Electromagnetic field of axially-symmetric conductor with charge in the vicinity of Kerr black hole

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Introduction

Plasma accretion onto supermassive black hole is the main mechanism of X-rays and gamma rays of most astrophysical sources.

Рис.:

Introduction: fixed and moving charges

1. Analytical solution for the vector potential A_j for the charged particle that is **fixed in space** was firstly obtain by E. Copson, 1928.
2. This solution was verified and explained by B. Linet, 1976.
3. The solution for the fixed particles in terms of multipole expansion firstly was obtained by R. Hanni and R. Ruffini, 1973. It was shown, that this field can be described by only moments $l = 0, 1, 2, 3$ with high level of accuracy.
4. The solution of **stationary** problem of determining electric and magnetic fields of currents in the vicinity of rotating black hole was developed in works of J. Bicak, 1976; D. Kofron, 2022.

1. The radiation of a radially **falling** into Schwarzschild black hole charged particle was firstly considered by Ross, 1971. Energy calculated here by using special relativistic formula:

$$\mathrm{d}P^i = \frac{2q^2}{3}\frac{\mathrm{d}^2x^k}{\mathrm{d}\tau^2}\frac{\mathrm{d}^2x_k}{\mathrm{d}\tau^2}u^i\mathrm{d}\tau\,; \qquad (1)$$

2. Formula (1) is used for the calculation of the radiated energy from geodesically falling particle in the paper by A. A. Shatskiy, I. D. Novikov and L. N. Lipatova, 2013;
3. Fully relativistic treatment of the potential of the radiation of a particle in terms of multipole expansion is given in Zerilli, 1970, R. Ruffini, 1971, R. Ruffini 1972. Here radiation given by first 3 multipols are calculated. Statement that radiation from other multipols are negligible is made;
4. In the paper by A. Shoom, 2015 it is shown that formula (1) can be used only in cases where scale of the trajectory change much smaller than the scale of the gravitational field inhomogeneity.

Charge in the vicinity of Schwarzschild black hole

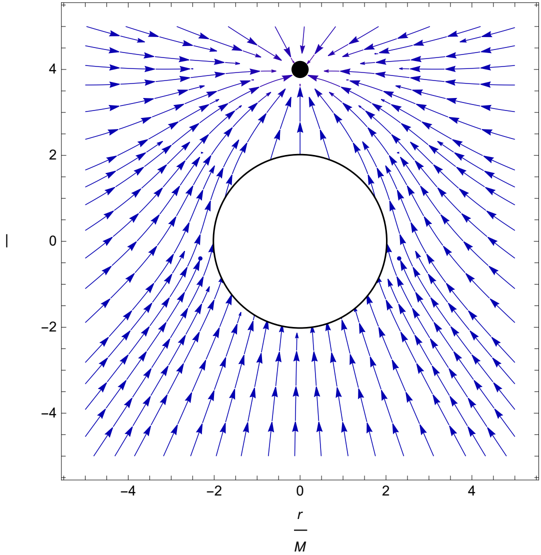

Рис.: Lines of electric force for the charge at a distance $r_0 = 4M$ from black hole

Charge in the vicinity of Schwarzschild black hole

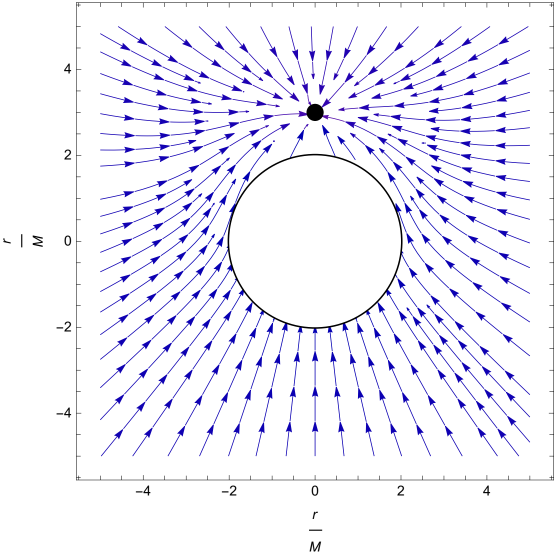

Рис.: Lines of electric force for the charge at a distance $r_0 = 3M$ from black hole

Charge in the vicinity of Schwarzschild black hole

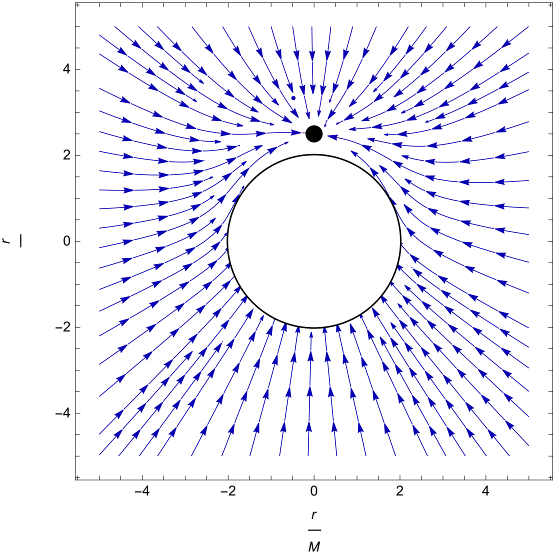

Рис.: Lines of electric force for the charge at a distance $r_0 = 2,5M$ from black hole

Charge in the vicinity of Schwarzschild black hole

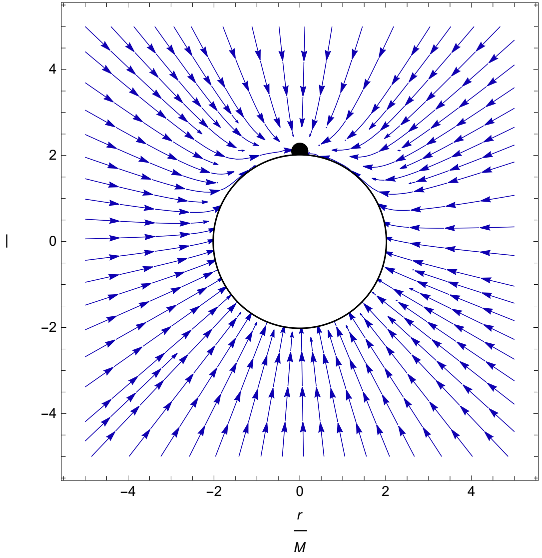

Рис.: Lines of electric force for the charge at a distance $r_0 = 2,1M$ from black hole

Charge in the vicinity of Schwarzschild black hole

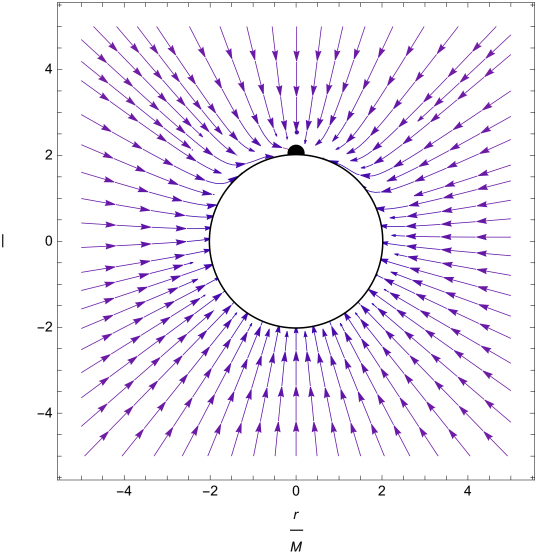

Рис.: Lines of electric force for the charge at a distance $r_0 = 2,05M$ from black hole

In the case, when charge falling into black hole, electromagnetic field goes to spherically symmetric also

Komarov, S. O. Electromagnetic field of a charge asymptotically approaching a spherically symmetric black hole / Komarov, S. O., Gorbatsievich, A. K. and Vereshchagin, G. V. // Physical Review D. – 2023. – Vol. 108. – P. 104056 (8 pages).

Wald solution

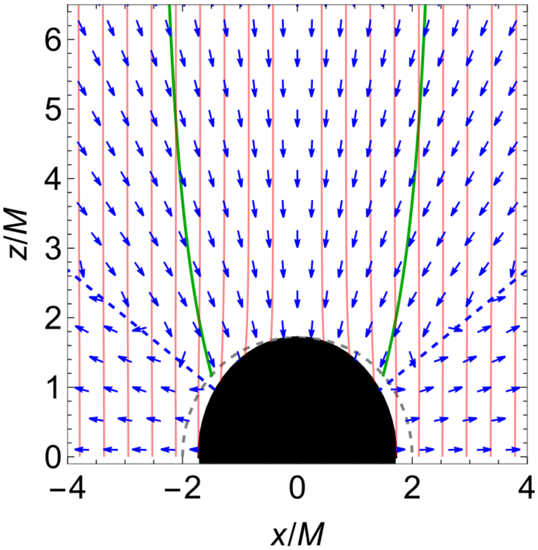

Рис.:

The tensor of electromagnetic field F_{ij} satisfies general covariant Maxwell equations:

$$\begin{cases} F^{ls}{}_{;s} = 4\pi j^l \,, \\ F_{[ij,k]} = 0 \,. \end{cases} \tag{2}$$

it is follows,

$$F_{ij} = A_{j;i} - A_{i;j} \,. \tag{3}$$

4-current j^l has the form:

$$j^l(x^k) = q\frac{\delta(r-r_0)\delta(\theta-\theta_0)}{2\pi\sqrt{-g}\,u^4}u^l(x^0, \tilde{x}^\alpha(x^0)) \,. \tag{4}$$

$$\frac{1}{\Delta}\frac{\partial}{\partial r}\left[\Delta^2\frac{\partial\Phi_0}{\partial r}\right]+2\Phi_0+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi_0}{\partial\theta}\right)-\frac{\Phi_0}{\sin^2\theta}=2J(r,\theta), \tag{5}$$

$$(r-ia\cos\theta)\frac{\partial\Phi_1}{\partial r}+2\Phi_1-\frac{1}{\sqrt{2}}\left(\frac{\partial\Phi_0}{\partial\theta}+\cot\theta\Phi_0\right)+$$

$$\frac{ia\sin\theta}{(r-ia\cos\theta)\sqrt{2}}\Phi_0=(r-ia\cos\theta)j_k l^k\,. \tag{6}$$

$$\tag{7}$$

Here

$$J(r,\theta)=\left(\frac{\partial}{\partial r}+\frac{1}{r-ia\cos\theta}\right)\rho^2 j_k m^k-$$
$$\frac{1}{\sqrt{2}}\left(\frac{\partial}{\partial\theta}+\frac{ia\sin\theta}{r-ia\cos\theta}\right)(r-ia\cos\theta)\,j_k l^k\,.$$

Equation (5) is called Teukolsky equation

The right-hand side

Right-hand side of the Teukolsky equation has the form:

$$\sqrt{2}\delta(r-r_0)\left[\frac{1}{r_0}\left(\frac{Q}{2\pi}-aJ\right)\frac{2l+1}{2}P_l(\cos\theta_0)+\right.$$
$$\left.+iJ\frac{2l+1}{2l(l+1)}P_l'(\cos\theta_0)\right]+$$
$$+\frac{i\sqrt{2}}{r_0}\delta'(r-r_0)\left[(r_0^2+a^2)J-a\frac{Q}{2\pi}\right]\frac{2l+1}{2l(l+1)}P_l'(\cos\theta_0)\,. \qquad (8)$$

Solution

Write the solution in the form:

$$R_1(r) = \begin{cases} ia_1 \left[\dfrac{r_0 - 1}{r_0^2 + a^2 - 2r_0} - \dfrac{1}{2K} \ln \dfrac{r_0 - 1 + K}{r_0 - 1 - K} \right] & \text{for } r < r_0; \\ ib_1 \left[\dfrac{r - 1}{r^2 + a^2 - 2r} - \dfrac{1}{2K} \ln \dfrac{r - 1 + K}{r - 1 - K} \right] & \text{for } r > r_0\,. \end{cases} \tag{9}$$

$$R_2(r) = \begin{cases} a_2(1 - r) \left[2 + \dfrac{(r_0 - 1)^2}{r_0^2 + a^2 - 2r_0} + \dfrac{3(r_0 - 1)}{2K} \ln \dfrac{r_0 - 1 - K}{r_0 - 1 + K} \right] \\ \text{for } r < r_0\,; \\ b_2(1 - r_0) \left[2 + \dfrac{(r - 1)^2}{r^2 + a^2 - 2r} + \dfrac{3(r - 1)}{2K} \ln \dfrac{r - 1 - K}{r - 1 + K} \right] \\ \text{for } r > r_0\,. \end{cases} \tag{10}$$

Here

$$K = \sqrt{1 - a^2}\,. \tag{11}$$

Solution

Obtain for the coefficients:

$$b_1 = \frac{3\sqrt{2}}{4(a^2-1)}\left[-a\frac{Q}{2\pi} + \left(\frac{r_0^2+a^2}{2} - \frac{a^2}{r_0}\right)J\right]; \tag{12}$$

$$a_1 = b_1 - \tag{13}$$

$$-\frac{3\sqrt{2}}{4r_0}\frac{(r_0^2+a^2)J - a\frac{Q}{2\pi}}{r_0^2+a^2-2r_0}\left[\frac{r_0-1}{r_0^2+a^2-2r_0} - \frac{1}{2K}\ln\frac{r_0-1+K}{r_0-1-K}\right]^{-1}.$$

Obtain for the coefficients:

$$b_1 = \frac{3\sqrt{2}}{4(a^2-1)}\left[-a\frac{Q}{2\pi} + \left(\frac{r_0^2+a^2}{2} - \frac{a^2}{r_0}\right)J\right]; \qquad (14)$$

$$a_1 = b_1 - \qquad (15)$$

$$-\frac{3\sqrt{2}}{4r_0}\frac{(r_0^2+a^2)J - a\frac{Q}{2\pi}}{r_0^2+a^2-2r_0}\left[\frac{r_0-1}{r_0^2+a^2-2r_0} - \frac{1}{2K}\ln\frac{r_0-1+K}{r_0-1-K}\right]^{-1}.$$

For $Q \neq 0$ and $r_0 \to 1+\sqrt{1-a^2}$ b_1 is finite!
This means that the dipole component (magnetic) of the field is finite

For the ZAMO observer, for the 4-velocity u^k we have: $\xi_k u^k = 0$, where $\xi^k = \{0,\, 0,\, 0,\, 1\}$ — azimuthal Killing vector field.

$$h^i{}_{(1)} = \left\{\frac{0, \sqrt{\Delta}}{\rho},\, 0,\, 0\right\}\,;$$
$$h^i{}_{(2)} = \left\{0,\, 0,\, \frac{1}{\rho},\, 0,\, 0\right\}\,;$$
$$h^i{}_{(3)} = \left\{0,\, 0,\, 0,\, \frac{\rho}{\Sigma\sin\theta},\, 0\right\}\,;$$
$$h^i{}_{(0)} = \left\{\frac{\Sigma}{\rho\sqrt{\Delta}},\, 0,\, 0,\, \frac{2ar}{\Sigma\rho\sqrt{\Delta}}\right\}\,.$$

Conponents of filed in orhtonormal reference frame:

$$E_{\hat{r}} = h^i{}_{(1)} h^j{}_{(0)} F_{ij}\,;$$
$$E_{\hat{\theta}} = h^i{}_{(2)} h^j{}_{(0)} F_{ij}\,;$$

The electric field lines can be found from the equation

$$\frac{\mathrm{d}r}{r\mathrm{d}\theta} = \frac{E_{\hat{r}}}{E_{\hat{\theta}}} = \frac{1}{\sqrt{\Delta}}\frac{F^{10}}{F^{20}} = \sqrt{\Delta}\frac{\Sigma^2 F_{10} + 2arF_{13}}{\Sigma^2 F_{20} + 2arF_{23}}\,.$$

Magnetic field lines in the ZAMO reference frame

Components of magnetic field

$$B_{\hat{r}} = h^i{}_{(1)} h^j{}_{(0)} e_{ijkl} F^{kl}\,;$$
$$B_{\hat{\theta}} = h^i{}_{(2)} h^j{}_{(0)} e_{ijkl} F^{kl}\,;$$

$$\frac{\mathrm{d}r}{r\mathrm{d}\theta} = \frac{B_{\hat{r}}}{B_{\hat{\theta}}}\,.$$

Numerical electric field lines in ZAMO ($J = 2Qc/M$)

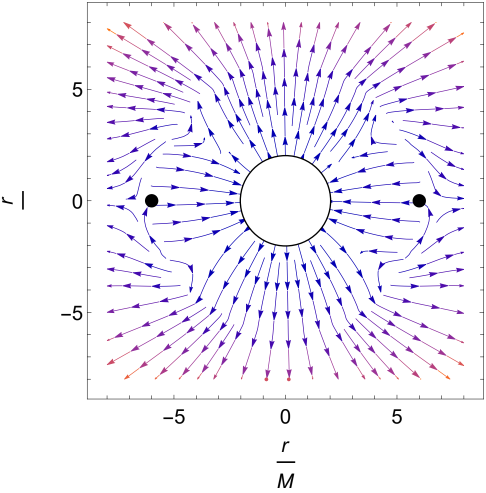

Рис.: $r_0 = 6M$, $a = 0,1M$

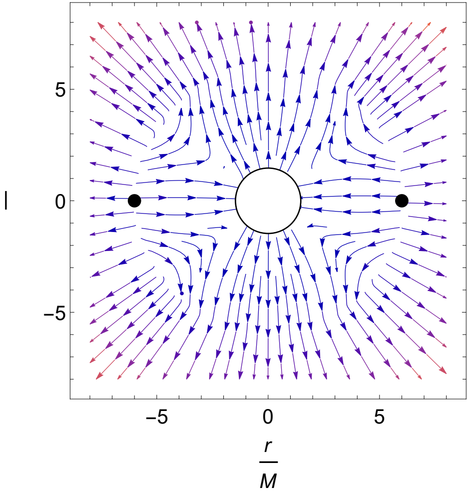

Рис.: $r_0 = 6M$, $a = 0,9M$

Numerical magnetic lines in ZAMO ($J = 2Qc/M$)

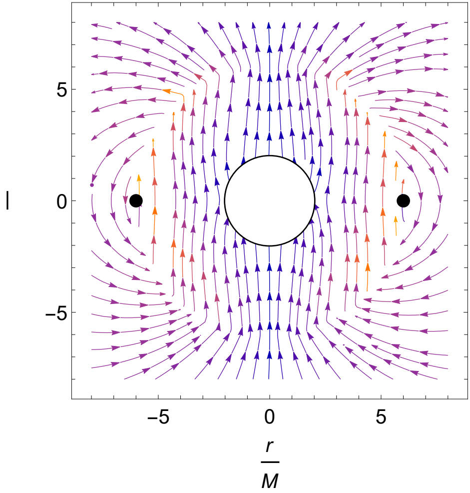

Рис.: $r_0 = 6M$, $a = 0,1M$

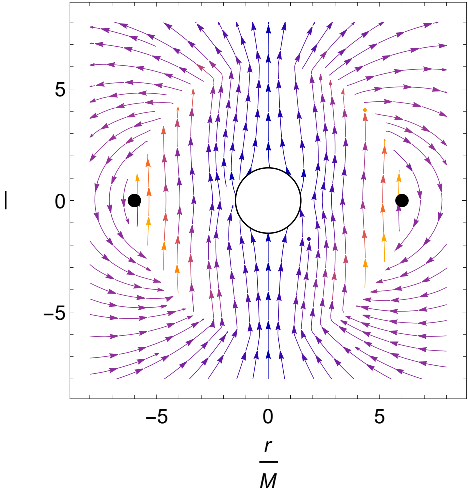

Рис.: $r_0 = 6M$, $a = 0,9M$

Conclusion

1. The computer program in Wolfram Mathematica for the calculating of electric and magnetic lines of a circular ring is created.
2. By using analytical formulas it is shown, that the dipolar magnetic field of a ring is not vanishes while putting the ring near event horizon of black hole.
3. It is follows from the numerical results, that in the axially-symmetric case magnetic lines near black hole are very symilar to those for the Wald solution.

Thank you for your attention!