

The interaction of gravitational waves with matter

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- If an electromagnetic wave passes through a plasma, there is a critical wavelength λ_c : if the EM wave has $\lambda \ll \lambda_c$ then it is minimally affected by the plasma, but for $\lambda > \lambda_c$ the wave can be almost completely reflected. Can something similar happen to GWs?
- It has been known since 1966¹ that if a GW propagates from r to $r + \Delta$ through a dissipative medium with shear viscosity η

$$(h_+, h_\times)(r + \Delta) = (h_+, h_\times)(r) \exp(-8\pi\eta\Delta)$$

However, in SI units,

$$8\pi\eta\Delta = \frac{8\pi\eta\Delta G}{c^3} = 8\pi\eta\Delta \times 2.477 \times 10^{-36}$$

Thus, while energy is transferred from GWs to viscous matter, the effect is treated as negligibly small.

¹E.g., S.W. Hawking, Perturbations of an expanding universe, *Astrophys. J.* **145**, 544 (1966)

Coordinates are based on outgoing null hypersurfaces, (u, r, x^A) ; u labels the null hypersurface, r is a radial coordinate, and x^A are angular coordinates θ, ϕ . Then

$$ds^2 = - \left(e^{2\beta} (1 + W_c r) - r^2 h_{AB} U^A U^B \right) du^2 - 2e^{2\beta} du dr \\ - 2r^2 h_{AB} U^B du dx^A + r^2 h_{AB} dx^A dx^B .$$

Minkowski spacetime has $W_c = \beta = U^A = 0$, $h_{AB} = q_{AB}$ with q_{AB} a unit sphere metric, $q_{AB} dx^A dx^B = d\theta^2 + \sin^2 \theta d\phi^2$. We define perturbations

$$\beta = \Re \left(\beta^{[2,2]}(r) e^{i\nu u} \right) Z_{2,2}, \quad W_c = \Re \left(W_c^{[2,2]}(r) e^{i\nu u} \right) Z_{2,2}, \\ U = \Re \left(U^{[2,2]}(r) e^{i\nu u} \right) {}_1Z_{2,2}, \quad J = \Re \left(J^{[2,2]}(r) e^{i\nu u} \right) {}_2Z_{2,2},$$

where $U = U^A q_A$, $J = h^{AB} q^A q^B / 2$ with $q_A = (1, i \sin \theta)$ and where ${}_s Z_{2,2}$ are spin-weighted spherical harmonics. This “separation of variables” ansatz leads to the linearized Einstein equations reducing to ODEs in r with solutions polynomials in $1/r$.

$$\begin{aligned}
 \beta^{[2,2]} &= 0, \\
 W_c^{[2,2]} &= \frac{12\nu^2 C_{40}}{r^2} - \frac{12i\nu C_{40}}{r^3} - \frac{6C_{40}}{r^4} - C_{in0} \exp(2ir\nu) \frac{3}{r^4}, \\
 U^{[2,2]} &= -\frac{2\nu^2 \sqrt{6} C_{40}}{r^2} \\
 &\quad - \frac{4i\nu \sqrt{6} C_{40}}{r^3} - \frac{3\sqrt{6} C_{40}}{r^4} - C_{in0} \exp(2ir\nu) \sqrt{6} \left(i \frac{\nu}{r^3} - \frac{3}{2r^4} \right), \\
 J^{[2,2]} &= -\frac{2\nu^2 \sqrt{6} C_{40}}{r} + \frac{2\sqrt{6} C_{40}}{r^3} \\
 &\quad + C_{in0} \exp(2ir\nu) \sqrt{6} \left(\frac{1}{r^3} - 2i \frac{\nu}{r^2} - \frac{\nu^2}{r} \right).
 \end{aligned}$$

Defining the rescaled GW strain by $\mathcal{H}_0 = r(h_+ + ih_\times)$, we find

$$\mathcal{H}_0 = \Re(H_0 \exp(i\nu u)) {}_2Z_{2,2} \text{ with } H_0 = -2\sqrt{6}\nu^2 C_{40}.$$

Note that, for large r , $J = h_+ + ih_\times$, and that C_{in0} represents the magnitude of incoming GWs.

The model: GW source surrounded by a spherical matter shell

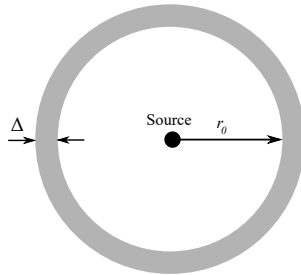


Figure: GW source inside a spherical shell of mass M_S between $r = r_0$ and $r = r_0 + \Delta$.

- Construct solutions (a) Exterior to the shell, (b) Within the shell, and (c) Interior to the shell.
- Enforce continuity of the metric and first derivatives at the boundaries of the shell, and so construct a complete solution.

$$H = H_0 \left[1 + \frac{2M_S}{r_0} - \frac{iM_S\lambda}{r_0^2\pi} + \frac{iM_S\lambda e^{-4ir_0\pi/\lambda}}{4r_0^2\pi} + \dots \right].$$

Each of the terms containing M_S represents a correction to the wave strain in the absence of the shell.

- The first correction, $2M_S/r_0$ represents the (well-known) gravitational red-shift effect.
- The second correction term, $iM_S\lambda/(\pi r_0^2)$, is out of phase with the leading terms $1 + 2M_S/r_0$ and hence represents a phase shift of the GW without changing the GW energy.
- The third correction term represents a partial reflection of the GW by the shell, and leads to the possibility of a matter shell causing a GW echo.

- There has been previous work on whether the astrophysical environment might modify a GW signal including the possibility of an echo².
- The magnitude of the echo term can be written

$$\frac{2M_S}{r_0} \times \frac{\lambda}{8\pi r_0}$$

where λ is the wavelength, and the echo delay is $2r_0$.

²E.g., Barausse, E., Cardoso, V., Pani, P.: Can environmental effects spoil precision gravitational-wave astrophysics? Phys. Rev. D **89**(10), 104059 (2014); Konoplya, R.A., Stuchlík, Z., Zhidenko, A.: Echoes of compact objects: new physics near the surface and matter at a distance. Phys. Rev. D **99**, 024007 (2019); Abedi, J., Dykaar, H., Afshordi, N.: Echoes from the Abyss: tentative evidence for Planck-scale structure at black hole horizons. Phys. Rev. D **96**(8), 082004 (2017)

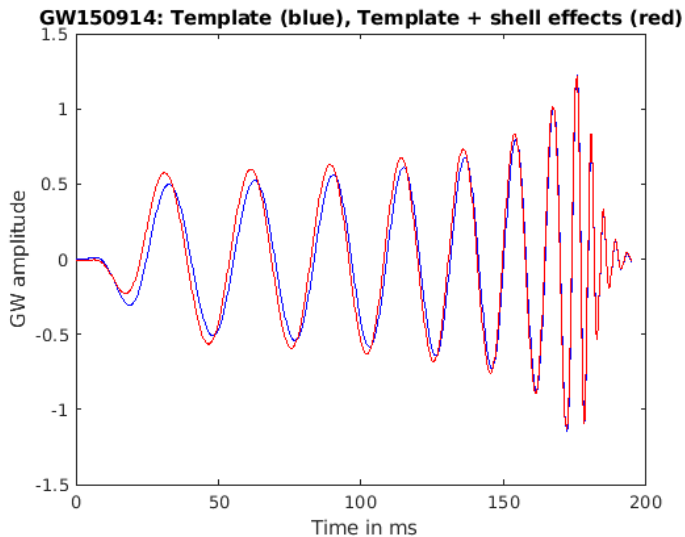


Figure: The effect of a matter shell of radius 900km and mass $60M_{\odot}$ on the signal of GW150914. The original signal is in blue, and the original signal plus modifications is in red.

Using the conservation conditions $\nabla_a T_{bc} g^{ab} = 0$, the velocity field is found, from which the shear tensor σ_{ab} and shear scalar $\sigma^2 = \sigma_{ab} \sigma^{ab}$ can be evaluated. We use the formula that the rate of energy loss per unit volume is $\dot{E}_\eta = -2\eta\sigma^2$. This quantity is evaluated, then integrated over a shell of radius r and thickness Δr and then time-averaged (denoted as $\langle \rangle$) over a wave period. We find

$$\langle \dot{E}_\eta \rangle = -12\eta C_{40}^2 \nu^6 \Delta r \left(1 + \frac{\lambda^2}{2r^2\pi^2} + \frac{9\lambda^4}{2^4 r^4 \pi^4} + \frac{45\lambda^6}{2^6 r^6 \pi^6} + \frac{315\lambda^8}{2^8 r^8 \pi^8} \right).$$

The rate of energy output as GWs is $\langle \dot{E}_{GW} \rangle = 3C_{40}^2 \nu^6 / (4\pi)$ so that

$$\langle \dot{E}_\eta \rangle = -16\pi\eta\Delta r \langle \dot{E}_{GW} \rangle \left(1 + \frac{\lambda^2}{2r^2\pi^2} + \frac{9\lambda^4}{2^4 r^4 \pi^4} + \frac{45\lambda^6}{2^6 r^6 \pi^6} + \frac{315\lambda^8}{2^8 r^8 \pi^8} \right).$$

Energy conservation means that energy absorbed by the viscous fluid is balanced by a reduction in the GW energy. Thus

$$\langle \dot{E}_{GW} \rangle (r + \Delta r) = \langle \dot{E}_{GW} \rangle (r) \times \left[1 - 16\pi\eta\Delta r \left(1 + \frac{\lambda^2}{2r^2\pi^2} + \frac{9\lambda^4}{2^4 r^4 \pi^4} + \frac{45\lambda^6}{2^6 r^6 \pi^6} + \frac{315\lambda^8}{2^8 r^8 \pi^8} \right) \right],$$

which can be expressed as an ODE in r .

Since $\dot{E}_{GW} \propto H^2$ where $H = r(h_+ + ih_\times)$, the ODE in $\dot{E}_{GW}$ becomes one in H yielding

$$H(r_o) = H(r_i) \exp \left[-8\pi\eta \left(r_i(\alpha - 1) - \frac{\lambda^2}{2r_i\pi^2}(1 - \alpha^{-1}) - \frac{3\lambda^4}{2^4 r_i^3 \pi^4}(1 - \alpha^{-3}) - \frac{9\lambda^6}{2^6 r_i^5 \pi^6}(1 - \alpha^{-5}) - \frac{45\lambda^8}{2^8 r_i^7 \pi^8}(1 - \alpha^{-7}) \right) \right],$$

where r_i, r_o are the inner and outer radii of the shell and $r_o = \alpha r_i$ with $\alpha > 1$.

If $r_i \gg \lambda$ Hawking's result from 1966 is recovered

$$H(r_o) = H(r_i) \exp(-8\pi\eta(r_o - r_i)),$$

whereas for $\lambda \gg r_i$

$$H(r_o) = H(r_i) \exp \left(-\frac{45\eta\lambda^8}{32r_i^7\pi^7} (1 - \alpha^{-7}) \right).$$

This formula is often insensitive to the value of α : $(1 - \alpha^{-7})$ is 0.99 for $\alpha = 2$ and is 0.5 for $\alpha = 1.104$.

- Energy lost by GWs = Energy gained by viscous fluid.
- The calculation of changes to the temperature T needs
 - Specific heat capacity C
 - Density ρ
 - Energy of GWs ΔE_{GW}
 - Thermal diffusivity α .
- Let

$$\psi = \frac{2\pi r}{\lambda}$$

- Then

$$T - T_0 = \frac{2G\eta\Delta E_{GW}}{r^2 c^3 C\rho} [D_0 Y_{0,0} + D_2 Y_{2,0} + D_4 Y_{4,0}]$$

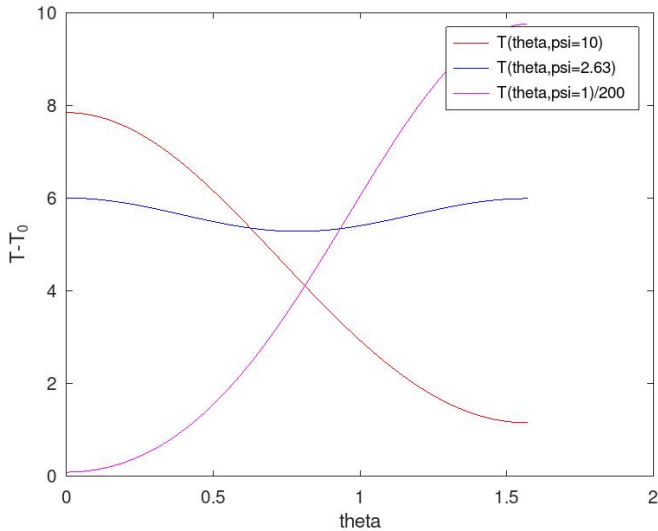
with D_0, D_2, D_4 having the same functional form

- When there is good heat flow in the shell ($\alpha \Delta u / r^2 \gg 1$), or if the GW signal is on average isotropic, then

$$T - T_0 = \frac{2G\eta\Delta E_{GW}}{r^2 c^3 C\rho} D_0 Y_{0,0} = \frac{G\eta\Delta E_{GW}}{\sqrt{\pi} r^2 c^3 C\rho} \left(\frac{315}{\psi^8} + \frac{45}{\psi^6} + \frac{9}{\psi^4} + \frac{2}{\psi^2} + 1 \right)$$

- For large r , $T - T_0$ falls off as r^{-2} , but for small r/λ fall-off goes as r^{-10} .

The heating effect as a function of θ for different $\psi = 2\pi r/\lambda$



- Formulas for the GW damping and heating effects have been obtained for $\ell > 2$.
- For long wavelengths, the effects are enhanced, so in astrophysical cases where the $\ell = 2$ mode is damped, higher order ℓ modes are unlikely to be detected.
- Since higher ℓ modes transport less energy than the $\ell = 2$ mode, the heating effect of higher ℓ modes is unlikely to be significant.

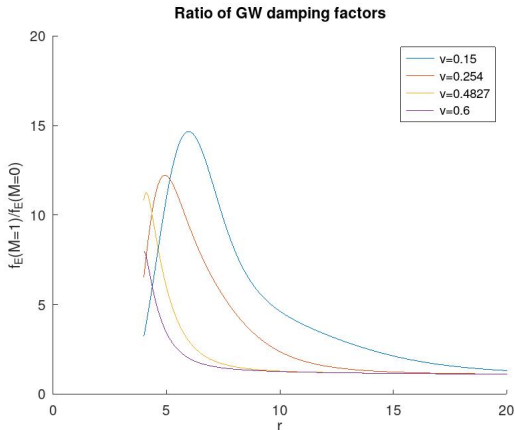
- Making the ansatz of a small perturbation about a static spherically symmetric spacetime leads to

$$\begin{aligned}\beta &= B_0(r) + \Re(\beta^{[2,2]}(r)e^{i\nu u})_0 Z_{2,2}, \quad U = \Re(U^{[2,2]}(r)e^{i\nu u})_1 Z_{2,2}, \\ W_c &= W_0(r) + \Re(W_c^{[2,2]}(r)e^{i\nu u})_0 Z_{2,2}, \quad J = \Re(J^{[2,2]}(r)e^{i\nu u})_2 Z_{2,2}.\end{aligned}$$

where $B_0(r)$, $W_0(r)$ are easily determined from the background density and pressure.

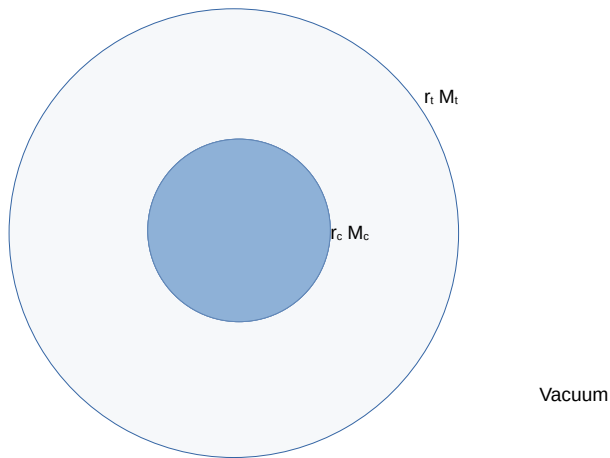
- For Schwarzschild, $B_0(r) = 0$, $W_0(r) = -2M/r^2$.
- Manipulation of the Einstein equations leads to a master equation which has simple analytic solutions in the Minkowski case, but in general has to be solved numerically.
- Then the conservation conditions $\nabla_a T_{bc} g^{ac} = 0$ lead to the perturbed velocity components, from which the shear tensor σ_{ab} can be evaluated and thus the GW damping and heating effects.

The damping/heating effects on a Schwarzschild background



Using a Minkowski background underestimates the damping/heating effect by a factor 9 for matter at $r = 6M$ for GWs with frequency $\nu = 0.254$ (which is the frequency at the time of merger of two black holes).

General spherically symmetric, static matter distribution



GW interactions with matter may be significant for:

- Core collapse supernovae (CCSNe)
- Post-merger signal from a binary neutron star merger
- Primordial GWs from the inflationary era
- And perhaps other cases

Some further details are in the next talk by Monos Naidoo, and in the Bibliography.

- Measurable changes occur when GWs pass through a dust shell.
- If the shell is viscous, then GW damping and heating occur.
- The effects are enhanced when the background is Schwarzschild and near the ISCO.
- GW damping and heating can be astrophysically significant.
- **Key point:** GW interactions with matter have long been regarded as being too small to be significant, but that may not be the case when $\lambda \gg r_i$.

1. N.T. Bishop, M. Naidoo, and P.J. van der Walt, *Effect of a low density dust shell on the propagation of gravitational waves* Gen.Rel. Grav. **52** 92 (2020)
2. M. Naidoo, N.T. Bishop, and P.J. van der Walt, *Modifications to the signal from a gravitational wave event due to a surrounding shell of matter* Gen.Rel. Grav. **53** 77 (2021)
3. N.T. Bishop, P.J. van der Walt and M. Naidoo, *Effect of a viscous fluid shell on the propagation of gravitational waves* Phys. Rev. D **106** 084018 (2022)
4. V. Kakkat, N.T. Bishop and A.S. Kubeka, *Gravitational wave heating* Phys. Rev. D **109** 024013 (2024)
5. N.T. Bishop, V. Kakkat, A.S. Kubeka, M. Naidoo and P.J. van der Walt *The interaction of gravitational waves with matter* Int. Jnl. Mod. Phys. **33** 2441012 (2024)
6. N.T. Bishop, V. Kakkat, A.S. Kubeka, M. Naidoo and P.J. van der Walt *Astrophysical and cosmological scenarios for gravitational wave heating* Phys. Rev. D **110** 084003 (2024)
7. N.T. Bishop *The interaction between gravitational waves and a viscous fluid shell on a Schwarzschild background* Phys. Rev. D **110** 104062 (2024)
8. N.T. Bishop, V. Kakkat and M. Naidoo *Gravitational wave interactions with a viscous fluid: Core collapse supernova, binary neutron star merger, and accretion around a black hole merger* Phys. Rev. D **113** 064047 (2026)
9. U.K. Beckering Vinckers and N.T. Bishop *Gravitational wave interactions with matter: beyond quadrupolar perturbations* arXiv: 2605.25950 (2026)