

A New Gravitational Instanton:

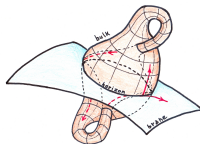
Shadow of an Extra Dimension?

*"understanding the processes in the vicinity of a black hole is
understanding quantum gravity"*

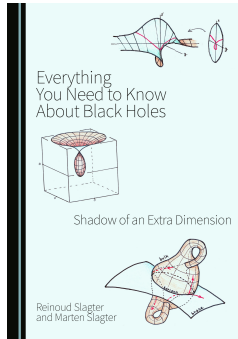
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Many details can be found in my book
Cambridge Scholar Publishing



Also: [Universe](#), [Slagter\(2025;2026\)](#)

Proposition:

- There exists an exact solution of a **gravitational instanton** of the 5D conformal invariant warped (Randall-Sundrum) Riemannian 5D brane world manifold, with topology $S^3 \times \mathbb{R}^3 / \mathbb{Z}_2$
- It can be described by the effective 4D brane **conformal Kähler manifold** $\mathbb{C}^1 \times \mathbb{C}^1$.
- We applied a **double cover of S^3 via stereographic projection to $\mathbb{CP}^1 \times \mathbb{CP}^1$** of the effective 4D manifold and by using the **Klein surface**, in order to utilize the $\mathbb{Z}_2$ symmetry.
- It can analytically transformed to a **complex locally conformally related Kählerian** manifold with a **Kähler potential**.
- **Self-duality is manifest, so represents a gravitational instanton**
- Questions:
 - A, Can a PBH originate from a gravitational instanton : "bumb" in Riemannian space?
 - B,.What will remain after the evaporation of a BH?
 - C. Can it possess quantum properties? [Bekenstein 1974; 't Hooft 2022)

The conformal invariant action

$$S = \int d^d x \sqrt{-\tilde{g}} \left[-\frac{1}{2} \xi (\Phi \Phi^* + \omega^2) \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} (\mathcal{D}_\mu \Phi (\mathcal{D}_\nu \Phi)^* + \partial_\mu \omega \partial_\nu \omega) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\Phi, \omega) - \Lambda \kappa \frac{4}{d-2} \xi \frac{d}{d-2} \omega \frac{2d}{d-2} \right], \quad \xi = (d-2)/(4(d-1)) \quad (1)$$

We applied 't Hooft's trick:

$$(5) g_{\mu\nu} = \omega^{4/3(5)} \tilde{g}_{\mu\nu}, \quad (5) \tilde{g}_{\mu\nu} = (4) \tilde{g}_{\mu\nu} + n_\mu n_\nu, \quad (4) \tilde{g}_{\mu\nu} = \tilde{\omega}^2 (4) \tilde{g}_{\mu\nu}$$

- n_μ normal to the brane
- Note that $\omega \rightarrow 0$ (sometimes called a dilaton field) describes the small distance limit—No singular behavior
- When scalar field is incorporated, ω and Φ behave at equal footing as quantum field on small scales
- Black holes are a kind of "exotic" states of matter
- It is conceivable that they are large-size limit of gravitational instantons at small scales
- $(\omega, \tilde{\omega})$ are used by in-falling observer to describe his experience of the vacuum and by the outside observer (using different Ω to fix the conformal gauge)
- We redefined $\omega^2 \rightarrow -6 \frac{\omega^2}{\kappa^2}$, with $\kappa = 8\pi G_N$, to ensure that the field ω has the same unitarity and positivity properties as the scalar field Φ .
- The Lagrangian is conformally invariant: $g_{\mu\nu} \rightarrow \Omega^{4/(d-2)} g_{\mu\nu}$, $\omega \rightarrow \Omega^{(2-d)/2} \omega$, $\Phi \rightarrow \Omega^{(2-d)/2} \Phi$
- The conformal invariance is spontaneously broken by a mass term in the potential. It spoils tracelessness of $T_{\mu\nu}$
- Conjecture: spacetime singularities and horizons disappear and causality and locality survive!

$$ds^2 = \omega(t, r, y_5)^{4/3} y_0 \left[-N(t, r)^2 dt^2 + \frac{1}{N(t, r)^2} dr^2 + dz^2 + r^2 (d\varphi + N^\varphi(t, r) dt)^2 + dy_5^2 \right] \quad (2)$$

- The RS model results in the 5D equations and the on the effective 4D brane.
- They must be solved simultaneously!
- We will apply the Wick rotation $t \rightarrow i\tau$, $N^\varphi \rightarrow iN^\varphi$

The field equations

- The system becomes

$$\text{Einstein : } \omega^{2(5)} G_{\mu\nu} - {}^{(5)}T_{\mu\nu}^{(\omega)} + \frac{3}{4\sqrt[3]{12}} \Lambda_5 \kappa_5^{4/3} \omega^{10/3(5)} g_{\mu\nu} = 0 \quad (3)$$

$${}^{(4)}G_{\mu\nu} - \frac{1}{\omega^2} \left[{}^{(4)}T_{\mu\nu}^{(\omega)} - \frac{1}{6} \Lambda_{\text{eff}} \kappa_4^2 \omega^{4(4)} g_{\mu\nu} \right] + \mathcal{E}_{\mu\nu} = 0 \quad (4)$$

$$\text{dilaton : } \xi \omega R - \tilde{g}^{\mu\nu} \nabla_\mu \nabla_\nu \omega - \frac{2n}{n-2} \Lambda \kappa \frac{4}{n-2} \xi \frac{n}{n-2} \omega^{\frac{n+2}{n-2}} = 0 \quad (5)$$

with

$$T_{\mu\nu}^{(\omega)} = \tilde{\nabla}_\mu \tilde{\nabla}_\nu \omega^2 - \tilde{g}_{\mu\nu} \tilde{\nabla}^2 \omega^2 + \frac{1}{\xi} \left(\frac{1}{2} \tilde{g}_{\alpha\beta} \tilde{g}_{\mu\nu} - \tilde{g}_{\mu\alpha} \tilde{g}_{\nu\beta} \right) \partial^\alpha \omega \partial^\beta \omega. \quad (6)$$

- $\mathcal{E}_{\mu\nu}$ the Weyl contribution from the bulk.
- Without a scalar field we obtain

$$\ddot{\omega} = -N^4 \omega'' + \frac{d}{\omega(d-2)} (N^4 \omega'^2 + \dot{\omega}^2) \quad (7)$$

$$\ddot{N} = \frac{3\dot{N}^2}{N} - N^4 \left(N'' + \frac{3N'}{r} + \frac{N'^2}{N} \right) - \frac{d-1}{(d-3)\omega} \left[N^5 \left(\omega'' + \frac{\omega'}{r} + \frac{d}{2-d} \frac{\omega'^2}{\omega} \right) + N^4 \omega' N' + \dot{\omega} \dot{N} \right] \quad (8)$$

- So the spin part decouples: $N^\varphi = \int \frac{dr}{r^3 \omega^{\frac{d-1}{d-3}}} + F_d(t)$. Further: $N(z)^2 = \frac{4}{z^2} \int z(z-a)^3 dz$

The exact solution becomes [NOTE : The dilaton equation is **superfluous!!** (see later)]

$$N^2 = \frac{C_2}{5r^2((t-t_0)^4 + C_3)} (4r^5 - 15a^2 r^4 + 20a^2 r^3 - 10a^3 r^2 + C_1), \quad \omega = \left(\frac{b_i}{(r-a)(t-t_0)} \right)^{\frac{1}{2}d-1} \quad (9)$$

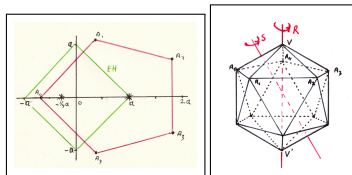
Properties

- The solution is invariant under $t \rightarrow it$. So one obtains Euclidean signature!
- C_1 is determined by the (complex) **topology**.
- So N is the same for the 4D and 5D cases, apart from the constants.
- We are interested here in the **stationary case**. Then N is determined by a **first order DE**

$$rN \frac{\partial N}{\partial r} + N^2 = \frac{k+1}{2} (r-a)^k \rightarrow N^2 = \frac{1}{r^2} \left[\frac{(r-a)^{k+1} (r(k+1) + a)}{k+2} + C_1 \right] \quad (10)$$

- $k=3$ delivers our quintic (k integer?). C_1

From the constraints: $\Lambda_{eff} = \frac{3}{8} \sqrt[3]{18} \frac{\kappa_5^{4/3} b_5^2}{\kappa_4^2 b_4^2} \Lambda_5$, $C_4 = \frac{C_5 b_4^3}{b_5^2}$



Cauchy theorem

- In the special case $C_1 = 0$, we noticed that zeros are determined by the quintic [this complex z not to confuse with coordinate z]

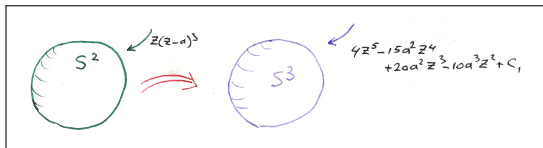
$$\frac{1}{5z^2} (4z^5 - 15a^2z^4 + 20a^2z^3 - 10a^3z^2 + C_1) = \frac{4}{z^2} \int z(z-a)^3 dz = \frac{4}{z^2} \oint_S f(z) dz = 0 \quad (11)$$

with $f(z)$ holomorphic.

One applies **Cauchy's formula** in our situation

If $f(z)$ is analytic on the boundary S^2 , then it determines its values inside, S^3 by $N^2(z)$:

$$N^2(a) = \frac{2}{z^2 \pi i} \oint_{S^2} \frac{f(z)}{z-a} dz, \quad \forall a \in S^3 \quad (12)$$



The **"rigidity" of analytic functions**: Information on a boundary (S^2) can deliver information for the inside (S^3) (see [Needham, 1996](#))

Complex transformation

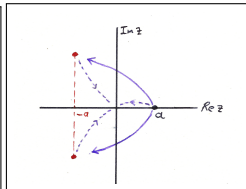
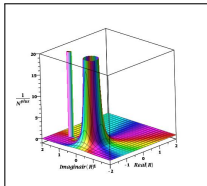
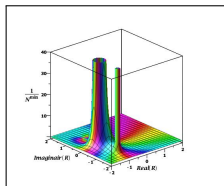
Let us apply the transformation on our metric [compare EH: $z^4 - a^4 \rightarrow z'^4$]

$$(z - a)^4 \rightarrow (z' + a)^4$$

(13)

There are 2 possibilities: $z = z' + 2a$, $z = a \pm i(z' + a)$. So the zero $z = a$ is shifted to two **complex values**

- When a decreases, then the $z = 0$ singularity is **approached from $-z$** !.

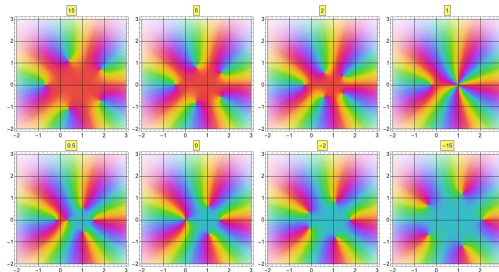


$$\bullet H_{comp}(z') = \pm \frac{(z' + a)^4 \sqrt{16z'^2 + 32z'a + 41a^2}}{z'^2 + 2z'a + 2a^2} e^{i\chi}, \quad \chi = \arctan\left(\frac{4z'^2 + z'a + 10a^2}{a(z' + a)^3(3z'^2 + 6z'a + 8a^2)}\right)$$

$$ds^{2(5)} = H_{comp}(z') dV d\bar{V} + dW d\bar{W} + (a + i(z' + a))^2 d\psi^2, \quad V = \tau + iz', W = z + i\psi$$

"Dance of the roots"

- ▶ Bold idea: (Bekenstein,1974;t Hooft,2022): compare a primordial black hole with a hydrogen atom?
- ▶ Would a BH possess a discrete spectrum for example, the surface area A_n ?
- ▶ Our quintic: $N^2 = 4z^5 - 15az^4 + 20a^2z^3 - 10a^3z^2 + c$. We vary for $a = 1$ and c varies from $c = -15..15$
- ▶ The dots represents the zeros and the colors $\arg(z)$.



- ▶ The case $c = 0$: zeros at $z = 0$ with multiplicity 2, one real zero $z \approx 1.6$ and two complex zeros.
- ▶ It seems to be an unstable 'ground state'. When c switches from a small negative to a positive value, then two complex zeros switch to two real values and immediately afterwards to one real negative zero point.
- ▶ What means integer k ??
 $k = 2 (c < a^4/4)$: 4 zeros;
 $k = 1 (c < -a^3/3)$ 1 real zero
 $k = 0$ 2 zeros at $\pm a$ (Schwarzschild). $\omega = \text{constant}$

New topology

- We have the Riemannian stationary effective metric for the special case $C_1 = 0$

$$ds_{\text{eff}}^2 = N_1(R)^2 [d\tau^2 + dR^2] + dz^2 + r(R)^2 d\varphi^{*2}, \quad R = \frac{4}{125a^2} \ln \left[\frac{(4r+a)}{(r-a)} \right] + \frac{1}{75a} \frac{17a^2 - 39ar - 3r^2}{(r-a)^3} \quad (14)$$

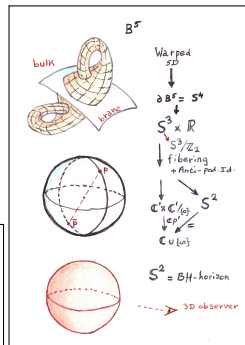
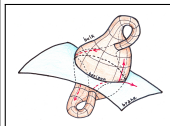
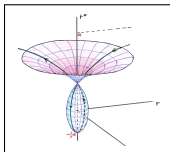
or

$$ds_{\text{eff}}^2 = \left[\frac{(a+4r)(r-a)^4}{5r^2} d\tau^2 + dr^{*2} + dz^2 + r^2 d\varphi^{*2} \right],$$

$$r^* = \pm \frac{1}{5\sqrt{a}} \left[6 \operatorname{arctanh} \left(\sqrt{\frac{a+4r}{5a}} \right) + \frac{\sqrt{5a(a+4r)}}{r-a} \right] \quad (15)$$

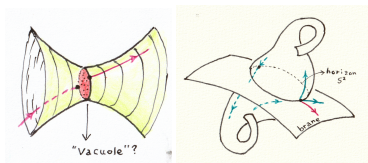
with $-\frac{a}{4} < r < a$. It describes the **interior of the black hole**

- Use a two fold Riemannian cover $\mathbb{C}P^2$ by the Klein topology and Hopf fibration to get S^2 as the horizon.



New topology: antipodal identification

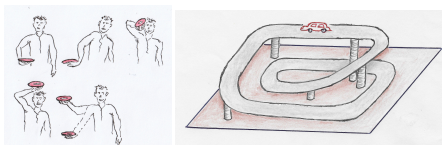
- ▶ Already introduced by Schrödinger (Book: "Universe", 1957!)
- ▶ Goal: how to treat gravitational back reaction between in and out particle during Hawking radiation: particles become hard!
- ▶ maintain unitarity, locality and purity of Hawking radiation and solve the information paradox
- ▶ This means that a curve crossing the horizon from region I to region II, will NOT enter the interior: we are at "the other side of the BH" (the antipode region). Keeps the wavefunctions pure
- ▶ The mapping must preserve the metric, so an element of $O(3)$ with **eigenvalues only -1** ! (no fixed points) $+1$ generates conical singularity.
- ▶ $-I \in SO(3, 1)$; in Euclidean space homotopically the identity. So analytically continuation!
- ▶ Connection between in- and out-going observer: **one-to-one map between in- and out-going radiation.**
- ▶ NOTE: Klein surface is direct sum of two projected planes $\mathbb{R}P^2 \# \mathbb{R}P^2$
- ▶ Path of the Hawking particle in 4D (Möbius band) and 5D (Klein bottle):



- ▶ The identification is **symmetric under CPT**
- ▶ 't Hooft: cut out a S^3 , with topology $\mathbb{R} \times S^3 / \mathbb{Z}$
- ▶ in our model it is there already

The related issues

- ▶ **A. Dirac plate:** $O(3)$ in 3-space is a **double cover of S^3** of quaternions in 4-space: $SO(3) \rightarrow S^3/\mathbb{Z}_2$
- ▶ and also double cover of projective space for antipodality!



- ▶ Fermions possess unusual property: after a rotation of 4π it returns to its original state: **topological effect!**
- ▶ NOTE: Dirac spinors can be constructed on non-orientable Riemann surfaces
- ▶ Antipodal identification comes in, i.e., the topology of the rotation group: $\mathbb{R} \times SO(3)$ as configuration space. The real 3-space $\mathbb{R}^3$ arises from S^3 in $\mathbb{R}^4$ by identifying antipodal points on the boundary. And $\mathbb{R}P^3 \sim S^3$.
- ▶ Because $\mathbb{R}P^3$ is compact and a continuous image of S^3 , then is $SO(3)$ homeomorphic to $\mathbb{R}P^3$. And $\pi(SO(3)) \cong \pi(\mathbb{R}P^3) \cong \mathbb{Z}_2$
- ▶ This happens also in our double Klein bottle model!

The related issues

- ▶ **B. RS model:** the range of the graviton modes are extended to the brane : "KK-modes"
- ▶ One says that ω is part of the **gravitational sector** . It fixes the **scale** at which **quantum gravity** becomes important
- ▶ For $\Lambda = 0$, the Lagrangian contains **no intrinsic mass or length scale** (Compare with Higgs mechanism.)
- ▶ Now we include the **RS model** : The renormalized counter terms in S^{eff} will cause unitarity problems, **only in the bulk** at the large extra dimension-scale
- ▶ This means that in a **EFT** , the **subtraction point** will be in the **UV** as seen by the observer in the bulk.
- ▶ **This is possible by the KK contribution in the 4D equations.**
- ▶ The 4D Planck-scale $M_4 \sim 10^{19} GeV$ is **no longer the fundamental scale**, which is M_{4+d} . It becomes an effective scale, describing gravity on scales much larger than the extra dim, $M_{pl}^2 \sim M_{4+d}^{2+d} l^d$
- ▶ If the extra dim is much larger than Planck scale, then the true fund scale M_{4+d} can be much less than the eff scale $M_{pl} \sim 10^{19} GeV$
- ▶ For $l \sim 1 mm$ ($d=1$), the **fundamental Planck scale can be $\sim 1 TeV$!**
- ▶ In RS model we have $M_5^3 = \frac{M_{pl}}{l}$ with l the effective length scale of the infinite large extra dimension
- ▶ So the effective size of the extra dimension, probed by the 5D graviton, is l .
- ▶ **Now $\bar{g}_{\mu\nu}$ will interact with the "quantum" field ω**
- ▶ **It turns out that ω equations decouple from $\bar{g}_{\mu\nu}$ equations and they can be solved **exact****
- ▶ **So for low energy scale, it prevents gravity leaking into the extra dimension.**

Going complex: needed to proof Gravitational Instanton

► A. The notion of instantons

- was introduced by 't Hooft and represents a **finite action topological invariant** in field theory
- First developed in the non-abelian **YM**, in particular in $SU(n)$ gauge theories.
- classical field equations with **self-dual field strength** gave rise to non-perturbative treatment
- after **Wick rotation** $t \rightarrow i\tau$, with τ parameterization of the favoured tunneling path in configuration space
- quantization is then unambiguous
- solutions are around a point in **4D Euclidean** space and at one instant there is a 'bump' in field strength

► Counterpart in GR:

• We need here the most famous ones: **axi-symm Eguchi-Hanson and Fubini-Study**

The EH: In the basis 1-forms

$$ds_{EH}^2 = \frac{1}{f(r)^2} dr^2 + \frac{r^2}{4} (\sigma_x^2 + \sigma_y^2 + f(r)^2 \sigma_z^2) \quad (16)$$

- The solution $f^2 = r^4 / (r^4 - a^4)$ is determined by a **first order differential equation**.
- in **Euler angles** (ψ, θ, φ), one of the angles **must run from 0, 2π in stead of 0, 4π** , in order to remove the $r = 0$ singularity [in our model also!].
- Defining $\rho^4 = r^4 - a^4$, the metric is then geodesically complete and **without true singularities**.
- One writes the manifold in **Kähler form** K on $\mathbb{C}^2 / \{0\}$ (with Kähler potential κ)

$$K = \frac{\rho^2}{\sqrt{\rho^4 + a^4}} (dz_1 d\bar{z}_1 + dz_2 d\bar{z}_2) + \frac{a^4}{\sqrt{\rho^4 + a^4}} \partial \bar{\partial} \log \rho^2 = \partial \bar{\partial} \log \left(\frac{\rho^2 e^{\sqrt{\rho^4 + a^4}}}{a^2 + \sqrt{\rho^4 + a^4}} \right) \quad (17)$$

with $(z_1, z_2) = (x + iy, z + it)$, $\rho = \sqrt{z_i \bar{z}_i}$.

- The singular point $\rho = 0$ is removed by the **antipodal identification** $(z_1, z_2) \rightarrow (-z_1, -z_2)$. Near $r = a$ the manifold is $\mathbb{R}^2 \times S^2$.
- At each point of the two sphere (θ, ψ) there is attached an $\mathbb{R}^2$ which shrinks to a point for $r \rightarrow a$.
- The manifold is so homotopic to S^2 . For **large r** , the metric approaches a flat metric and the constant- r hypersurfaces are **3-spheres**, with **antipodal points identified**.
- The boundary is the $SO(3) = \mathbb{R}P^3$ = group manifold for which $S^3 = SU(2)$ is the double cover. So the EH is **locally Euclidean**, i.e., $\mathbb{R}P^3 = S^3 / \mathbb{Z}_2$ but not globally.

Going complex: needed to proof Gravitational Instanton

► The FS:

- There exist interesting compact Einstein self-dual (SD) and anti-self-dual (ASD) manifolds, isomorphic to the Euclidean 4-sphere or the **complex projected plane** $\mathbb{CP}^1 \# \mathbb{CP}^1$.
- One can use several routes to study these manifolds, such as Penrose twister model and the **Plebanski-chiral method**(1977).
- By demanding that the curvature is self-dual as a 2-form, one obtains a second order differential equation in the metric components, yielding two integration constants, one of which can be the **cosmological constant**. One prefers, of course, a first order equation, as was the case of the EH manifold.
- Let us consider the FS manifold, which we will also use in our model.

$$ds_{FS}^2 = \frac{1}{S(r)^2} \left(dr^2 + \frac{r^2}{4} \sigma_x^2 + \frac{r^2 S(r)}{4} (\sigma_y^2 + \sigma_z^2) \right) \quad (18)$$

with $S(r) = 1 + \Lambda r^2$.

- One can also use the Euler angles, $\sigma_x = \sin \psi \partial_\theta - \cos \psi \sin \theta \partial_\varphi$, $\sigma_y = \cos \psi \partial_\theta + \sin \psi \sin \theta \partial_\varphi$, $\sigma_z = \partial_\psi + \cos \theta \partial_\varphi$.
- **One needs, however, a term $\alpha g_{\mu\nu}$ on the right hand side of the Einstein equations.**
- One can also use the pure connection formalism on $\mathbb{CP}^2$. The FS is Kähler, i.e., it is of the form ($i = (1, 2)$)

$$ds_{FS}^2 = \frac{\partial^2 \kappa}{\partial z^i \partial \bar{z}^{\bar{j}}} dz^i d\bar{z}^{\bar{j}}, \quad \kappa = \frac{1}{\Lambda} \log(1 + \Lambda z^i \bar{z}^{\bar{j}}) \quad (19)$$

with κ the **Kähler potential** (up to a real part of a holomorphic function!).

- The metric is **local conformal** to

$$ds_{FS}^2 = (1 - |z^1|^2)(dz^1 d\bar{z}^1 + dz^2 d\bar{z}^2) - \bar{z}^1 z^2 dz^1 d\bar{z}^2 - z^1 \bar{z}^2 d\bar{z}^1 dz^2 \quad (20)$$

- Conformal Kähler means $d(\Omega^2 K) = 0$, $g_{ij} = \Omega^2 \hat{g}_{ij} = \partial_{z^i} \partial_{\bar{z}^j} \kappa$

The final step using Plebanski method

- To prove that the FS represents a gravitation instanton, one can use the Plebanski self-dual method, by computing the ASD (SD)-1-form $SO(3)$ connections Σ^i , the 2-forms $d\Sigma^i$ and curvature forms F^i

$$\Sigma^i / \bar{\Sigma}^i = ie^0 \wedge e^i \pm \frac{1}{2} \epsilon_{jk}^i e^j \wedge e^k, \quad d^A \Sigma^i \equiv d\Sigma^i + \epsilon_{jk}^i A^j \Sigma^k, \quad F^i(A) = dA^i + \frac{1}{2} \epsilon_{jk}^i A^j A^k \quad (21)$$

The Einstein equations follow from the action (Ψ^{ij} Lagrange multipliers)

$$S = \int \Sigma^i F_i - \frac{1}{2} \left(\Psi^{ij} + \frac{\Lambda}{3} \delta^{ij} \right) \Sigma^i \Sigma^j, \Rightarrow F^i = \left(\Psi^{ij} + \frac{\Lambda}{3} \delta^{ij} \right) \Sigma^j, \quad \Sigma^i \Sigma^j \sim \delta^{ij}, \quad d^A \Sigma^i = 0 \quad (22)$$

- One calls the connections perfect, if $F^i F^j \sim \delta^{ij}$. They describe ASD Einstein Ricci flat metrics of non-zero scalar curvature Λ !

NOTE: in this method, one includes a cosmological constant Λ [in our model replaced by $T_{\mu\nu}^{(\omega)}$!!]

- The solution follow from FIRST-order DE, and satisfy also the SECOND-order DE ! [just as ours!!]
- (anti-) self-duality is provided by the spin connections $\omega_{ab} = \pm \tilde{\omega}_{ab} \equiv 1/2 \epsilon_{abcd} \omega_{cd}$
- Or by the F^i , expressed as combination of the Σ^i 's
- When we have $\omega_j^i = \pm 1/2 \epsilon_{ijk} \omega_k^j$ then the curvature two forms are (anti-) self-dual
- After some algebra, one finds $F^i = -\frac{\alpha}{3} \Sigma^i$, which represents the correct Plebanski equation $\Psi^{11} = 0$
- So the FS represents a gravitational instanton, in the sense of one of the two halves of its spin connections is non-vanishing
- After writing the F^i in terms of Σ^i and $\bar{\Sigma}^i$, one obtains the Einstein equations by equating to zero all the anti-self dual components.

The equivalent route is to prove the manifold is conformal Kähler!

Example: Kerr. The Janis-Newman-Winicour(JNW) "trick"

- The Kerr can be written in local conformal Kähler block form. We prove it with Maple

restart:with(grtensor):

*******define the Kerr spacetime in complex coordinates**

$$dz_0 := dt - \frac{(a^2 + r^2)}{\Delta} dr - ia \sin \theta d\theta; \quad dz_1 := d\varphi - \frac{a}{\Delta} dr - \frac{i}{\sin \theta} d\theta;$$

$$d\bar{z}_0 := dt + \frac{(a^2 + r^2)}{\Delta} dr + ia \sin \theta d\theta; \quad d\bar{z}_1 := d\varphi + \frac{a}{\Delta} dr + \frac{i}{\sin \theta} d\theta;$$

$$g_{tt} := (\Delta - a^2 \sin^2 \theta) \Sigma^{-1}; \quad g_{\varphi\varphi} := \frac{a^2 \Delta \sin^2 \theta - (a^2 + r^2)^2}{\Sigma}$$

$$g_{t\varphi} := \frac{2aMr \sin^2 \theta}{r^2 + a^2 \cos^2 \theta}; \quad \Delta = r^2 - 2Mr + a^2, \Sigma = r^2 + a^2 \cos^2 \theta$$

spacetime(Kahler, coord=[t, r, ϕ, θ], ds = (g_{tt}dz₀d̄z₀ + g_{ϕϕ}dz₁d̄z₁ + g_{tϕ}(dz₀d̄z₁ + dz₁d̄z₀));

grcalc(g(dn,dn),R(dn,dn));gralter(R(dn,dn),simplify);R(dn,dn);

R_{ab}= all components are zero.

- Consider the general conformal Kähler Plebanski-Demianski class

$$\frac{1}{\Pi \Sigma} \begin{pmatrix} \Delta_r - a^2 \Delta_x & a(r^2 + a^2) \Delta_x - a(1 - x^2) \Delta_r & 0 & 0 \\ a(r^2 + a^2) \Delta_x - a(1 - x^2) \Delta_r & a^2(1 - x^2)^2 \Delta_x - (r^2 + a^2)^2 \Delta_x & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \frac{\Sigma}{\Pi} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\Delta_x} & 0 \\ 0 & 0 & 0 & -\frac{1}{\Delta_r} \end{pmatrix} \quad (23)$$

with $(\Delta_x, \Delta_r, \Pi)$ the Plebanski-Demianski metric functions and $\Sigma = r^2 + a^2 x^2$. They contain several parameters, such as the mass M, angular J momentum and Nut charge N. The Kerr solution is a special case:

$$\hat{ds}_{kerr}^2 = \frac{i}{(r - iax)^2} \left[d\varphi \wedge \left(a(x^2 - 1)dr + i(r^2 + a^2)dx \right) + d\tau \wedge (dr - iadx) \right], \quad x = \cos \theta \quad (24)$$

with $\kappa_{kerr} = 4 \left[-\frac{r - iax \cos \theta}{2M} + \log \sin \theta \right]$ and $\Omega^2 = \Pi / (r - iax)^2$, Π arbitrary function in (r, x)

Example Kerr $\Leftrightarrow$ Schwarzschild

- One observes that by a COMPLEX shift (found by Janis, et al.(1965)

$$r \rightarrow r - ia \cos \theta \quad (25)$$

on the Schwarzschild, delivers the Kerr. It is also LCK

$$ds_{Schw}^2 = \frac{1}{r^2} \left[\left(1 - \frac{2M}{r}\right) d\tau^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 \right] + d\theta^2 + \sin^2 \theta d\varphi^2 \quad (26)$$

The Kähler potential is

$$\kappa_{kerr} = 4 \left[-\frac{r - ia \cos \theta}{2M} + \log \sin \theta \right] \quad (27)$$

In Eddington-Finkelstein

$$ds_{Schw}^2 = \left(1 - \frac{2M}{r}\right) du^2 - 2du dr - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (28)$$

On uses $\bar{u} = u - i \cos \theta$, $\bar{r} = r + i \cos \theta$, Weyl tensor transforms as
 $\Psi = -M/r^3 \rightarrow -M/(r - ia \cos \theta)^3$.

$$\begin{aligned} l^\mu &= \delta_1^\mu, \quad n^\mu = \delta_0^\mu - \frac{1}{2} \left(1 - \frac{2mr'}{r'^2 + a^2 \cos^2 \theta}\right) \delta_1^\mu, \\ m^\mu &= \frac{1}{\sqrt{2}(r' + ia \cos \theta)} \left(ia \sin \theta (\delta_0^\mu - \delta_1^\mu) + \delta_2^\mu + \frac{i}{\sin \theta} \delta_3^\mu \right) \\ \bar{m}^\mu &= \frac{1}{\sqrt{2}(r' - ia \cos \theta)} \left(-ia \sin \theta (\delta_0^\mu - \delta_1^\mu) + \delta_2^\mu - \frac{i}{\sin \theta} \delta_3^\mu \right) \end{aligned} \quad (29)$$

This is easily checked in GRTENSOR.

- **That is strange: spin from staticity?** Related to the **extra parameter** showing up in solution
- Obtaining intrinsic spin from a transformation is very well known in physics!!
- There are several "explanations", such has the use of the general **Kerr-Schild**. But there is more.
- There is a more physical "**heavenly**" explanation (related to Penrose twistors): go to 4-dim complex manifold (Flaherty,1976; Krasnov 2022).

Intermezzo: JWN-trick explained

Janis-Newman-Winicour:

'system of relativistic particles: one can be interpret intrinsic spin as arising from a center of mass world line displaced from real into complex manifold!'

- Including charge: The Dirac particles has gyromagnetic ratio $e/mc!$
- One can find spectrum of bound states of Dirac spin $1/2$ particles in the vicinity of rotating cosmic strings

$$ds^2 = -(dt + 4GJ_t d\varphi)^2 + dr^2 + (1 - 4GM)^2 r^2 d\varphi^2 + (dz + 4GJ_z d\varphi)^2 \quad (30)$$

If one boost in the (t, z) plane, (J_t, J_z) transform as a 2D vector.
Orthonormal basis

$$E_a^\mu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 4GJ/(1 - 4GM)r & 0 & 1/(1 - 4GM)r & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (31)$$

- Then for spin half field: $(i\gamma E_a^\mu D_\mu - \mu)\Psi = 0$

Expansion in eigenfunctions $\Psi_{E,p,z,m}(x^\mu) = e^{i(-Et+p_z+m\varphi)} \Psi_{E,p,m}(r)$

- There is a relation between their Weyl curvature spinors, i.e., $\Psi_{Kerr}(x) = \Psi_{Schw.}(x + ia)$ and between their Kähler potentials.
- Then there is the striking analogue between the Kerr solution with the singular rotating ring of mass about its symmetry axis and the Maxwell's equations of rotating ring of charge, just like the analogue of the Schwarzschild field and Coulomb field.
- There is a relation with the double copy of the gauge potential of the self-dual dyon in electromagnetism, the Dirac string and the curious behavior of the semi-infinite line mass.
- **CONJECTURE:**

The JNW trick is replaced in our model by the complex transformations $[\varphi \rightarrow \varphi + iN^\varphi t, z \rightarrow a \pm i(z + a)]$

Our new Gravitational Instanton

We will use the topology $S^3 \times \mathbb{R}/\mathbb{Z}_2$.

- It is conjectured that a metric on Klein bottle K^2 , as part of $H^2 \times S^1$ is self-dual Kähler, with H^2 a compact manifold with boundary S^1 .
- It can be proven that any complex manifold that can be embedded in projective space, is Kähler.
- If a Hermitean manifold with metric $g_{\mu\nu}$ and integrable almost complex, then it is conformally related to a Kählerian $\Omega^2 g_{\mu\nu}$
- NOTE: Conformal Kähler only possible in **even dimension!** Happily, we have also the 4D effective manifold!
- One proves (Calabi) that a 2-dimensional manifold (M,g) admits a **local Kähler immersion into a Hilbert space**, i.e., a real 3-dimensional Kähler manifold N, $f: M \rightarrow N$, given by $(z_1, z_2) \rightarrow (z_1, z_2, f(z))$, with f **holomorphic with no terms of degree less than 2**.
Our N fulfills !!

We write :

$$K = \partial_{\bar{\zeta}} \partial_{\zeta} \kappa d\zeta d\bar{\zeta}, \quad \zeta = \{\xi(\tau, r), \chi(z, \varphi)\}, \quad \xi = \rho(r)e^{i\tau}, \quad \chi = z + i\varphi^* \quad (32)$$

with the κ the Kähler potential. The Kähler potential is not unique. One can perform transformations.

- We write $K = K_1 + K_2$, with K_1 the first axisymmetric block written as

$$ds_{2D}^2 = N(r)^2 d\tau^2 + \frac{1}{N(r)^2} dr^2 = \partial_{\xi} \partial_{\bar{\xi}} \kappa d\xi \wedge d\bar{\xi},$$

$$C(r) \equiv \log |\rho(r)| = \int \frac{1}{N^2} dr = \frac{4}{125a^2} \ln\left(\frac{4r+a}{r-a}\right) + \frac{17a^2 - 39ar - 3r^2}{a(r-a)^3} \quad (33)$$

with $\kappa = J(\rho) = J(\sqrt{\xi\bar{\xi}})$ the Kähler potential. One finds finally

$$\frac{dC}{dr} = \frac{1}{N^2} \quad \frac{d^2 J}{dC^2} = 4N(C)^2 \quad (34)$$

One needs the inverse $r = r(C)$

So the first block is conformal Kähler: $g_{ij} = \Omega^2 \hat{g}_{ij} = \partial_{z^i} i \partial_{\bar{z}^j} \kappa, d(\Omega^2 K) = 0$

Our result

- We write the second block $(dz, d\varphi^*)$

$$dz^2 + r^2 d\varphi^{*2} = r^2 \left(\frac{1}{r^2} (dz^2 + d\varphi^{*2}) \right) = r^2 \left(\frac{\tan^2 \theta}{z^2} dz^2 + d\varphi^{*2} \right) = r^2 \left(\tan^2 \theta dz^{*2} + d\varphi^{*2} \right) \quad (35)$$

where we isolated a conformal factor r^2 . Further, $z \equiv e^{-z^*}$ and θ constant, because we work in the plane.

- Finally we arrive at our 2-form

$$\tilde{K} = \Omega(r)^2 K = \Omega(r)^2 \left[\partial_\xi \partial_{\bar{\xi}} \mathcal{K}_1 d\xi \wedge d\bar{\xi} + \partial_\chi \partial_{\bar{\chi}} \mathcal{K}_2 d\chi \wedge d\bar{\chi} \right] \quad (36)$$

We isolated the r^2 -term and absorbed it in the conformal factor. Further, $d(\Omega^2 K) = 0$.

- **We have completed the proof of the proposition**

- One can also use the form

$$ds_2^2 = \frac{(a+4r)(r-a)^4}{5r^2} d\tau^2 + dr^{*2}, \quad r^* = \pm \frac{1}{5\sqrt{a}} \left[6 \operatorname{arctanh} \left(\sqrt{\frac{a+4r}{5a}} \right) + \frac{\sqrt{5a(a+4r)}}{r-a} \right] \quad (37)$$

$$\frac{d^2 J}{dC^2} = 4N(r^*)^2, \quad \left(\frac{dC}{dr^*} \right)^2 \frac{d^2 J}{dC^2} = 4, \quad \rightarrow \quad \frac{dC}{dr^*} = \frac{1}{N(r^*)} \quad (38)$$

Further, one can evaluate $d^2 J / dC^2 = 4N(r^*)^2$

Our result



$$\frac{dJ}{dC} = \int N(r^*)^2 dC = \int N(r^*)^2 \frac{dC}{dr^*} dr^* = \int N(r^*) dr^* \quad (39)$$

The Kähler metric possesses an isometry for Killing vectors k^ζ ,

$$\zeta \rightarrow \zeta + k^\zeta, \quad \bar{\zeta} \rightarrow \bar{\zeta} + k^{\bar{\zeta}}, \quad k^\zeta(\zeta) = ig^{\zeta\bar{\zeta}} \partial_{\bar{\zeta}} P, \quad k^{\bar{\zeta}}(\bar{\zeta}) = -ig^{\bar{\zeta}\zeta} \partial_\zeta P \quad (40)$$

for some real $P(\zeta, \bar{\zeta})$

$$P = -\frac{1}{2}i \left(k^\zeta \partial_\zeta \mathcal{K} - k^{\bar{\zeta}} \partial_{\bar{\zeta}} \mathcal{K} \right) \quad (41)$$

The Kähler potential is invariant under the isometry. There is also an isometry of $\tau \rightarrow \tau + a$, generated by $k^\zeta = i\zeta, k^{\bar{\zeta}} = -i\bar{\zeta}$. We then have

$$P = \rho \frac{dJ}{d\rho} = \frac{dJ}{dC} \quad (42)$$

In the 'plane' case, we have for a translation, $k^\xi = 1$. So again

$$P = -\frac{1}{2}i \left(k^\xi \partial_\xi \mathcal{K} - k^{\bar{\xi}} \partial_{\bar{\xi}} \mathcal{K} \right) = \frac{dJ}{dC} \quad (43)$$

So in this 'plane' case, we conclude that

$$N(r^*) = \frac{d}{dr^*} P(r^*) \quad (44)$$

where we should invert $r(r^*)$.

- One conclude that N^2 of our solution is the derivative with respect to r^* of the 'momentum-map' of the Killing vector

Our result: compared with JNW solution (not to confuse with their "trick")

- ▶ A different route to the gravitational instanton is Plebanski's self-dual method (as used for the FS manifold)
- ▶ It has to do with the epic solution found by JNW(1968) an interesting link between the central singularity of the Schwarzschild solution and the solution with a massless scalar field ϕ .

$$ds_{JNW}^2 = -N(R)dt^2 + \frac{1}{N(R)}dR^2 + r(R)^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (45)$$

with field equations $G_{\mu\nu} = -\kappa T_{\mu\nu}, \quad T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi, \quad \nabla^2\phi = 0 \quad (46)$

with solution (Maple+GRTENSOR!)

$$N(r) = \left(\frac{1 + \alpha_2(R - 2M)}{1 - \alpha_2(R - 2M)} \right)^{1/\sqrt{\alpha_2}}, \quad r(R) = \frac{1}{\alpha_2^2} \left(\alpha_2^2(R - 2M)^2 - 1 \right) \left(\frac{1 + \alpha_2(R - 2M)}{1 - \alpha_2(R - 2M)} \right)^{-1/\sqrt{\alpha_2}} \quad (47)$$

$$\phi(R) = \frac{\sqrt{\alpha_2 - 1}}{\sqrt{2\kappa\alpha_2}} \ln \left(\frac{\alpha_2(R - 2M) - 1}{\alpha_2(R - 2M) + 1} \right) + \alpha_1$$

α_1, α_2 constants ($\alpha_2 \geq 1$). $\alpha_2 = 1$ delivers Schwarzschild

- ▶ **NOTE:** (N, r, ϕ) are obtained solely from the Einstein equations! **Just as in our case!**
- ▶ There is a strange anomaly. The surface area of a sphere of constant R is $\sim r^2$. When $R = \frac{1}{\alpha_2} + 2M = 0$, i.e., $r = 0$, there is a singular point.
- ▶ It turns out that the sphere $R = 2M$ is no longer a curvature singularity, but merely a point then a sphere.
- ▶ I. 'All asymptotically flat static solutions with a point singularity and with a mass and some asymmetry, approach this kind of 'truncated Schwarzschild solution' as the asymmetry vanishes.
 II. 'If a non-singular event horizon develops during the collapse a kind of asymmetry, it must radiate away all its multipole moments (otherwise trapped surfaces)'
 III. 'It is conjectured, due to the unusual behavior of the JNW black hole, that the collapse beyond the Schwarzschild radius, must be reviewed by a topology change'

Our result: Plebanski's self-dual method (see Krasnov,2022)

- We can write our 4D manifold

$$G_{\mu\nu} - \mathcal{E}_{\mu\nu} = \frac{1}{\omega^2} \left[T_{\mu\nu}^{(\omega)} - \lambda \kappa^2 \omega^4 g_{\mu\nu} \right] \quad (48)$$

- Left we have already the trace-free part $\mathcal{E}_{\mu\nu}$
- If we omit λ , we split off the trace part

$$T_{\mu\nu}^{\omega} = T_{\mu\nu}^{0(\omega)} - 1/4 T^{(\omega)} g_{\mu\nu}, \quad T^{(\omega)} = \frac{6}{r\gamma_0} (rN^2 \omega \omega')' \quad (49)$$

- Next one forms the pure ASD 2 forms $T_{\mu\nu}^i \equiv \Sigma_{[\mu}^{i\rho} T_{\rho\nu]}^{0(\omega)}$
- In our case we have the SD two forms (and in the same way the ADS)

$$\begin{aligned} \Sigma^1 &= \frac{\sqrt{N^2 + r^2(N^\varphi)^2}}{N} dt dr - \frac{r^2 N^\varphi}{N \sqrt{N^2 + r^2(N^\varphi)^2}} d\varphi dr + \frac{rN}{\sqrt{N^2 + r^2(N^\varphi)^2}} dz d\varphi, \\ \Sigma^2 &= \sqrt{N^2 + r^2(N^\varphi)^2} dt dz - \frac{r^2 N^\varphi}{\sqrt{N^2 + r^2(N^\varphi)^2}} d\varphi dz - \frac{r}{\sqrt{N^2 + r^2(N^\varphi)^2}} dr d\varphi, \quad \Sigma^3 = -rN dt d\varphi - \frac{1}{N} dr dz \end{aligned}$$

writing our $d^A \Sigma^i$ one obtains the A^i (see FS case)

- Next one can add a multiple of the ASD part of the 'stress-energy' tensor $T_{\mu\nu}^i$ to the Plebanski equation,

$$F_i = \left(\Psi_{ij} + \frac{\lambda - 2\pi G T}{3} \delta_{ij} \right) \Sigma^j + 8\pi G T_i \quad (51)$$

- Just as the Euclidean Kerr, Petrov type $D \otimes D$, is conformal to two different Kähler metric, ours will also!
- To find the Einstein equations, we write again F^i in terms of $(\Sigma^i, \bar{\Sigma}^i)$

NOTES: Imaginary displacement and intrinsic spin :

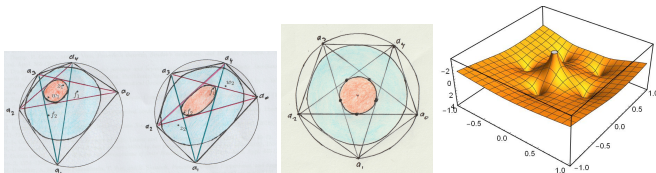
- ▶ An other illuminating approach is the use of the Kerr-Schild (KS) spacetime, where one writes $g^{\mu\nu} = \eta^{\mu\nu} + \lambda^2 l^\mu l^\nu$, with l_μ null-geodesics.
- ▶ One can make the connection with electrodynamics(ED): one "replaces" $r_S = \kappa M/4\pi$ by $gq/4\pi$.
- ▶ In ED one has a gauge field by some current configuration.
- ▶ A KS metric is then the result also of a source: For the Kerr, a disk whose mass distribution shows the ring singularity; for Schwarzschild the NJW "phantom" source.
- ▶ Remember, the FS needs a cosmological constant.
- ▶ Further, Kerr was obtained by complexifying Schwarzschild and to perform a shift, introducing spin effects.
- ▶ In the same way, a minimal coupled scalar field can be transformed into a spinning particle (cosmic strings!!).
- ▶ Further, we mention the link between the Dirac string and its line singularity. The dipole moment with the gyro-magnetic ratio e/mc , is transformed to a **complex center of charge line**.
- ▶ Note that the complex center of mass lines for Schwarzschild and Kerr differ by an imaginary transformation.
- ▶ A momentum vector and an angular momentum tensor can be defined in "Heaven" (quoting Newman) from the curvature data and null infinity in the original manifold.
- ▶ The imaginary displacement of the center of mass line signals the presence of intrinsic spin. The angular momentum of the Kerr corresponds then to intrinsic spin in the new complex system. The Kerr spacetime looks like a spinning particle.

Conclusions

- ▶ A new vacuum (*instanton*) solution is found on a warped 5D Kerr-type conformally-invariant spacetime. The zero's (singularities) are determined by a *quintic*, related to the Laplacian on a *Klein surface*
- ▶ The *anti-podal map* between region I and II in Penrose diagram is accompanied by a continuous transition in the *bulk* using the *non-orientable Klein surface* $\sim \mathbb{C}^1 \times \mathbb{C}^1$
- ▶ For a *dilaton-scalar* field model, the fields equations for the dilaton and the scalar field obey comparable Klein-Gordon-type PDE's. Both can be considered as a *quantum field* close to the horizon.
- ▶ The *geometric quantization* of our Klein surface can be constructed using a *two level Hilbert space* on $\mathbb{C}^1 \times \mathbb{C}^1$
- ▶ *Hawking particles* stay a while on Klein surface as seen by the outside observer. The time delay could provide an *estimate for the extra dimension* y_0
- ▶ The *information paradox* was solved by the antipodicity, i.e. by *removing the inside* of the black hole. The result was, that the black hole is *not thermal*. Because there is *no "hidden sector"* to produce the *thermodynamically mixed state* of the Hawking radiation. All states remain *pure* for the outside observer.
- ▶ In our model, the antipodal map on the brane is accompanied by the $\mathbb{Z}_2$ *symmetry in the bulk* in the RS2 model. This means that there is *no infinitely fast transport* to the other side of the hemisphere.
- ▶ The solution represents a *gravitational instanton*

Extra: Relation with icosahedron-Poncelet theorem and self-duality

- ▶ Let us consider the relevant term which determines the zero's ($k=3$):
- ▶
$$r^2 N_1(r)^2 = \frac{(r-a)^{k+1} (r(k+1)+a)}{k+2} + C = 4r^5 - 15ar^4 + 20a^2r^3 - 10a^3r^2 + C$$
- ▶ **first derivative: zero's $(0, a)$. They are the **foci of ellipse!****
- ▶ Right: plot of the logarithm of $1/N^2$. Left: **Poncelet ellipses**



- ▶ Note: **static instantons in 4D YM** could be **dynamical** particle-like solitons in $(4 + 1)$ YM!
- ▶ Our instantons live on S^4 embedded in $\mathbb{R}^5$. The ellipse in the interior
- ▶ There is an connection with the **self-duality** and the **polyhedral symmetry groups**
- ▶ **A gravitational instanton is a 4-dimensional spacetime M , everywhere regular, with self-dual curvature and asymptotically locally Euclidean:**

$$M \rightarrow S^3/\Gamma \times \mathbb{R} \text{ with } \partial M = S^3/\Gamma \text{ and } \Gamma \text{ a discrete subgroup of } SO(4), \text{ i.e. one of the 5 polyhedral groups.}$$
 In our case the icosahedron.

▶

Extra Notes on the quintic and the icosahedron

- ▶ Note that we have $\frac{1}{N_1^2} = \frac{dr^*}{dr} = \frac{d}{dr} \left[\sum r_i \frac{r_i \log(r-r_i)}{4(r_i-a)^3} \right] = \sum r_i \frac{r_i}{4(r_i-a)^3} \frac{1}{(r-r_i)}$
- ▶ Further: $N_1^2(z) = \frac{P(z)}{Q(z)}$ is a **meromorphic Laurent series** and holomorphic outside the zero's of Q , and with $P' = z(z-a)^3$
- ▶ It can be seen as a map from the **Riemann sphere** into itself via **stereographic projection**.
- ▶ **Gauss-Lucas**: *If F is a polynomial with a set of roots in the complex plane, the roots of $F^{(n)}$ all lie within the convex hull of the roots of F*
- ▶ The **topological degree (or algebraic degree)** of N_1 is the number of preimages of a point c (with multiplicities) to be found by solving $P(z) - cQ(z) = 0$. In our case it is in general 5.
- ▶ One calls this equation the **polyhedral eq.** associated with the finite Möbius group. In our case it is the **icosahedron group A_5** .
- ▶ the cases $c = (0, 1, \infty)$ correspond with the singularities, the regular case and the $r = 0$ pole.
- ▶ In our case we have then one pole and 5 rotations over $2\pi/5$

Extra Black hole as virtual intermediate "particle"?

- ▶ Ideally: describe black hole's entire history in terms of a **wavefunction**
- ▶ consider entire "event" as a **tunneling event** , i.e. an **instanton**
- ▶ The tunneling amplitudes can be derived by **classical equations on Euclidean space**
- ▶ Euclidean space is eminently suitable for topological **non-trivial boundary conditions** , i.e the **antipodal on its boundary** .
- ▶ The instanton is then related to the black hole there where a region is **cut-out** .
- ▶ **Wick-rotate back** to pseudo-Riemannian: antipodicity then in space and **time** !
- ▶ **unitary evolution S-matrix** by the antipodal boundary condition
- ▶ Treatment of the gravitational **back reaction** by transverse spherical coordinates (θ, φ) :
$$Y_{l,m}(\pi - \theta, \pi + \varphi) = (-1)^l Y_{l,m}(\theta, \varphi)$$

And: One avoids so the back reaction of "hard" particles by **Shapiro-time shift** .
The interaction is replaced by footprints they leave in the spectrum of **soft** particles
- ▶ Information paradox resolved: Hawking radiation is in a **pure state** (maximally entangled)
- ▶ **No firewall** needed; no **ER=EPR**
- ▶ Information **retrieval process** : out-going particles at the antipodal points carry the information of the ingoing particles.
- ▶ the map is **one-to-one** . No summation over "unseen states"
- ▶ Variables $u^\pm$ and $p^\pm$ all switch signs
- ▶ **Concerns**: 1. cut-and-past 2. infinite fast transport? 3. maximum of l ?