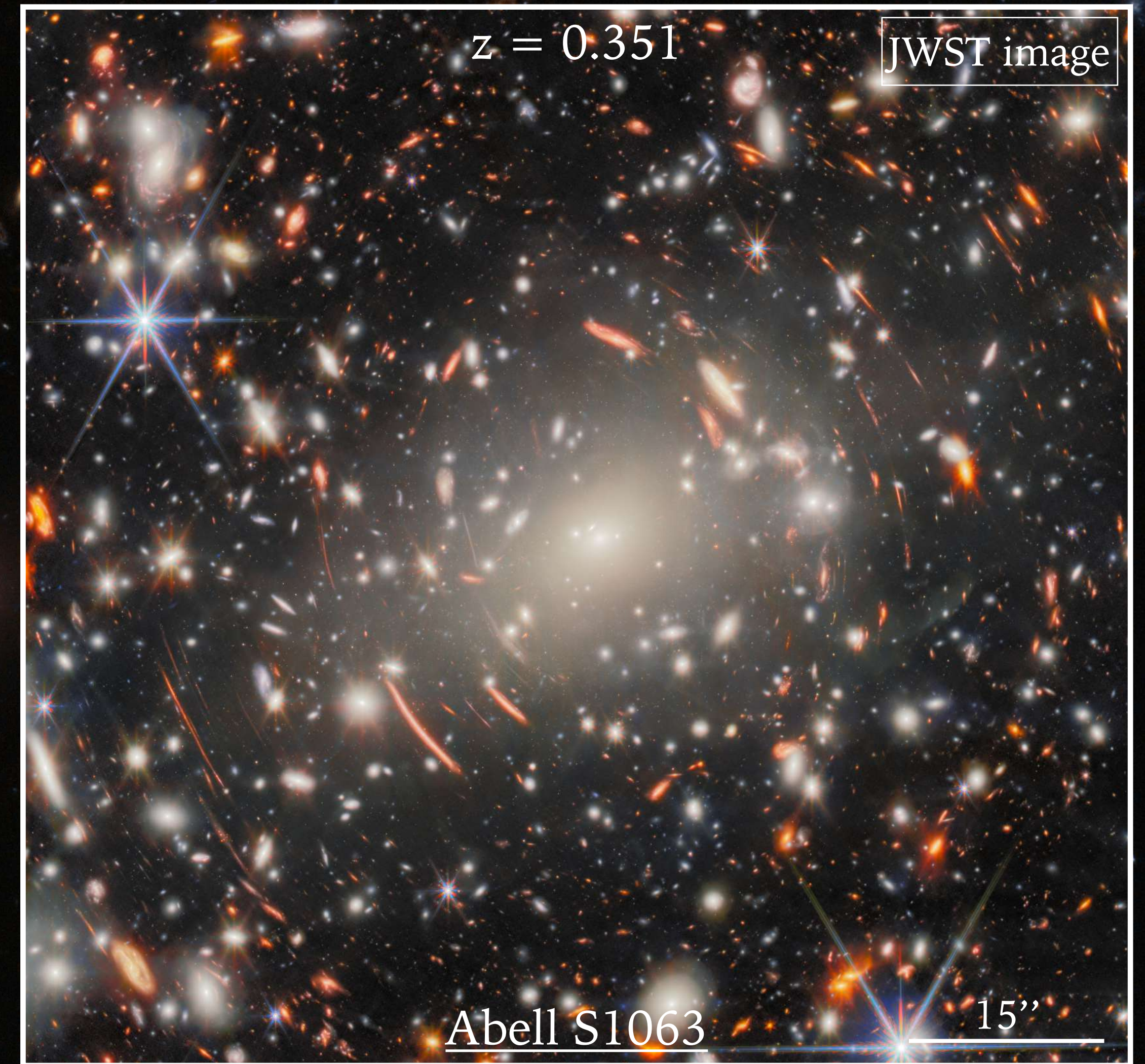
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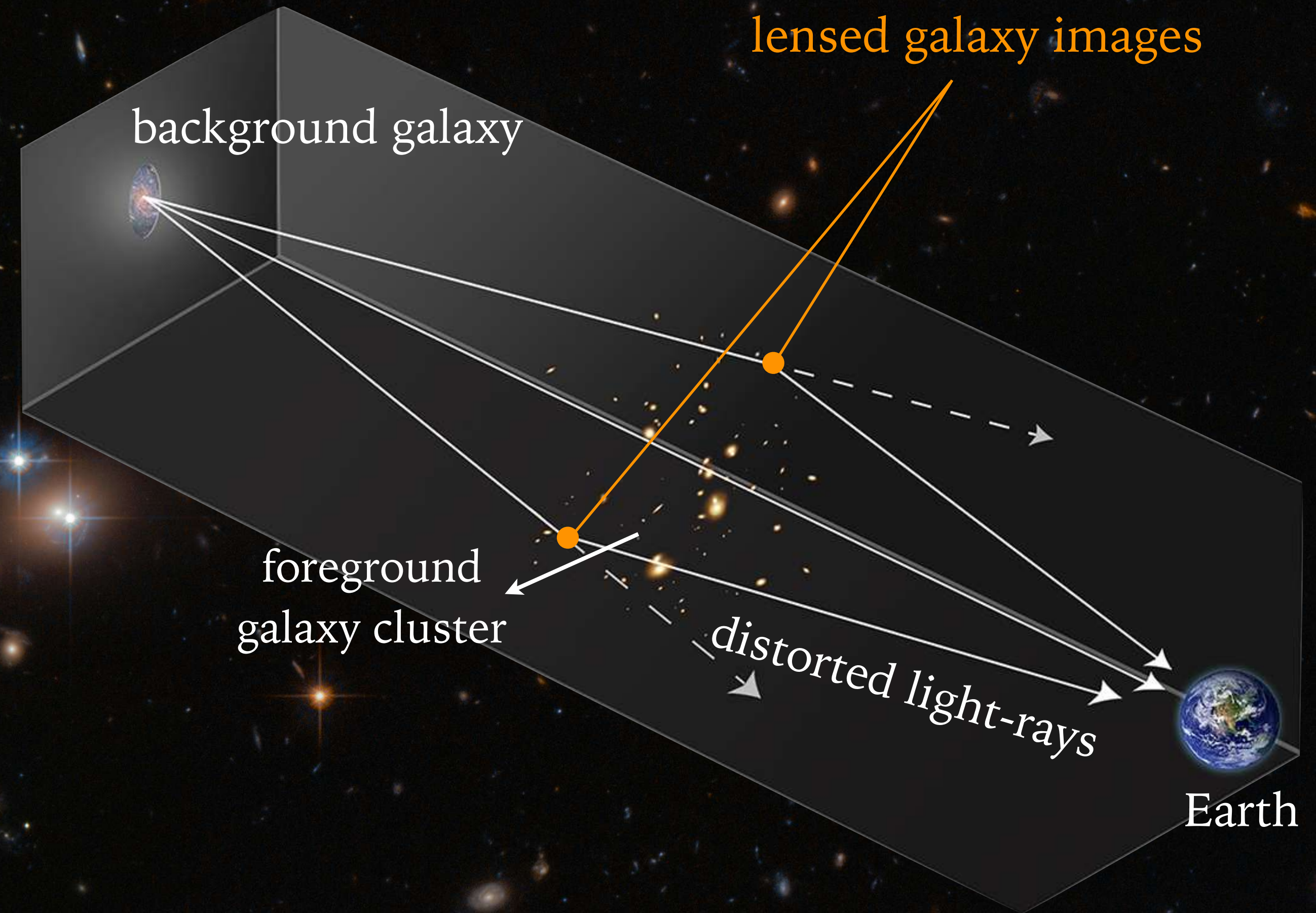
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- production of distinct and resolvable multiple images of the same background source
- amplification, magnification and distortion of the multiple images →
- time delays between multiple images
- reconstruct the **total mass distribution** of the lens
- detect and study **distant and faint sources**
- estimate cosmological parameters



# Time-delay cosmography

## ON THE POSSIBILITY OF DETERMINING HUBBLE'S PARAMETER AND THE MASSES OF GALAXIES FROM THE GRAVITATIONAL LENS EFFECT\*

*Sjur Refsdal*

(Communicated by H. Bondi)

(Received 1964 January 27)

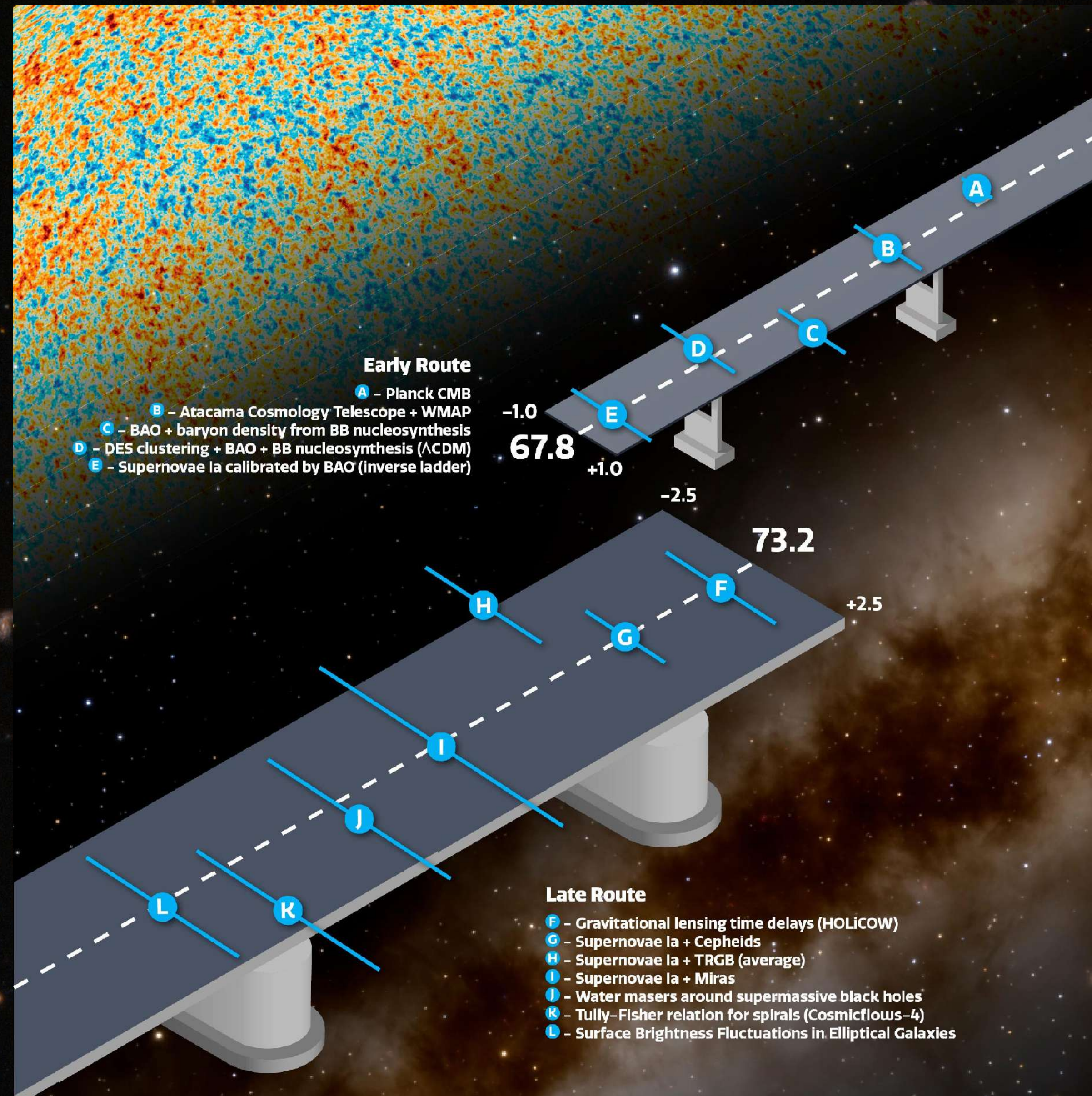
### *Summary*

The gravitational lens effect is applied to a supernova lying far behind and close to the line of sight through a distant galaxy. The light from the supernova may follow two different paths to the observer, and the difference  $\Delta t$  in the time of light travel for these two paths can amount to a couple of months or more, and may be measurable. It is shown that Hubble's parameter and the mass of the galaxy can be expressed by  $\Delta t$ , the red-shifts of the supernova and the galaxy, the luminosities of the supernova "images" and the angle between them. The possibility of observing the phenomenon is discussed.



# Time-delay cosmography

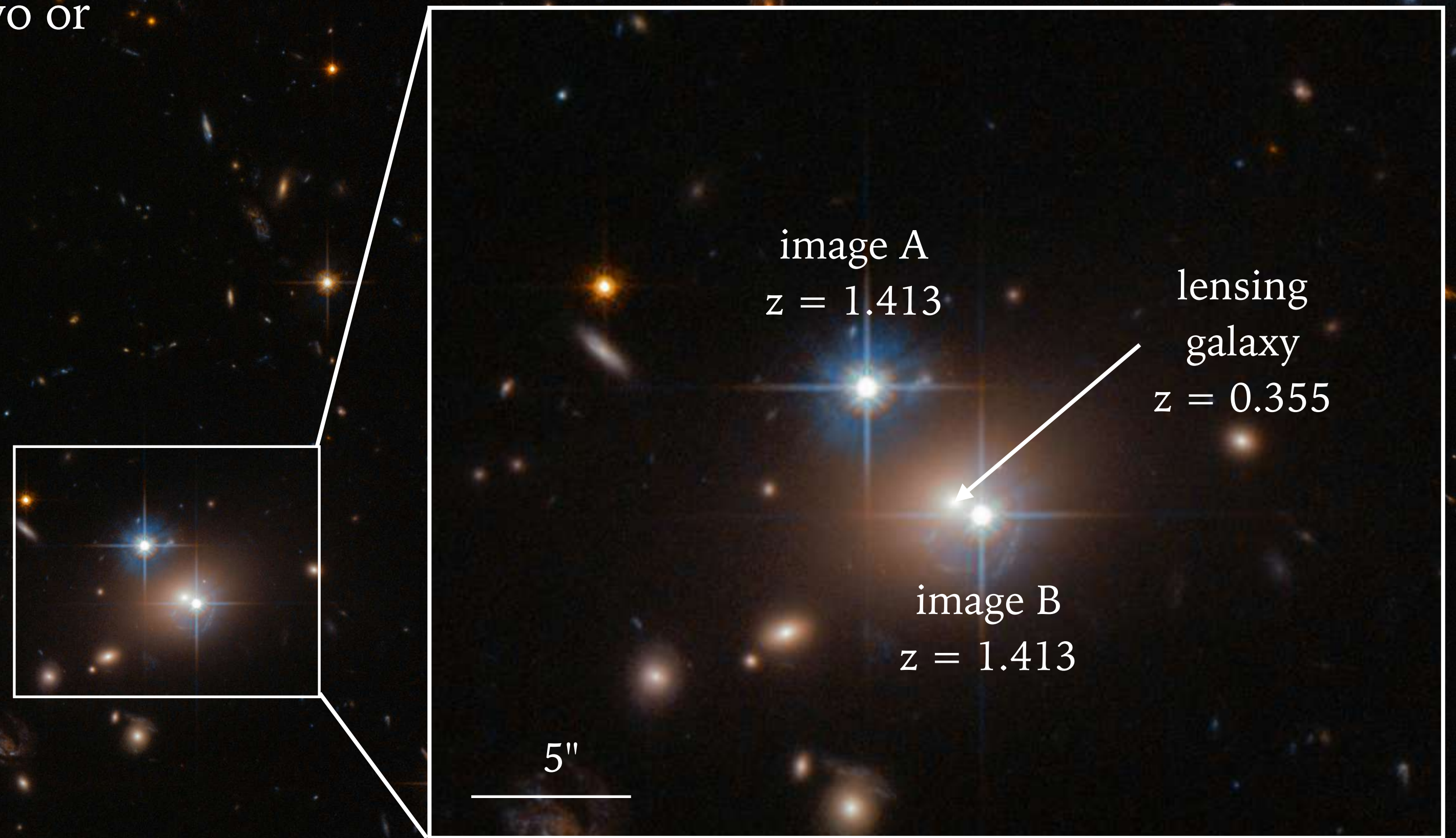
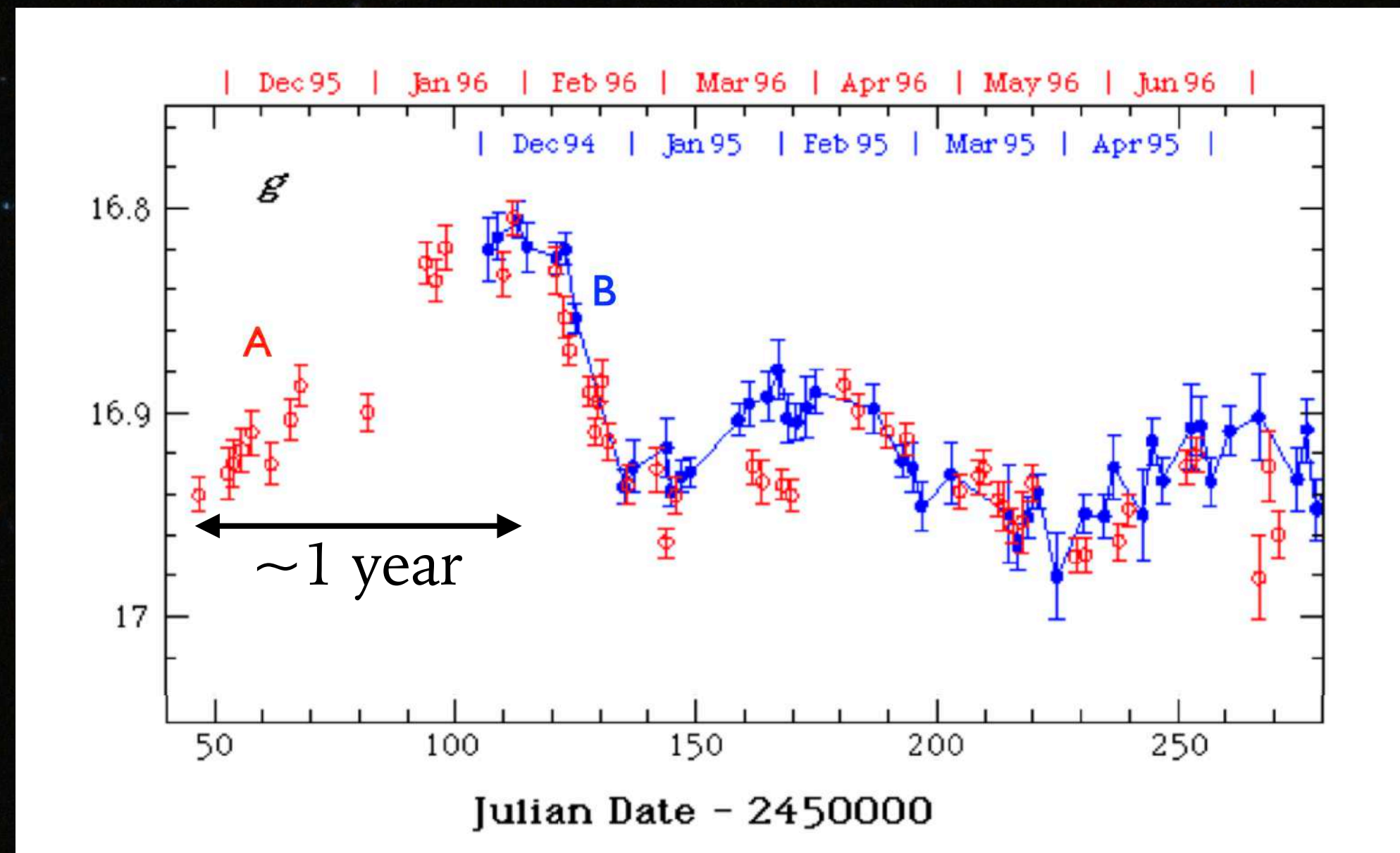
## The Hubble tension





# Time-delay cosmography

If a variable background source is lensed into two or more images, we can measure the time-delay between their light curves.



Twin Quasar Q0957+561  
[Walsh et al., 1979]



# Time-delay cosmography

**Time delays** between multiple images of variable background sources allow the geometry and the expansion rate ( $H_0$ ) to be constrained:

$$\Delta t = \frac{D_{\Delta t}}{c} \Delta \left[ \frac{|\vec{\theta} - \vec{\beta}|^2}{2} - \psi(\vec{\theta}, \vec{\beta}) \right]$$

geometrical + gravitational (Shapiro) time delay

$$\Delta t_{ij}(\vec{\theta}, \vec{\beta}) = \frac{1}{H_0} f(\Omega_M, \Omega_{DE}, w, z_l, z_s) \Delta \Phi_{Fij}(\vec{\theta}, \vec{\beta})$$

$\downarrow$  this can be measured       $\downarrow$  this can be inferred!       $\downarrow$  this depends on  $\psi$  and  $\psi'$

AN ACCURATE AND PRECISE  
MASS MODEL IS REQUIRED FOR  
TIME-DELAY COSMOGRAPHY!



# Strong Lensing Modelling

→ from the images to the mass  $\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$

$$\vec{\alpha} = \frac{D_{\text{ds}}}{D_{\text{s}}} \frac{2}{c^2} \int \vec{\nabla}_{\perp} \phi dl$$

- Parametric approach

The total mass distribution of the lens cluster is described as the sum of different contributions:

- cluster-scale halos (DM + intra-cluster medium)
- galaxy-scale halos (DM + baryons)
- additional components (external shear, foreground/background galaxies)

$$\phi_{\text{tot}} = \sum_i \phi_i^{\text{halo}} + \sum_j \phi_j^{\text{gal}} + \phi_{\text{ext}}$$

Each component is described by a certain profile specified by some parameters.



# Strong Lensing Modelling

→ from the images to the mass  $\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$

- Model optimization

Parameter inference is typically performed in a Bayesian framework, with MCMC methods used to sample the posterior probability distribution. The analysis aims to identify the parameter set that maximizes the posterior, consistent with the specified priors and the likelihood function:

$$\mathcal{L} = P(D | x(\boldsymbol{\theta})) = \prod_{i=1}^N \frac{1}{\prod_j \sigma_{ij} \sqrt{2\pi}} \exp^{-\chi_i^2/2} \quad \chi_i^2 = \sum_{j=1}^{n_i} \frac{[x_{\text{obs}}^j - x^j(\boldsymbol{\theta})]^2}{\sigma_{ij}^2}$$



# Strong Lensing Modelling

→ from the images to the mass  $\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$

Multiple images provide observational constraints of different nature:

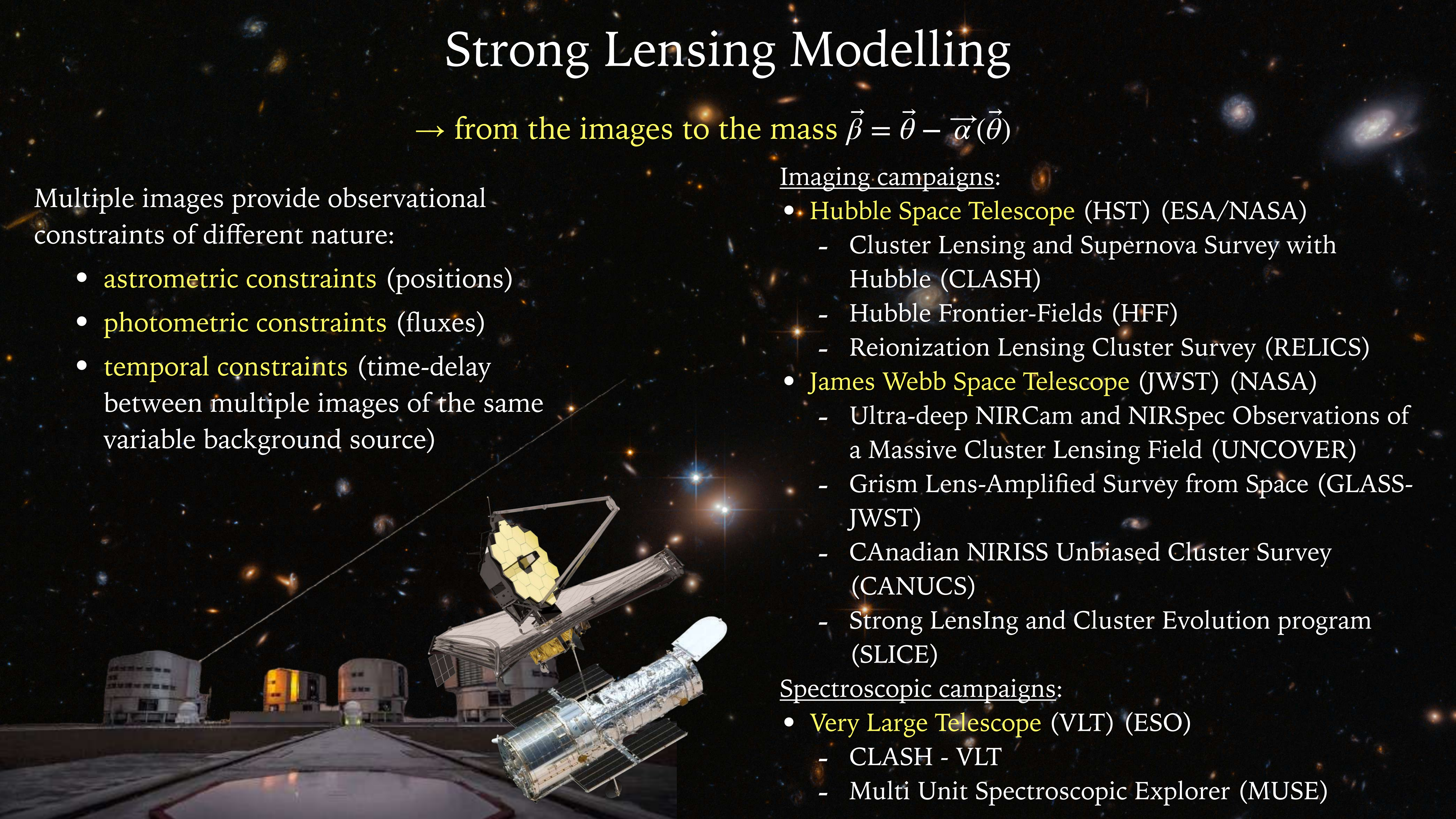
- **astrometric constraints** (positions)
- **photometric constraints** (fluxes)
- **temporal constraints** (time-delay between multiple images of the same variable background source)

Imaging campaigns:

- **Hubble Space Telescope** (HST) (ESA/NASA)
  - Cluster Lensing and Supernova Survey with Hubble (CLASH)
  - Hubble Frontier-Fields (HFF)
  - Reionization Lensing Cluster Survey (RELICS)
- **James Webb Space Telescope** (JWST) (NASA)
  - Ultra-deep NIRCam and NIRSpec Observations of a Massive Cluster Lensing Field (UNCOVER)
  - Grism Lens-Amplified Survey from Space (GLASS-JWST)
  - Canadian NIRISS Unbiased Cluster Survey (CANUCS)
  - Strong Lensing and Cluster Evolution program (SLICE)

Spectroscopic campaigns:

- **Very Large Telescope** (VLT) (ESO)
  - CLASH - VLT
  - Multi Unit Spectroscopic Explorer (MUSE)





# Strong Lensing Modelling

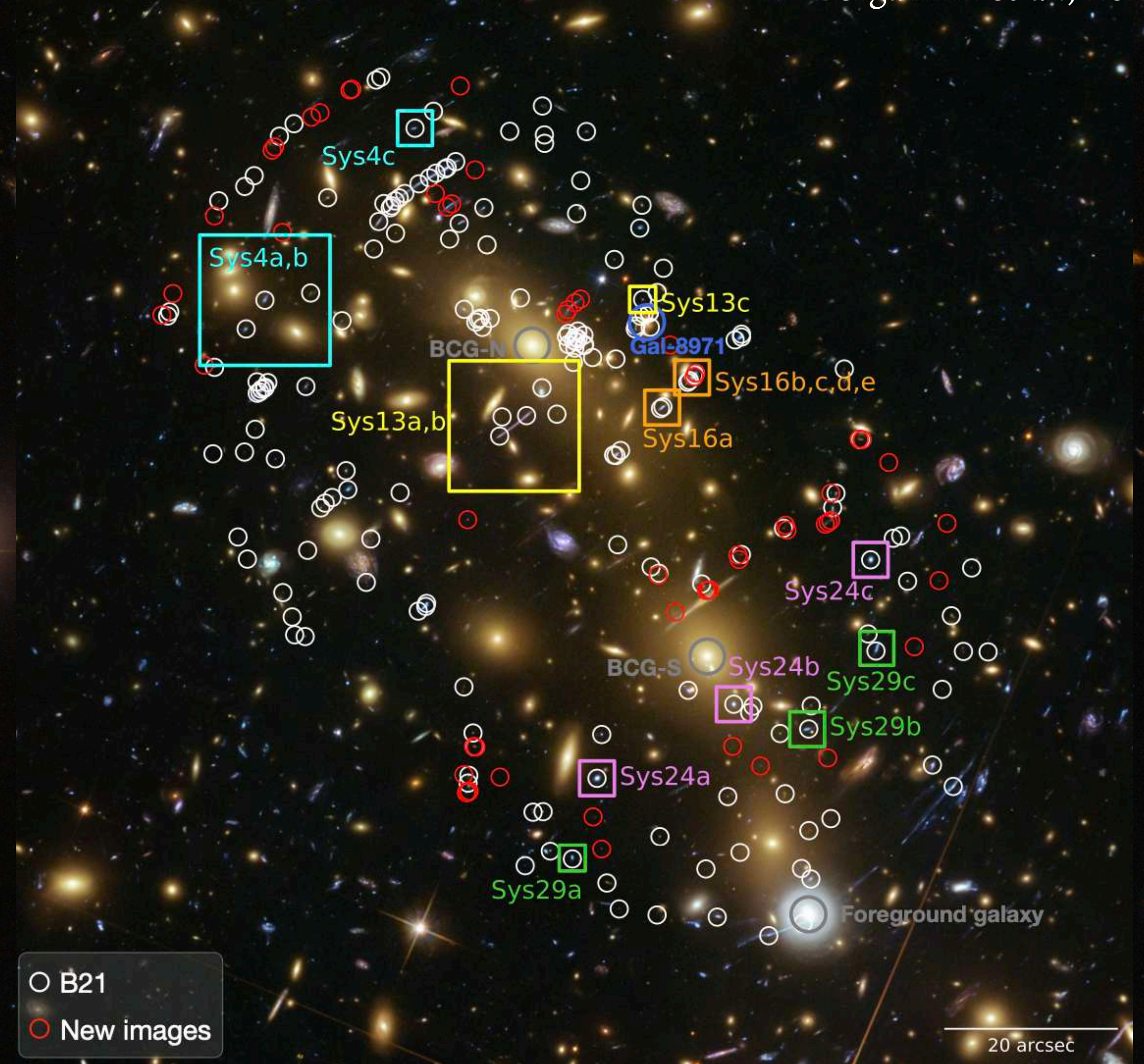
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MACS J0416.1 – 2403 (z=0.396)

Bergamini et al., 2022





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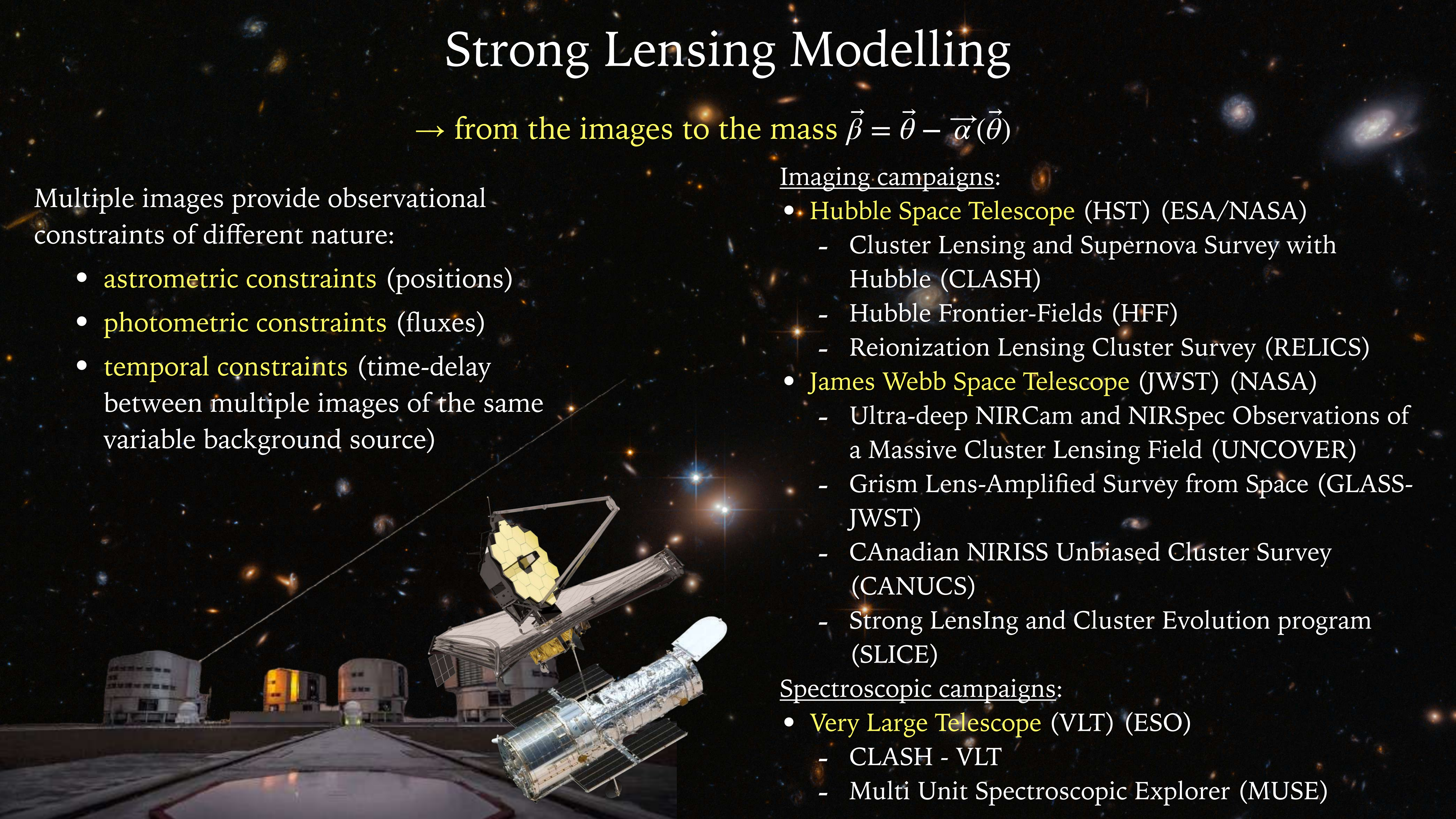
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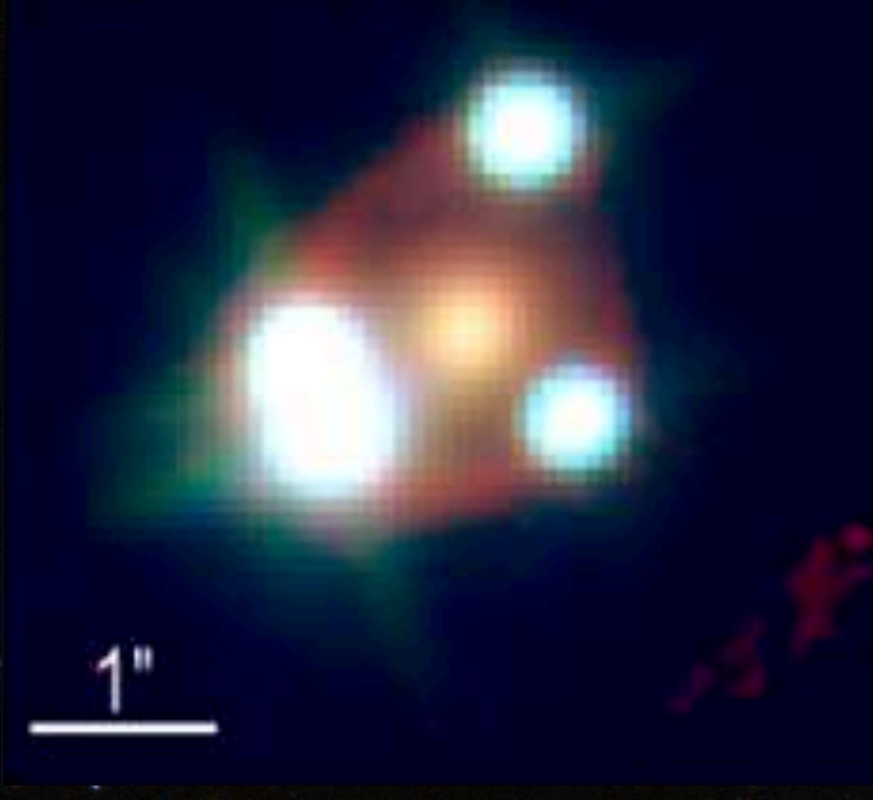
# Time-delay cosmography

GALAXY-SCALE

HE 0435-1223



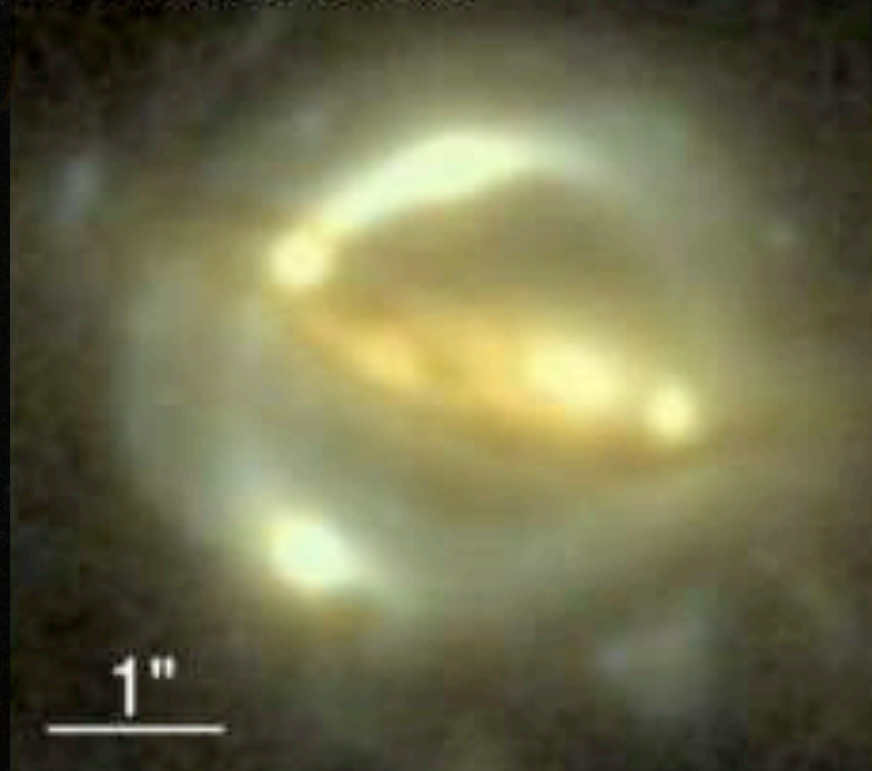
PG 1115+080



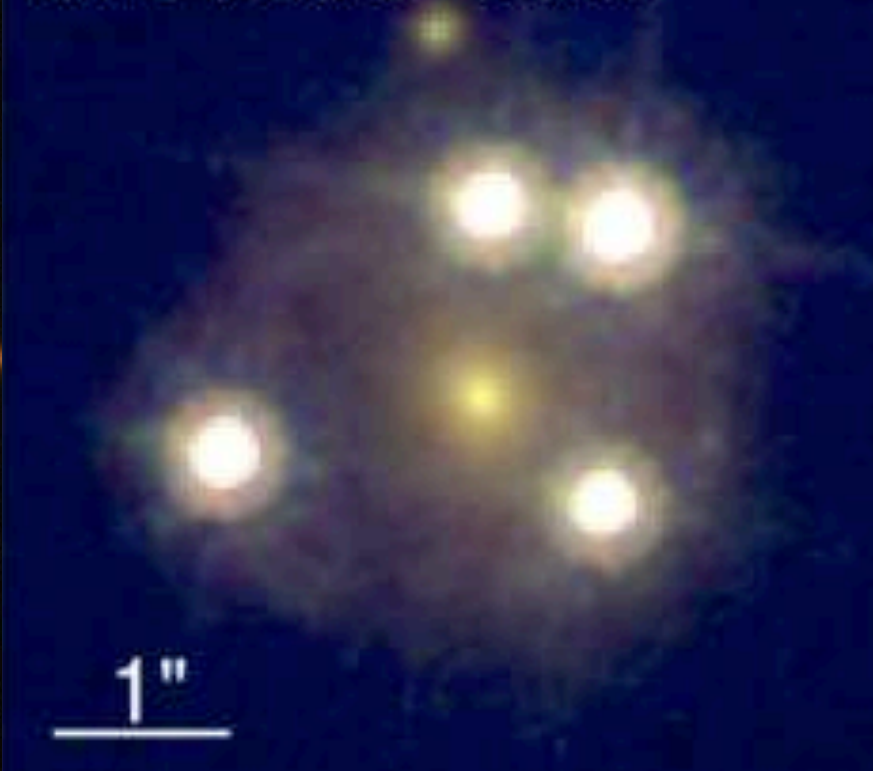
RX J1131-1231



B1608+656



WFI 2033-4723

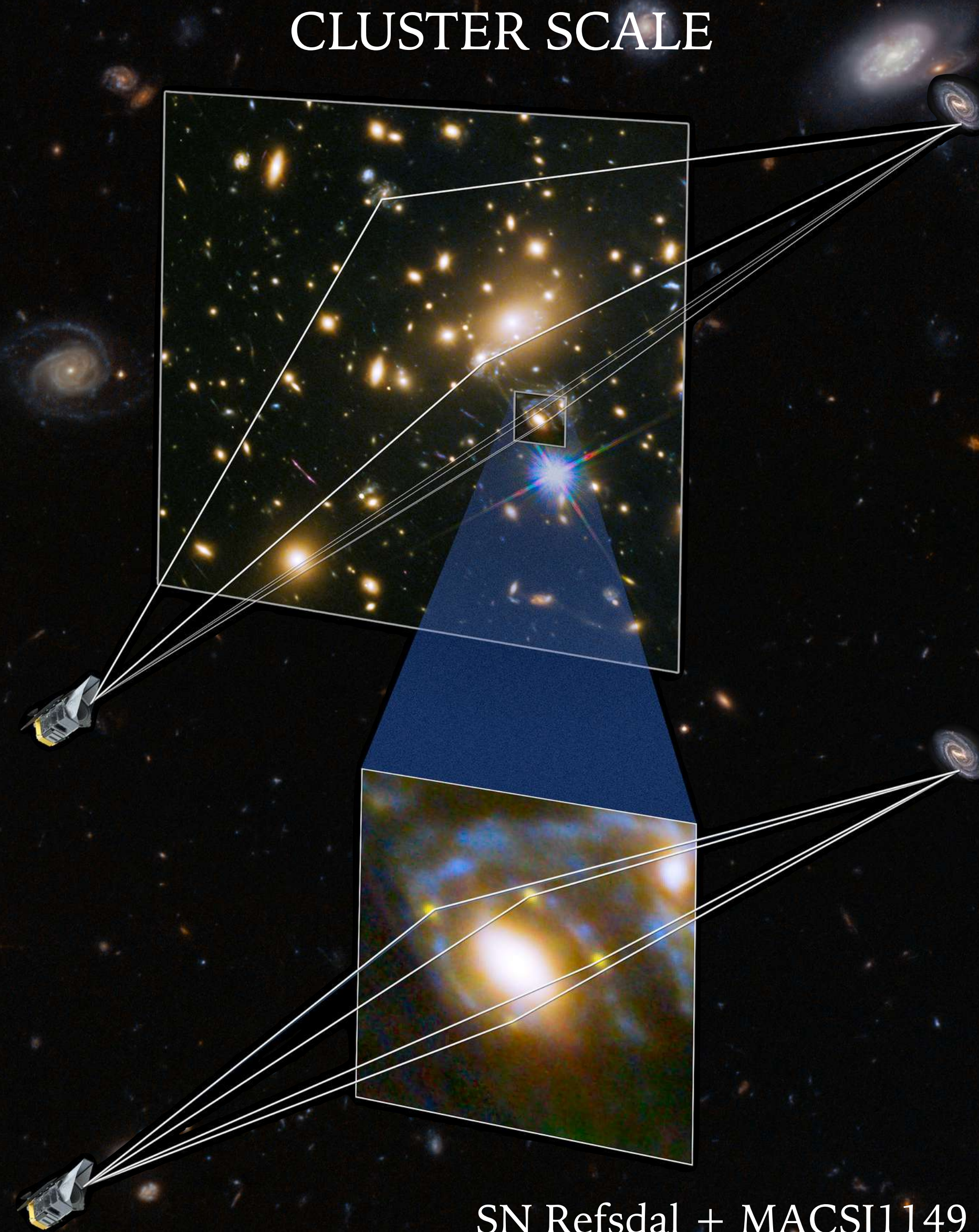


SDSS J1206+4332



H0LiCOW collaboration  
[Wang et al., 2019]

CLUSTER SCALE



SN Refsdal + MACSJ1149  
[Grillo et al., 2024]



The landmark case of SN Refsdal strongly  
lensed by the galaxy cluster MACSJ1149

CLASH+GLASS observations of  
MACSJ1149 [Kelly et al., 2015]

$z_{\text{lens}}=0.542$ ,  $z_{\text{host}}=1.489$



MACSJ1149.5 +2223  
( $z = 0.542$ )



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Supernova Refsdal ( $z=1.489$ )



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SN discovered in  
November 2014



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16 years  
before...  
(1998)

SN discovered in  
November 2014



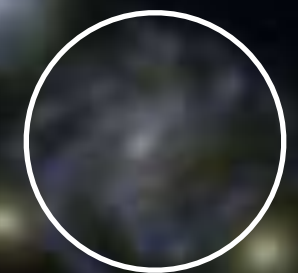
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16 years  
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(1998)



lens model challenge to  
predict reappearance



SN discovered in  
November 2014



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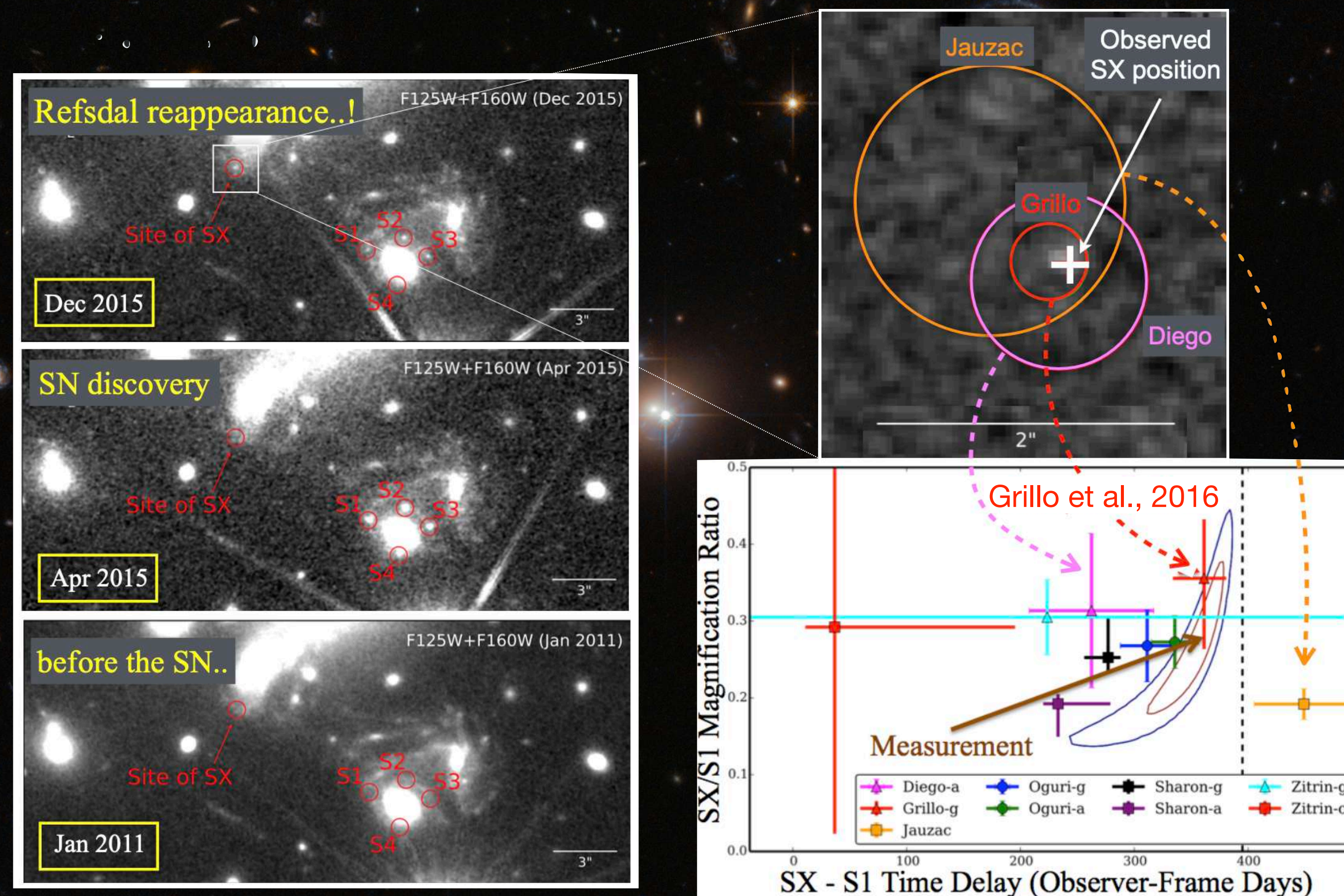
# Time-delay cosmography

The landmark case of SN Refsdal strongly lensed by the galaxy cluster MACSJ1149

**“REFSDAL” MEETS POPPER: COMPARING PREDICTIONS OF THE RE-APPEARANCE  
OF THE MULTIPLY IMAGED SUPERNOVA BEHIND MACSJ1149.5+2223**

T. Treu et al., 2016

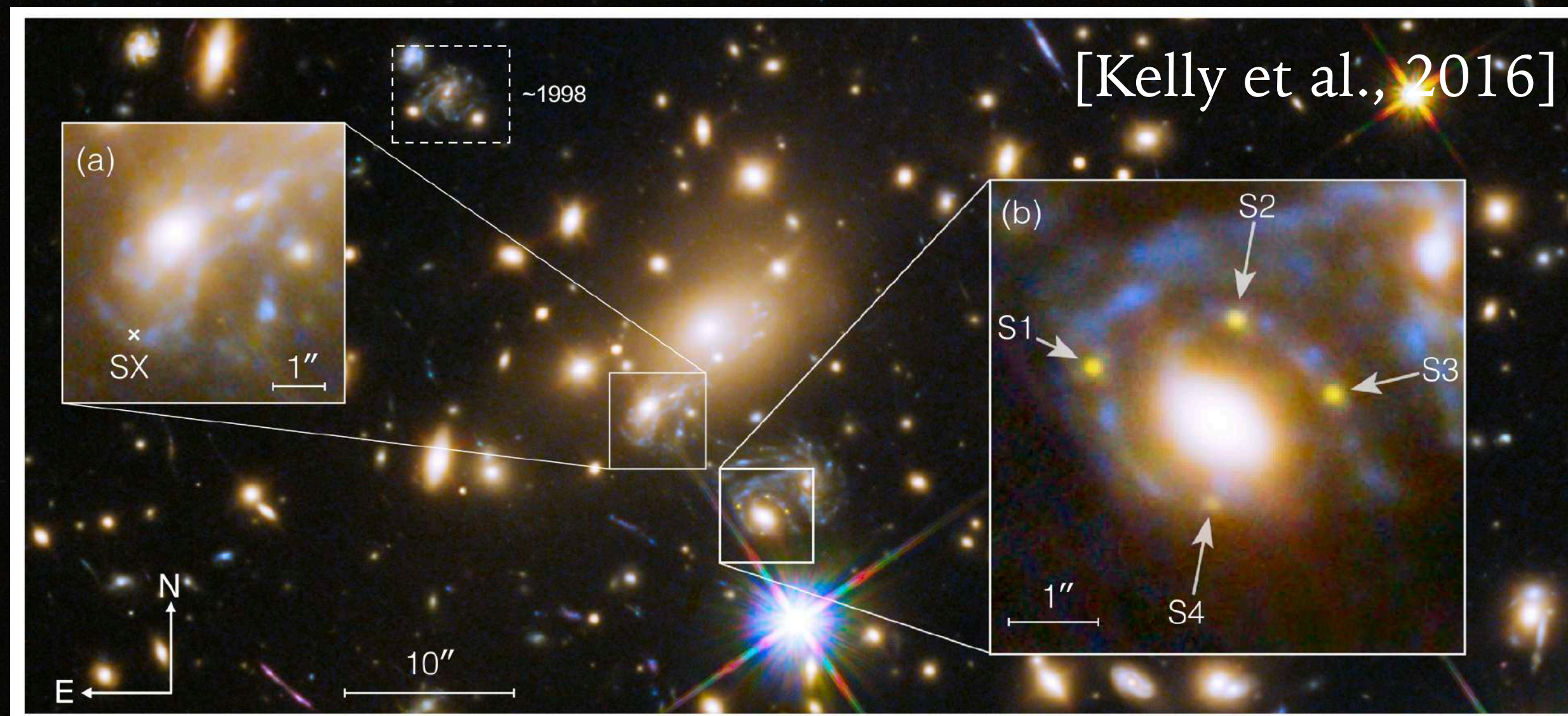
The Astrophysical Journal, Volume 817 (Jan 2016)





# Time-delay cosmography

The landmark case of SN Refsdal strongly lensed by the galaxy cluster MACSJ1149



HST + MUSE/VLT

## THE STORY OF SUPERNOVA “REFSDAL” TOLD BY MUSE

C. Grillo et al., 2016

The Astrophysical Journal, Volume 822 (May 2016)

## Measuring the Value of the Hubble Constant “à la Refsdal”

C. Grillo et al., 2018

The Astrophysical Journal, Volume 860 (Jun 2018)

## On the Accuracy of Time-delay Cosmography in the Frontier Fields Cluster MACS J1149.5+2223 with Supernova Refsdal

C. Grillo et al., 2020

The Astrophysical Journal, Volume 898 (July 2020)

## Cosmography with supernova Refsdal through time-delay cluster lensing: Independent measurements of the Hubble constant and geometry of the Universe

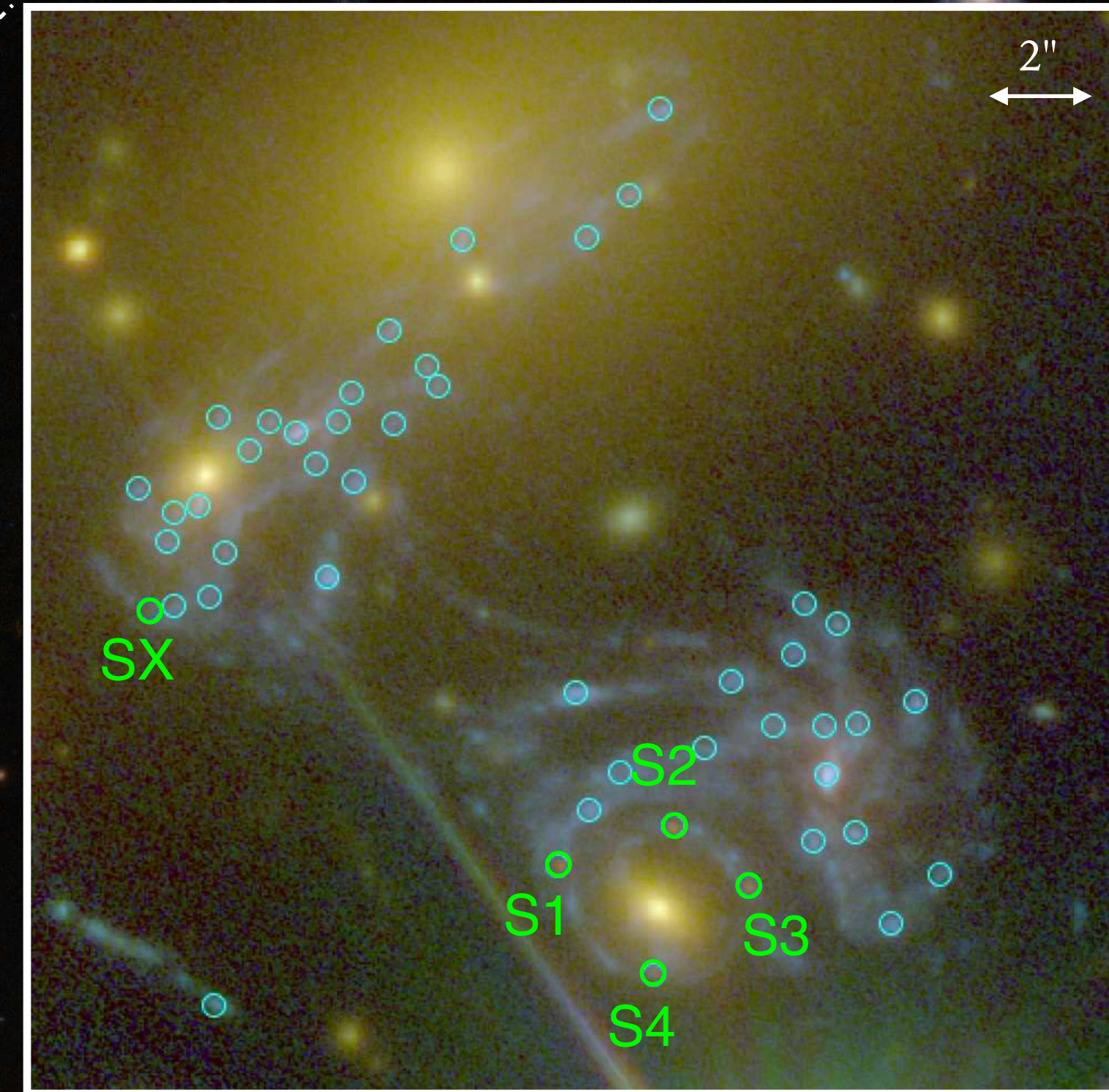
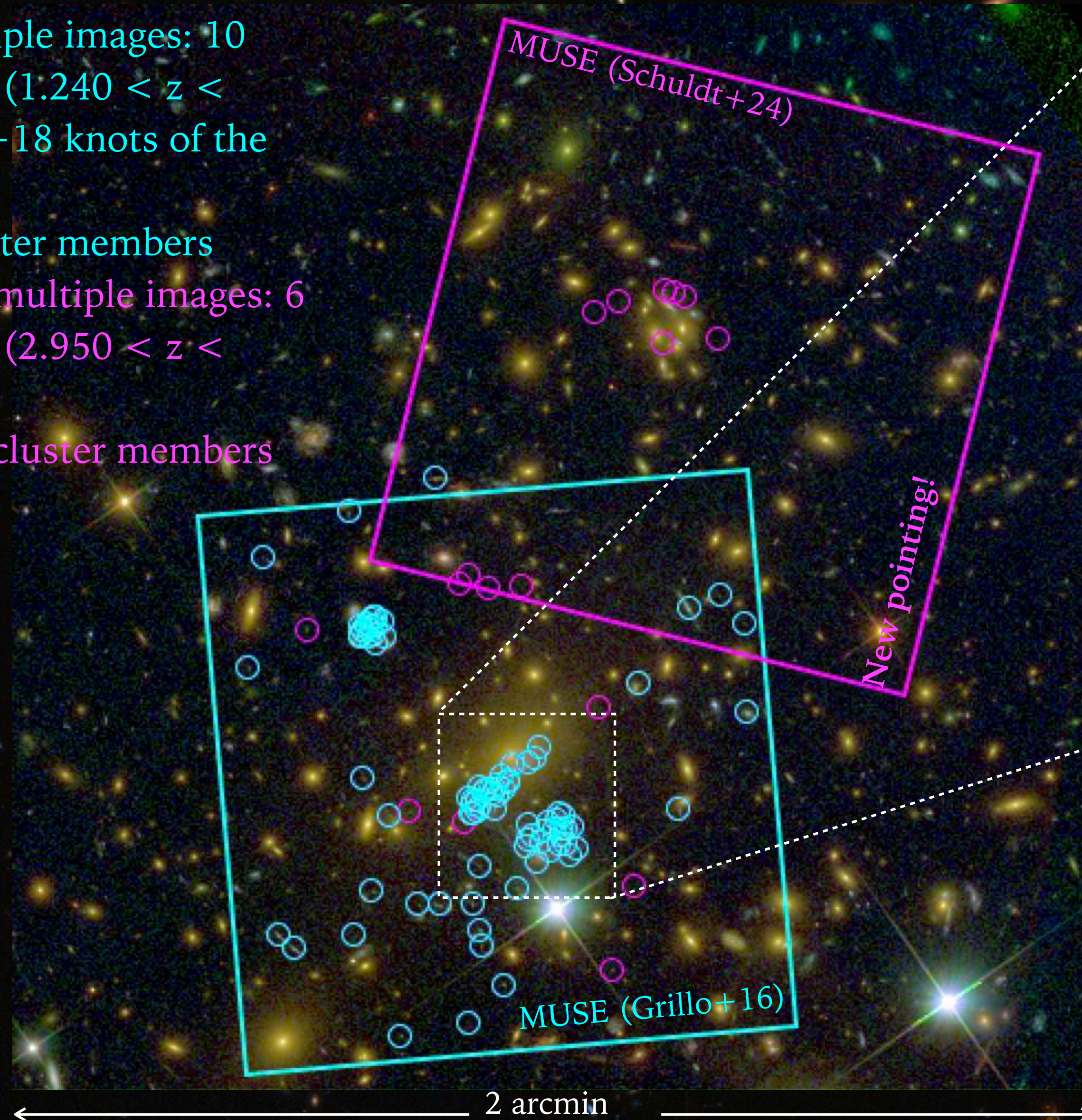
C. Grillo et al., 2024

Astronomy & Astrophysics, Volume 684 (Jun 2018)



# MACSJ1149: SL system

- 89 multiple images: 10 families ( $1.240 < z < 3.703$ ) + 18 knots of the SN host
- 300 cluster members
- 17 new multiple images: 6 families ( $2.950 < z < 5.983$ )
- 10 new cluster members



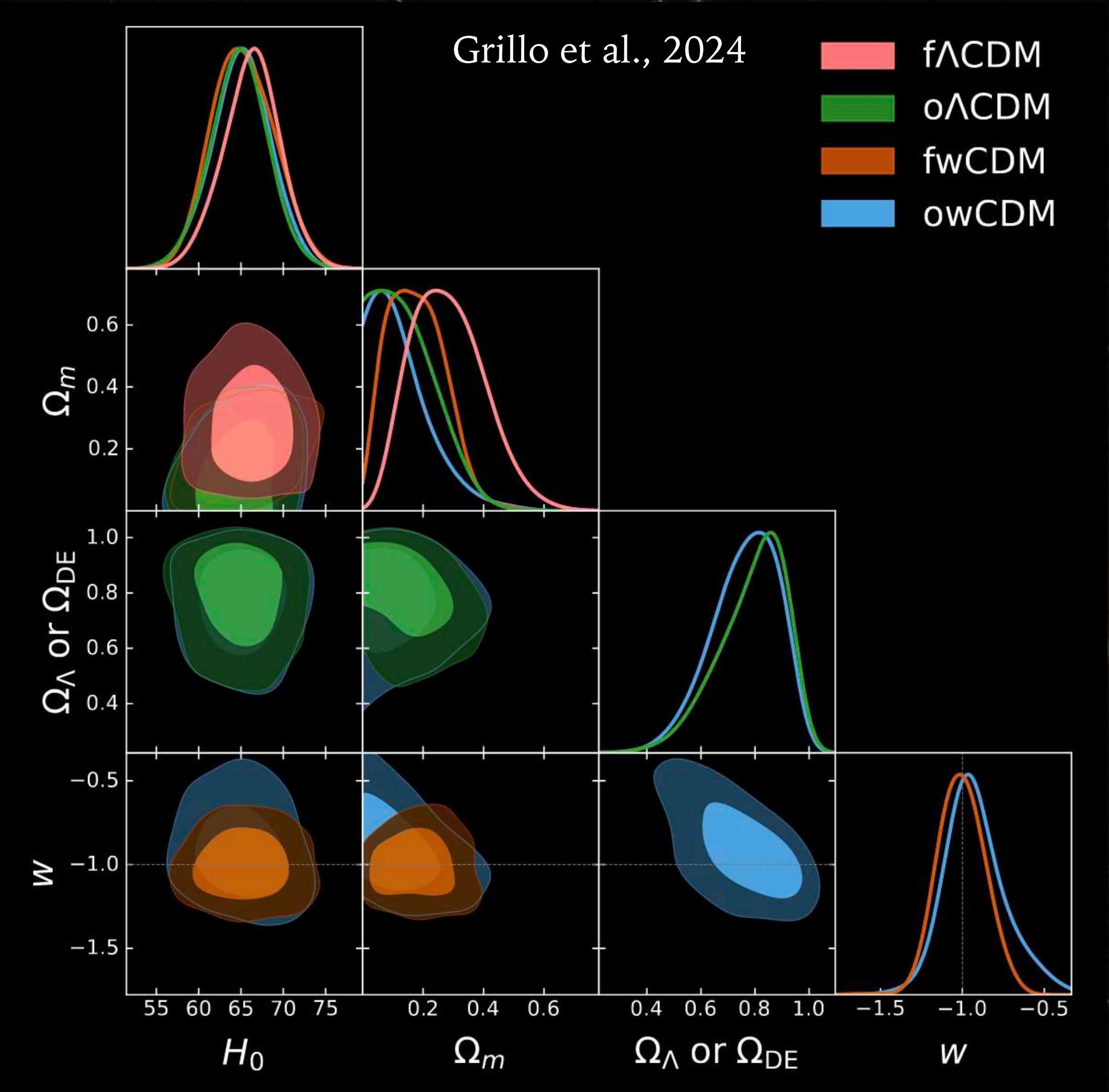
- Cluster redshift  $z=0.542$
- 5 multiple images of SN Refsdal ( $z=1.489$ ) (S1—S4 + SX)
- 4 measured time-delays [Kelly et al., 2023]



Cosmography with supernova Refsdal through time-delay cluster lensing:  
Independent measurements of the Hubble constant and geometry of the Universe

C. Grillo et al., 2024

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Model	$H_0$ (km s <sup>-1</sup> Mpc <sup>-1</sup> )	$\Omega_m$	$\Omega_\Lambda$ or $\Omega_{DE}$	$w$
f $\Lambda$ CDM	$66.2^{+3.5}_{-3.2}$	$0.28^{+0.10}_{-0.14}$	$0.72^{+0.14}_{-0.10}$	$\equiv -1$
o $\Lambda$ CDM	$64.8^{+3.5}_{-3.3}$	$< 0.34$	$0.79^{+0.16}_{-0.09}$	$\equiv -1$
fwCDM	$65.3^{+3.5}_{-4.1}$	$0.18^{+0.08}_{-0.11}$	$0.82^{+0.12}_{-0.08}$	$-1.00^{+0.14}_{-0.15}$
owCDM	$65.1^{+3.5}_{-3.4}$	$< 0.34$	$0.76^{+0.15}_{-0.10}$	$-0.92^{+0.15}_{-0.21}$

Time-delays + distance ratios

$H_0$

$\Omega_m, \Omega_\Lambda, w$

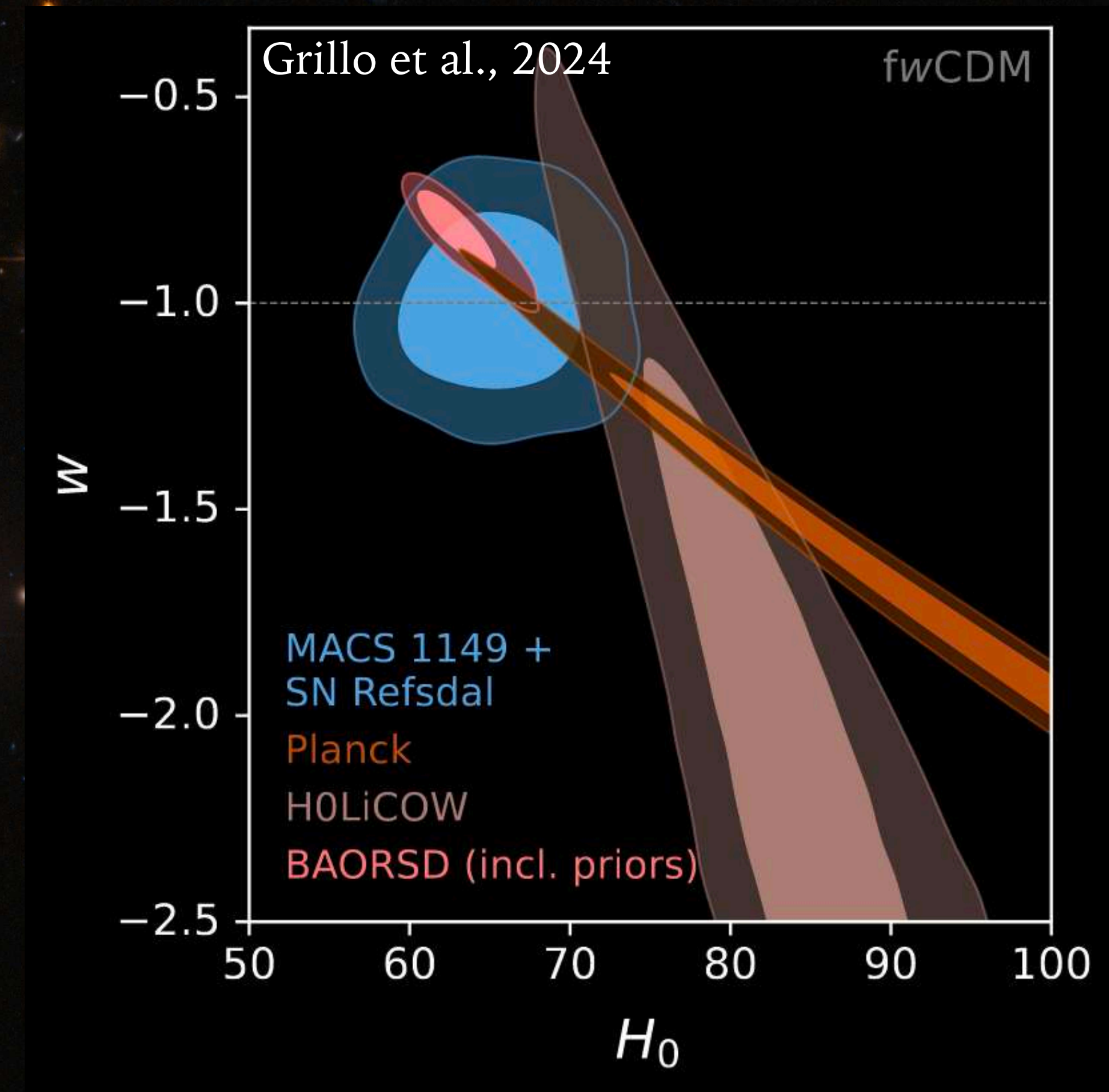
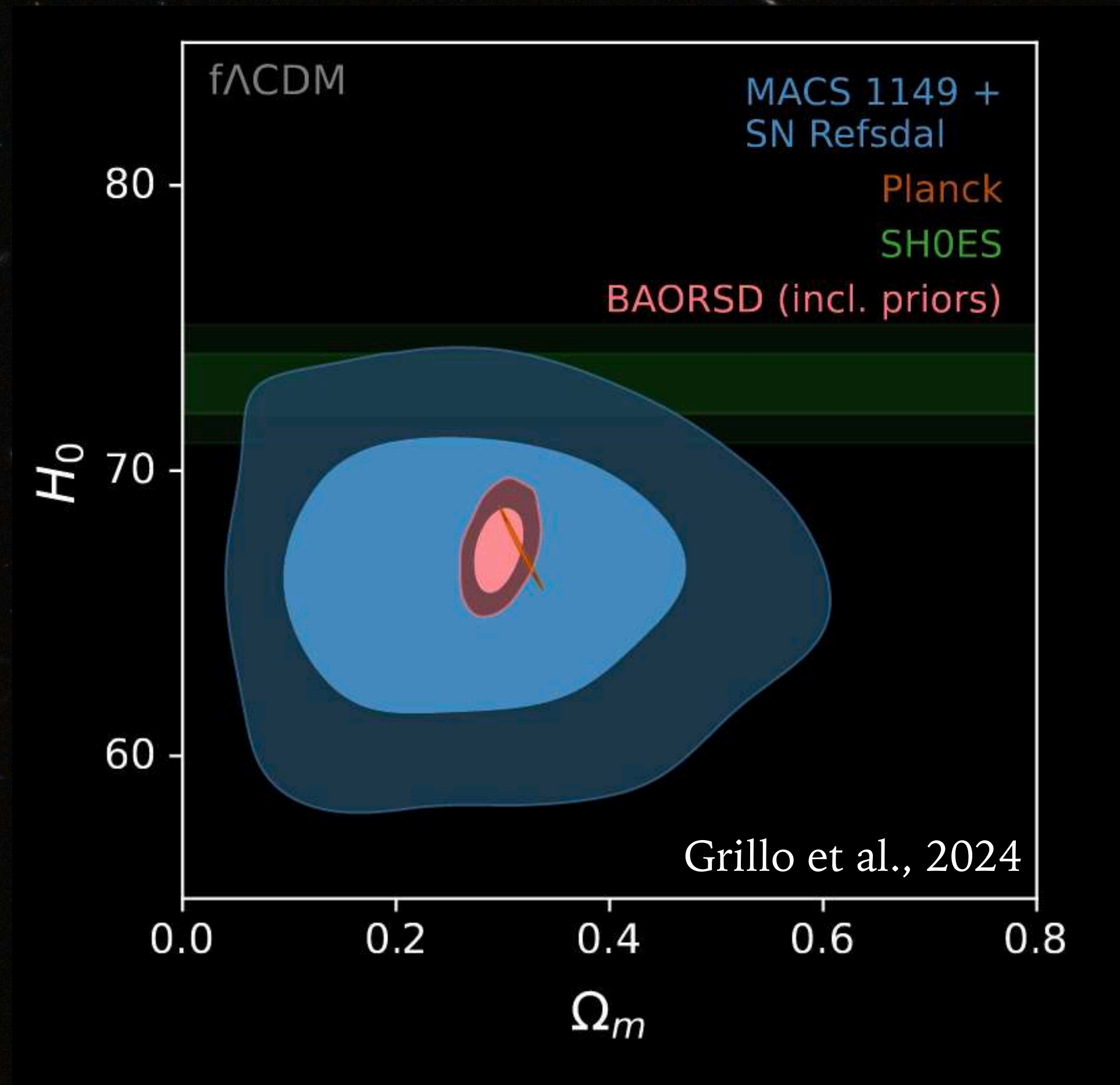
$$\eta = \frac{\left(\frac{D_{ds}}{D_s}\right)_{z_{S1}}}{\left(\frac{D_{ds}}{D_s}\right)_{z_{S2}}}$$



# Cosmography with supernova Refsdal through time-delay cluster lensing: Independent measurements of the Hubble constant and geometry of the Universe

C. Grillo et al., 2024

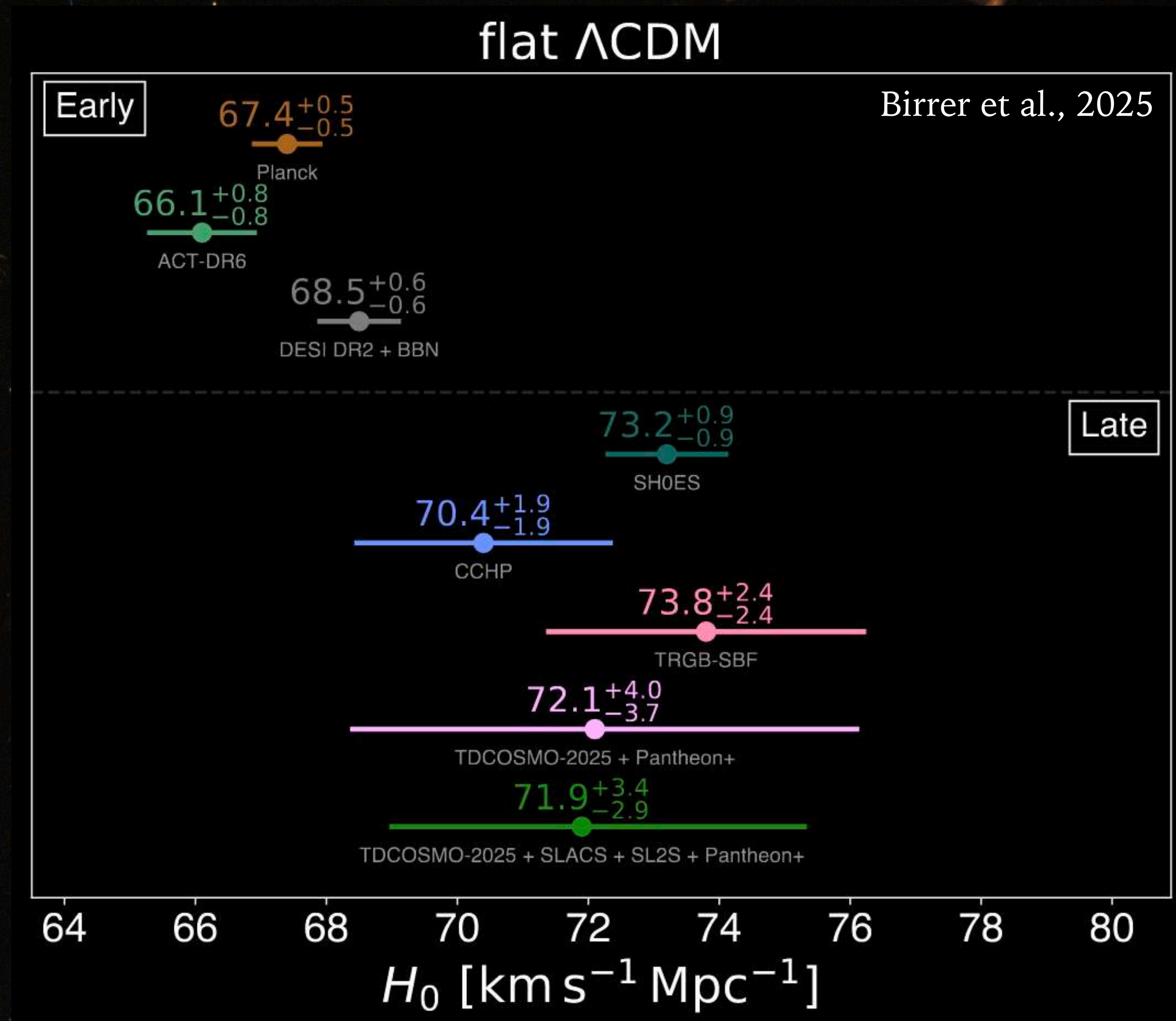
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# Time-delay cosmography

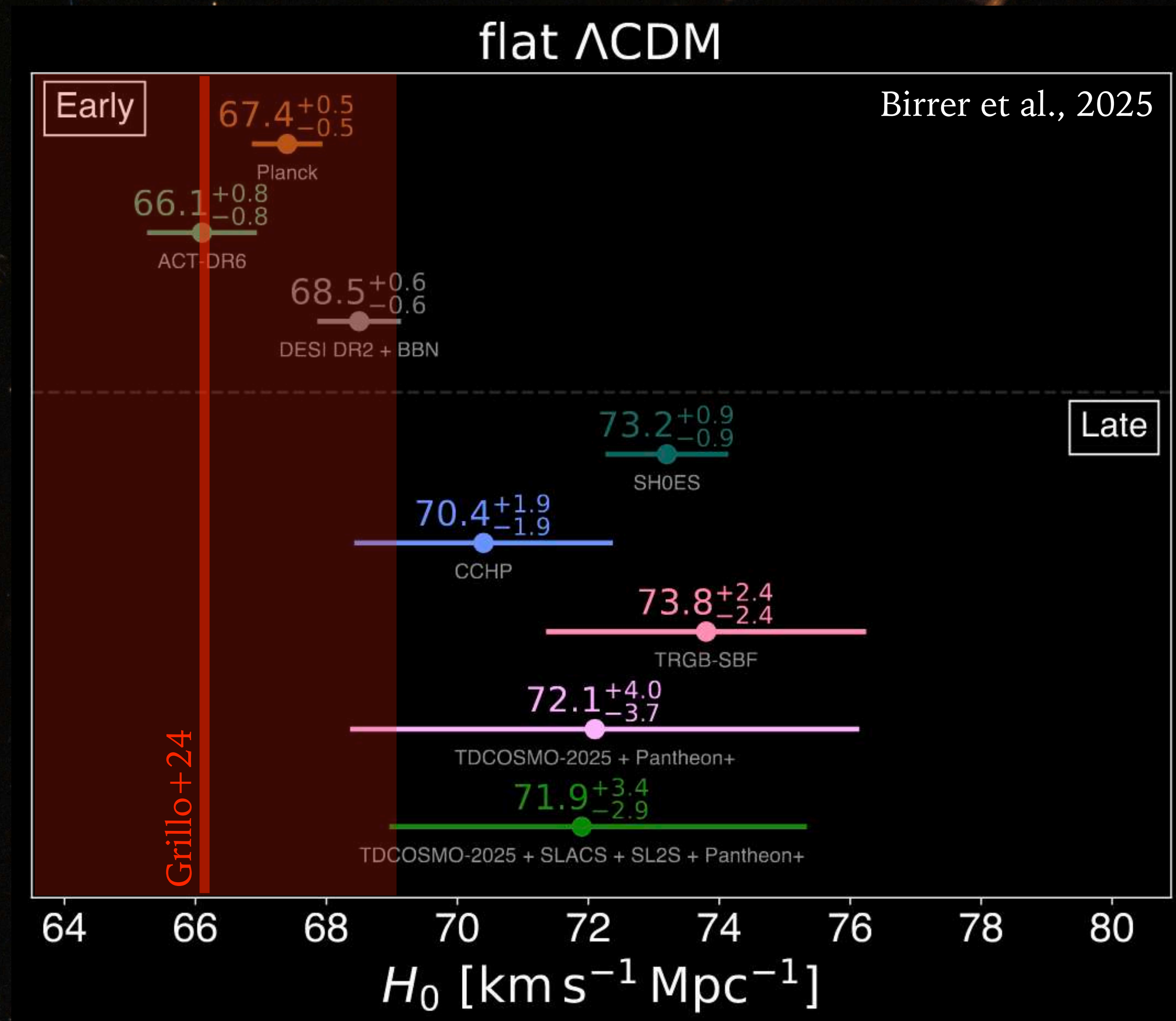
## The Hubble tension





# Time-delay cosmography

## The Hubble tension





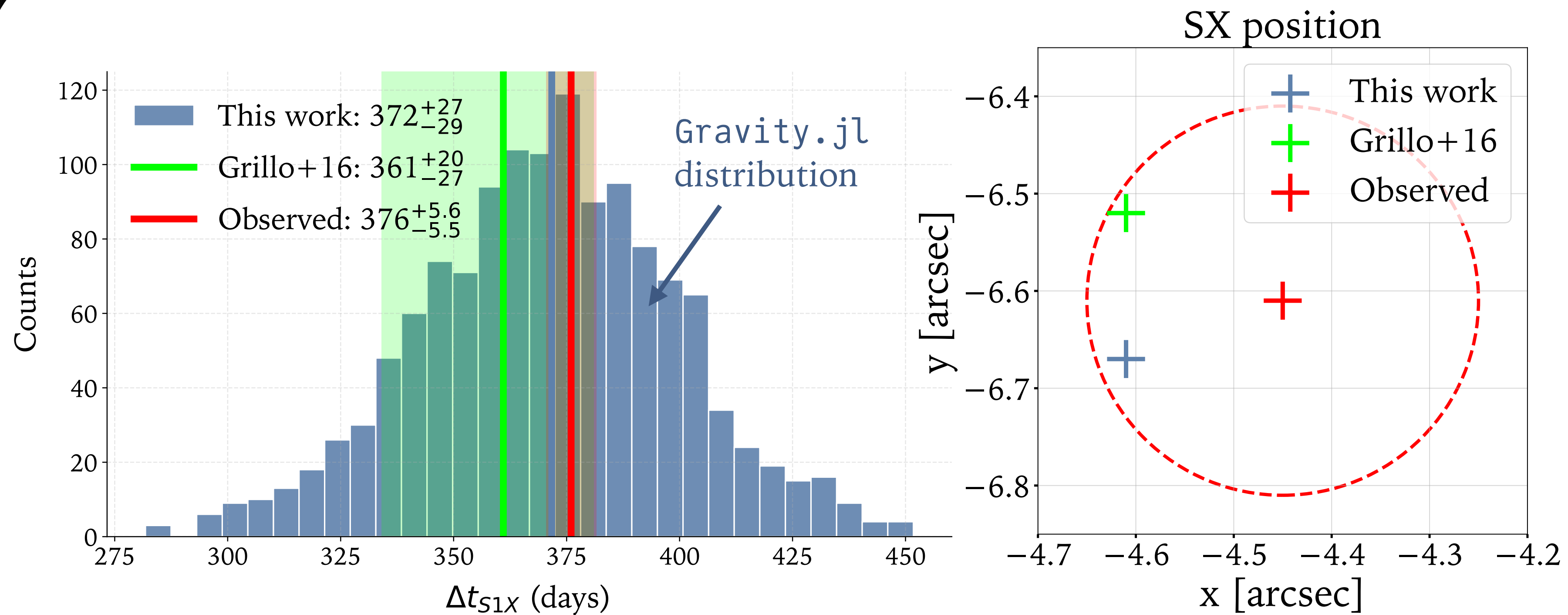
# MACSJ1149: SL model with Gravity.jl

- !! New software to perform SL modelling [M. Lombardi, 2024]
- !! Fast: implemented in Julia (high-performance programming language)
- !! General/flexible: multi-plane lensing equations, arbitrary cosmology, different sources, many different algorithms
- Goal: apply Gravity.jl to cluster-scale time-delay cosmography (TDC)
- Gravity.jl is fast and efficient, but it's new and must be tested
- MACSJ1149 as a representative test case: use the MACSJ1149 Refsdal lensing dataset to compare inferences on  $H_0$  with other studies using different SL tools.



# MACSJ1149: SL model with Gravity.jl

1. Trying to reproduce [Grillo et al., 2016](#) SL model (based on the SL tool GLEE)



Predictions on SX: position and time-delay

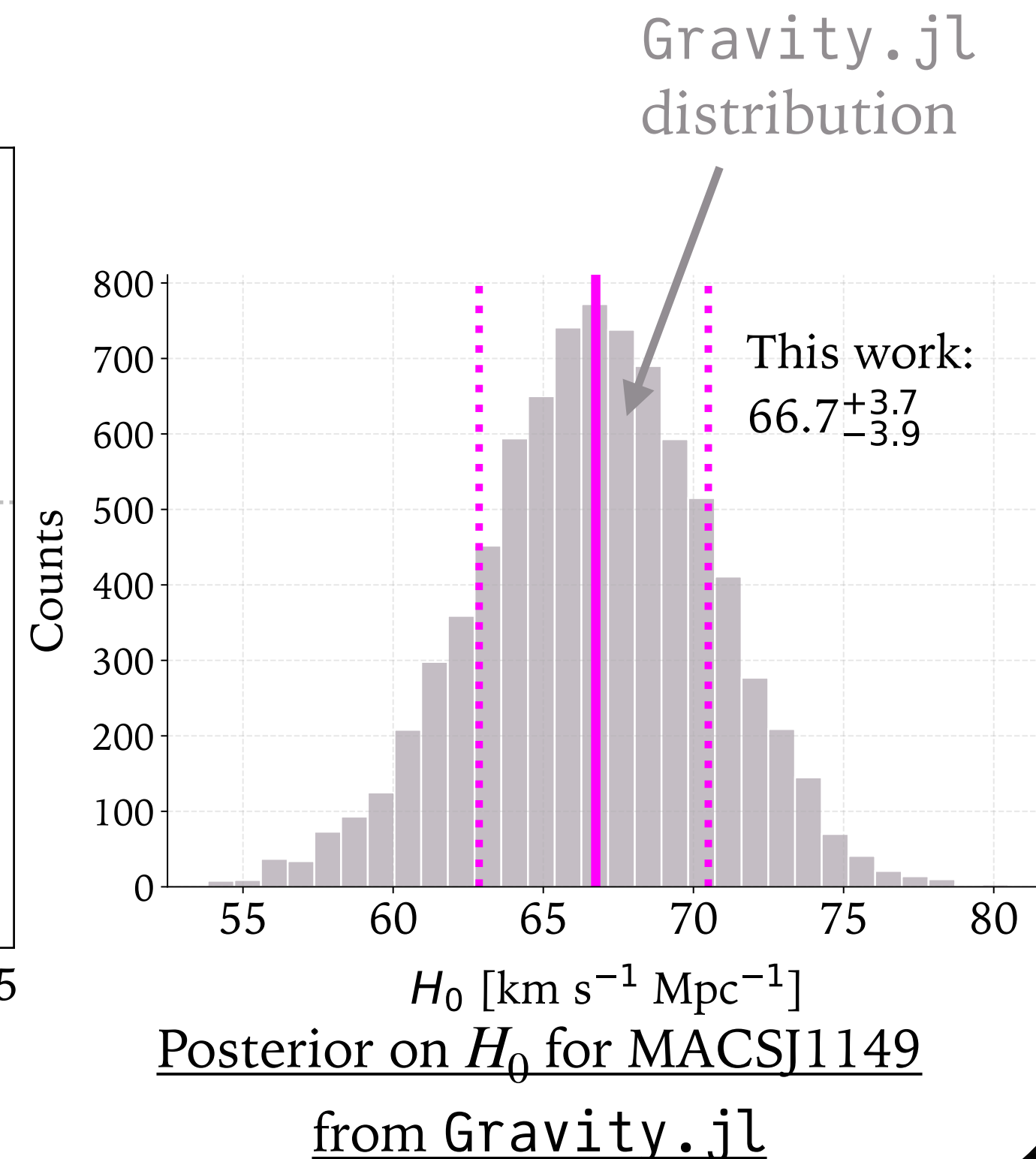
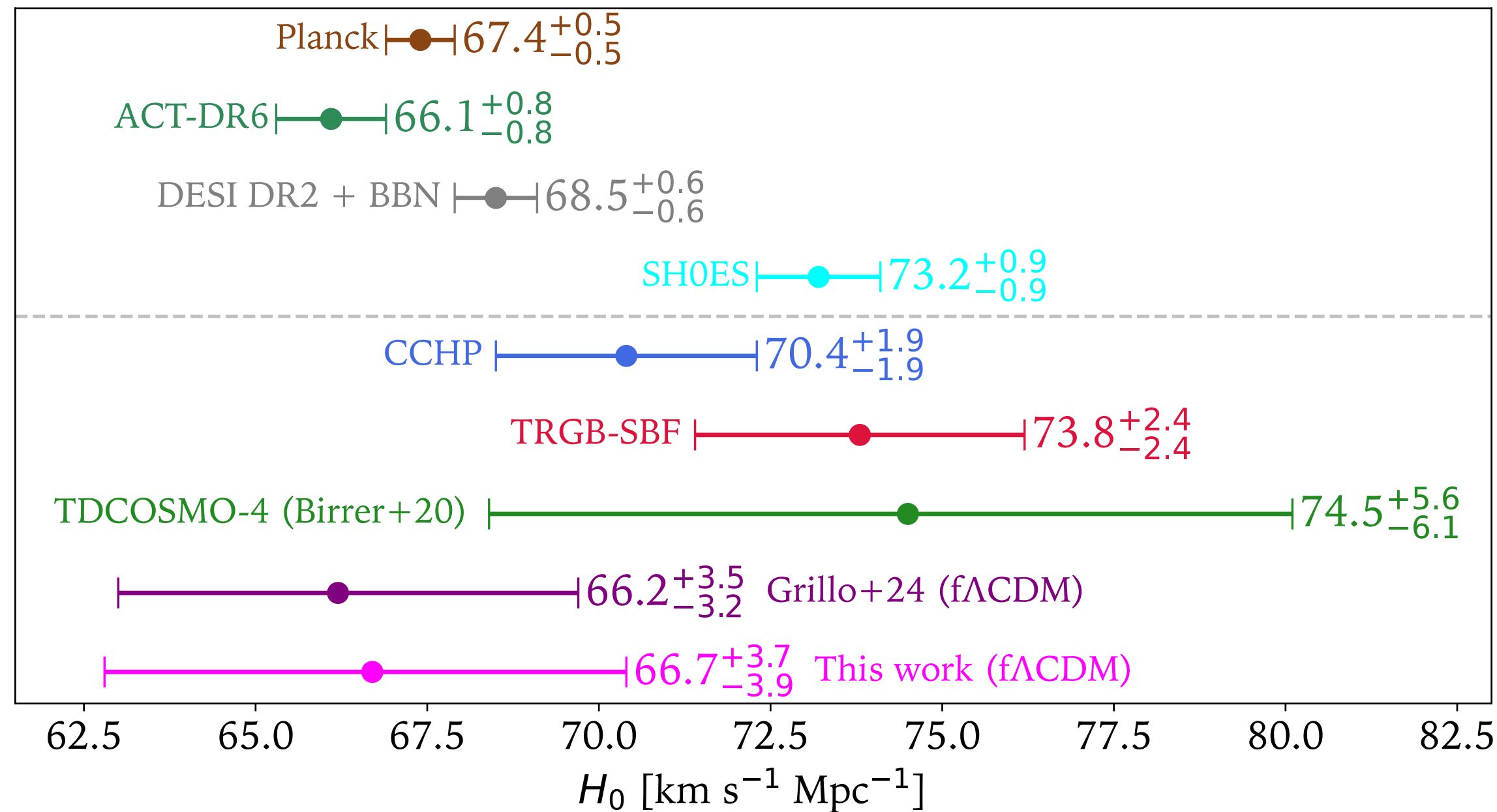
Our Gravity.jl model is able to reproduce [Grillo et al., 2016](#) results!



# MACSJ1149: SL model with Gravity.jl

2. With the same mass model, we include SX position and S1—SX time-delay measurements as additional constraints, leaving  $H_0$  as a free parameter (flat  $\Lambda$ CDM with  $\Omega_m = 0.3$ ).

## Inference of $H_0$



Our Gravity.jl model is able to reproduce [Grillo et al., 2024](#) results!



# Future prospects for cosmography

Gravity.jl promises to be a competitive strong lens modelling package in the upcoming era of high-volume, high-quality SL data.

- **James Webb Space Telescope** will provide a number of SL constraints  $\sim 1000$  in massive clusters vs  $\lesssim 100$  in the HST era
- **Euclid** (2023) and,
- **Nancy Grace Roman Space Telescope** (2027) will find thousands lenses (on galaxy and cluster scale) with multiply imaged quasars
- **Vera Rubin Observatory** (2025) will provide light curves of bright multiply imaged quasars, but also discover tens of lensed supernovae

*Thanks for the attention!*