The interaction of gravitational waves with matter

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- If an electromagnetic wave passes through a plasma, there is a critical wavelength λ_c: if the EM wave has $\lambda \ll \lambda_c$ then it is minimally affected by the plasma, but for $\lambda > \lambda_c$ the wave can be almost completely reflected. Can something similar happen to GWs?
- It has been known since 1966[1] that if a GW propagates from r to $r + \Delta$ through a dissipative medium with shear viscosity η

$$(h_+, h_\times)(r + \Delta) = (h_+, h_\times)(r) \exp(-8\pi\eta\Delta)$$

However, in SI units,

$$8\pi\eta\Delta = \frac{8\pi\eta\Delta G}{c^3} = 8\pi\eta\Delta \times 2.477 \times 10^{-36}$$

Thus, while energy is transferred from GWs to viscous matter, the effect is treated as negligibly small.

[1] E.g., S.W. Hawking, Perturbations of an expanding universe, Astrophys. J. **145**, 544 (1966).

Coordinates are based on outgoing null hypersurfaces, (u, r, x^A); u labels the null hypersurface, r is a radial coordinate, and x^A are angular coordinates θ, ϕ. Then

$$ds^2 = -\left(e^{2\beta}(1 + W_c r) - r^2 h_{AB} U^A U^B\right) du^2 - 2e^{2\beta} du dr \\ - 2r^2 h_{AB} U^B du dx^A + r^2 h_{AB} dx^A dx^B \,.$$

Minkowski spacetime has $W_c = \beta = U^A = 0$, $h_{AB} = q_{AB}$ with q_{AB} a unit sphere metric, $q_{AB} dx^A dx^B = d\theta^2 + \sin^2\theta d\phi^2$. We define perturbations

$$\beta = \Re\left(\beta^{[2,2]}(r) e^{i\nu u}\right) Z_{2,2}\,, W_c = \Re\left(W_c^{[2,2]}(r) e^{i\nu u}\right) Z_{2,2}\,, \\ U = \Re\left(U^{[2,2]}(r) e^{i\nu u}\right) {}_1Z_{2,2}\,, J = \Re\left(J^{[2,2]}(r) e^{i\nu u}\right) {}_2Z_{2,2}\,,$$

where $U = U^A q_A, J = h^{AB} q^A q^B/2$ with $q_A = (1, i\sin\theta)$ and where ${}_sZ_{2,2}$ are spin-weighted spherical harmonics. This "separation of variables" ansatz leads to the linearized Einstein equations reducing to ODEs in r with solutions polynomials in $1/r$.

$$
\begin{aligned}
\beta^{[2,2]} =& 0\,,\\
W_c^{[2,2]} =& \frac{12\nu^2 C_{40}}{r^2} - \frac{12 i\nu C_{40}}{r^3} - \frac{6C_{40}}{r^4} - C_{in0}\exp(2ir\nu)\frac{3}{r^4}\,,\\
U^{[2,2]} =& -\frac{2\nu^2\sqrt{6}C_{40}}{r^2}\\
& -\frac{4i\nu\sqrt{6}C_{40}}{r^3} - \frac{3\sqrt{6}C_{40}}{r^4} - C_{in0}\exp(2ir\nu)\sqrt{6}\left(i\frac{\nu}{r^3} - \frac{3}{2r^4}\right)\,,\\
J^{[2,2]} =& -\frac{2\nu^2\sqrt{6}C_{40}}{r} + \frac{2\sqrt{6}C_{40}}{r^3}\\
& + C_{in0}\exp(2ir\nu)\sqrt{6}\left(\frac{1}{r^3} - 2i\frac{\nu}{r^2} - \frac{\nu^2}{r}\right)\,.
\end{aligned}
$$

Defining the rescaled GW strain by $\mathcal{H}_0 = r(h_+ + ih_\times)$, we find

$$
\mathcal{H}_0 = \Re(H_0\exp(i\nu u))\,{}_2Z_{2,2} \text{ with } \; H_0 = -2\sqrt{6}\nu^2 C_{40}\,.
$$

Note that, for large r, $J = h_+ + ih_\times$, and that C_{in0} represents the magnitude of incoming GWs.

The model: GW source surrounded by a spherical matter shell

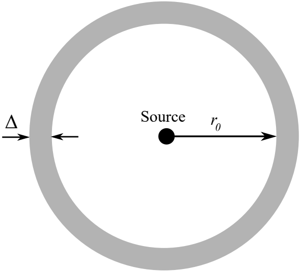


Figure: GW source inside a spherical shell of mass M_S between $r = r_0$ and $r = r_0 + \Delta$.

- Construct solutions (a) Exterior to the shell, (b) Within the shell, and (c) Interior to the shell.
- Enforce continuity of the metric and first derivatives at the boundaries of the shell, and so construct a complete solution.

The modified GW strain due to a dust shell

$$H = H_0\left[1 + \frac{2M_S}{r_0} - \frac{iM_S\lambda}{r_0^2\pi} + \frac{iM_S\lambda e^{-4ir_0\pi/\lambda}}{4r_0^2\pi} + \cdots\right].$$

Each of the terms containing M_S represents a correction to the wave strain in the absence of the shell.

- The first correction, $2M_S/r_0$ represents the (well-known) gravitational red-shift effect.
- The second correction term, $iM_S\lambda/(\pi r_0^2)$, is out of phase with the leading terms $1 + 2M_S/r_0$ and hence represents a phase shift of the GW without changing the GW energy.
- The third correction term represents a partial reflection of the GW by the shell, and leads to the possibility of a matter shell causing a GW echo.

- There has been previous work on whether the astrophysical environment might modify a GW signal including the possibility of an echo[2].
- The magnitude of the echo term can be written

$$\frac{2M_S}{r_0} \times \frac{\lambda}{8\pi r_0}$$

where λ is the wavelength, and the echo delay is $2r_0$.

[2] E.g., Barausse, E., Cardoso, V., Pani, P.: Can environmental effects spoil precision gravitational-wave astrophysics? Phys. Rev. D **89**(10), 104059 (2014); Konoplya, R.A., Stuchlík, Z., Zhidenko, A.: Echoes of compact objects: new physics near the surface and matter at a distance. Phys. Rev. D **99**, 024007 (2019); Abedi, J., Dykaar, H., Afshordi, N.: Echoes from the Abyss: tentative evidence for Planck-scale structure at black hole horizons. Phys. Rev. D **96**(8), 082004 (2017)

Astrophysical example: Shell around BBH merger

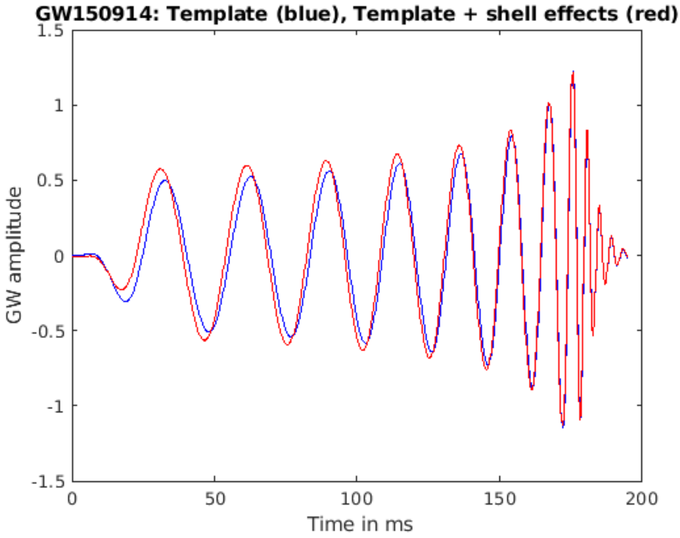


Figure: The effect of a matter shell of radius 900km and mass $60M_\odot$ on the signal of GW150914. The original signal is in blue, and the original signal plus modifications is in red.

Using the conservation conditions $\nabla_a T_{bc} g^{ab} = 0$, the velocity field is found, from which the shear tensor σ_{ab} and shear scalar $\sigma^2 = \sigma_{ab}\sigma^{ab}$ can be evaluated. We use the formula that the rate of energy loss per unit volume is $\dot{E}_\eta = -2\eta\sigma^2$. This quantity is evaluated, then integrated over a shell of radius r and thickness Δr and then time-averaged (denoted as $\langle\,\rangle$) over a wave period. We find

$$\left\langle \dot{E}_\eta \right\rangle = -12\eta C_{40}^2 \nu^6 \Delta r \left(1 + \frac{\lambda^2}{2r^2\pi^2} + \frac{9\lambda^4}{2^4 r^4 \pi^4} + \frac{45\lambda^6}{2^6 r^6 \pi^6} + \frac{315\lambda^8}{2^8 r^8 \pi^8} \right) .$$

The rate of energy output as GWs is $\left\langle \dot{E}_{GW} \right\rangle = 3C_{40}^2 \nu^6/(4\pi)$ so that

$$\left\langle \dot{E}_\eta \right\rangle = -16\pi\eta\Delta r \left\langle \dot{E}_{GW} \right\rangle \left(1 + \frac{\lambda^2}{2r^2\pi^2} + \frac{9\lambda^4}{2^4 r^4 \pi^4} + \frac{45\lambda^6}{2^6 r^6 \pi^6} + \frac{315\lambda^8}{2^8 r^8 \pi^8} \right) .$$

Energy conservation means that energy absorbed by the viscous fluid is balanced by a reduction in the GW energy. Thus

$$\begin{aligned} \left\langle \dot{E}_{GW} \right\rangle (r + \Delta r) &= \left\langle \dot{E}_{GW} \right\rangle (r) \times \\ & \left[1 - 16\pi\eta\Delta r \left(1 + \frac{\lambda^2}{2r^2\pi^2} + \frac{9\lambda^4}{2^4 r^4 \pi^4} + \frac{45\lambda^6}{2^6 r^6 \pi^6} + \frac{315\lambda^8}{2^8 r^8 \pi^8} \right) \right] , \end{aligned}$$

which can be expressed as an ODE in r.

Since $\dot{E}_{GW} \propto H^2$ where $H = r(h_+ + ih_\times)$, the ODE in $\dot{E}_{GW}$ becomes one in H yielding

$$H(r_o) = H(r_i)\exp\Bigg[-8\pi\eta\bigg(r_i(\alpha-1) - \frac{\lambda^2}{2r_i\pi^2}(1-\alpha^{-1}) - \frac{3\lambda^4}{2^4 r_i^3\pi^4}(1-\alpha^{-3}) - \frac{9\lambda^6}{2^6 r_i^5\pi^6}(1-\alpha^{-5}) - \frac{45\lambda^8}{2^8 r_i^7\pi^8}(1-\alpha^{-7})\bigg)\Bigg],$$

where r_i, r_o are the inner and outer radii of the shell and $r_o = \alpha r_i$ with $\alpha > 1$.
If $r_i \gg \lambda$ Hawking's result from 1966 is recovered

$$H(r_o) = H(r_i)\exp\left(-8\pi\eta(r_o - r_i)\right),$$

whereas for $\lambda \gg r_i$

$$H(r_o) = H(r_i)\exp\left(-\frac{45\eta\lambda^8}{32 r_i^7\pi^7}\left(1-\alpha^{-7}\right)\right).$$

This formula is often insensitive to the value of α: $(1-\alpha^{-7})$ is 0.99 for $\alpha = 2$ and is 0.5 for $\alpha = 1.104$.

- Energy lost by GWs = Energy gained by viscous fluid.
- The calculation of changes to the temperature T needs
  - Specific heat capacity C
  - Density ρ
  - Energy of GWs ΔE_{GW}
  - Thermal diffusivity α.
- Let

$$\psi = \frac{2\pi r}{\lambda}$$

- Then

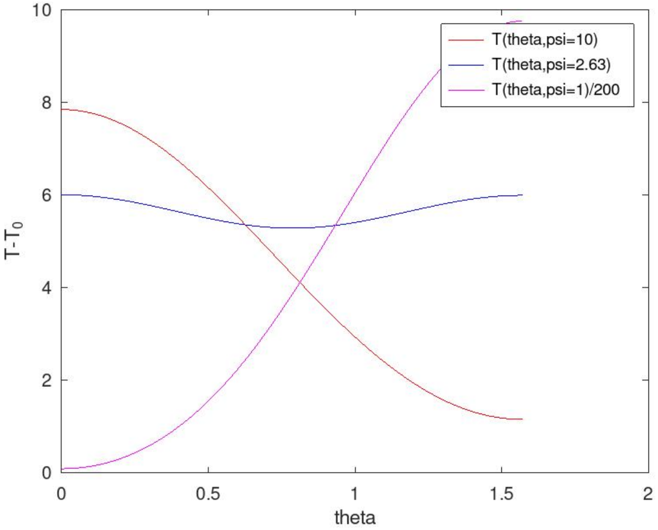
$$T - T_0 = \frac{2G\eta\Delta E_{GW}}{r^2 c^3 C\rho}\left[D_0 Y_{0,0} + D_2 Y_{2,0} + D_4 Y_{4,0}\right]$$

  with D_0, D_2, D_4 having the same funcional form
- When there is good heat flow in the shell ($\alpha\,\Delta u/r^2 \gg 1$), or if the GW sgnal is on average isotropic, then

$$T - T_0 = \frac{2G\eta\Delta E_{GW}}{r^2 c^3 C\rho} D_0 Y_{0,0} = \frac{G\eta\Delta E_{GW}}{\sqrt{\pi}r^2 c^3 C\rho}\left(\frac{315}{\psi^8} + \frac{45}{\psi^6} + \frac{9}{\psi^4} + \frac{2}{\psi^2} + 1\right)$$

- For large r, $T - T_0$ falls off as r^{-2}, but for small r/λ fall-off goes as r^{-10}.

The heating effect as a function of θ for different $\psi = 2\pi r/\lambda$



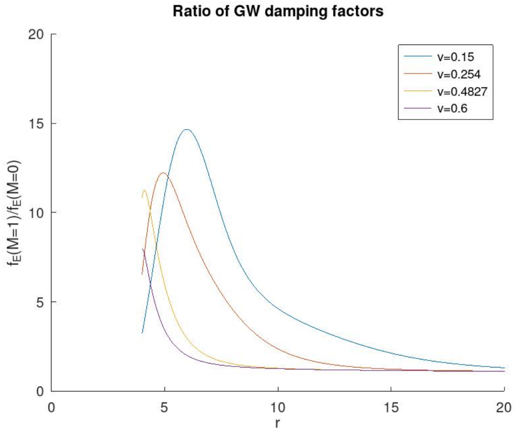
- Formulas for the GW damping and heating effects have been obtained for $\ell > 2$.
- For long wavelengths, the effects are enhanced, so in astrophysical cases where the $\ell = 2$ mode is damped, higher order ℓ modes are unlikely to be detected.
- Since higher ℓ modes transport less energy than the $\ell = 2$ mode, the heating effect of higher ℓ modes is unlikely to be significant.

The damping/heating effects on a spherically symmetric background

- Making the ansatz of a small perturbation about a static spherically symmetric spacetime leads to

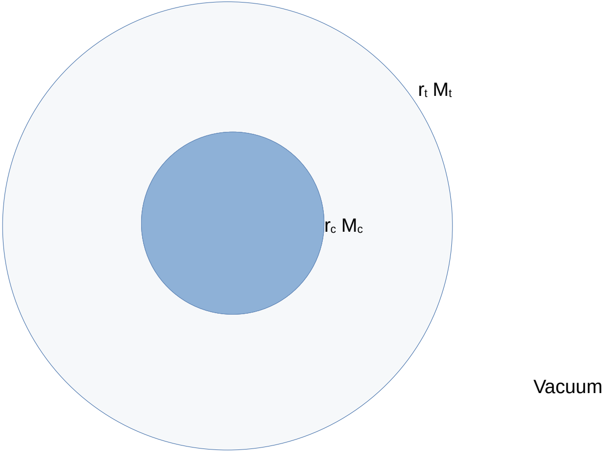
$$\beta = B_0(r) + \Re(\beta^{[2,2]}(r)e^{i\nu u})_0 Z_{2,2}\,, \quad U = \Re(U^{[2,2]}(r)e^{i\nu u})_1 Z_{2,2}\,,$$
$$W_c = W_0(r) + \Re(W_c^{[2,2]}(r)e^{i\nu u})_0 Z_{2,2}\,, \quad J = \Re(J^{[2,2]}(r)e^{i\nu u})_2 Z_{2,2}\,.$$

  where $B_0(r)$, $W_0(r)$ are easily determined from the background density and pressure.
- For Schwarzschild, $B_0(r) = 0$, $W_0(r) = -2M/r^2$.
- Manipulation of the Einstein equations leads to a master equation which has simple analytic solutions in the Minkowski case, but in general has to be solved numerically.
- Then the conservation conditions $\nabla_a T_{bc} g^{ac} = 0$ lead to the perturbed velcoity components, from which the shear tensor σ_{ab} can be evaluated and thus the GW damping and heating effects.



Using a Minkowski background underestimates the damping/heating effect by a factor 9 for matter at $r = 6M$ for GWs with frequency $\nu = 0.254$ (which is the frequency at the time of merger of two black holes).

General spherically symmetric, static matter distribution



Astrophysical applications

GW interactions with matter may be significant for:

- Core collapse supernovae (CCSNe)
- Post-merger signal from a binary neutron star merger
- Primordial GWs from the inflationary era
- And perhaps other cases

Some further details are in the next talk by Monos Naidoo, and in the Bibliography.

Conclusions

- Measurable changes occur when GWs pass through a dust shell.
- If the shell is viscous, then GW damping and heating occur.
- The effects are enhanced when the background is Schwarzschild and near the ISCO.
- GW damping and heating can be astrophysically significant.
- **Key point:** GW interactions with matter have long been regarded as being too small to be significant, but that may not be the case when $\lambda \gg r_i$.

References

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