Probing effective quantum gravity with fluid dynamics

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Center of Excellence "Fundamental Universe"

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Outline

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Problems in gravity and cosmology

- So far unexplained cosmological observations:
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  - Similarly: replacing flat spacetime by curved spacetime led to GR.
  - Replace perfect fluid model by velocity-dependent distribution of particles.
- Questions arising from new matter model:
  - ✓ How does a kinetic gas react to a gravitational field?
  - ? How does a kinetic gas create a gravitational field?

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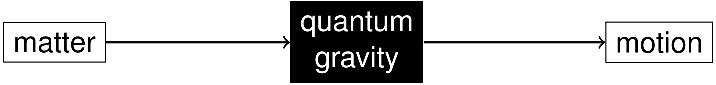
$\Rightarrow$ Here: effective quantum gravity phenomenology with gas dynamics near black holes.

How does quantum gravity phenomenology work?

- Basic operating principle of (quantum) gravity theory:



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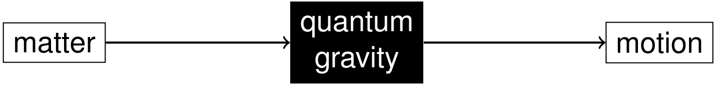

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⇝ Only need to study (all) possible quantum corrections!

Examples of fluids

- Perfect fluid:
  - Most general energy-momentum tensor compatible with cosmological symmetry.
  - No shear stress, no friction.
  - Characterized by density ρ and pressure p.
    - Dust, dark matter: $p = 0$.
    - Radiation: $p = \frac{1}{3}\rho$.
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- Hyperfluid:
  - Additional coupling to affine connection generates hypermomentum.
  - Intrinsic property of matter, e.g., spin.

Why study matter beyond fluids?

- Dynamical friction:
  - Massive object passes distribution of light objects.

  ⇒ Gravity of massive object changes positions of lighter objects.

  ⇒ Perturbation of light objects asserts gravity on massive object.
  - Example: globular cluster passing through galaxy.

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- Dynamics of intergalactic medium:
  - Cosmic gas highways: gas in and near filaments
  - Crossing sheets in collapse and structure formation.

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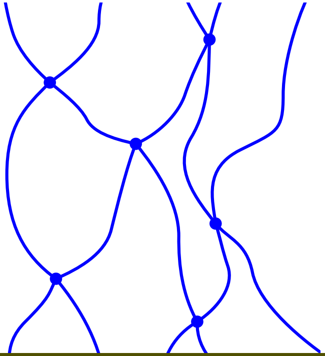
Definition of kinetic gas

- Single-component gas:
  - Constituted by classical, relativistic particles.
  - Particles have equal properties (mass, charge, . . .).
  - Particles follow piecewise geodesic curves.
  - Endpoints of geodesics are interactions with other particles.



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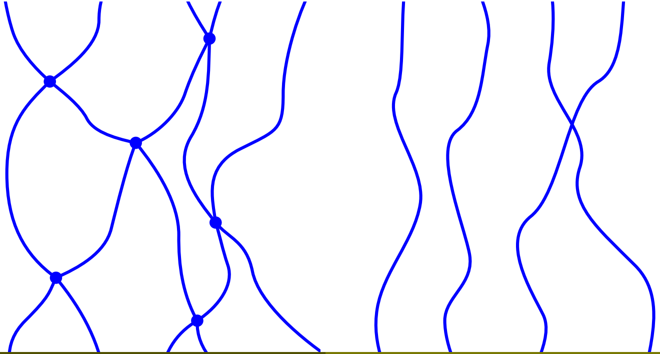

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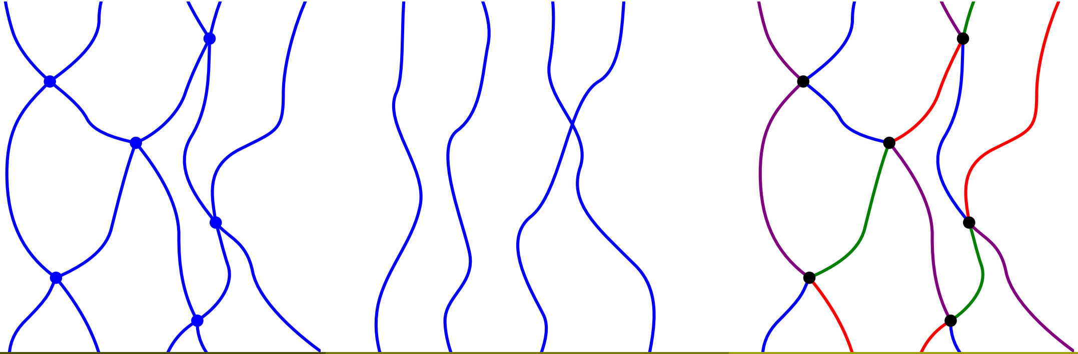
- Multi-component gas: multiple types of particles.



One-particle distribution function

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For every hypersurface $\sigma \subset O$,

$$N[\sigma] = \int_\sigma \phi\Omega$$

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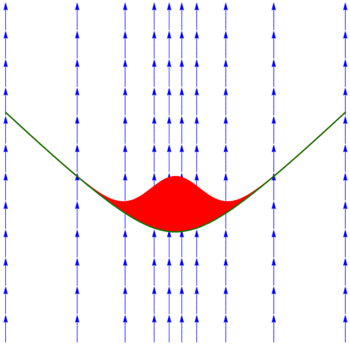

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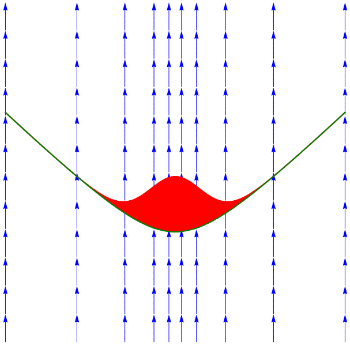

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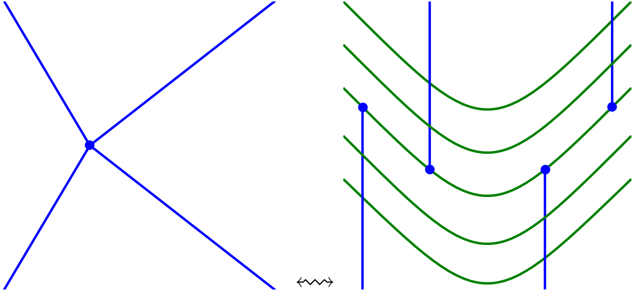
- For multi-component fluids: ϕ_i for each component i.

Collisions & the Liouville equation

- Collision in spacetime $\leftrightsquigarrow$ interruption in observer space.



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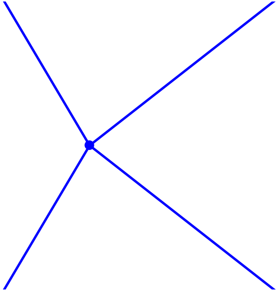

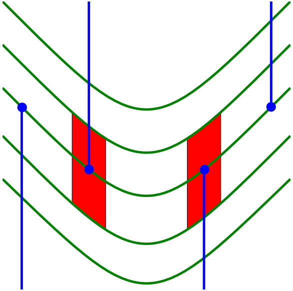

- For any open set $V \in O$,

$$\int_{\partial V} \phi\Omega = \int_V d(\phi\Omega) = \int_V \mathcal{L}_{\mathbf{r}}\phi\Sigma$$

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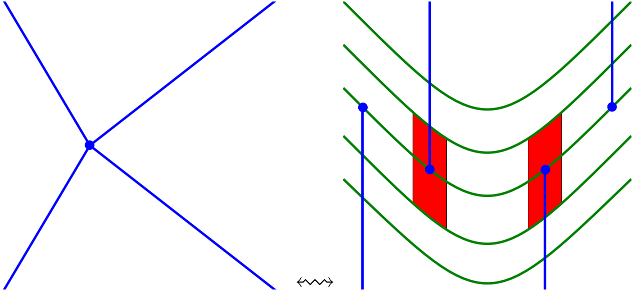

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- Collisionless fluid: trajectories have no endpoints, $\mathcal{L}_{\mathbf{r}}\phi = 0$.

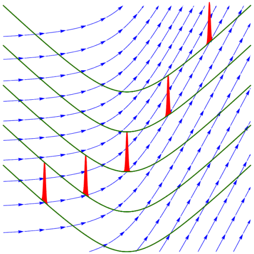
$\Rightarrow$ Simple, first order equation of motion for collisionless fluid.

$\Rightarrow$ ϕ is constant along integral curves of $\mathbf{r}$.

Some (very) pictorial examples

Geodesic dust fluid:

$$\phi(x, y) \sim \delta(y - u(x)) .$$



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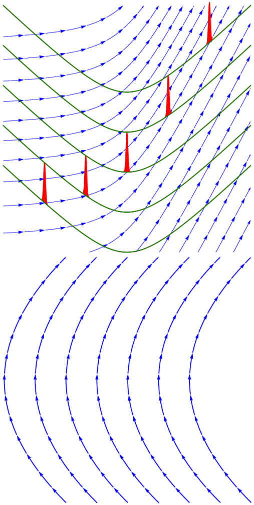

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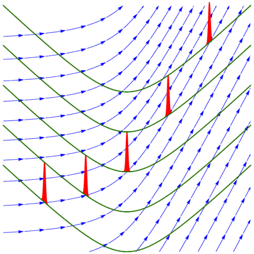

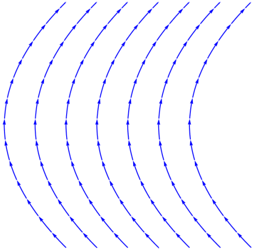

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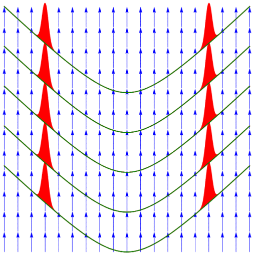

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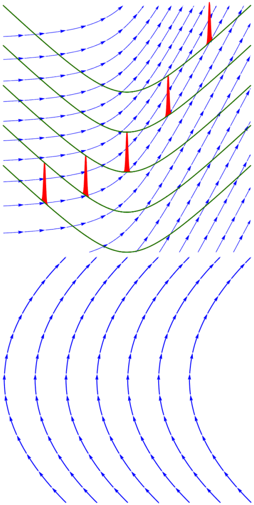

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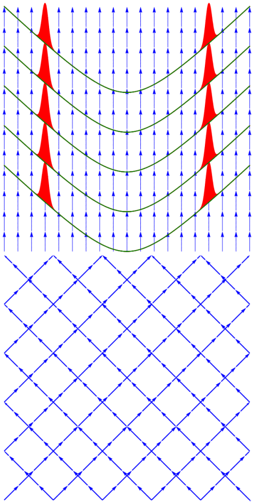

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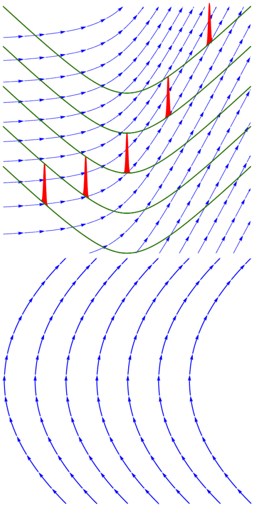

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Collisionless fluid:

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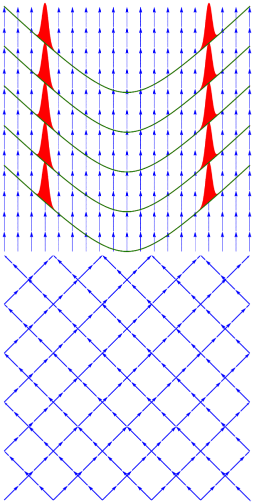

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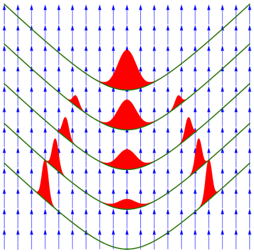

Some (very) pictorial examples

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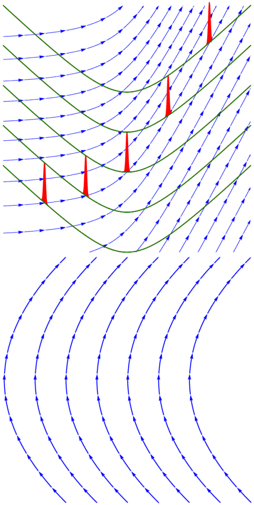

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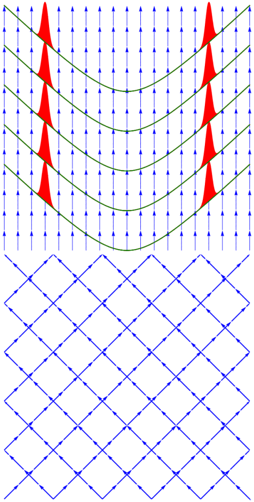

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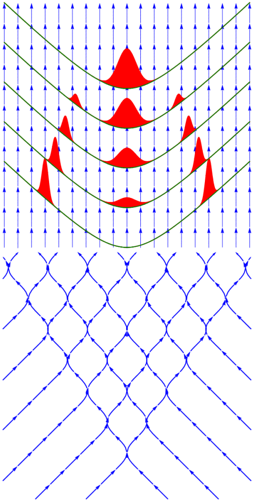

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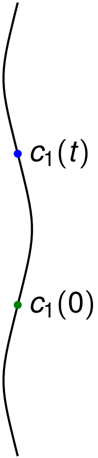
Outline

Action for a single point particle:

$$S = m \int_0^t (F \circ c_1)(\tau)\, d\tau\,.$$

Assume arc length parameter τ:

$$S = mt\,.$$



Action of a kinetic gas

Action for P point particles:

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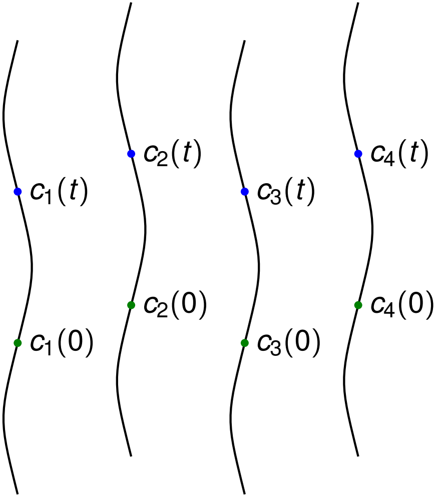

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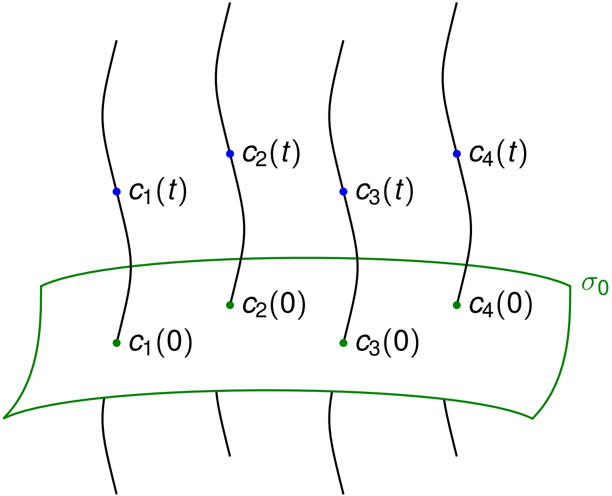

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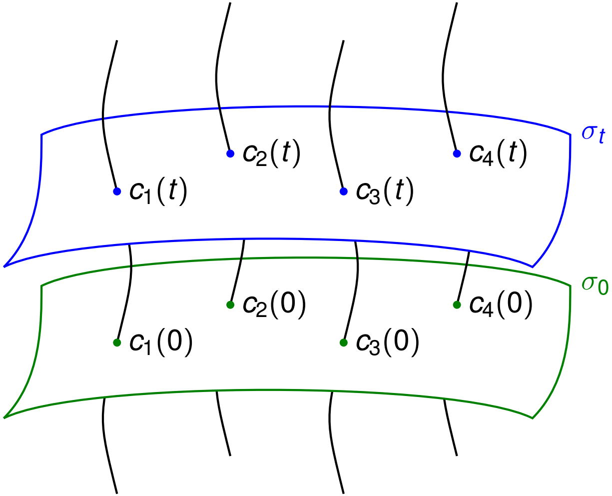

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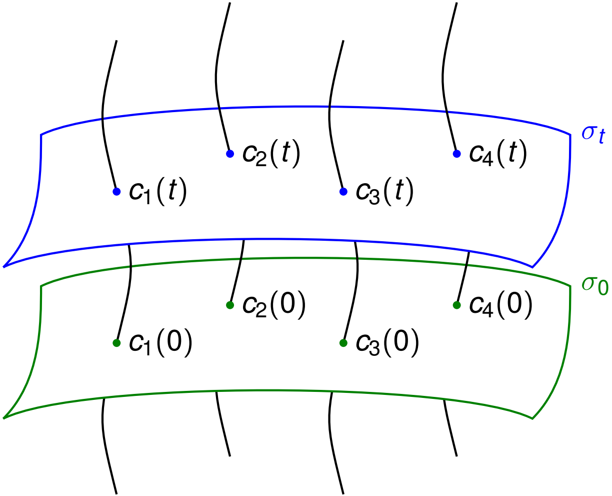

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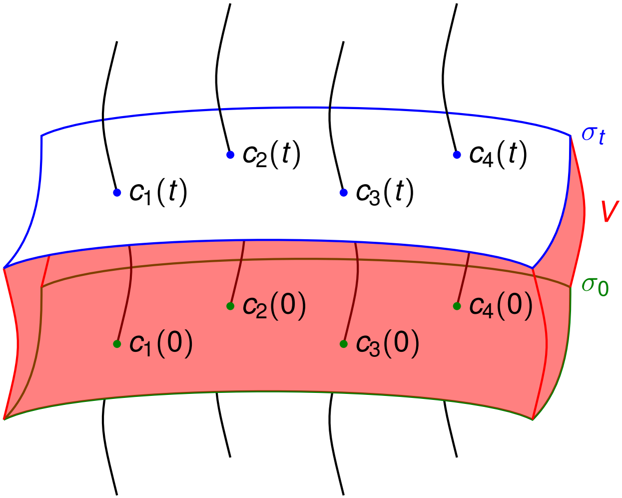

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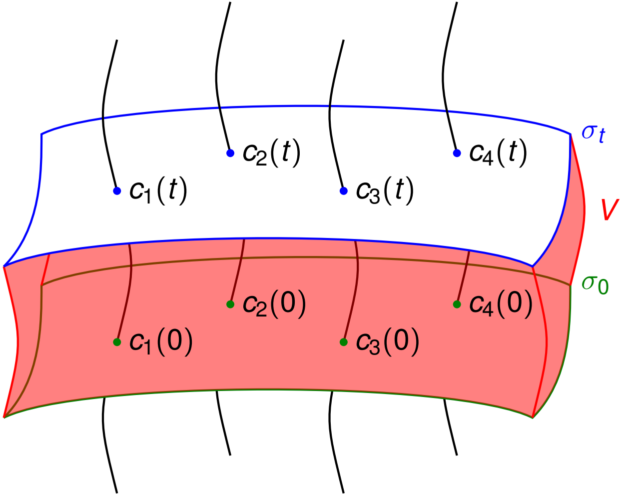

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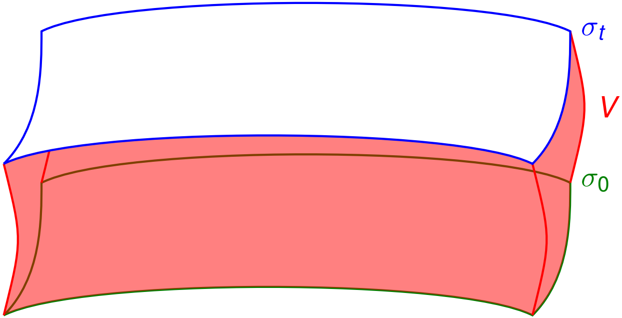

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$\Rightarrow$ Reduces to Einstein-Hilbert action for metric geometry.

Variation and field equations

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⇒ Gravitational field equations with kinetic gas matter [MH, Pfeifer, Voicu '19]:

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Physical implications

- There are no metric non-vacuum solutions to the field equations.
  - Field equations in case of a metric geometry $F^2 = g_{ab}(x)y^a y^b$:

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⇒ Gravitational field of a kinetic gas always depends on the velocity of the observer.
  - For observers whose velocity exceeds that of any gas particles:

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  - Solution of the differential equation still depends on ϕ via boundary conditions.

  ⇒ Observers at velocities beyond gas velocities are still affected, but differently.

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- Next task: solve cosmological field equations with kinetic gas.

Outline

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- Schwarzschild black hole:
  - Spherically symmetric spacetime.
  - Vacuum solution of Einstein's equations (general relativity).
  - Unique solution with these properties (Birkhoff theorem).

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  - Spherically symmetric spacetime.
  - Vacuum solution of Einstein's equations (general relativity).
  - Unique solution with these properties (Birkhoff theorem).
- κ-Poincaré modification of spacetime:
  - Interaction between particles and "quantum structure of spacetime".
  - Interaction depends on de Broglie wavelength (momentum).

  $\rightsquigarrow$ Distinguished time direction (vector field).

  $\Rightarrow$ κ-Minkowski spacetime has modified symmetry algebra.
  - Black hole spacetime: assume spherically symmetric vector field.

  $\Rightarrow$ Vector field may only have time and radial components.
  - Modification depends on a parameter ℓ (Planck length).
  - Spacetime approaches Schwarzschild for $\ell \to 0$.

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$\rightsquigarrow$ Consider gas $\phi \sim \delta(E)\delta(L)\delta(H)$ of identical energy, angular momentum, mass.

κ-Poincaré correction of Schwarzschild spacetime

- General κ-Poincaré modification of metric dispersion relation:

$$H = -\frac{2}{\ell^2}\sinh^2\left(\frac{\ell}{2}Z^\mu\bar{x}_\mu\right) + \frac{1}{2}e^{\ell Z^\mu\bar{x}_\mu}(g^{\mu\nu} + Z^\mu Z^\nu)\bar{x}_\mu\bar{x}_\nu\,. \tag{2}$$

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⇒ Minimal modification of Schwarzschild spacetime of mass M:

$$a^{-1} = b = c^{-2} = 1 - \frac{2M}{r}\,, \quad d = 0\,. \tag{4}$$

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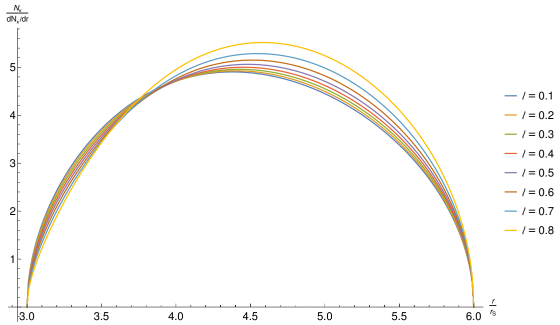

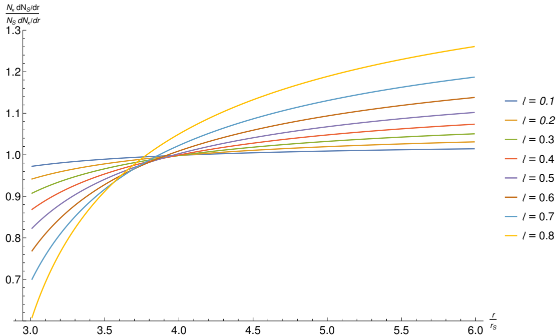

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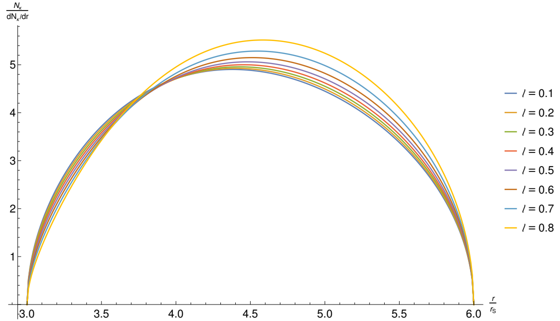

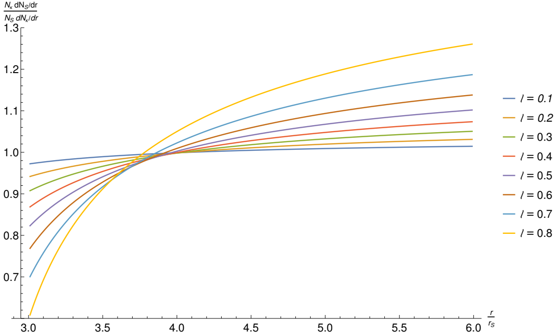

$\Rightarrow$ κ-Poincaré modification shifts particles inward.

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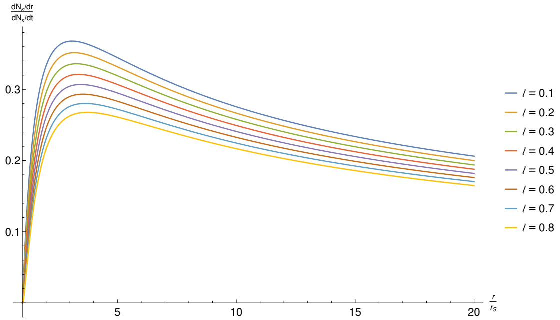

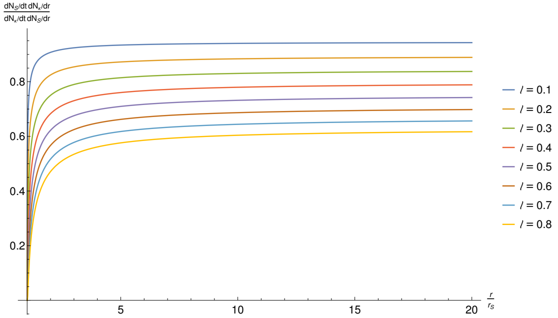

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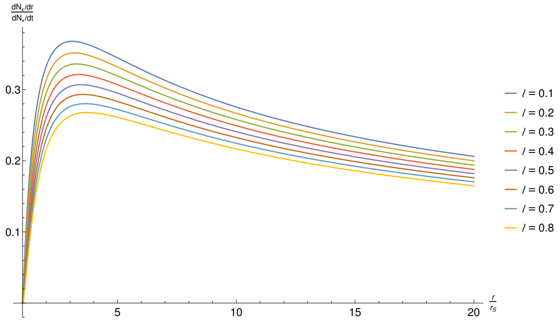

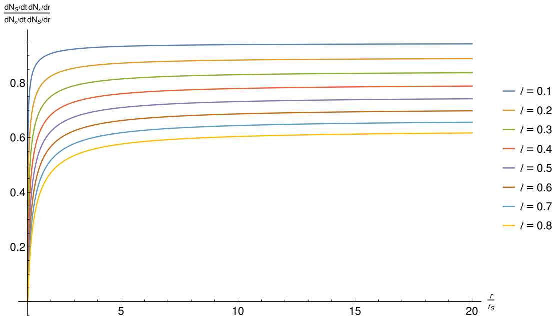

$\Rightarrow$ κ-Poincaré modification decreases particle density.

Outline

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- More general applications of kinetic gas model:
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  - Weak field limit: Newtonian, post-Newtonian…
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  - Dynamics of heterogeneous systems: stars + gas in galaxies?