Spacetime foam correlation renders cosmological constant (dark energy)

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Abstract. Wheeler's spacetime foams (wormholes) at the Planck length undergo quantum nucleation, oscillation and annihilation. Their collective excitations over foamy spacetime interact with field operators at large distances. We describe such collective excitation and interaction using an effective "foamon" field coupled with field operators. The Wilson renormalisation group approach shows that the foamon field theory evolves from an infrared scaling invariant domain to an ultraviolet one, when numerous particles are present. In these domains, the foamon field induces an effective action of field operators, and its correlation length sets a natural scale. Applying this to cosmology, we obtain the effective Einstein action for the Ricci scalar and the cosmological constant (dark energy), including its equation of state and interaction with matter.

Contents

1 Introduction

1.1 Infrared catastrophe of Euclidean quantum gravity

The laws and constants of nature, as seen by an ordinary observer inside a macroscopic universe, relate to a background of baby universes or wormhole operators which act on a wave-function space of universes [1–9]. Wormholes are gravitational fluctuations of

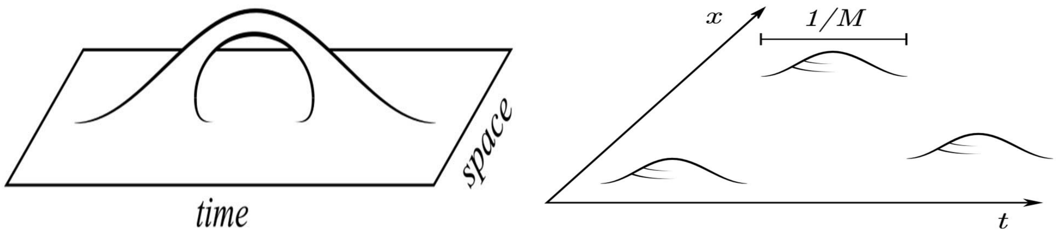

Figure 1. The left figure illustrates an Euclidean spacetime wormhole solution: an R^3 transits to an R^3 plus an S^3 baby universe, which is subsequently absorbed, becoming again an R^3. The right figure illustrates many wormholes in the spacetime x. The *time* in left and t in right indicate the Euclidean time, i.e., the tunnelling length $\beta \sim 1/M$, and M is the wormhole mass scale. We adopt these Figures from Figures 1 and 2 of the Reference [10].

topology changes and two-point connections of the spacetime, as schematically illustrated in Fig. 1. The Coleman formulation of this idea for the cosmological constant issue [7] is based on the Euclidean path integral approach of the effective actions,

$$I_E = \int \sqrt{g} d^4 x_1 \sqrt{g} d^4 x_2 O_i^\dagger(x_1) C_{ij}(x_1, x_2) O_j(x_2), \tag{1.1}$$

$$\sim \alpha_i^\dagger C_{ij}^{-1} \alpha_j + \alpha_i \int \sqrt{g} d^4 x O_i(x), \tag{1.2}$$

$$\Rightarrow \int \sqrt{g} d^4 x \Big[\rho_{\Lambda} - \frac{1}{16\pi G} R + \gamma R^2 + \cdots \Big]. \tag{1.3}$$

The two-point connection functional $C_{ij}(x_1, x_2)$ in the nonlocal action (1.1) represents the *background wormhole dynamics*, and its eigenvalues and eigenvectors relate to the auxiliary parameters $\{\alpha_i\}$ of the universes. The path integration (1.2) over $\{\alpha_i\}$ values yields the action (1.1). The various types "i" of local scalar operators $O_i(x)$ represent physical observables of the universes, relating to the cosmological constant $\rho_{\Lambda} = \Lambda/(8\pi G)$, gravitational coupling R/G, high-order γR^2 terms of the Ricci scalar R, and other local field operators in the local effective action (1.3).

The formulation [7] assumed the following *background wormhole dynamics*, as schematically illustrated in Fig. 2. (i) The Euclidean path integral amplitudes (probability) of different $\{\alpha_i\}$-parametrised universes (1.2) are dominated by the large-scale universe, connected by background wormholes $C_{ij}(x_1, x_2)$. (ii) The amplitude for a large-scale spherical universe is of order $\exp\{1/(G^2\rho_{\Lambda})\}$ for the case of small cosmological constant Λ, that has been *a priori* introduced in the effective action (1.3). (iii) After integrating all wormholes connecting any pair of points x_1 and x_2, the probability of different α_i-universes is given by [11]

$$\sim e^{-C_{ij}^{-1}\alpha_i\alpha_j} \exp\Big\{ \exp \frac{3}{8(\Lambda + \alpha_1)(G + \alpha_2)} + (\gamma + \alpha_3) + \cdots \Big\}, \tag{1.4}$$

where natural constants G, Λ and $\gamma, \cdots$ are functions in the $\{\alpha_i\}$ parameter space. Our Universe (1.3) with the observed Newton constant G, small cosmological constant Λ and negligible coefficients $\gamma, \cdots$ corresponds to the particular $\{\alpha_i\}$ values for which the probability (1.4) is maximal.

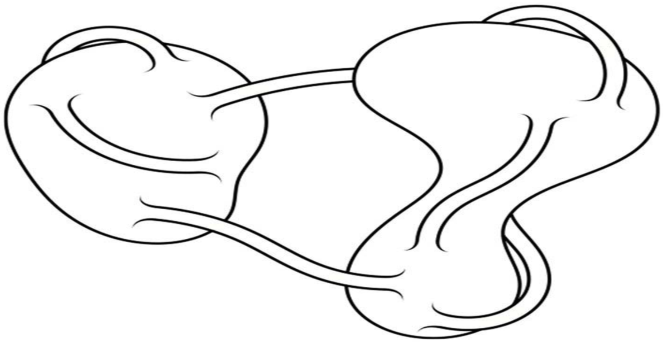

Figure 2. The Coleman scenario for background wormhole dynamics. Two (or many) large smooth geometries (spherical universes) connected by wormholes. Large universes connected by wormholes. We adopt this figure from Figure 1 of the Reference [12].

In such a formulation, however, the amplitude runs into the instability of the infrared catastrophe, due to the absence of physically sensible infrared regulators for the background wormhole dynamics C_{ij} and the unboundedness of the Euclidean action (1.3) of large-scale universes. The arbitrarily enormous numbers of macroscopic wormholes [11] and large-scale spherical universes [12] can be nucleated all the way up to the infrared scale $G^2\rho_{_\Lambda} \to 0$. Therefore, the amplitude runs away, and the parameters α_i are unbounded. The interpretations using the probability functional (1.4) are inconsistent. Thus, many alternative approaches studying this issue have been advocated, and they can be found in the review article [10].

Differing from the Coleman formulation, we will discuss the novel scenario of background wormhole dynamics with the physical ultraviolet and infrared regulators. As a result, we obtain *a posterior* the Euclidean actions (1.1) and (1.3) at long distances.

1.2 Wheeler spacetime foams and Planck lattice at short distances

As heuristically illustrated in Fig. 1, wormholes connect two spacetime points with a non-trivial topology, which gain quantitative support from a corresponding Euclidean classical solution [13]. The physical arena is of wormholes formed by gravitational (spacetime) fluctuations, connecting spacetime points with a non-trivial topology [14, 15]. Such a wormhole can be viewed as the process of the creation of an instanton (a half-wormhole [16]) and the absorption (annihilation) of an instanton, in other words, the pair creation, oscillation and annihilation of an instanton and an anti-instanton. It is analogous to the dynamics of electron and positron pairs in the QED vacuum. The gravitational instanton [1] is an energetically favourable configuration that can be spontaneously nucleated [17] in the local Euclidean quantum gravity (1.3) without the cosmological constant $\rho_{_\Lambda}$ term. Its nucleation (creation) and absorption (annihilation) [18, 19] can be viewed as a tunnelling solution with a semi-classical amplitude proportional to the gravitational instanton action $e^{-\beta M/2}$, the nucleation rate of numerous instantons is proportional to $e^{-\beta^2 M^2/2}$ [17]. Here M is the instanton mass (size) and β indicates the tunnelling length between two spacetime points.

[1]For many instantons, negative Coulomb energies are present among instantons.

Therefore, it is a plausible conjecture that precisely due to the violent quantum gravitational fluctuations at the Planck length $\ell_{\rm pl} = M_{\rm pl}^{-1}$, numerous Planck-size wormholes (instanton and anti-instanton pairs) of mass $M \sim M_{\rm pl}$ and the tunnelling length $\beta \sim \ell_{\rm pl}$ are nucleated without exponential suppression $e^{-\beta^2 M^2/2} \sim \mathcal{O}(1)$. In such a small length scale, Euclidean action well describes gravitational quantum fluctuations. As the nucleating processes are both energetically and entropically favourable, the number of Planck-size wormholes should be as large as permitted by spacetime geometry and topology, and the wormhole number density is about $\sim M_{\rm pl}^3$, namely the close packing density. The right of Figure 1 illustrates only a dilute density of wormholes. Such a configuration is the ground state of the spacetime that must exhibit the complex small-scale structure known as "foam" [20–23].

Thus, the physical spacetime made of fundamental "constituent" foams becomes a fluctuating foamy *Planck lattice* [24] endowed with a basic length $\ell_{\rm pl}$, rendering a natural ultraviolet cutoff of quantum field theories. The Planck lattice represents the ground state of Euclidean quantum gravity (1.3) without cosmological constant Λ. We studied it using the Wilson-loop area action, path-integral quantisation, and nonperturbative discretisation of a four-dimensional simplicial complex and Regge calculus [25, 26].

Such a speculative scenario describes the background wormholes of the spacetime. Its detailed quantum dynamics at short distances are unknown, except for providing a natural ultraviolet cutoff. However, we will discuss how long-range background wormhole dynamics couple to physical observables at long distances, which act as natural infrared regulators in the Wilson renormalisation approach.

1.3 Wormhole effects on field operators at large distances

In addition to the spacetime discretisation, instanton quantum nucleation and absorption are described by the creation and annihilation operators $\hat{\alpha}_i$, $\hat{\alpha}_i^\dagger$ and $[\hat{\alpha}_i, \hat{\alpha}_i^\dagger] = \delta_{ij}$ of type-i wormhole topological solution. Their interactions with field operators O_i at distances much larger than the wormhole size at the Planck scale are [7, 8, 27, 28]

$$\alpha_i \int \sqrt{g} d^4x O_i(x), \tag{1.5}$$

in the action (1.2). The ground state $|\alpha_i\rangle$ or α_i-vacuum, $(\hat{\alpha}_i + \hat{\alpha}_i^\dagger)|\alpha_i\rangle = \alpha_i|\alpha_i\rangle$, is determined by a set of eigenvalues $\{\alpha_i\}$, representing different Universes degenerated and tunneling to each other. The operators α_i and $\alpha_i^\dagger$ associate with the amplitudes $e^{-\beta M/2}$ of creation and annihilation [29], and we will adopt the approximation $e^{-\ell_{\rm pl} M_{\rm pl}/2} \sim \mathcal{O}(1)$ for Planck-size wormholes. It is analogous to the topological θ-vacuum in gauge field theories [30]. The field operator O_i subscript "i" indicates the type of operator associated with the type-i wormhole solution. As the consequences of the interaction (1.5), the spacetime wormholes may have effects on the natural constants and fundamental interactions of effective field theories of the field operators Q_i, as an example, the gravitational constant G, the cosmological constant Λ and Ricci scalar R of the Einstein theory. In the proposal [7, 31], the eigenvalues α_i possibility distribution and most probable value explain the vanishing of the cosmological constant, leading to an enormous spike of activity, see review [10].

Our proposal is as follows. Upon the ground state $|\alpha_i\rangle$ of the foamy spacetime (Planck lattice), we introduce the foamon field Φ_i or quantised "foamon", a quasi-particle, to describe the collective excitations of wormhole quantum nucleation, oscillation and absorption at short distances. This foamon field represents the long-range dynamics of background wormholes. It is analogous to the phonon excitations of collective vibration in the nuclear lattice or the magnon excitations of the collective spin wave of electrons in a crystal lattice. This differs from the usual spacetime description $g_{\mu\nu} + h_{\mu\nu}$ by the classical metric $g_{\mu\nu}$ and quantum fluctuation $h_{\mu\nu}$. The interactions (1.5) induce the foamon field Φ_i coupling to physical operators O_i as an infrared regulator at large distances. As a result, we show that the foamon field Φ_i correlation length renders the characteristic scale of the effective theories for the field operator Q_i. We apply this scenario to the Ricci scalar and cosmological constant of the Einstein theory for cosmology. As a result, we show how the long-range correlation length of background wormholes gives rise to the origin of the cosmological constant observed in the macroscopic universe.

Section 2 presents a general discussion on the foamon field theory, spectrum and correlation length. In Sec. 3, we generalise the wormhole interaction (1.5) to the foamon field interaction with the physical operators O_i, and discuss the renormalisation group equation by using O_i as an infrared regulator. Section 4 studies the scaling invariant domains of (i) the infrared fixed point in the symmetric phase and (ii) the ultraviolet fixed point in the symmetry-breaking phase, where numerous fermions are present. Sections 5 and 6 derive the foam-field induced effective action for the field operator O_i, in particular, Einstein's action for cosmology, focusing on dark energy, its equation of state and interaction with matter.

2 Euclidean foamon field theory for collective excitations

2.1 Foamon fields describe collective excitations of wormhole fluctuations

Apart from endowing the spacetime foam structure at the Planck length, wormholes undergo quantum fluctuations of creation, oscillation, and annihilation upon the ground state $|\alpha_i\rangle$. Due to their interactions, these quantum fluctuations have collective excitations at all length scales. In the four-dimensional Euclidean spacetime, we effectively describe the wormhole collective excitations by self-conjugated scalar foamon fields,

$$\Phi_i(x) = \hat{\alpha}_i(x) + \hat{\alpha}_i^\dagger(x), \tag{2.1}$$

where $\hat{\alpha}_i^\dagger(x)$ and $\hat{\alpha}_i(x)$ are type "i" wormhole creating and annihilating operators at the point $x = (\tau, \mathbf{x})$, following the equal-time commutator

$$[\hat{\alpha}_i(\tau, \mathbf{x}'), \hat{\alpha}_j^\dagger(\tau, \mathbf{x})] = (2\pi)^4 \delta_{ij} \delta^3(\mathbf{x}' - \mathbf{x}). \tag{2.2}$$

and other commutators vanish. The operators $\hat{\alpha}_j^\dagger(x)$ and $\hat{\alpha}_i(x)$ generalize the eigenvalues (1.5) to spacetime-dependent fields upon the ground state. It is equivalent to the third quantisation introduced in Ref. [18]. The high-energy modes of the foamon field probe complex topology and violent fluctuations of wormholes. The foamon field low-energy modes couple to the field operator O_i (1.5) of interest at long distances. They can be pseudo-scalar or other field kinds, depending on the types of field operators O_i to which they couple.

In this article, we consider the effective Euclidean Lagrangian of the foamon field as follow [2]

$$\mathcal{L}(\Phi_i, \partial_\mu \Phi_i) = \frac{1}{2}(\partial_\mu \Phi_i)^2 + V(\Phi_i), \tag{2.3}$$

and the effective potential $V(\Phi_i)$ represents foamon fields Φ_i self-interactions due to the complex dynamics of background wormholes at short distances. There are possibilities for the interactions between different types "i" of wormholes. The potential $V(\Phi_i)$ bounded from below, and its absolute minimum $V^{\rm min}$ is the ground state $\Phi_i^{\rm min}$ of the lowest energy in theory, thus, the true and stable ground state. It is located at: (i) zero field $\Phi_i^{\rm min} = 0$, (ii) finite field $\Phi_i^{\rm min} \neq 0$; (iii) more than two degenerate minima. We do not wander in the swamp land of possible potentials [32], but instead look at relevant and irrelevant operators in fixed points and scaling invariant domains with known interacting dynamics. Consider the simplest and $\Phi_i \Leftrightarrow -\Phi_i$ invariant potential,

$$V(\Phi_i) = \frac{1}{2} m_i^2 \Phi_i^2 + \frac{\lambda_i}{4} \Phi_i^4 + (\cdots), \tag{2.4}$$

which preserves both local and global symmetries. In four-dimensional spacetime, the quadratic Φ_i^2 is a $d = 2$ relevant operator, the quartic Φ_i^4 is a $d = 4$ marginal operator, and the last term $(\cdots)$ indicates high-dimensional irrelevant operators. The potential (2.4) minimises $V_{\rm min} = 0$ at a zero vacuum expectation value $\langle \Phi_i^2 \rangle = 0$, and the system is in a symmetric (disorder) phase.

2.2 Foamon fields effective action and energy density

The foamon field "vacuum-to-vacuum" transition amplitude is described by the Euclidean path integral

$$e^{-S_E^{\rm eff}(0)} \equiv W_E(0) = \prod_i \int \mathcal{D}\Phi_i \; e^{-\int_x \mathcal{L}(\Phi_i, \partial_\mu \Phi_i)}, \tag{2.5}$$

where $\int_x \equiv \int_\Omega d^4x = \int_\Omega d\tau d^3x \sqrt{g}$. We define functional integration as

$$\int \mathcal{D}\Phi = \prod_i \int_0^{M_{\rm pl}} \mathcal{D}\Phi_i = \prod_i \prod_{M_{\rm pl} \geq k > 0} \int d\Phi_i(k). \tag{2.6}$$

The functional integration (2.5) is finite and well-defined as a quantum field theory, and foamon fields $\Phi_i(k)$ fluctuate at all scales k (Euclidean momenta) up to the ultraviolet cutoff $M_{\rm pl}$.

[2]Gauge symmetries and covariant invariance preserved, and $g_{\mu\nu}$ is not explicit written.

In the momentum space, the effective Euclidean action is given by [3]

$$S^{\rm eff}_{E}(0) = -\frac{1}{2}\sum_i \int_{\rm states} \ln \Delta_i(\alpha_i, k), \tag{2.7}$$

where the diagonalized $\Delta_i(\alpha_i, k)$ represents the eigenvalues of the foamon field theory, the sum $\sum_i$ is over the topological sector i, and integration $\int_{\rm states}$ is

$$\int_{\rm states} \equiv \int_x \int_0^{M_{\rm pl}}, \quad \int_0^{M_{\rm pl}} \equiv \int_0^{M_{\rm pl}} \frac{d^4k}{(2\pi)^4}, \tag{2.8}$$

indicates the sum over all foamon field eigenstates in the momentum phase space ($M_{\rm pl} \geq k > 0$) of a finite spacetime volume Ω. Assuming the homogeneity at the macroscopic scale, one defines the energy $\mathcal{E} = -S^{\rm eff}_{E}(0) = \Omega\rho_{W}$ with the energy density

$$\rho_{W} = \frac{1}{2}\int_0^{M_{\rm pl}} \sum_i \ln \Delta_i(\alpha_i, k), \tag{2.9}$$

contributed from foamon fields' fluctuations at all length scales. It is an overall physically irrelevant constant unless foamon fields' fluctuations couple to observable field operators (3.1) at given physical scales.

2.3 Foamon fields correlation function and length

The two-state mixing and two-point connected Green (correlation) function of foamon fields,

$$\begin{aligned} G^{(2)}_{ij}(x_2 - x_1) &= \langle \Phi_i(x_2)\Phi^\dagger_j(x_1)\rangle - \langle \Phi_i(x_2)\rangle\langle \Phi^\dagger_j(x_1)\rangle \\ \langle \cdots \rangle &= W^{-1}_E(0)\int_0^{M_{\rm pl}} \mathcal{D}\Phi(\cdots)e^{-\int_x \mathcal{L}(\Phi_i, \partial_\mu\Phi_i)}. \end{aligned} \tag{2.10}$$

Here, we use the translation invariance at the macroscopic length scale, which is much larger than the wormhole size, the Planck lattice spacing. In the space of the foamon field eigenstates, the diagonalized two-point function (2.10) is

$$G^{(2)}_{ij}(x_2 - x_1) \approx \delta_{ij}\int \frac{d^4k}{(2\pi)^4}\frac{e^{ik_E(x_2-x_1)}}{k^2 + \tilde m_i^2} \propto \delta_{ij}e^{-|x_2-x_1|/\xi_i}. \tag{2.11}$$

The the two-point correlation (2.11) is exponentially suppressed if $|x_2 - x_1| \gg \xi_i$, where $\xi_i \sim \tilde m_i^{-1}$ is the foamon field correlation length in the scaling invariant domain of fixed points of the foamon field theory. This characteristic scale $\tilde m \sim \alpha_i$ relates to the eigenvalue α_i in (1.5). Analogously, we have n-state mixing and n-point connected Green functions,

$$G^{(2n)}_{i,\cdots,j}(x_1, \cdots, x_n) = \langle \Phi_i(x_1), \cdots, \Phi^\dagger_j(x_n)\rangle_{\rm conn}, \tag{2.12}$$

[3] In the coordinate space, $S^{\rm eff}_{E}(0) = -\sum_i \int_{\rm states} \ln \Delta_i(\alpha_i, x, y)$ and $\int_{\rm states} \equiv \int_x \int_y$.

where the even number $n = 2, 4, \cdots$.

The fluctuating foamon fields Φ_i are associated with the ground state α_i-vacuum. Their correlation functions (2.10) and (2.11) contain the information about the mixings (tunnelling) of different α_i vacuum states and correlations in different spacetime points. By definition, the foamon fields' correlation functions (2.10) and (2.12) differ from the connection function $C_{ij}(x_1, x_2)$ (1.1) of wormholes connecting two spacetime points, as shown in Fig. 2.

3 Foamon field regularization and renormalization

3.1 Foamon fields interact with field operators at large distances

At short distances, quantum foamon field theories possess complex interactions, which could be generally described by a Lagrangian density $\mathcal{L}$ (2.3) of quadratic and quartic interactions and high-dimensional operators. The interaction (1.5) is generalised to the interaction between the quantum foamon fields $\Phi_i(x)$ (2.1) and their associated field operators $O_i(x)$ at long distances,

$$\Phi_i(x)O_i(x) = [\hat{\alpha}_i(x) + \hat{\alpha}_i^\dagger(x)]O_i(x), \tag{3.1}$$

where $O_i(x) = O_i^\dagger(x)$ and $O_i(-k) = O_i^\dagger(k)$ are diffeomorphism and gauge invariant operators of fundamental fields at larger length scales compared with wormhole scales. Moreover, we consider the interactions that mix different sectors of the foamon fields Φ_j and the operators O_i

$$y^{ij}\Phi_i O_j = \Phi_i O_i + y^{i\neq j}\Phi_i O_j = \Phi O + y\Phi\bar{\Psi}_L\Psi_R + \cdots \tag{3.2}$$

where the sector "i" is fixed, the coupling $y^{i=j} = 1$ is for the same sector $i = j$ and mixing $y = y^{i\neq j} < 1$ is for different sectors $i \neq j$.

This article will study the dynamics of the foamon field Φ that couples to the operator $Q = Q(R)$ of the Riemann curvature scalar R, and mixes with the bilinear operator $\bar{\Psi}_L\Psi_R$ of chiral fermion fields $\bar{\Psi}_L$ and Ψ_R. The dimensionless mixing parameter $y < 1$ is a coupling of Yukawa type.

3.2 Field operator at large distance acts as an infrared regulator

The interaction (3.1) and Lagrangian (2.3) can be incorporated as a generating Euclidean functional of the

$$W_E(O_i) = \int_0^{bM_{\rm pl}}\int_{bM_{\rm pl}}^{M_{\rm pl}} \mathcal{D}\Phi_i e^{-\int_x[\mathcal{L}(\Phi_i,\partial_\mu\Phi_i)+\Phi_i O_i]}. \tag{3.3}$$

Here, introducing an infrared scale $bM_{\rm pl} < M_{\rm pl}$ and a scaling parameter $b < 1$, we formally split the functional modes integration $\int_0^{M_{\rm pl}} = \int_0^{bM_{\rm pl}}\int_{bM_{\rm pl}}^{M_{\rm pl}}$ of the foamon field Φ_i. The infrared scale $bM_{\rm pl} < M_{\rm pl}$ represents the characteristic scale of the low-energy field operator O_i at large distances. The scaling parameter b_i depends on the field operator O_i sector, and we omit its subscript "i" to simplify the notation.

The scale $bM_{\rm pl}$ is much smaller than the Planck scale $M_{\rm pl}$. This means that the operator $O_i(x) = \int \frac{d^4k}{(2\pi)^4} O_i(k) e^{ikx}$ varies slowly over the Planck length scale $M_{\rm pl}^{-1}$, and its momentum components are approximately

$$O_i(k) = \begin{cases} \hat{O}_i(k) \approx 0, & M_{\rm pl} > k > bM_{\rm pl} \\ O_i(k) \neq 0, & bM_{\rm pl} > k > 0 \end{cases} . \tag{3.4}$$

The main contributions $O_i(k)$ are around $k \sim bM_{\rm pl}$, and $O_i(k)$ vanishes for $k \gg bM_{\rm pl}$ and $k \ll bM_{\rm pl}$. Correspondingly, we split the foamon field modes as,

$$\Phi_i(k) = \begin{cases} \hat{\Phi}_i(k) \neq 0, & M_{\rm pl} > k > bM_{\rm pl} \\ \phi_i(k) \neq 0, & bM_{\rm pl} > k > 0 \end{cases} . \tag{3.5}$$

This is equivalent to separating the classical low-energy modes $\phi_i^{\rm cl} \propto \phi_i(k)$ from quantum high-energy modes $\eta_i \propto \hat{\Phi}_i(k)$, i.e., $\Phi_i = \phi_i^{\rm cl} + \eta_i$ in usual notation.

The foamon and field operator interaction (3.1) is then given by

$$\int_x \Phi_i O_i \approx \int_k \phi_i(k) O_i(k) = \int_0^{bM_{\rm pl}} \frac{d^4k}{(2\pi)^4} \phi_i(k) O_i(k), \tag{3.6}$$

indicating that the long-distance foamon field modes $\phi(k)$ dominantly strongly interact with the field operator O_i and short-distance foamon field modes $\hat{\Phi}(k)$ weakly interact to the field operator O_i. We approximately write the generating function (3.3) as

$$W_E(O_i) \approx \int_0^{bM_{\rm pl}} \mathcal{D}\phi_i e^{-\int_x \phi_i O_i} \int_{bM_{\rm pl}}^{M_{\rm pl}} \mathcal{D}\hat{\Phi}_i e^{-\int_x \mathcal{L}(\Phi_i, \partial_\mu \Phi_i)}. \tag{3.7}$$

At short distances, the wormhole Planck scale $M_{\rm pl}$ provides an ultraviolet cutoff of effective quantum foamon field theories. At long distances, the physical length scales $bM_{\rm pl}$ of field operators O_i provide infrared cutoff regulators or renormalisation scales on foamon field theories. We follow the effective potential method [33] and the Wetterich approach [34] to give the general formulation of how field operators O_i render infrared regulators to define an effective action of renormalised foamon fields and couplings. Then, we adopt the Wilson renormalisation approach [35] for calculations.

3.3 General renormalization group equation

Using generating functional $W_E(O_i)$ (3.7) for low-energy fields $\phi_i(k)$ and $O_i(k)$, which depend on the scaling parameter b or $t = \ln b$, we define the averaged (classical) foamon fields as

$$\phi_i^{\rm cl}(k) \equiv \langle \phi_i(k) \rangle = -\frac{\delta W_E(O_i)}{\delta O_i(k)}, \tag{3.8}$$

and the effective action $\Gamma(\phi_i^{\rm cl})$ by the Legendre transformation

$$\Gamma(\phi_i^{\rm cl}) = -W_E(O_i) - \int_k O_l \phi_l^{\rm cl}, \tag{3.9}$$

where the index l is summed. The spacetime dependent classical field $\phi_i^{\rm cl}(x)$ and potential $\Gamma(\phi_i^{\rm cl})$ are functional of the operator $Q_i(x)$. Equation (3.8) shows actually the classical field $\phi_i^{\rm cl}$ (3.8) is an average of over low-energy modes $\phi_i(k)$ coupling to the operator O_i at large distances. The classical field $\phi_i^{\rm cl}(x)$ obeys the equation of motion

$$O_i(k) = -\frac{\delta\Gamma(\phi_i^{\rm cl})}{\delta\phi_i^{\rm cl}(k)}, \tag{3.10}$$

with "external source" $Q_i(x)$.

Two-state mixing and two-point correlation function and its inverse are given by

$$G_{ij}^{(2)}(k_1,k_2) = \frac{\delta^2 W_E(O_i)}{\delta O_i(k_1)\delta O_j^\dagger(k_2)}, \quad [G_{ij}^{(2)}(k_1,k_2)]^{-1} = \frac{\delta^2\Gamma(\phi_i^{\rm cl})}{\delta\phi_i^{\rm cl}(k_1)\delta\phi_j^{\rm cl\dagger}(k_2)}, \tag{3.11}$$

and

$$\int_{k_3} G_{il}^{(2)}(k_1,k_3)[G_{lj}^{(2)}(k_3,k_2)]^{-1} = \delta_{ij}\delta_{k_1k_2},$$

where $\delta_{k_1k_2} = (2\pi)^4\delta^4(k_1-k_2)$. In the coordinate space, these formulae are the same, replacing $k\to x$, $\int_k \to \int_x$ and $\delta_{k_1k_2}\to\delta_{x_1x_2} = \delta^4(x_1-x_2)$.

The classical field $\phi_i^{\rm cl}$ is the low-energy modes coupling to the operator O_i at long distances. The classical field $\phi_i^{\rm cl}$ and potential $\Gamma(\phi_i^{\rm cl})$ are functions of spacetime x. We can assume that they very slowly over distances $\sim 1/(bM_{\rm pl})$ can be approximated as constant background fields, independent of x. In this case, they do not correlate different points in spacetime, and their kinetic terms vanish. The effective action and potential are equal $\Gamma(\phi_i^{\rm cl}) = V(\phi_i^{\rm cl})$ (2.3).

The change in the infrared cutoff scale $bM_{\rm pl}$ explicitly causes the field operator $O_i(k)$ variation. It results in the variations of the classical field $\phi_i^{\rm cl}$ and effective action $\Gamma(\phi_i^{\rm cl})$, which thus implicitly depend on the infrared cutoff $bM_{\rm pl}$. We consider the infrared scale variation $bM_{\rm pl}$, the effective action $\Gamma(\phi_i^{\rm cl})$ variation is given by

$$\begin{aligned}\frac{d\Gamma(\phi_i^{\rm cl})}{dt} &= -\frac{dW(O_i)}{dt} - \frac{d}{dt}\int_k \phi_l^{\rm cl} O_l \\ &= -\int_k \frac{\delta W(O_i)}{\delta O_l(k)}\frac{dO_l(k)}{dt} - \frac{d}{dt}\int_k \phi_l^{\rm cl} O_l \\ &= \int_k \phi_l^{\rm cl}\frac{dO_l}{dt} - \frac{d}{dt}\int_k \phi_l^{\rm cl} O_l = -\int_k O_l\,\frac{d}{dt}\phi_l^{\rm cl},\end{aligned} \tag{3.12}$$

where $dt = d\ln b$. Using Eqs. (3.8)-(3.11), we have the relations:

$$\frac{d}{dt}\phi_l^{\rm cl}(k_1) = -\int_{k_2}[G_{lj}^{(2)}(k_1,k_2)]\,\frac{d}{dt}O_j(k_2) \tag{3.13}$$

$$\frac{d}{dt}O_j(k_1) = -\int_{k_2}[G_{jl}^{(2)}(k_1,k_2)]^{-1}\frac{d}{dt}\phi_l^{\rm cl}(k_2). \tag{3.14}$$

As a result, the effective action $\Gamma(\phi_i^{\rm cl})$ evolution follows

$$\begin{aligned}\frac{d\Gamma(\phi_i^{\rm cl})}{dt} &= \int_{k_1k_2} O_l(k_1)G^{(2)}_{lj}(k_1,k_2)\,\frac{d}{dt}O_j(k_2)\\ &= \frac{\Omega}{2}\sum_j\int_k G^{(2)}_j(k)\,\frac{d}{dt}O^2_j(k).\end{aligned} \tag{3.15}$$

In the last step, we use the diagonalized two-point function $G^{(2)}_{lj}(k_1,k_2)\propto\delta_{lj}(4\pi)^4\delta(k_2-k_1)$ by assuming the foamon field theory is translation invariant at large distances of field operators Q_i. This equation (3.15) is similar to the Wetterich equation [34] for effective potential evolution, there a quadratic term $\phi^\dagger(k)\mathcal{R}\phi(k)$ is added into the Lagrangian (2.3) as an infrared regulator, and $\mathcal{R}$ replaces O^2 in Eq. (3.15).

Since the low-energy (long-wavelength) field $\phi_i(x)$ varies smoothly over short distances, we approximately adopt the classical field $\phi_i^{\rm cl}(x)\approx\phi_i(x)$ as a slowly-varying background field. We will remove the superscript $^{\rm cl}$ from $\phi_i^{\rm cl}$ and use ϕ_i, unless otherwise specified. The general evolution equation (3.15) for the effective potential $\Gamma(\phi_i^{\rm cl})$ is a kind of renormalisation group (RG) equation. It will be helpful, provided we know the scaling invariant domains of fixed points of foamon field theory (2.3).

3.4 Wilson approach to foamon field renormalisation

Henceforth, we discuss only one type of the foamon field and suppress the subscript "i" to simplify the notations. Using the Wilson renormalization approach to integrate high-energy modes $\hat\Phi$ (3.5) in the generating function (3.7), we can formally write

$$e^{-\int_x\mathcal{L}_{\rm eff}(\phi,\partial_\mu\phi)}=\int_{bM_{\rm pl}}^{M_{\rm pl}}\mathcal{D}\hat\Phi e^{-\int_x\mathcal{L}(\Phi,\partial_\mu\Phi)}, \tag{3.16}$$

where $\mathcal{L}_{\rm eff}(\phi,\partial_\mu\phi)$ is the effective Lagrangian for low-energy field modes ϕ. The integration of high-energy field modes proceeds by continuous n-steps of integrating a thin momentum shell $(b^{n+1},b^n)M_{\rm pl}$ $(b<1)$ in the momentum space. This iterative procedure is like rescaling the spacetime x and the momentum space k as follows [36],

$$x'=bx,\quad k'=k/b,\quad b<1, \tag{3.17}$$

indicating short distances x' (large momenta k') scale to long distances x (small momenta k). In the coordinate space, the Wilson renormalisation approach at each step "n" is to obtain the mean "coarse" field $\bar\phi^{n+1}_{\rm coarse}=\frac{1}{V_b}\sum\phi^n_{\rm fine}$ and interactions by averaging (smearing) all "fine" fields $\phi^n_{\rm fine}$ and interactions in small coordinate cells $[(b^{n+1},b^n)M_{\rm pl}]^{-4})$ over the volume $V_b=1/(bM_{\rm pl})^4$.

Continuous steps $n=1,2,3,\cdots$ generate the RG flow of the effective Lagrangian $\mathcal{L}_{\rm eff}$ transformation as a function of running energy scale $\mu=b^nM_{\rm pl}$ in the momentum or coordinate space. In the vicinity (scaling invariant domain) of the fixed points (critical points) of the foamon field Lagrangian (2.3), the foamon field correlation length becomes very large (2.11), the relevant operators remain in the same form, and irrelevant operators are suppressed. After renormalization, the effective actions $\Gamma'(\phi') = \int d^dx' \mathcal{L}_{\rm eff}(\phi')$ and $\Gamma(\phi) = \int d^dx \mathcal{L}_{\rm eff}(\phi)$ follow the Wilson scaling invariance $\Gamma'(\phi') \approx \Gamma(\phi)$. In the running energy scale $\mu = b^n M_{\rm pl}$, $b^n \ll 1$ for a fixed value $b < 1$ and step number $n \gg 1$. This is formally equivalent to considering the rescaling parameter $b < 1$, and we will thus use varying b, instead of b^n, fixed b and varying n, consistently with Eq. (3.17).

4 Fixed points and scaling invariant domains

4.1 Infrared fixed point and scaling invariant domain

The field theory (2.4) upon the symmetric ground state at the zero field $\phi = 0$ has an infrared (IR) stable fixed point $\lambda_{\rm ir}^{\rm fix} = 0$. In its scaling invariant domain $\mathcal{D}_{\rm ir}$, after renormalised short distance modes, quasi-free field $\phi_{\rm ir}$ action and Lagrangian are physical measurable in the phase space (x, k)

$$\Gamma(\phi_{\rm ir}) = \int d^dx \mathcal{L}(\phi_{\rm ir}) = \int_x \left(\frac{1}{2}(\partial_\mu \phi_{\rm ir})^2 + \frac{1}{2} m_{\rm ir}^2 \phi_{\rm ir}^2 + \frac{\lambda_{\rm ir}}{4} \phi_{\rm ir}^4 \right), \tag{4.1}$$

$m_{\rm ir} \gtrsim 0$ and $\lambda_{\rm ir} \gtrsim \lambda_{\rm ir}^{\rm fix}$ are RG invariant. At the running renormalisation scale $\mu = bM_{\rm pl}$, one obtains the effective action and Lagrangian in the phase space (x', k') [36]

$$\Gamma(\phi) = \int d^dx' \mathcal{L}_{\rm eff} = \int_{x'} \left[\frac{1}{2}(\partial'_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4} \phi^4 + (\cdots) \right], \tag{4.2}$$

and $\partial'_\mu = \partial / \partial x'_\mu$, where the field wave-function, mass and coupling are scaled as

$$\begin{aligned} \phi &= [b^{2-d}(1+\delta Z)]^{1/2} \phi_{\rm ir} \approx b^{\frac{2-d}{2}} \phi_{\rm ir}, \\ m^2 &= (1+\delta Z)^{-1} b^{-2} (m_{\rm ir}^2 + \delta m^2) \approx m_{\rm ir}^2 b^{-2} \\ \lambda &= (1+\delta Z)^{-2} b^{d-4} (\lambda_{\rm ir} + \delta\lambda) \approx \lambda_{\rm ir} b^{d-4}. \end{aligned} \tag{4.3}$$

The $(\cdots)$ indicates the counterterms and irrelevant high-dimensional operators suppressed by $\mathcal{O}(b^l)$ and $l \geq 1$. In the scaling invariant domain, the field ϕ wave-function, mass and coupling corrections δZ, δm^2 and $\delta\lambda$, as well as irrelevant operators $(\cdots)$ can be ignored. It gives the Wilson scaling invariance $\Gamma(\phi) \approx \Gamma(\phi_{\rm ir})$ with $\phi \approx b^{\frac{2-d}{2}} \phi_{\rm ir}$ and $m^2 \approx m_{\rm ir}^2 b^{-2}$ and $\lambda \approx \lambda_{\rm ir} b^{d-4}$, because the correlation length $\xi = 1/m_{\rm ir}$ is much larger than the scale variation (3.17).

In the four dimension ($d = 4$), the mass operator $m^2\phi^2$ is relevant, interacting operator $\lambda\phi^4$ is marginal, and perturbative calculations give the renormalised coupling λ and β-function,

$$\lambda = \lambda_{\rm ir} + \beta_\lambda \ln b, \quad \beta_\lambda = b \frac{\partial \lambda}{\partial b}, \quad \beta_\lambda^{(1)} = \frac{3\lambda_{\rm ir}^2}{16\pi^2}. \tag{4.4}$$

The one-loop result $\beta_\lambda^{(1)} > 0$ shows when the running energy scale $\mu = bM_{\rm pl}$ decreases and renormalised coupling λ decreases, the RG flows approach an infrared stable fixed point $\lambda \to \lambda^{\rm fix}_{\rm ir} = 0$, where the β-function vanishes. Up to the seven-loop $\beta^{(7)}_\lambda > 0$ function, Ref. [37] confirms this infrared fixed point without the evidence of ultraviolet fixed points at strong scalar field couplings.

In the regime of weak $\lambda\phi^4$ interacting and $y\bar\psi_L\psi_R$ Yukawa coupling, using the $\beta^{(n)}_\lambda(\lambda, y)$ and $\beta^{(n)}_y(\lambda, y)$ functions up to two-loop ($n = 2$) expansions of small couplings λ and y, one studies the RG flows of dimensionless couplings,

$$\lambda(b) = \lambda_{\rm ir} + \beta_\lambda \ln b, \quad y(b) = y_{\rm ir} + \beta_y \ln b, \tag{4.5}$$

where the β_y function is

$$\beta_y = b\partial y/\partial b, \quad \beta^{(1)}_y = N_f 5y^3_{\rm ir}/(32\pi^2) > 0, \tag{4.6}$$

in the symmetric phase without spontaneous symmetry breakings.

The studies show the scaling invariant domain $\mathcal{D}_{\rm ir}$, where RG flows $\lambda(b)$ and $y(b)$ starts at initial values $\lambda_{\rm ir} < 1$ and $y_{\rm ir} < 1$ evolves to an infrared fixed point,

$$\lambda^{\rm fix}_{\rm ir} = 0, \quad y^{\rm fix}_{\rm ir} = 0; \quad \beta_{\lambda,y}(\lambda^{\rm fix}_{\rm ir}, y^{\rm fix}_{\rm ir}) = 0 \tag{4.7}$$

and mass $m \propto m_{\rm ir}/b \to M_{\rm pl}$, as the energy scale $\mu = bM_{\rm pl}$ decreases. This is known as the divergent mass and triviality of scalar field theories in four-dimensional spacetime.

Moreover, as the energy scale $\mu = bM_{\rm pl}$ increases, the RG flows (4.5) approach an ultraviolet (UV) fixed point,

$$\lambda^{\rm fix}_{\rm uv} \neq 0, \quad y^{\rm fix}_{\rm uv} \neq 0; \quad \beta_{\lambda,y}(\lambda^{\rm fix}_{\rm uv}, y^{\rm fix}_{\rm uv}) = 0 \tag{4.8}$$

and scaling invariant domain $\mathcal{D}_{\rm uv}$. This is not conclusive because the nontrivial $\lambda^{\rm fix}_{\rm uv}$ and $y^{\rm fix}_{\rm uv}$ are not in the regime of weak couplings where the perturbative calculations are reliable. Readers are referred to Refs. [38, 39], and detailed discussions and literature are therein.

4.2 Fermion coupling and nonperturbative effective action

Due to mixing between wormhole sectors in the interaction (3.2), the foamon field Φ couples to a bilinear operator $\bar\Psi_L\Psi_R$ of N_f chiral fermion fields,

$$y\Phi\bar\Psi_L\Psi_R + {\rm h.c.}. \tag{4.9}$$

The mixing parameter (Yukawa coupling) $y < 1$ is smaller than unity. Integration over the high-energy modes of fermion fields $\Psi = \Psi_L + \Psi_R$ yields

$$\int^{M_{\rm pl}}_{bM_{\rm pl}} \mathcal{D}\bar\Psi\mathcal{D}\Psi\, e^{-\bar\Psi\Delta\Psi} \approx \exp\left\{N_f b^4 \int_{x'}\int^{M_{\rm pl}}_{bM_{\rm pl}} \frac{d^4k}{(2\pi)^4} \ln\Delta(k^2)\right\}, \tag{4.10}$$

where $\Delta = [k^2 + (y\Phi)^2]$ and $y\Phi$ is treated as an external mean field. Following the Wilson approach discussed below (3.17), all Φ modes at short distances smaller than

$(bM_{\rm pl})^{-1}$ are summed and averaged over the volume $V_b = (bM_{\rm pl})^{-4}$. This gives the factor $b^4 \int_{x'}$ in (4.10). The momentum integral leads to

$$\frac{N_f b^4}{32\pi^2}\int_{x'}\Big\{[k^4-(y\Phi)^4]\ln[k^2+(y\Phi)^2]-\frac{k^4}{2}+(y\Phi)^2k^2\Big\}_{bM_{\rm pl}}^{M_{\rm pl}}, \tag{4.11}$$

whose leading order contribution is

$$\frac{N_f b^4}{32\pi^2}\int_{x'}\Big\{[-(y\Phi)^4]\ln[M_{\rm pl}^2+(y\Phi)^2]+(y\phi)^4\ln(y\Phi)^2\Big\}, \tag{4.12}$$

neglecting high-order contributions $\mathcal{O}(b^l)$ for $b<1$, and using $\Phi \approx \phi/b$ (4.3). Expanding (4.12) in powers of $(y\Phi)/M_{\rm pl} < 1$ and using $\Phi \approx \phi/b$ (4.3), we obtain the approximate but nonperturbative mean-field contribution

$$\frac{N_f}{32\pi^2}\int_{x'}\Big[(y\phi)^4\ln\frac{(y\phi)^2}{(bM_{\rm pl})^2}\Big], \tag{4.13}$$

for small Yukawa coupling y and large fermion number N_f. It is worthwhile to compare it with the perturbative fermion-loop contribution $\sim \frac{N_f}{16\pi^2}\int_{x'}(y\phi)^4\ln(1/b)$. The fermion-loop contribution has an opposite sign to the boson-loop contribution (4.4).

We include the mean-field contribution (logarithmic term) (4.13) to the effective action (4.2) at the scale $bM_{\rm pl}$,

$$\Gamma(\phi) = \int_{x'}\Big[\frac{1}{2}(\partial'_\mu\phi)^2+\frac{1}{2}m^2\phi^2+\frac{\lambda}{4}\phi^4+N_f\frac{(y\phi)^4}{32\pi^2}\ln\frac{(y\phi)^2}{(bM_{\rm pl})^2}+(\cdots)\Big]. \tag{4.14}$$

As $b\to 0$, the effective action $\Gamma(\phi)$ approaches a quasi-free theory without interactions $\lambda \approx 0$ and $y \approx 0$ in the infrared scaling domain (4.7). The ground state is located at the minimum $\phi = 0$ of the positive potential $\frac{1}{2}m^2\phi^2$. However, the logarithmic term explicitly depending on b slightly violates the scaling invariance. We attempt to study the effective action ground state and SSB dynamics due to the logarithmic term, when the renormalisation scale $bM_{\rm pl}$ increases, departing from the infrared fixed point (4.7).

4.3 Spontaneous symmetry breaking at renormalization scale

The one-loop result $\beta_y^{(1)}$ (4.6) and the logarithmic term (4.14) are proportional to the fermion number N_f. As the scale $\mu = bM_{\rm pl}$ increases, the Yukawa coupling can increase more rapidly than the quartic coupling $\lambda(b)$ for $N_f \gg 1$. The SSB can occur when the negative logarithmic term prevails over the positive quadratic $m^2\phi^2$ and quartic $\lambda\phi^4$ terms in the effective action $\Gamma(\phi)$ (4.14). This can be shown by the effective potential $V(\phi)$ and the first derivative $V'(\phi)$

$$V(\phi) = \frac{1}{2}m^2\phi^2+\frac{\lambda}{4}\phi^4+N_f\frac{(y\phi)^4}{32\pi^2}\ln\frac{(y\phi)^2}{(bM_{\rm pl})^2}, \tag{4.15}$$

$$V'(\phi) = m^2\phi+\Big[\lambda+N_f\frac{y^4}{16\pi^2}\Big]\phi^3+N_f\frac{y^4\phi^3}{8\pi^2}\ln\frac{(y\phi)^2}{(bM_{\rm pl})^2}. \tag{4.16}$$

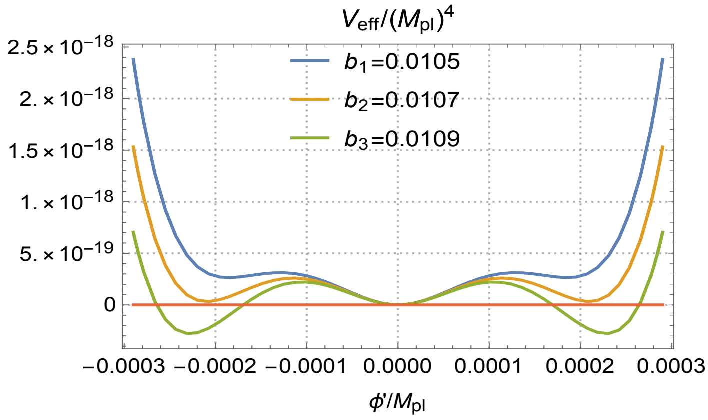

Figure 3. We use one-loop RG flows $\lambda(b)$ and $y(b)$ (4.5), where $\lambda_{\rm ir} = 10^{-3}$ and $y_{\rm ir} = 0.3$, to illustrate the effective potential (4.15) in the unit of the Planck mass $M_{\rm pl}$ for three different b values, $N_f = 10^2$ and $m/M_{\rm pl} = 1.135 \times 10^{-7}$. In the symmetric phase, the ground state is at the absolute minimum $\phi_c = 0$ (blue line) for small couplings λ and y. As couplings increase with b, the orange line shows two minima of degenerated vacuum states, indicating a phase transition to a symmetry-breaking phase. The SSB develops a symmetry-breaking ground state at the absolute minimum $\phi_c \neq 0$ (green line). The negative logarithmic term has caused a new minimum to appear away from the origin. This behaviour is analogous to loop corrections inducing spontaneous symmetry breaking [40].

In Fig. 3, we use one-loop RG flows $\lambda(b)$ and $y(b)$ (4.5) to illustrate the effective potential $V(\phi)$ (4.15) in the unit of the Planck mass $M_{\rm pl}$ for three different values $b_1 > b_2 > b_3$. It shows the following features.

In the symmetric phase, the ground state is at the absolute minimum $\phi'_c = 0$ (blue line) for small values of b. The RG flows $\lambda(b)$ and $y(b)$ are well described by perturbative loop-expansion results. Starting with initial values $\lambda_{\rm ir} \ll 1$ and $y_{\rm ir} \ll 1$ (4.7), as the energy scale $bM_{\rm pl}$ increases, RG flows $\lambda(b)$ and $y(b)$ (4.5) to the critical couplings λ_c and y_c, where two minima $\phi_c = 0$ and $\phi_c \neq 0$ are degenerate with $V(\phi) = 0$ (black line). The SSB phase transition occurs from the symmetric ground state $\phi_c = 0$ (disorder phase) to the symmetry-breaking ground state (order phase) $\phi_c \neq 0$. The SSB solutions $y_c\phi_c$ and m_c are uniquely determined by the equations of $V(\phi_c) = 0$ and $V'(\phi_c) = 0$,

$$(y_c\phi_c)^2 = (bM_{\rm pl})^2 e^{-\zeta_c - 1}, \quad \zeta_c \equiv \frac{8\pi^2\lambda_c}{N_f y_c^4}. \tag{4.17}$$

This is a type of gap equation for the nontrivial expectation value ϕ_c and the SSB mass scale $y_c\phi_c$. It produces fermion masses $M_f = y_c\phi_c$ and scalar boson mass $m_{\rm eff} = \sqrt{2}\, m_c$,

$$m_c^2 = \frac{N_f y_c^4}{16\pi^2}\phi_c^2. \tag{4.18}$$

The SSB mass is the physical scale, which should be relevant to the renormalisation scale $bM_{\rm pl}$, namely $y_c\phi_c \sim bM_{\rm pl}$. This means the parameter $\zeta_c \sim \mathcal{O}(1)$, $\lambda_c < 1$, $y_c < 1$ and $N_f \gg 1$. Given the physical scale M_f, the ζ_c value is fixed, and critical couplings λ_c and y_c are related.

In the symmetry-breaking phase, the critical couplings λ_c and y_c evolve with the scale $\mu = bM_{\rm pl}$. At the phase transition, they should continue with the counterparts (4.5) in the symmetric phase, namely $\lambda_c = \lambda$ and $y_c = y$. However, their derivatives, namely, the β-functions

$$\beta_y^{(c)} = b\frac{\partial y_c}{\partial b}, \quad \beta_\lambda^{(c)} = b\frac{\partial \lambda_c}{\partial b}, \tag{4.19}$$

discontinue from β_y and β_λ at the phase transition. The SSB solutions (4.17) depend on the arbitrary scale μ_c, which reflects the arbitrariness of the definition of renormalised critical couplings λ_c and y_c. The physical scale $M_f = y_c\phi_c$ is unambiguous, following the Callan-Symanzik equation

$$b\frac{\partial M_f}{\partial b} = 0 \quad \Rightarrow \quad \beta_y^{(c)} = -\frac{N_f y_c^5}{16\pi^2\lambda_c} + \frac{y_c}{4\lambda_c}\beta_\lambda^{(c)} < 0. \tag{4.20}$$

From the gap equation (4.17), we find that the coupling y_c (λ_c) should decrease (increase) to maintain M_f, as the scale $bM_{\rm pl}$ increases. This implies the negative $\beta_y^{(c)}$ and positive $\beta_\lambda^{(c)}$ for $N_f \gg 1$. This situation is analogous to the negative β-function of the non-linear Sigma model of the $O(N)$ symmetry in the large N limit.

Due to the negative logarithmic term, the effective potential (4.15) undergoes the phase transition even for $m^2 > 0$. At the critical point (4.17), we find positive curvature m_c^2 at $\phi = 0$, see Fig. 3. It implies a weak first-order phase transition, because the fields ϕ *jump* from the zero expectation value $\phi = 0$ ground state of the symmetric phase to the nonzero expectation value $\phi \neq 0$ ground state of the symmetry-breaking phase.

4.4 Ultraviolet fixed point and scaling invariant domain

In general, the effective mass and quartic coupling relate to,

$$V''(\phi) = m^2 + \left[3\lambda + N_f\frac{7y^4}{16\pi^2}\right]\phi^2 + N_f\frac{3y^4\phi^2}{8\pi^2}\ln\frac{(y\phi)^2}{(bM_{\rm pl})^2}, \tag{4.21}$$

$$V''''(\phi) = \left[6\lambda + N_f\frac{17y^4}{8\pi^2}\right] + N_f\frac{3y^4}{4\pi^2}\ln\frac{(y\phi)^2}{(bM_{\rm pl})^2}. \tag{4.22}$$

In the symmetry-breaking phase, using the gap equation (4.17) for $y_c\phi_c \neq 0$, we obtain

$$m_{\rm uv}^2 \equiv V''(\phi_c) = m_c^2 + \frac{N_f y_c^4}{16\pi^2}\phi_c^2 = 2m_c^2, \tag{4.23}$$

$$\lambda_{\rm uv} \equiv V''''(\phi_c) = \frac{11N_f}{8\pi^2}y_c^4, \tag{4.24}$$

which are functions of the energy scale $\mu = bM_{\rm pl}$ and critical parameter $\zeta_c(\lambda_c, y_c)$. Upon the symmetry-breaking ground state, the field $\phi = \phi_c + \phi_{\rm uv}$ and the effective Lagrangian of low-energy quantum fields of massive boson $\phi_{\rm uv}$ and fermions $\psi_{\rm uv}$

$$\begin{aligned}\mathcal{L}(\phi_{\rm uv}) = &\frac{1}{2}(\partial_\mu\phi_{\rm uv})^2 + \frac{1}{2}m_{\rm uv}^2\phi_{\rm uv}^2 + \frac{\lambda_{\rm uv}}{4}\phi_{\rm uv}^4 \\ &+ \bar\psi_{\rm uv}(i\partial\!\!\!/ + M_f)\psi_{\rm uv} + y_c\phi_{\rm uv}(\bar\psi_{\rm uv}\psi_{\rm uv}) + (\cdots).\end{aligned} \tag{4.25}$$

The $(\cdots)$ represents the counterterms and high-dimensional ($d > 4$) operators. The scalar field $\phi_{\rm uv}$ becomes massive, and massless Goldstone bosons $\pi_{\rm uv}$ become nonlinear Sigma model or longitudinal modes of massive gauge bosons coupling to fermions, which will be discussed in a separate article.

From the β-function $\beta_y^{(c)}$ (4.24) of the Yukawa coupling, we obtain the effective β_λ-function

$$\beta_\lambda^{\rm uv} = b\frac{\partial\lambda_{\rm uv}}{\partial b} = \frac{11N_f}{2\pi^2}y_c^3\beta_y^{(c)} < 0. \tag{4.26}$$

In the symmetry-breaking phase of the effective theory (4.27), the β-functions $\beta_y^{(c)} < 0$ and $\beta_\lambda^{\rm uv} < 0$ are negative. In the symmetric phase of the quasi-free theory (4.2) coupling to massless fermions $y_{\rm ir}\bar\Psi_L\Psi_R$, the β-functions $\beta_\lambda^{(1)} > 0$ and $\beta_y^{(1)} > 0$ are positive. This implies the effective β-functions to vanish around the SSB phase-transition critical point $\zeta_c(\lambda_c, y_c)$ (4.17). Therefore, we expect that the critical point should be a candidate for a UV fixed point and scaling invariant domain $\mathcal{D}_{\rm uv}$ (4.8) at small couplings $\lambda_c < 1$ and $y_c < 1$ for the large fermion number $N_f \gg 1$. However, we cannot determine the entire RG flow from the IR fixed point to the UV one.

Analogously to the effective Lagrangian (4.1) and scaling laws (4.3)-(4.3) in the IR scaling invariant domain $\mathcal{D}_{\rm ir}$, the effective Lagrangian $\mathcal{L}_{\rm uv}$(4.25) is realised in the UV scaling invariant domain $\mathcal{D}_{\rm uv}$. At the renormalization scale $bM_{\rm pl}$, the effective action is $\Gamma(\phi,\psi) = \int d^dx'\mathcal{L}_{\rm eff}$ and

$$\mathcal{L}_{\rm eff} = \frac{1}{2}(\partial'_\mu\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4 + \bar\psi(i\partial\!\!\!/' + M_f)\psi + y\phi(\bar\psi\psi) + (\cdots). \tag{4.27}$$

The relevant wave-function, mass and couplings are scaled as,

$$\begin{aligned} \phi &\approx \phi_{\rm uv}b^{\frac{2-d}{2}}, \quad \psi \approx \psi_{\rm uv}b^{\frac{1-d}{2}}, \\ m^2 &\approx m^2_{\rm uv}b^{-2}, \quad \lambda \approx \lambda_{\rm uv}b^{d-4}, \quad y \approx y_c b^{d-4}, \end{aligned} \tag{4.28}$$

when b increases. All irrelevant operators $(\cdots)$ are suppressed by $\mathcal{O}(b^l)$ and $l > 1$. The effective theory is the Wilson scaling invariant $\Gamma(\phi,\psi) \approx \Gamma(\phi_{\rm uv},\psi_{\rm uv})$ with the correlation length $\xi = 1/m_{\rm uv}$.

5 Spacetime foam effects on field effective actions

In the infrared and ultraviolet scaling invariant domains $\mathcal{D}_{\rm ir}$ and $\mathcal{D}_{\rm uv}$, the correlation length ξ sets the physically relevant scale,

$$\xi = \tilde m^{-1}, \quad \tilde m = m_{\rm ir}, m_{\rm uv}. \tag{5.1}$$

All physical quantities are RG invariant (scaling) when a length scale change is much smaller than ξ. Following scaling laws (4.3) and (4.28), we have approximate scaling laws for the interacting operator $\int_x \phi O$ (3.6),

$$\begin{aligned} \phi &\approx \tilde\phi b^{-\left(\frac{d-2}{2}\right)}, \quad O \approx \tilde O b^{-\left(d-\frac{d-2}{2}\right)} \\ \tilde\phi &= \phi_{\rm ir}, \phi_{\rm uv}, \quad \tilde O = O_{\rm ir}, O_{\rm uv} \end{aligned} \tag{5.2}$$

where $\frac{d-2}{2}$ and $d-\frac{d-2}{2}$ are the canonical dimensions of the field ϕ and operator O in energy scale. The field $\tilde{\phi} = \phi_{\rm ir}, \phi_{\rm uv}$ and operator $\tilde{O} = O_{\rm ir}, O_{\rm uv}$ are in the scaling invariant domains.

5.1 Effective action for field operators at large distances

Here we resume the subscript "i" for types of foamon field. In terms of the effective Lagrangian $\mathcal{L}_{\rm eff}$ (4.2) and (4.27) in the scaling invariant domain, the Euclidean generating function (3.7) becomes

$$e^{-S_E^{\rm eff}(O_i)} = W_E(O_i) \approx \int_0^{bM_{\rm pl}} \mathcal{D}\phi_i e^{-\int_{x'}[\mathcal{L}_{\rm eff}+\phi_i O_i]}, \tag{5.3}$$

which yields an effective action $S_E^{\rm eff}(O_i)$ for field operators O_i. Based on the two-state mixing and two-point correlation functions (2.10), we recast (5.3) as

$$\begin{aligned} e^{-S_E^{\rm eff}(O_i)} \equiv W_E(O_i) &\approx \int_0^{bM_{\rm pl}} \mathcal{D}\phi_i \; e^{-\frac{1}{2}\phi_i\Delta_{ij}\phi_j+\phi_i O_i} \\ &= [\det \Delta_{ij}]^{-1/2} e^{(1/2)O_i(\Delta_{ij})^{-1}O_j^\dagger}, \end{aligned} \tag{5.4}$$

where diagonalized $\Delta_{ij} = \delta_{ij}(\partial'^2 + m_i^2)$ and $\partial' = \partial/\partial x'$. In the coordinate space, the indexes (i,j) stands for (x_1', i) and (x_2', j). In the momentum space, the indexes (i,j) stands for (k_1', i) and (k_2', j). The eigenvalues $\Delta_i(k', m_i)$ are approximate for the eigenvalues of the general two-point correlation function $G_{ij}^{(2)}(x_2' - x_1')$ (2.11) of the foamon field theory.

5.2 Foamon field energy densities relevant to field operators

The approximate foamon field correlation function Δ_{ij} (5.4) in the scaling domain is diagonalised to two sets of eigenvalues of discrete modes m_i and continuous modes k' of spectrum $\Delta_i(k'^2) = k'^2 + m_i^2$ of the foam field ϕ. The first part of the effective action (5.4) is

$$[\det \Delta_i(k'^2)]^{-1/2} = \exp\left\{-\frac{1}{2}\int_{x'}\int_0^{bM_{\rm pl}} \frac{d^4k'}{(2\pi)^4} \ln \Delta_i(k'^2)\right\}, \tag{5.5}$$

and

$$\begin{aligned} \int_0^{bM_{\rm pl}} \frac{d^4k'}{(2\pi)^4} \ln \Delta_i(k'^2) &= \frac{1}{32\pi^2}\Big\{[(bM_{\rm pl})^4 - m_i^4]\ln[(bM_{\rm pl})^2 + m_i^2] \\ &\quad - \frac{(bM_{\rm pl})^4}{2} + m_i^2(bM_{\rm pl})^2 + m_i^4 \ln m_i^2\Big\}\Big|_{b\ll 1} \\ &\approx \frac{1}{32\pi^2} m_i^2 (bM_{\rm pl})^2 = \frac{1}{32\pi^2}\tilde{m}_i^2 M_{\rm pl}^2. \end{aligned} \tag{5.6}$$

As a result, we arrive at

$$[\det \Delta_i(k'^2)]^{-1/2} = \exp\left(-\frac{1}{2}\int_{x'} \frac{1}{32\pi^2}\tilde{m}_i^2 M_{\rm pl}^2\right), \tag{5.7}$$

and obtain the foamon field energy density

$$\tilde{\rho}_i = \frac{1}{2}\frac{1}{32\pi^2}\tilde{m}_i^2 M_{\rm pl}^2. \tag{5.8}$$

This energy density is contributed from the relevant foamon field ϕ_i modes associated with the operator O_i at the energy scale $\tilde{m}_i$. It differs from the today energy density $\sim M_{\rm pl}^4$ (2.9) contributed from all foamon field Φ_i modes from short distances $M_{\rm pl}^{-1}$.

5.3 Nonlocal field action originated from spacetime foam correlations

We turn to study the second part of the effective action (5.4) in the scaling domain. Approximately adopting the diagonalised two-point correlation function (2.11) in the scaling domain, we have in coordinate space,

$$e^{(1/2)O_i(\Delta_{ij})^{-1}O_j^\dagger} \approx \exp\left\{\frac{1}{2}\int_{x_1',x_2'} O_i(x_1')O_i^\dagger(x_2')\int_0^{bM_{\rm pl}}\frac{d^4k'}{(2\pi)^4}\frac{e^{ik'(x_2'-x_1')}}{k'^2+m_i^2}\right\}. \tag{5.9}$$

This is a non-local operator, since the foamon field ϕ_i propagation $(\Delta_{ij})^{-1} = \delta_{ij}(\partial'^2 + m_i^2)^{-1}$ couples two operators O_i at different spacetime points. This effective nonlocal action is attributed to the correlation functions (2.10) and (2.12) of the foamon fields in different spacetime points, in contrast with the connection function $C_{ij}(x_1,x_2)$ (1.1) of wormholes connecting two spacetime points. The two-point function (2.11) is exponentially suppressed for $|x_2'-x_1'| > (m_i)^{-1}$, indicating that $O_i(x_1')$ and $O_i^\dagger(x_2')$ are not correlated if they are separated beyond $(m_i)^{-1}$. It measures the size of the operator O_i locality.

In the neighbourhood of the point x_1', we approximately have the four-dimensional volume $\int_{x_2'} \approx (m_i)^{-4}$ in Eq. (5.9) [4], and expand the operator $O_i(x_2') = O_i[x_1' + (m_i)^{-1}]$ in the powers of operators' derivatives,

$$\begin{aligned} O_i(x_2') &= O_i(x_1') + (m_i)^{-2}\partial'^2 O_i(x_1') + \cdots \\ &= O_i(x_1') + b_i^2(\tilde{m}_i)^{-2}\partial'^2 O_i(x_1') + \cdots. \end{aligned} \tag{5.10}$$

The high-dimensional derivative operators $(\cdots)$ represent nonlocal contributions, which are suppressed for $b < 1$ in the scaling domains (4.2) and (4.27). We will henceforth neglect the nonlocal contributions $(\cdots)$. Thus, in Eq. (5.9) we approximately obtain the local operator at the leading order,

$$\frac{1}{2}(m_i)^{-4}\int_{x'} O_i^2(x')\int_0^{bM_{\rm pl}}\frac{d^4k'}{(2\pi)^4}\frac{1}{k'^2+m_i^2}, \tag{5.11}$$

where $O_i^2 = O_iO_i^\dagger$ and $x_1' \to x'$. The dimensional scaling laws (4.3) and (5.2) lead to

$$(m_i)^{-4}\int_{x'} O_i^2 \to \frac{1}{b^2}\int_{x'}(\tilde{m}_i)^{-4}\tilde{O}_i^2. \tag{5.12}$$

[4]Note that the phase factor $e^{ik'(x_2'-x_1')} \sim 1$ and $k'(x_2'-x_1')$ is invariant under the b scaling (3.17).

The momentum integration for $b<1$ gives

$$\frac{1}{b^2}\int_0^{bM_{\rm pl}}\frac{d^4k'}{(2\pi)^4}\frac{1}{k'^2+m_i^2}=\frac{1}{8\pi^2 b^2}\left[(bM_{\rm pl})^2-\frac{1}{2}m_i^2\ln\left(1+\frac{(bM_{\rm pl})^2}{m_i^2}\right)\right]_{b<1}$$
$$=\frac{1}{8\pi^2}\left(M_{\rm pl}^2-\frac{1}{2}\tilde m_i^2\right)\approx\frac{M_{\rm pl}^2}{8\pi^2}. \tag{5.13}$$

As a result, we obtain the local field operator

$$e^{(1/2)O_i[\Delta^{(2)}_{ij}]^{-1}O_j^\dagger}\Big|_{b<1}\approx\exp\left(\frac{M_{\rm pl}^2}{16\pi^2}\int_{x'}\frac{\tilde O_i^2}{(\tilde m_i)^4}\right), \tag{5.14}$$

which is the leading-order contribution of the nonlocal effective action (5.9).

In the scaling invariant domain (4.2) or (4.27), where the renormalisation flow approaches the stable fixed point (4.7) or (4.8), we approximately obtain the effective local action of relevant field operators $\tilde O_i$ at low energies $\tilde m_i\ll M_{\rm pl}$,

$$S_E^{\rm eff}(\tilde O_i)=-\frac{1}{2}\int_{x'}\frac{1}{32\pi^2}\tilde m_i^2M_{\rm pl}^2+\frac{M_{\rm pl}^2}{16\pi^2}\int_{x'}\frac{\tilde O_i^2}{(\tilde m_i)^4}+c_n\int_{x'}\frac{\tilde O_i^{2n}}{(\tilde m_i)^{4n}}, \tag{5.15}$$

up to an overall irrelevant constant, where $\tilde m_i=\xi_i^{-1}$ sets the characteristic scale of the effective theories. The high-dimensional terms $\tilde O_i^{2n}$ $(n>1)$ come from the contribution of n-point connect Green functions (2.12), yielding operators R^n, suppressed by $c_n\propto\mathcal{O}[(\ell_{\rm pl}/\xi_i)^{2n}]$ and $\xi_i\gg\ell_{\rm pl}$, in the scaling invariant domains of fixed points. They are irrelevant operators in the sense of the asymptotic safety [41].

By the Wick rotation on the effective Euclidean action (5.15) at long distances, we obtain the corresponding effective action $S_M^{\rm eff}(\tilde O_i)$ in the Minkowski space. We emphasise that the Wick rotation acts on the effective Euclidean action (2.3) and path integral (3.3) of the foamon field Φ_i coupling to the field operator O_i at large distances, *rather than* the Euclidean gravity action I_E (1.3) and path integral $\int\mathcal{D}ge^{-I_E}$.

6 Effective Einstein action for cosmology

6.1 Ricci scalar and cosmological constant

In the effective Einstein theory for the Freeman cosmology, the Ricci scalar R plays the role of an infrared regulator O for the quantum foamon field Φ through the interaction (3.1) or (1.5). From the effective action (5.15), we obtain that the characteristic scale $\tilde m$ of low-energy relevant field ϕ relates to the cosmological term $\Lambda/(8\pi G)$ and Ricci scalar curvature $R/(16\pi G)$:

$$\tilde m^2=8\pi\Lambda,\qquad \tilde O^2=\pi(\tilde m)^4R, \tag{6.1}$$

where the Newton constant $G=M_{\rm pl}^{-2}$. The correlation length $\xi=\tilde m^{-1}$ is about the cosmic horizon size H^{-1}. The Planck length $\ell_{\rm pl}$ and the horizon size H^{-1} determine the foamon field energy density (dark-energy density)

$$\rho_\Lambda=\frac{\Lambda}{8\pi G}=\frac{\xi^{-2}}{(8\pi)^2G}=\frac{\xi^{-2}\ell_{\rm pl}^{-2}}{(8\pi)^2}\sim\frac{H^2M_{\rm pl}^2}{(8\pi)^2}, \tag{6.2}$$

which is consistent with the one derived from simply counting degrees of freedom [42]. The result (6.2) is many orders of magnitude smaller and larger than the naive estimation $\ell_{\rm pl}^{-4}$ (2.9) and H^4 of foamon field energy density.

In the effective action (5.15), the foamon field energy term $\int_{x'} \tilde m_i^2 M_{\rm pl}^2 \sim (\xi_i/\ell_{\rm pl})^2$ is a holographical constant, depending on the spacetime area rather than volume $(\xi_i/\ell_{\rm pl})^3$. It mixes the UV scale $\ell_{\rm pl}$ of the quantum foam field Φ_i and IR scale ξ_i of the field operators O_i. It does not explicitly depend on the field operator O_i, but on the length scale ξ_i. All short-distance foamon modes irrelevant to the large-length scale ξ_i of the field operator O_i have been integrated away as a trivial constant in the path integration, thus relevant degrees of freedom become small.

6.2 Equation of state and interaction with matter

As the Universe volume $V \propto H^{-3}$ adiabatically expands $\delta V > 0$, conserving the total number of degrees of freedom $\delta N = 0$, the positive particle's repulsive energy E (mean kinetic energy) decreases $\delta E < 0$, i.e., energy density ρ and pressure p decrease. It obeys the first law of thermodynamics in the comoving frame,

$$p\delta V + \delta E = 0, \quad p = -(\delta E/\delta V) > 0, \quad E = \int dV \rho. \tag{6.3}$$

Particles' spectra and distributions determine the particles' pressure p and energy density ρ. It leads to the equation of state $p = (\gamma - 1)\rho$ with thermal index $\gamma > 1$ and $\gamma = 4/3$ for free relativistic particles.

Unlike particles kinematically moving upon the spacetime manifold, spacetime foams and their correlations are geometrically embedded in the stretching manifold and thus have a different equation of state [43]. The gravitational energy of the spacetime foamons is positive

$$E_\Lambda = \int dV \rho_\Lambda = \frac{M_{\rm pl}^2}{16\pi}\xi, \quad dV = 4\pi\xi^2 d\xi, \tag{6.4}$$

increasing with the volume expansion of the Universe, namely $\delta E_\Lambda > 0$. Using Eq. (6.3) $p_\Lambda = -(\delta E_\Lambda/\delta V) < 0$, we obtain the negative pressure p_Λ and equation of state

$$p_\Lambda = -(\delta E_\Lambda/\delta V) = \omega_\Lambda \rho_\Lambda, \quad \omega_\Lambda = -1, \tag{6.5}$$

of the foamon field accounting for the dark energy. The reason is that a stretched manifold increases the energy of spacetime foamons, which are collective excitations of Wheeler's foamy spacetime, the Planck lattice.

The foamon field energy density (6.2) can be formally written as $\rho_\Lambda = M_{\rm pl} n_w$ in terms of the effective foam number density $n_w = \ell_{\rm pl}^{-1}\xi^{-2}/(8\pi)^2$, as if the relevant modes of the spacetime foamon field were holographically distributed as a thin layer of width $4\pi\xi^2\ell_{\rm pl}$ near to the Universe horizon $\xi \sim H^{-1}$. The holographic configuration minimises the total energy and number of spacetime foamon modes, whose wavelength is of the order of the Universe horizon. The possible interpretation is that in the macroscopic average, the local foamon field fluctuations (oscillations) in the homogeneous space cancel each other up to the horizon. Here, the local foamon field oscillation exhibits similar dynamics to the "microcyclic universes" at small scales, shown by local scale factor oscillation in Figure 1 of Refs. [44, 45].

Via the foamon-particle interaction $y\phi\bar{\psi}\psi$ (3.2) or (4.27), the local foamon-field violent fluctuations can result in the massive particle-antiparticle pairs' production [46]. These pairs oscillate coherently with the foamon field fluctuation and the formation of a holographic layer of massive particle-antiparticle pairs near the horizon. The foamon field (dark) energy density (6.2) converts the massive pairs' energy density. On the other hand, massive pairs' backreaction on the foamon field converts their energy to the foamon field. Such a back-and-forth processes lead to the interaction between matter M and dark energy Λ that obeys the Bianchi identities for the total energy conservation. The foamon field (dark) energy density (6.2) at the inflation scale $H \sim 10^{-(4\sim 5)}M_{\rm pl}$ drives the inflation. It vanishes in reheating by producing matter in massive particle-antiparticle pairs. Afterwards, in standard cosmology, the dark-energy density increases and achieves today's value due to the backreaction from radiation- and matter-energy densities. The detailed studies are in Refs. [47–49].

6.3 Scale factor and renormalisation group flow

In the Friedmann–Lemaître–Robertson–Walker (FLRW) cosmology, the scale factor $a(t)$ stretches the physical spacetime $x_{\rm phys} = a(t)x_{\rm com}$ and $k_{\rm phys} = k_{\rm com}/a(t)$, where $x_{\rm com}$ and $k_{\rm com}$ are in the comoving coordinate. On the physical spacetime, the renormalised foam fields, masses, and couplings of the effective Lagrangian (4.1) or (4.25) should correspond to observables at long distances. It implies that as the physical spacetime length scale stretches in the universe expansion, the small-scale foamon field modes are integrated, and renormalised fields, masses, couplings and effective Lagrangian (4.2) should follow the Wilson renormalisation group equations in the scaling invariant domain of physical infrared or ultraviolet fixed point. From this point of view, we identify the correspondences between the scaling parameter b (3.17) and the scale factor $a(t)$,

$$b \Leftrightarrow a(t), \quad (x', k') \Leftrightarrow (x_{\rm phys}, k_{\rm phys}); \quad (x, k) \Leftrightarrow (x_{\rm com}, k_{\rm com}). \tag{6.6}$$

The infrared fixed point $b \sim 0$ corresponds to $a(t) \sim 0$ for the early Universe. The ultraviolet fixed point $b \sim 1$ corresponds to $a(t_0) \sim 1$ for the present t_0 epoch of the Universe. The scaling parameter b plays the same role of the scale factor $a(t)$ in a stretching FLRW Universe.

We recall the preliminary studies by using the Wilson-loop variables and their correlation functions in the simplicial complex (lattice) Euclidean quantum gravity [26, 50, 51], which show the gravitational coupling $g(b) = G(b)M^2_{\rm pl}$ depending on the running scale. Its critical behaviours are: (i) $g \sim g^*_{\rm ir}b^2$ and $\xi \sim \ell_{\rm pl}/b$ in the IR domain ($b \to 0, g \to g^*_{\rm ir} \approx 0$) for the early Universe; (ii) $g \sim g^*_{\rm uv}b^2$ and $\xi \sim \ell_{\rm pl}/(g^*_{\rm uv} - g)^{1/2}$ in the UV domain of $b \to 1, g \to g^*_{\rm uv} \neq 0$ and $G(b) \to G$ for the later Universe. The $g^*_{\rm ir}$ and $g^*_{\rm uv}$ are gravitational coupling g values at IR and UV fixed points. More investigation is necessarily required.

7 Summary and remarks

Spacetime foam structure, as the quantum gravity ground state, naturally provides the ultraviolet cutoff $M_{\rm pl}$ for fundamental quantum field theories, whose short-distance field fluctuations of virtual particles contribute the constant zero-point energy density $M^4_{\rm pl}$. Physically real particles are energetic excitations above this overall constant, defined as the ground state, i.e., the vacuum. Therefore, this overall constant does not contribute to the matter energy-momentum tensor $8\pi G T_{\mu\nu}$ in the RHS of the Einstein equation, as the energetic excitations of physical particles do. Subtracting this overall constant from physical particle energy spectra, long-distance zero-point fluctuations relevant to the infrared cutoff $H \ll M_{\rm pl}$ give a zero-point energy density H^4 of Casimir type, whose contribution $8\pi G H^4$ is negligible. The zero-point energy density $M^4_{\rm pl}$ (H^4) is conceptually and quantitatively irrelevant to the cosmological constant Λ in the LHS of the Einstein equation [43]. The foamon field (dark) energy density $\rho_{_\Lambda} \propto M^2_{\rm pl} H^2$ (6.2) is purely gravitational (spacetime) origin, interacting with the energy-momenta of physical particle excitations upon the overall constant of the vacuum. Nevertheless, it is worth noting that several recent proposals [44, 45, 52–55] study the attractive (negative) energy of gravitational field oscillating modes at small scales, compensating for the positive vacuum energy of matter (particle) fields at larger scales, as heuristically discussed [20, 23] using Wheeler's "spacetime foam".

Our scenario is as follows. Upon the spacetime ground state, spacetime foams at the Planck length undergo creation, oscillation and annihilation at short distances $\mathcal{O}(M^{-1}_{\rm pl})$, while their collective modes interact with the field operators at long distances $\mathcal{O}(H^{-1})$. We describe such a collective fluctuation and interaction by using a foamon field theory and its renormalisation flow, for which the field operators play the role of an infrared regulator. Approximately using the Wilson approach, our studies show that as the energy scale increases, the foamon field theory evolves from an infrared scaling invariant domain in the symmetry-preserving phase to an ultraviolet scaling invariant domain in the symmetry-breaking phase, when numerous particles are present. In these domains, the foamon field correlation length sets the characteristic scale for the effective action of the field operators at long distances. In cosmology, we derive the effective Einstein action of the Ricci scalar and the cosmological constant at the horizon scale, which plays the role of dark energy interacting with matter. Further studies are necessary for verification.

We end this article with a few speculations on this scenario for dark energy. In local universes, the matter distributions are not homogeneous, and the dark energy distributions should not be homogeneous. The reason is that the foamon field dark energy, the first term in (5.15), depends on the Ricci scalar R of self-gravitational systems of matter and dark energy, which obey the Einstein equation. The induced matter and dark energy interactions result in matter and dark energy inhomogeneous distributions in an equilibrium configuration. The foamon field dark energy in a galaxy relates to the Ricci scalar, which depends on galactic matter. It is a positive repelling energy bound inside a negative attractive potential due to self-gravitating matter. Thus, the interactions between matter and dark energy establish the gravitational potentials for stable galactic systems. Consequently, these interactions and resultant gravitational potential must lead to the flattened galaxy's rotation curve and the smoothed gravitational lensing effects, since the contributions from matter and dark energy are opposite. These dynamics and effects deserve studies. We note the studies [56, 57] using phenomenological models for dark energy.

In the sector of the foamon field Φ_f associated with the fermion operator $O_f = (\bar{\psi}_L \psi_R)$, from the effective action (5.15), we obtain the effective four-fermion operator of Einstein-Cardan type

$$\frac{M_{\rm pl}^2}{16\pi^2 M_f^4}(\bar{\psi}_L \psi_R)(\bar{\psi}_L \psi_R), \tag{7.1}$$

and the energy density term $M_f^2/(64\pi^2 G)$, determined by the characteristic length scale $\xi_f = M_f^{-1}$. It can be an effective operator beyond the Standard Model of particle physics. One conceives that the natural constants of field theories should be consequences of spacetime wormhole fluctuations [7, 16, 31]. Studying how these occur through foamon fields, collective excitations of spacetime wormhole fluctuations, and interaction with the field operators is interesting.

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