MOND: An alternative to particle dark matter

Federico Lelli

INAF – Arcetri

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Ministero dell'Università e della Ricerca

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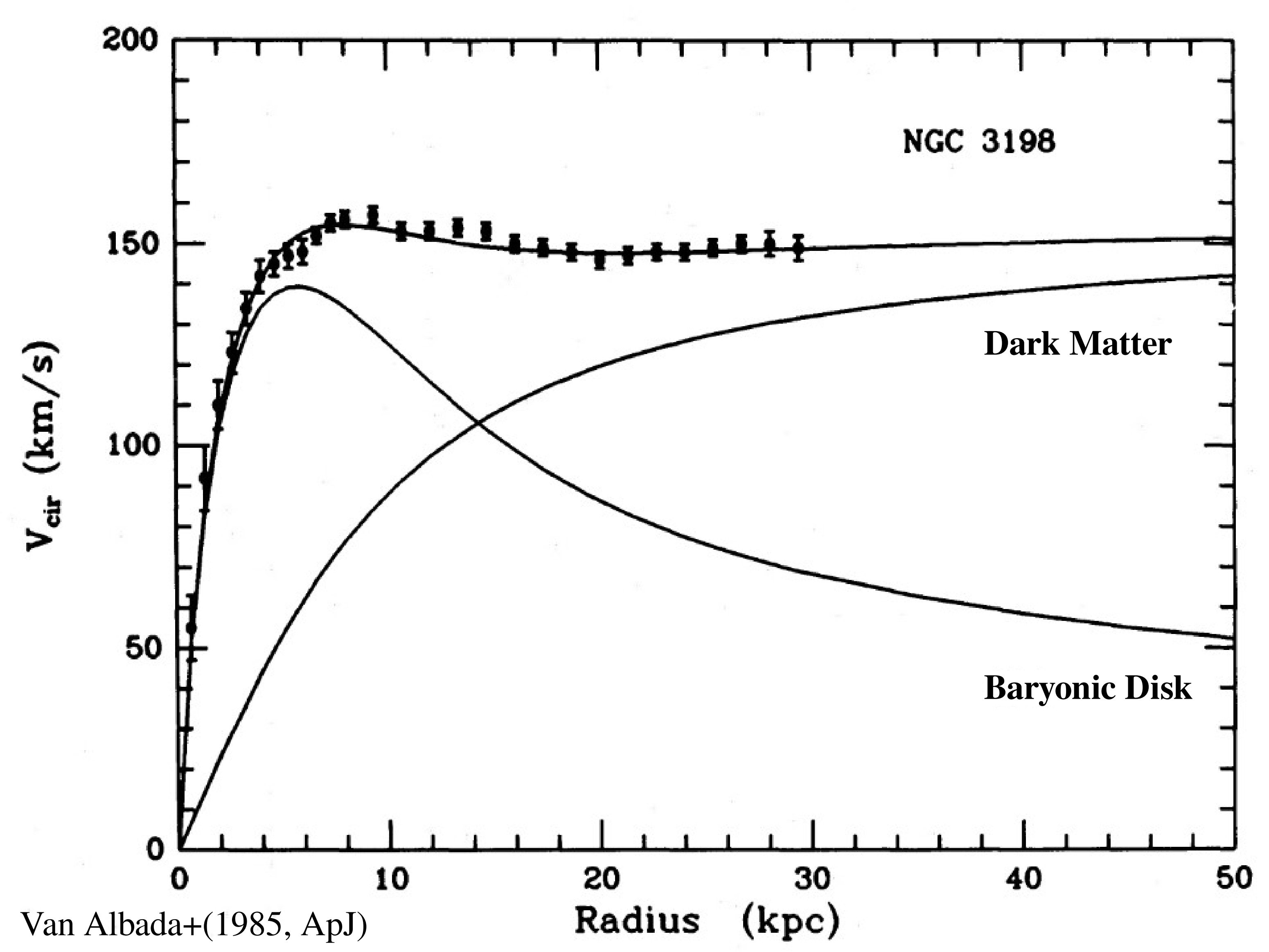
ISTITUTO NAZIONALE DI ASTROFISICA
OSSERVATORIO ASTROFISICO DI ARCETRI

Federico Lelli
INAF – Arcetri → University of Cagliari (Sardinia)

Rotation curves of disk galaxies: baryon-halo conspiracy



$$V_c(R) = \sqrt{V_b^2(R) + V_{DM}^2(R)} = cost$$

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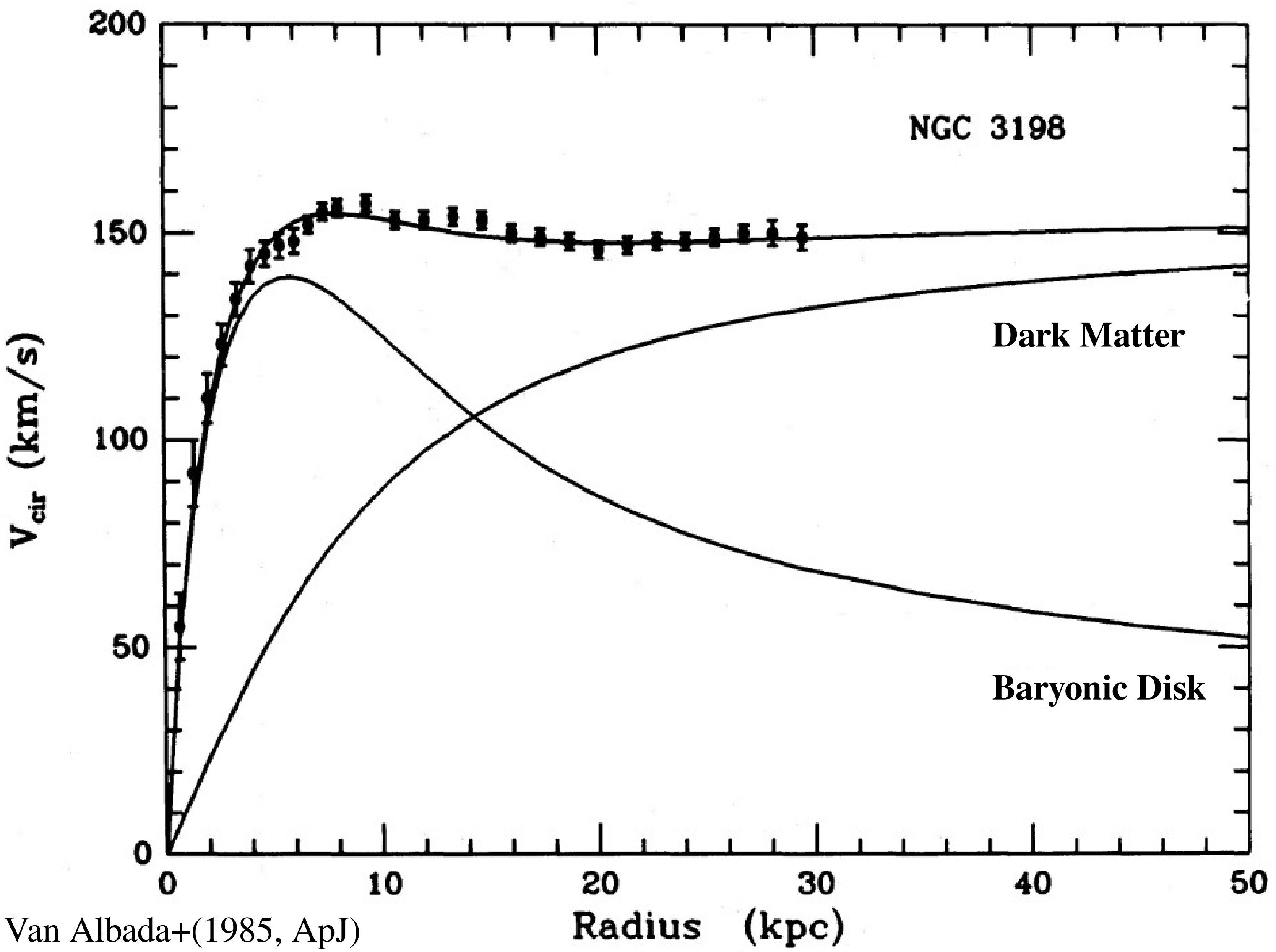

Van Albada+(1985, ApJ)

$$V_c(R) = \sqrt{V_b^2(R) + V_{DM}^2(R)} = cost$$

"Luminous and dark matter seem to conspire to produce the flat observed rotation curves in the outer regions. It seems unlikely that this coupling [...] results from the large-scale gravitational interaction between the two components."

Van Albada & Sancisi (1986, RSPTA)

It's still a problem for ΛCDM!

See Desmond (2017, MNRAS)

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$\Rightarrow$ $a = \sqrt{a_0 g_N}$ a_0 constant with dimensions of acceleration

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⇒ $a = \sqrt{a_0 g_N}$ $\quad$ a_0 constant with dimensions of acceleration

⇒ $\frac{V_c^2}{\cancel{R}} = \sqrt{\frac{a_0 G M_b}{\cancel{R^2}}}$ $\quad$ Flat rotation curve *<u>at large radii</u>*

→ Main empirical motivation of MOND

MOND = MOdified Newtonian Dynamics

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MOND = MilgrOmiaN Dynamics

Proposed by Moti Milgrom (1983a, b, c)

MOND is not a new idea. Over the past 40+ years, it has evolved into a general paradigm that includes different relativistic & non-relativistic theories.

MOND postulates at the non-relativistic level (Milgrom 2009, ApJ)

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Equations of motion do not change under $(\vec{x}, t) \rightarrow (\lambda \vec{x}, \lambda t)$

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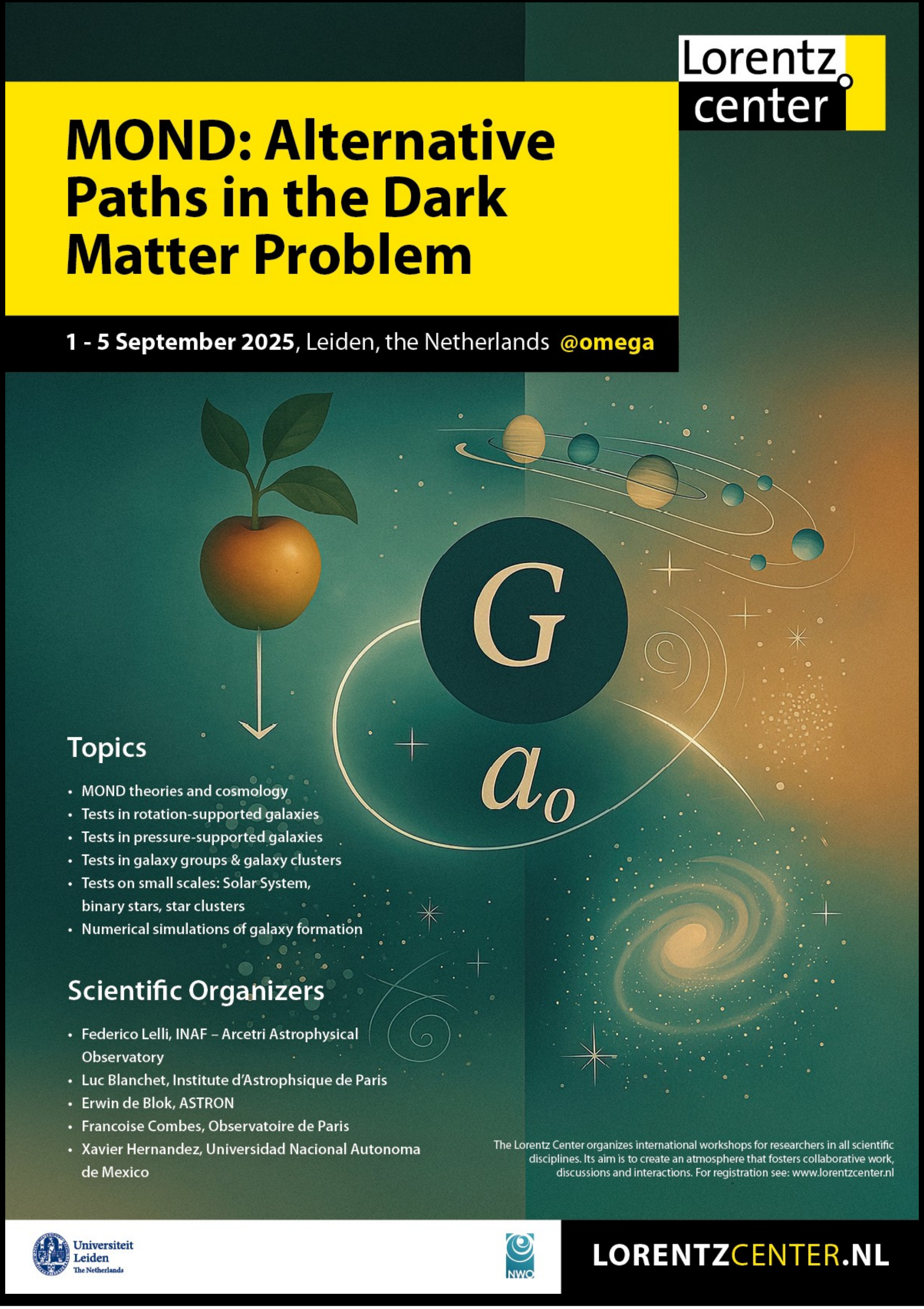
⇨ $a = \sqrt{a_0 g_N}$ $\quad V_c = invariant$

The MOND white paper

(50+ authors, 300+ pages)



The MOND research program:
A white paper on status and prospects

General coordinator: Federico Lelli

Section coordinators: Luc Blanchet, Françoise Combes, Erwin De Blok, Harry Desmond, Antonaldo Diaferio, Benoit Famaey, Jonathan Freundlich, Stacy S. McGaugh, Mordehai Milgrom, Tobias Mistele, Marcel Pawlowski

Contributors: Elena Asencio, Will Barker, Michal Bílek, Anthony Brown, Graeme Candlish, Kyu-Hyun Chae, Øyvind Christiansen, Pierfrancesco Di Cintio, Francis Duey, Amel Durakovic, Stephan Eijt, Stefano Ettori, Eanna Flanagan, Hosein Haghi, Christopher Harwey-Hawes, Akram Hasani Zoonozi, Konstantin Haubner, Aurelien Hees, Xavier Hernandez, Michael Hilker, Zichen Hua, Mark Huisjes, Mariana Júlio, Tahere Kashfi, Anastasiia Lazutkina, Steffen Mieske, Cezary Migaszewski, Giacomo Monari, Oliver Müller, Srikanth Nagesh, Roman Nagy, Luisa Ostorero, Jan Pflamm-Altenburg, Tom Richtler, Nick Samaras, Emeric Seraille, Sofia Splawska, Will Sutherland, Yong Tian, Andreea Varasteanu, Paul Visser, David M. J. Vokrouhlický, Richard Woodard

Endorsers: TBD

Content of the MOND white paper:

1. Introduction / Non-relativistic Theories
2. Tests in Galaxies
3. Tests in Galaxy Groups & Clusters
4. Tests on Small Scales
5. Relativistic Theories and Cosmology
6. Numerical Simulations

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Tests in Galaxies

A MODIFICATION OF THE NEWTONIAN DYNAMICS: IMPLICATIONS FOR GALAXIES[1]

M. MILGROM
Department of Physics, Weizmann Institute, Rehovot, Israel; and The Institute for Advanced Study
Received 1982 February 4; accepted 1982 December 28

ABSTRACT

I use a modified form of the Newtonian dynamics (inertia and/or gravity) to describe the motion of bodies in the gravitational fields of galaxies, *assuming that galaxies contain no hidden mass*, with the following main results.

1. The Keplerian, circular velocity around a finite galaxy becomes independent of r at large radii, thus resulting in asymptotically flat velocity curves.

2. The asymptotic circular velocity (V_∞) is determined only by the total mass of the galaxy (M): $V_\infty^4 = a_0 GM$, where a_0 is an acceleration constant appearing in the modified dynamics. This relation is consistent with the observed Tully-Fisher relation if one uses a luminosity parameter which is proportional to the observable mass.

3. The discrepancy between the dynamically determined Oort density in the solar neighborhood and the density of observed matter disappears.

4. The rotation curve of a galaxy can remain flat down to very small radii, as observed, only if the galaxy's average surface density Σ falls in some narrow range of values which agrees with the Fish and Freeman laws. For smaller values of Σ, the velocity rises more slowly to the asymptotic value.

5. The value of the acceleration constant, a_0, determined in a few independent ways is approximately $2 \times 10^{-8}(H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1})^2$ cm s^{-2}, which is of the order of $CH_0 = 5 \times 10^{-8}(H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1})$ cm s^{-2}.

The main predictions are:

1. Rotation curves calculated on the basis of the *observed* mass distribution and the modified dynamics should agree with the observed velocity curves.

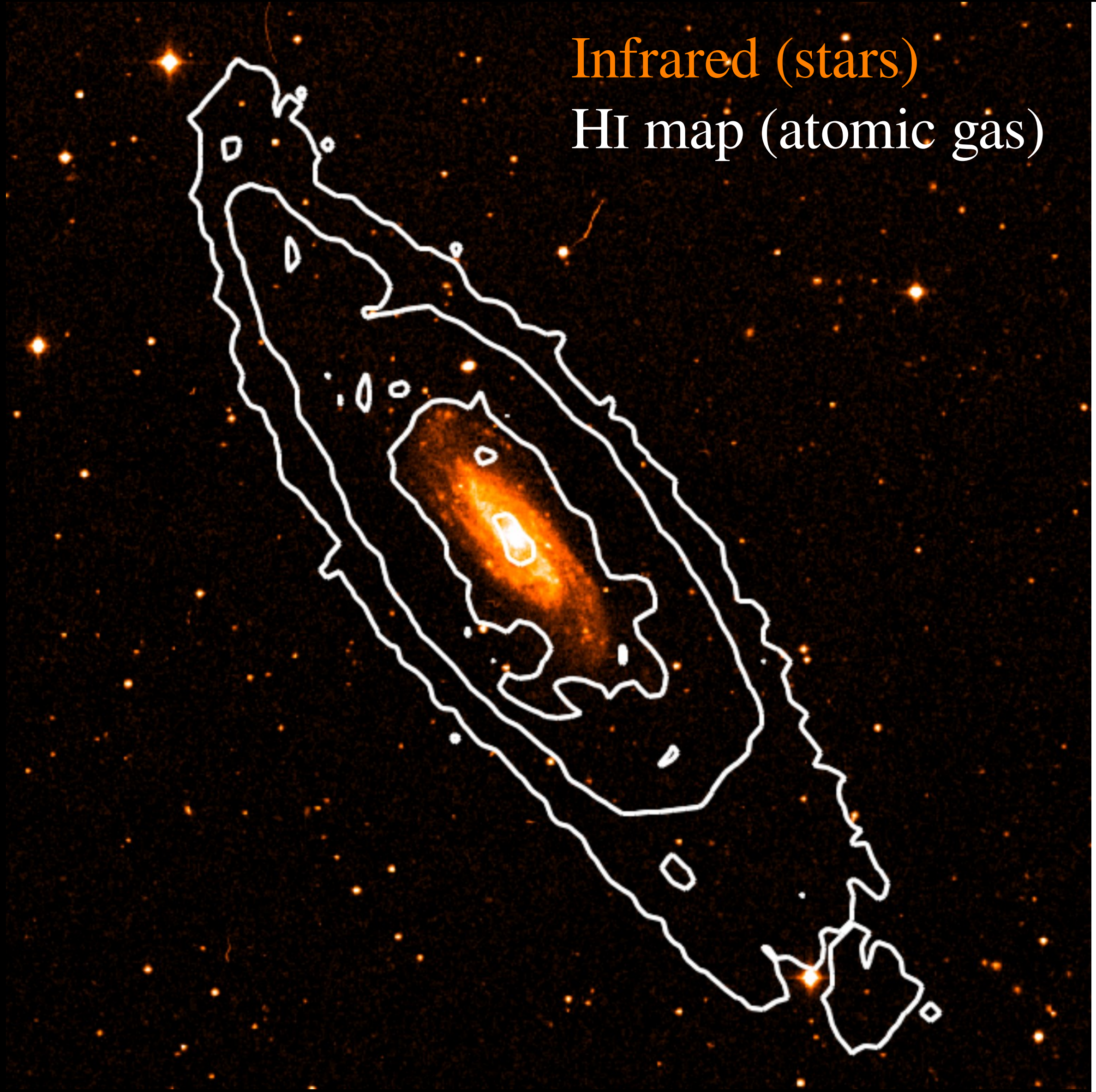
2. The $V_\infty^4 = a_0 GM$ relation should hold exactly.

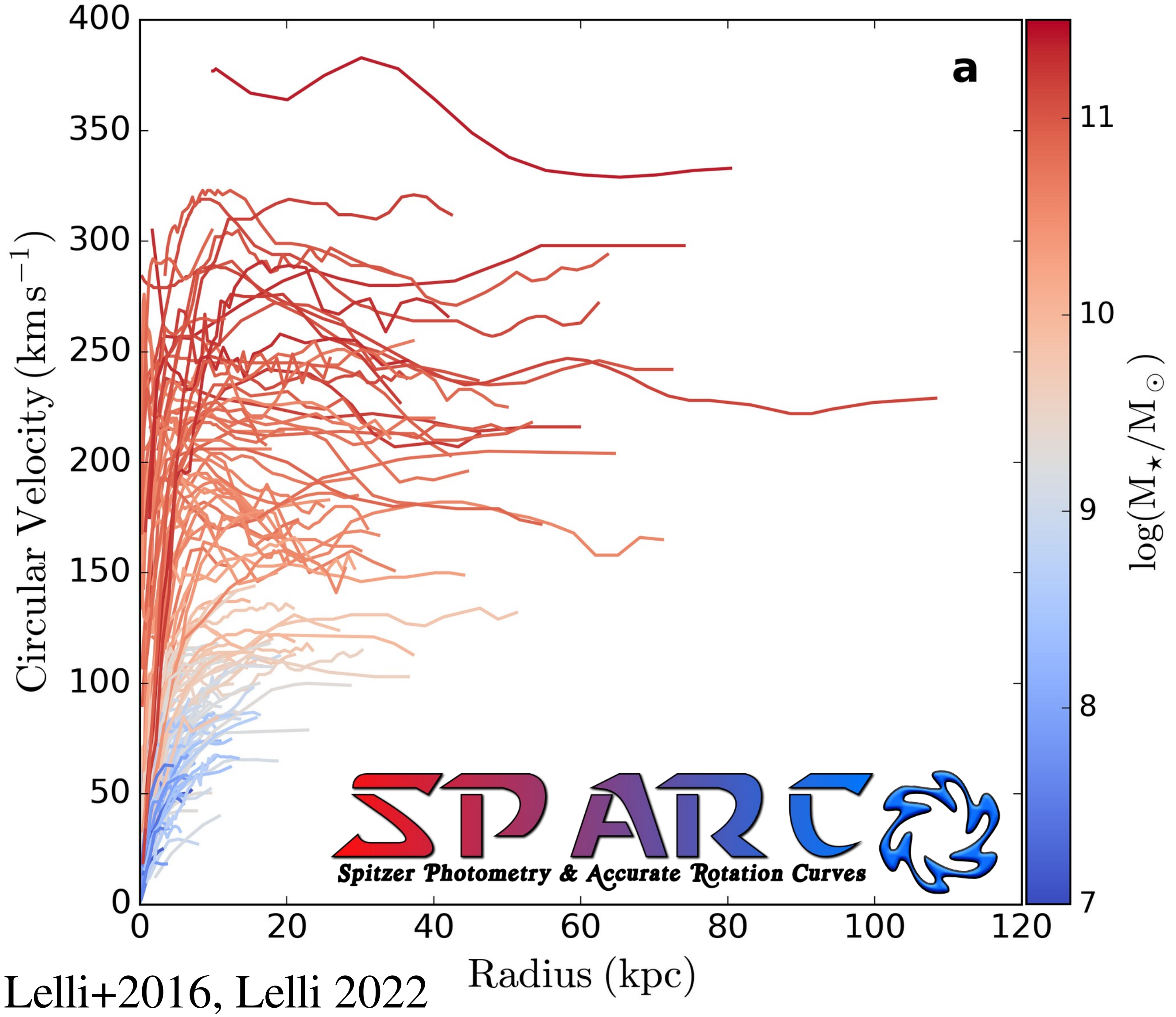
3. An analog of the Oort discrepancy should exist in all galaxies and become more severe with increasing r in a predictable way.

A-priori predictions from 1983!

(1) $V_c(R) = \text{const}$ at large R for isolated objects

(1) $V_c(R)$ = const at large R for isolated objects → HI line at 21 cm





Lelli+2016, Lelli 2022

(1) $V_c(R)$ = const at large R for isolated objects → Weak Lensing

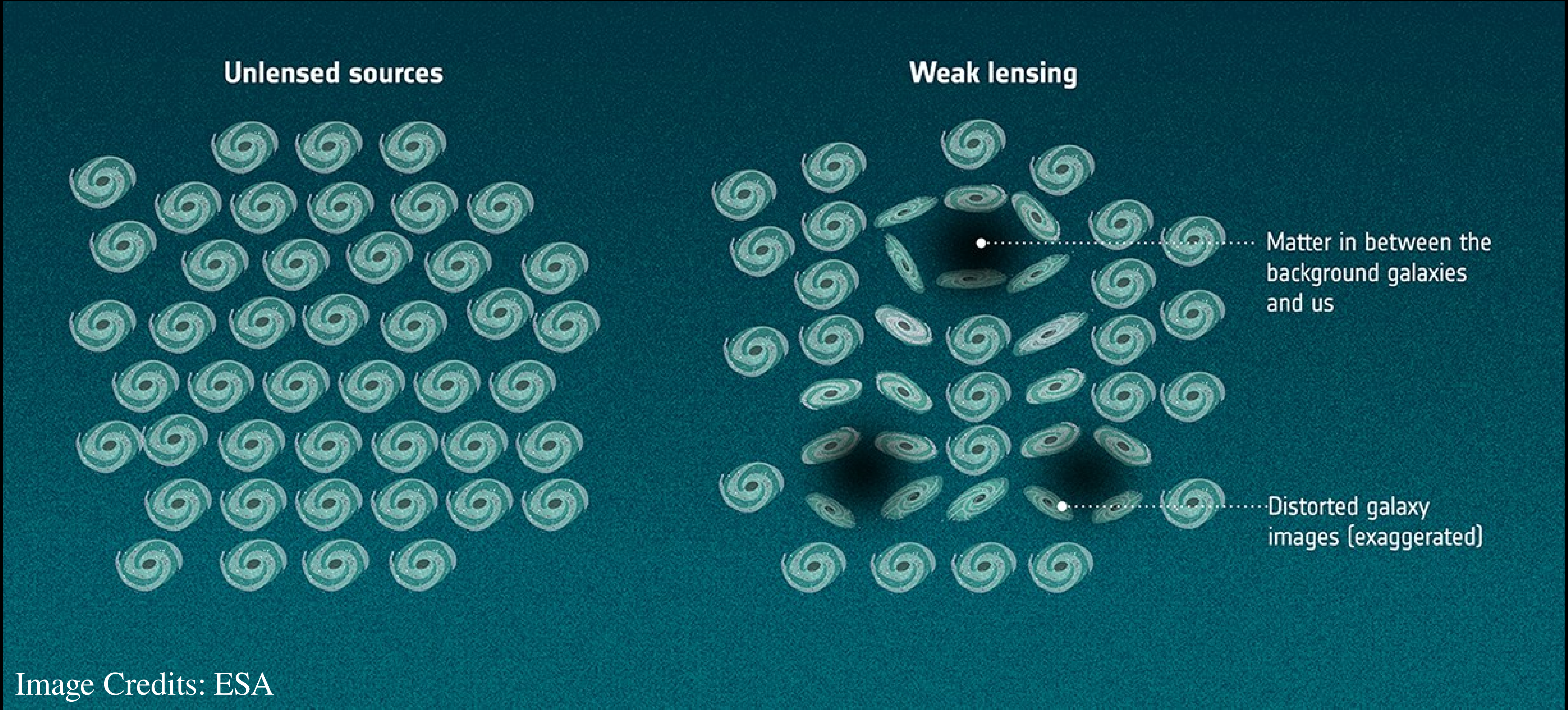


Image Credits: ESA

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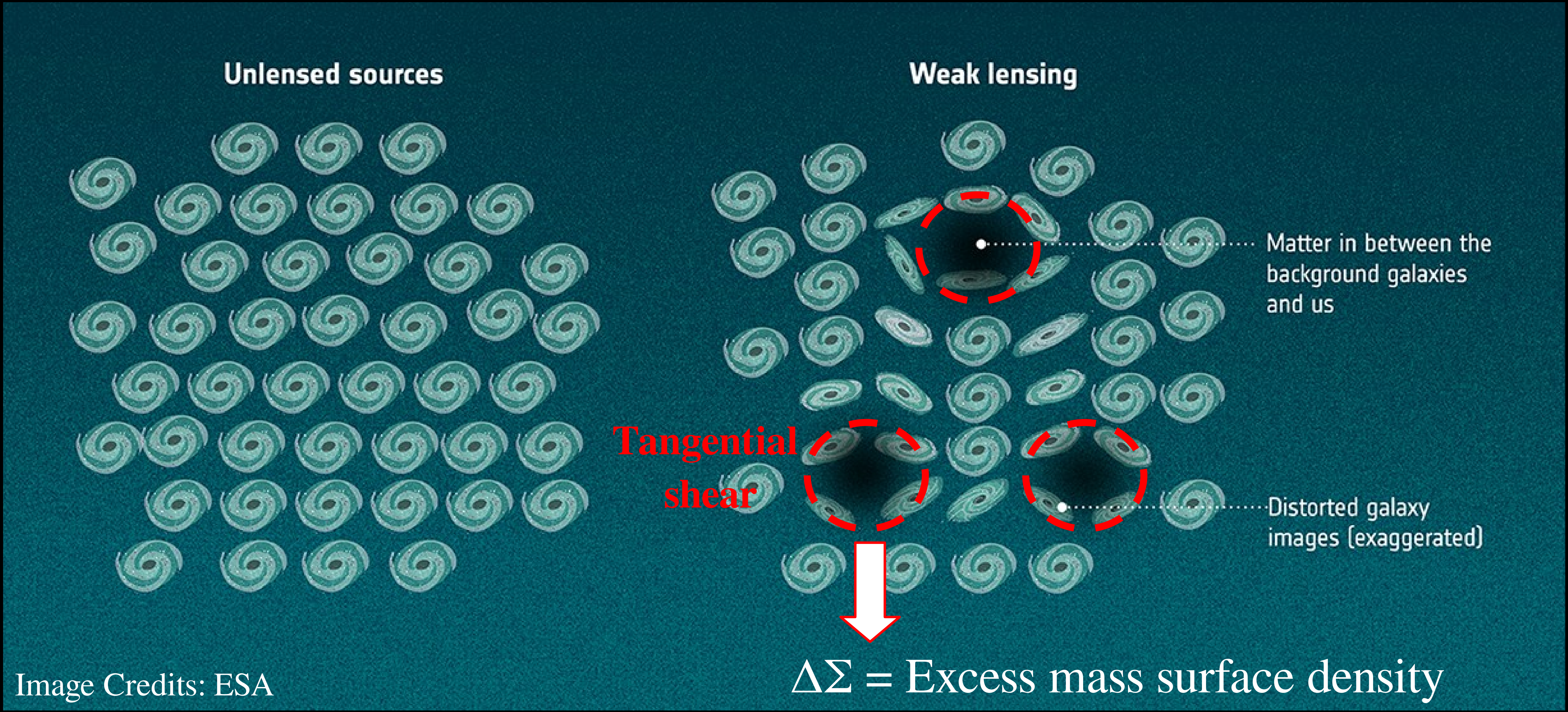

(1) $V_c(R)$ = const at large R for isolated objects → Weak Lensing

$$\frac{V_c^2}{r} = 4G \int_0^{\pi/2} \Delta\Sigma\left(\frac{r}{\sin(\theta)}\right) d\theta$$

From $\Delta\Sigma$ to $g = V_c^2/r$ in spherical symmetry

Application to isolated galaxies from KiDS

(Mistele+2024a, JCAP; Mistele+2024b, ApJ)

Tobias Mistele

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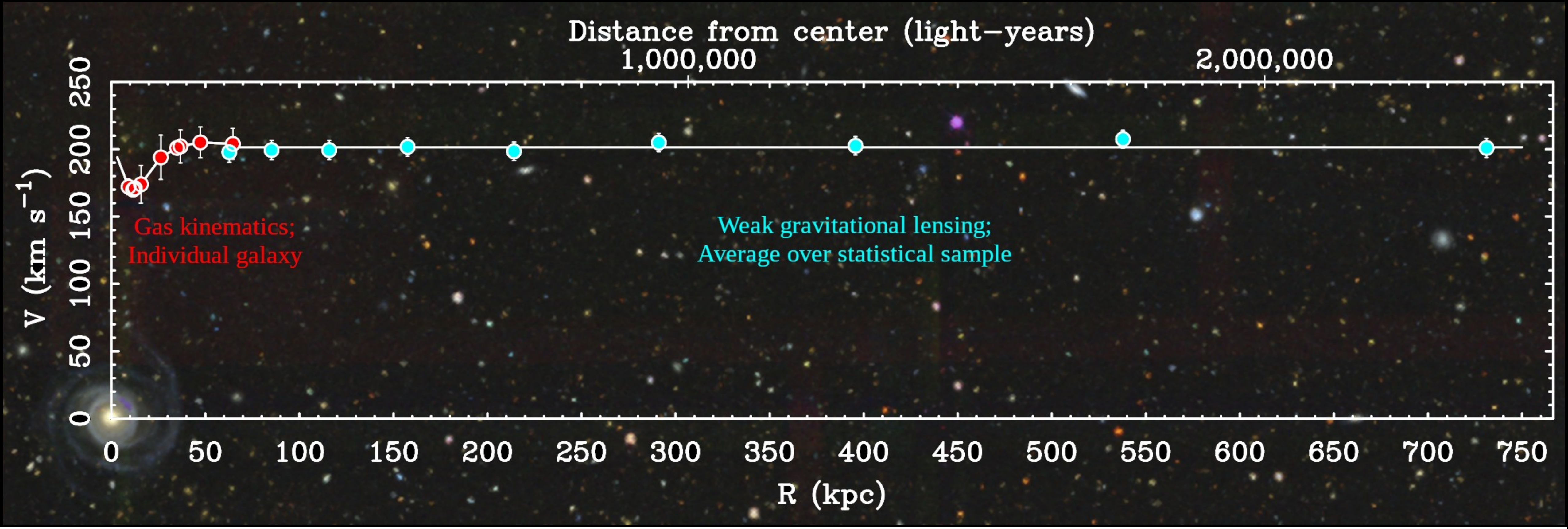

(2) $V_f^4 = a_0 G M_b$ for isolated systems

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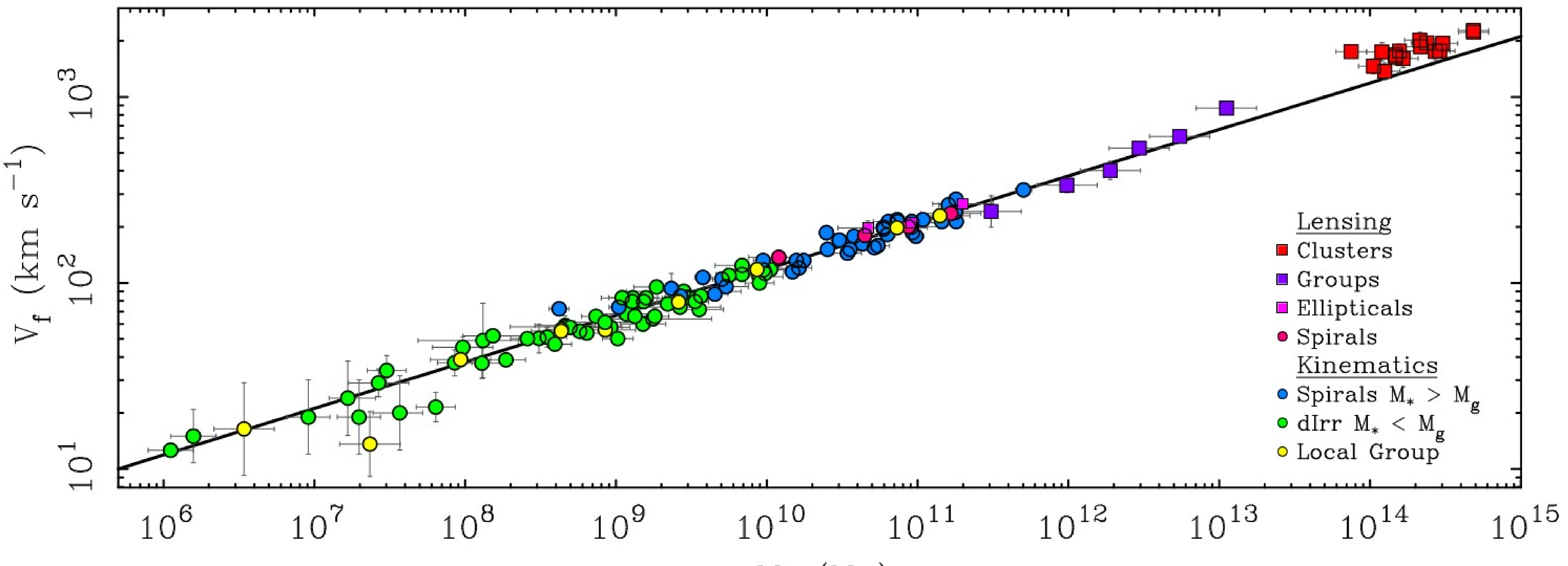

MOND White Paper, Section 1.1

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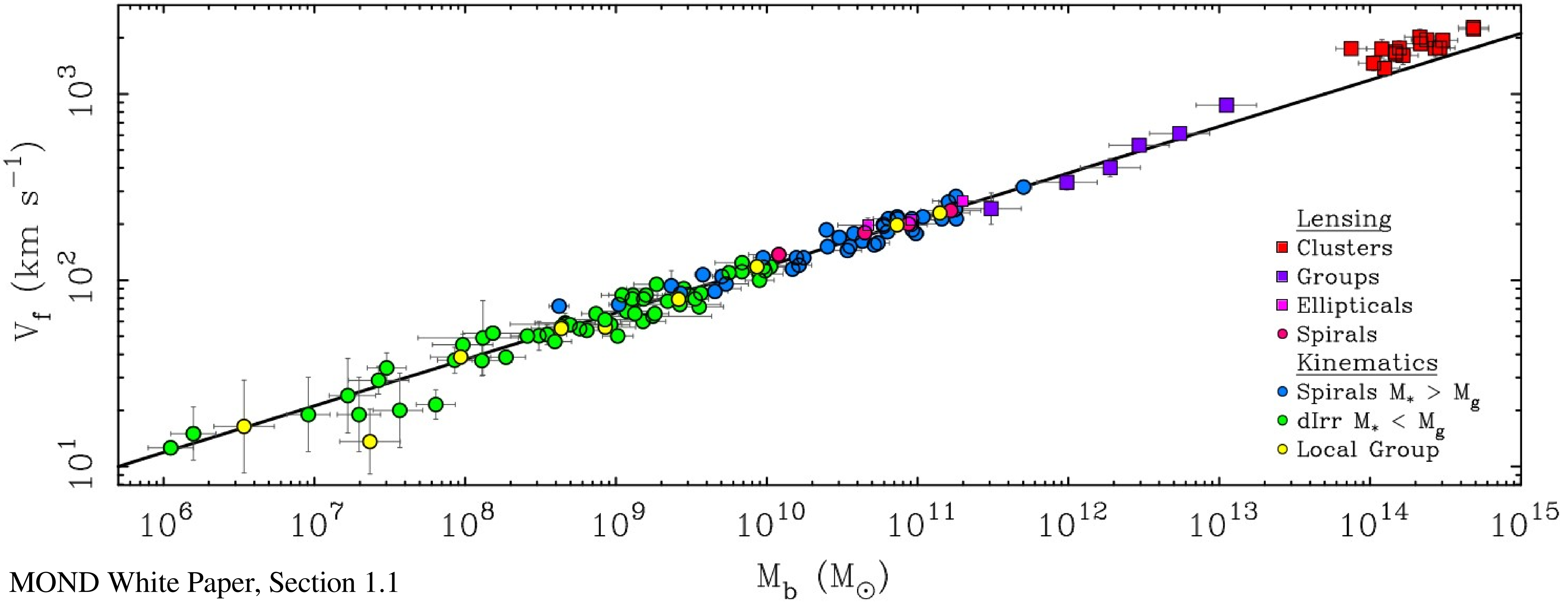

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NOT A FIT TO THE DATA!

Predictions: slope = ¼; intercept = a_0G; no intrinisc scatter; no residual dependencies

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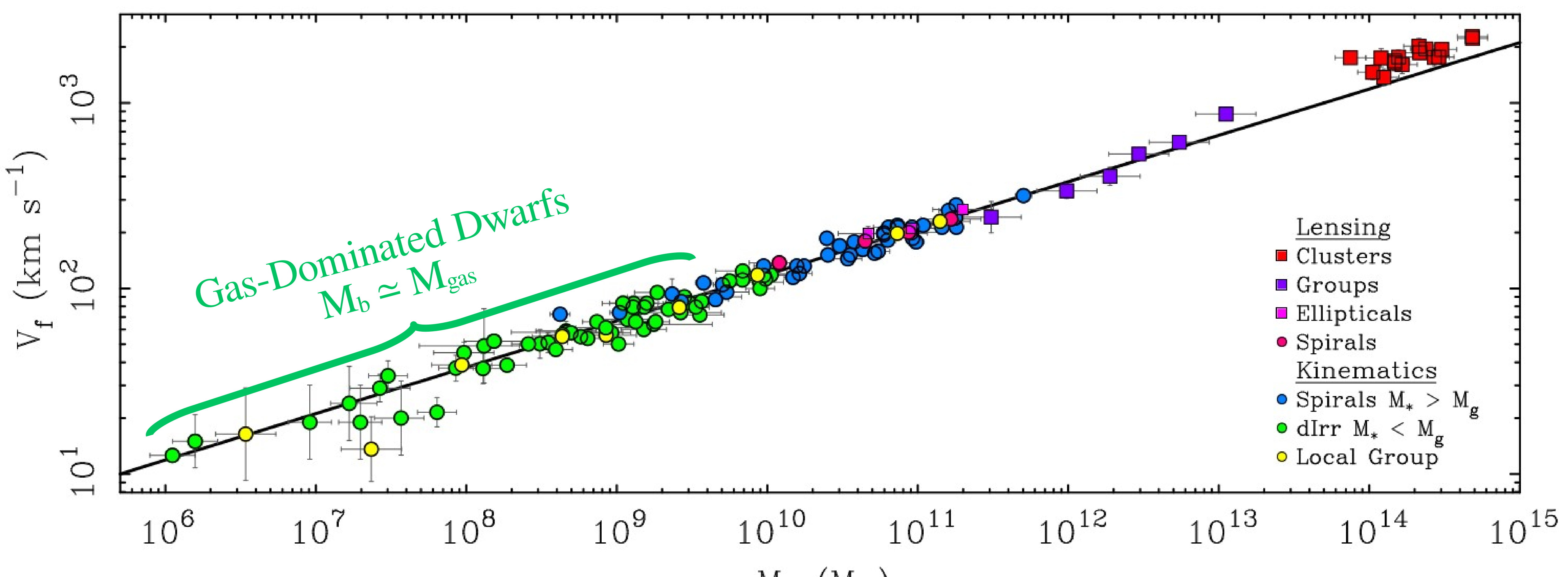

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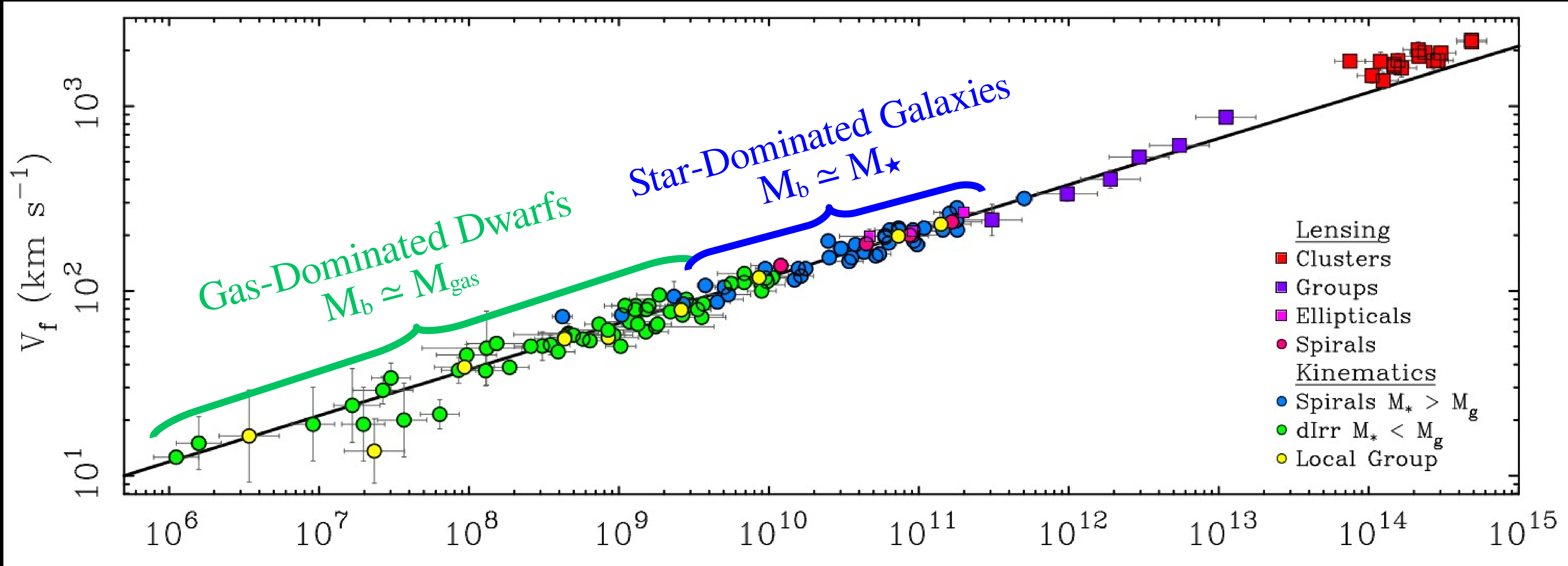

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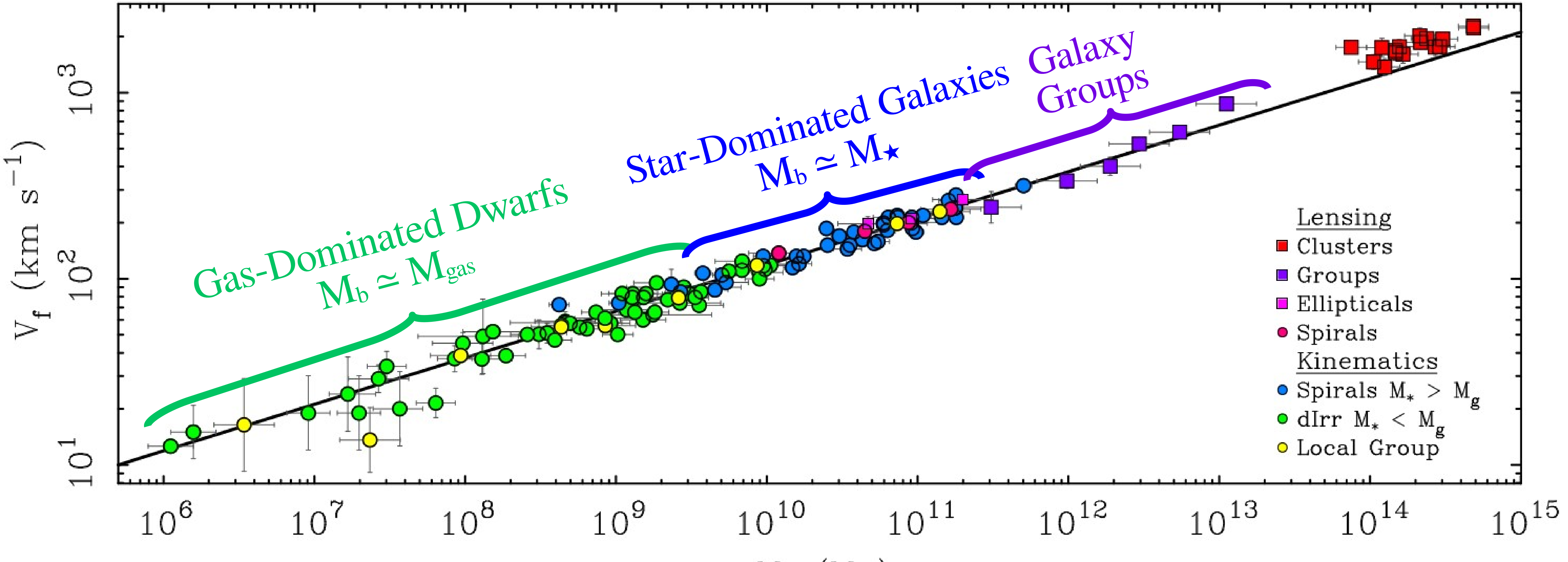

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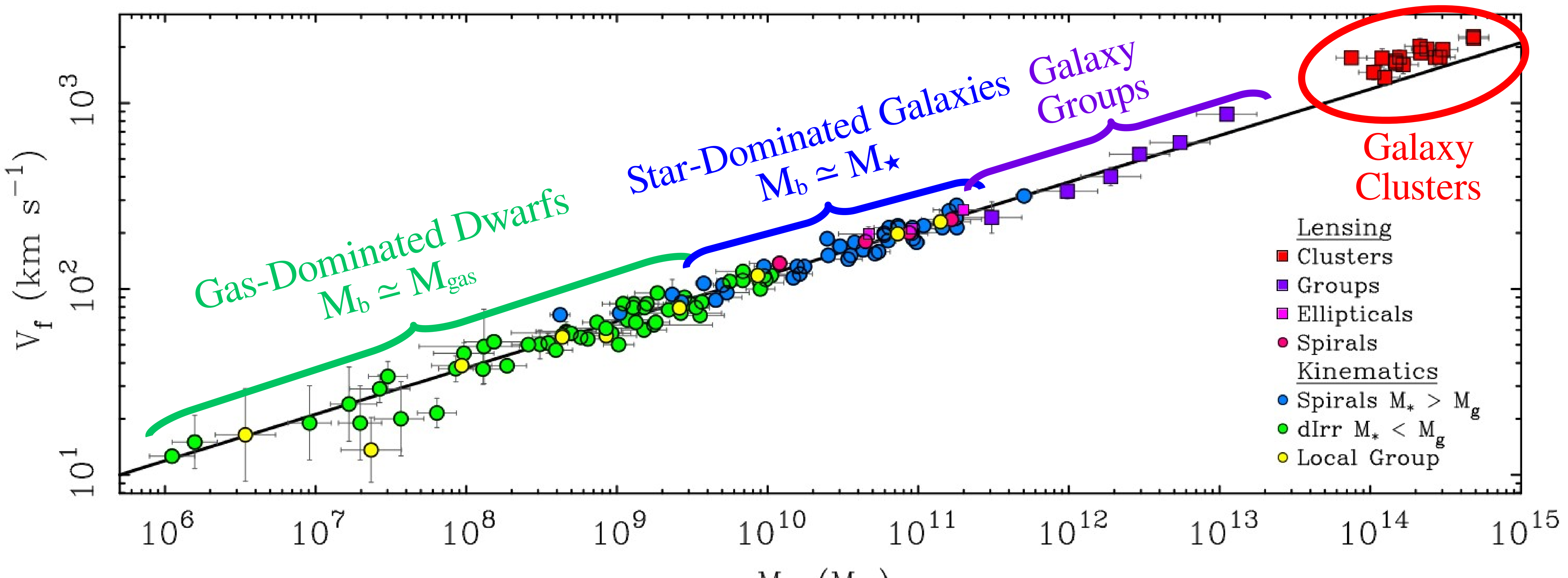

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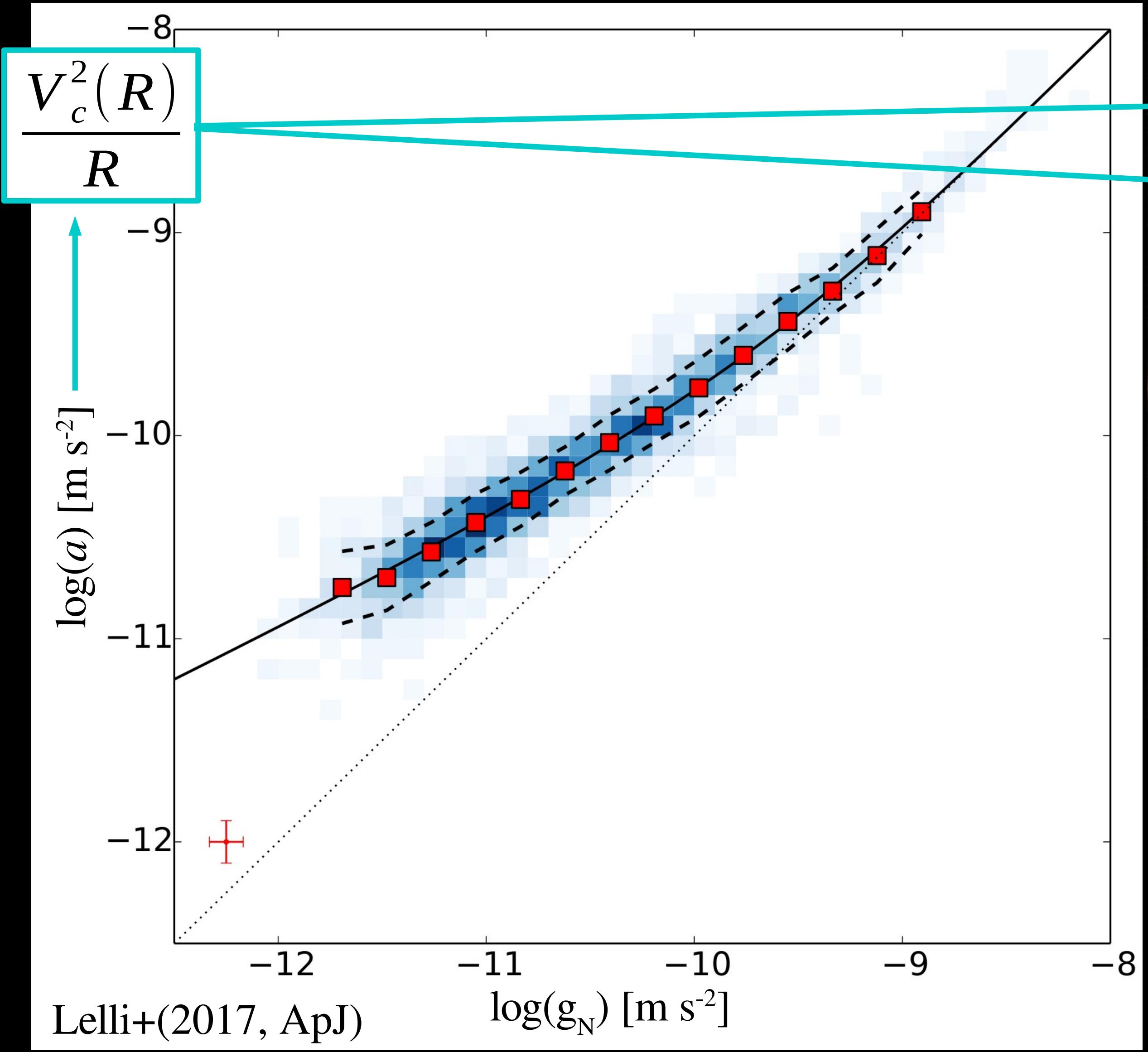

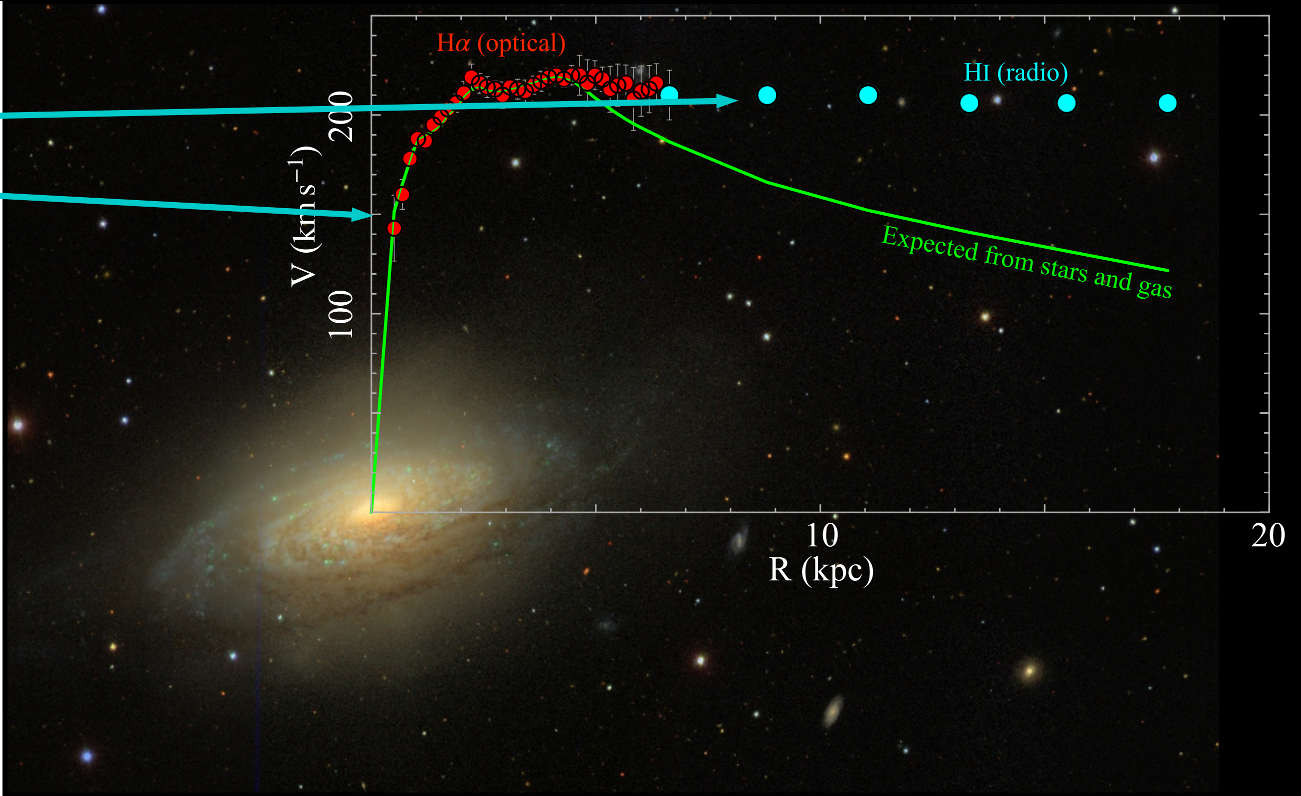

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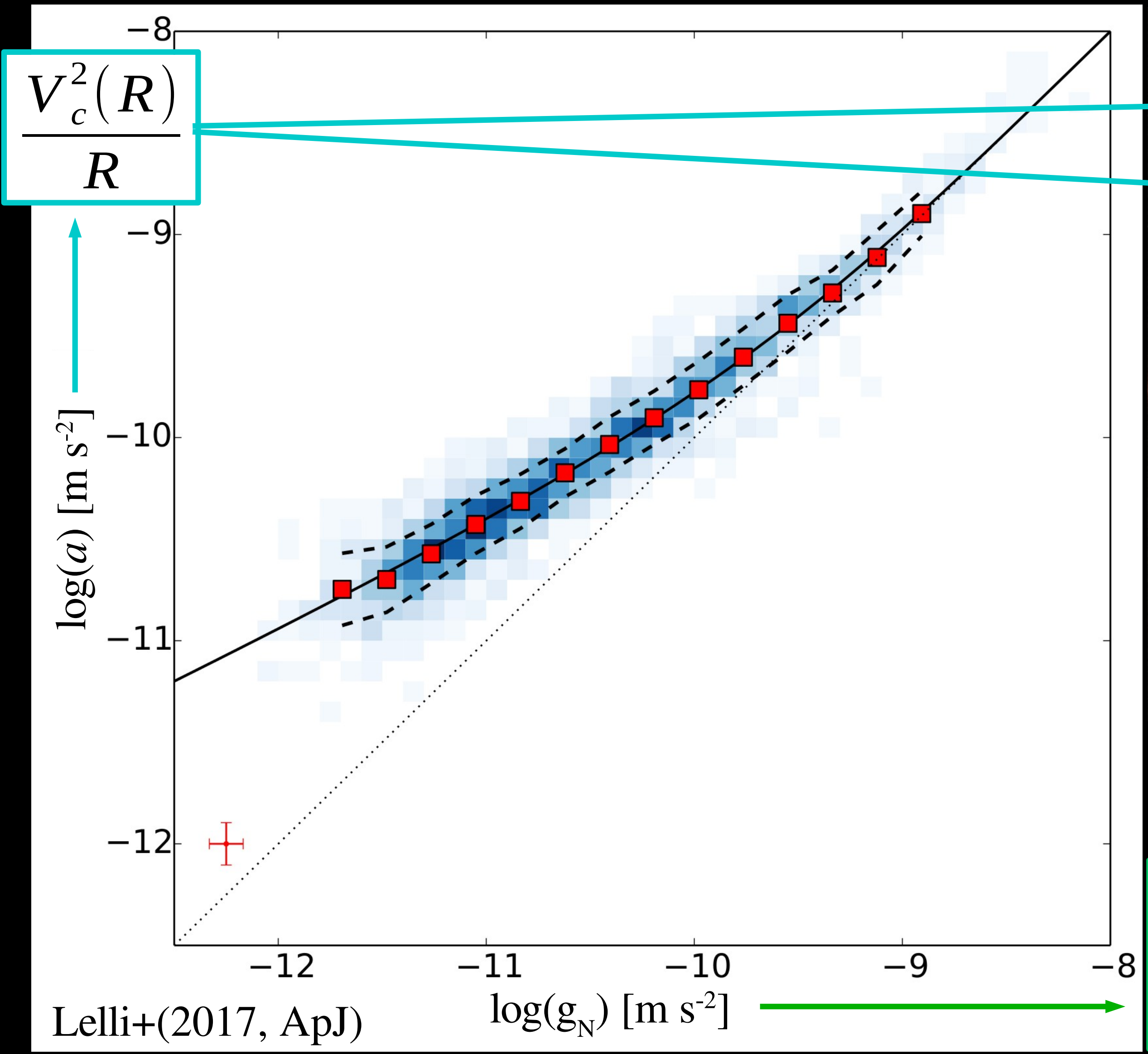

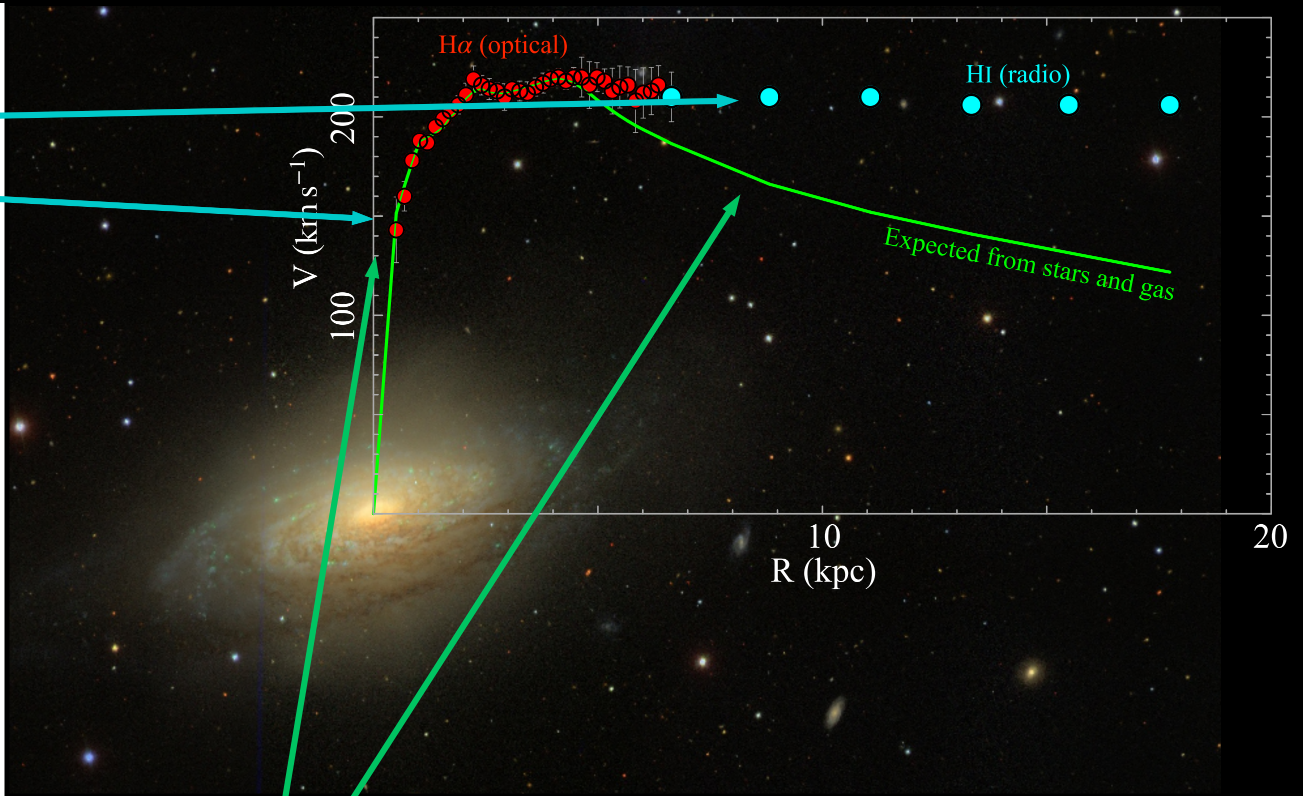

$$\nabla^2 \Phi_N = 4\pi G \rho_b$$
$$g_N = -\nabla_R \Phi_N$$

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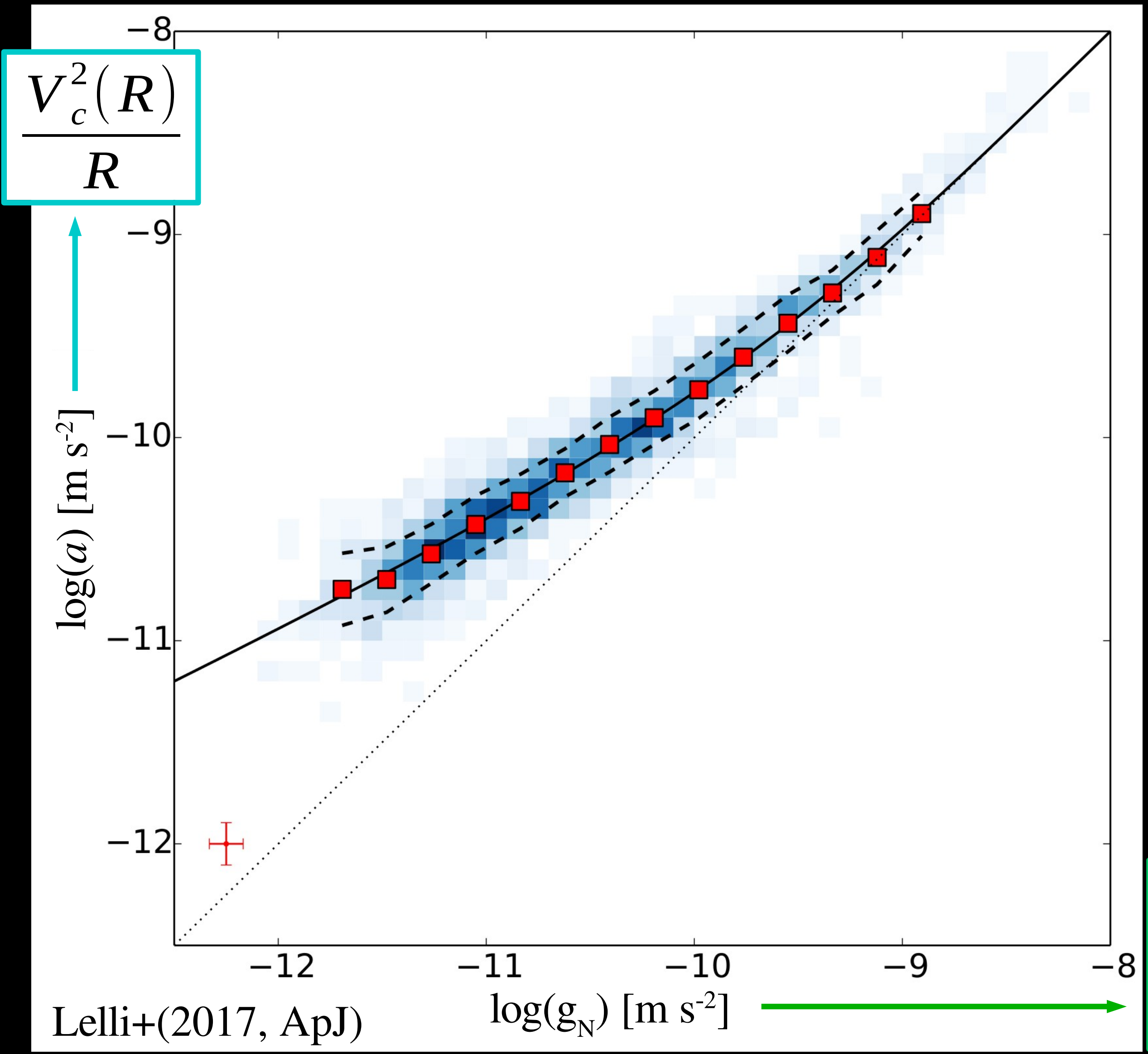

Radial Acceleration Relation (RAR)

✔ Acceleration scale $a_0 \simeq 10^{-10}$ m/s^2

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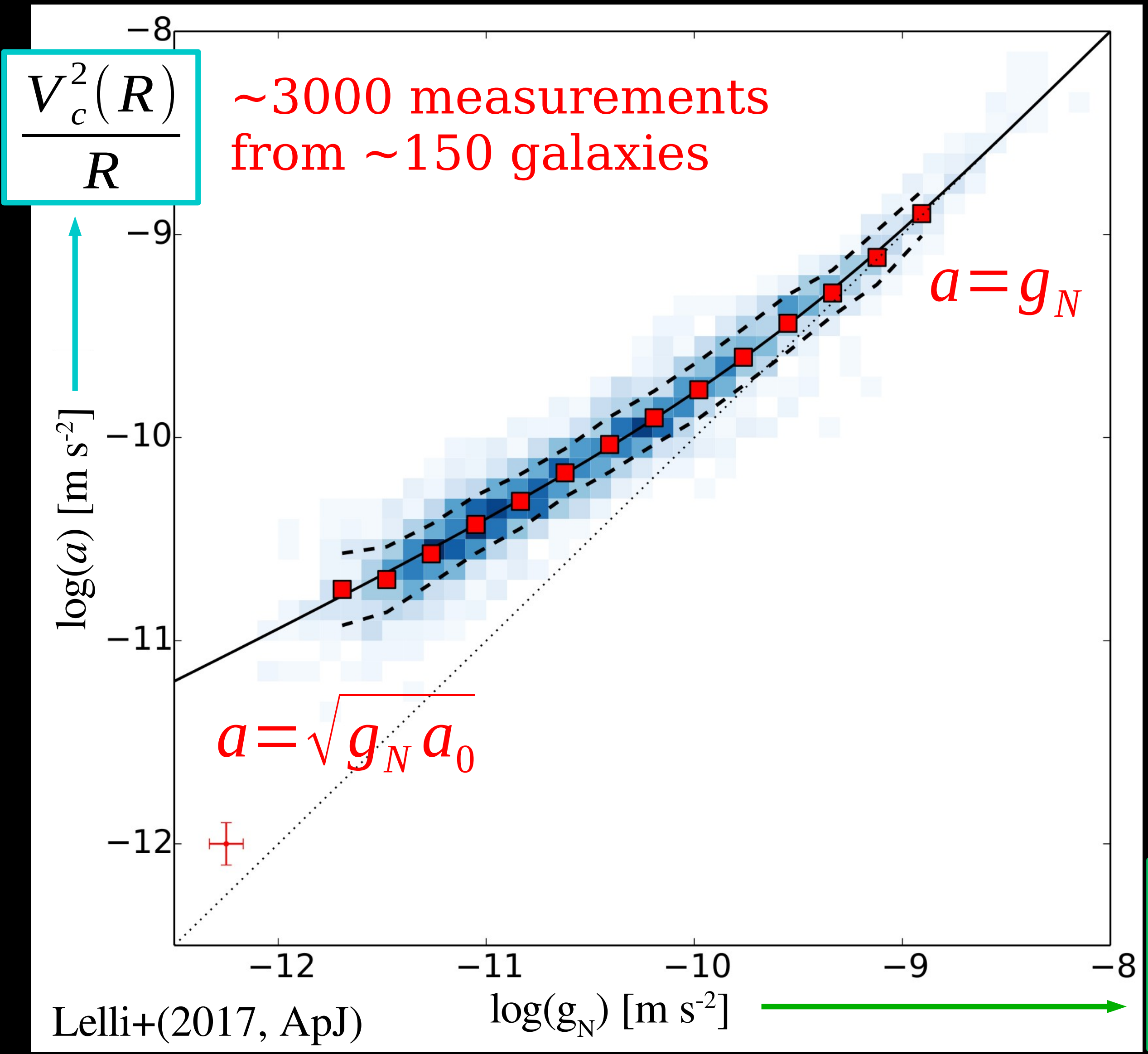

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- ✔ Acceleration scale $a_0 \simeq 10^{-10}$ m/s^2
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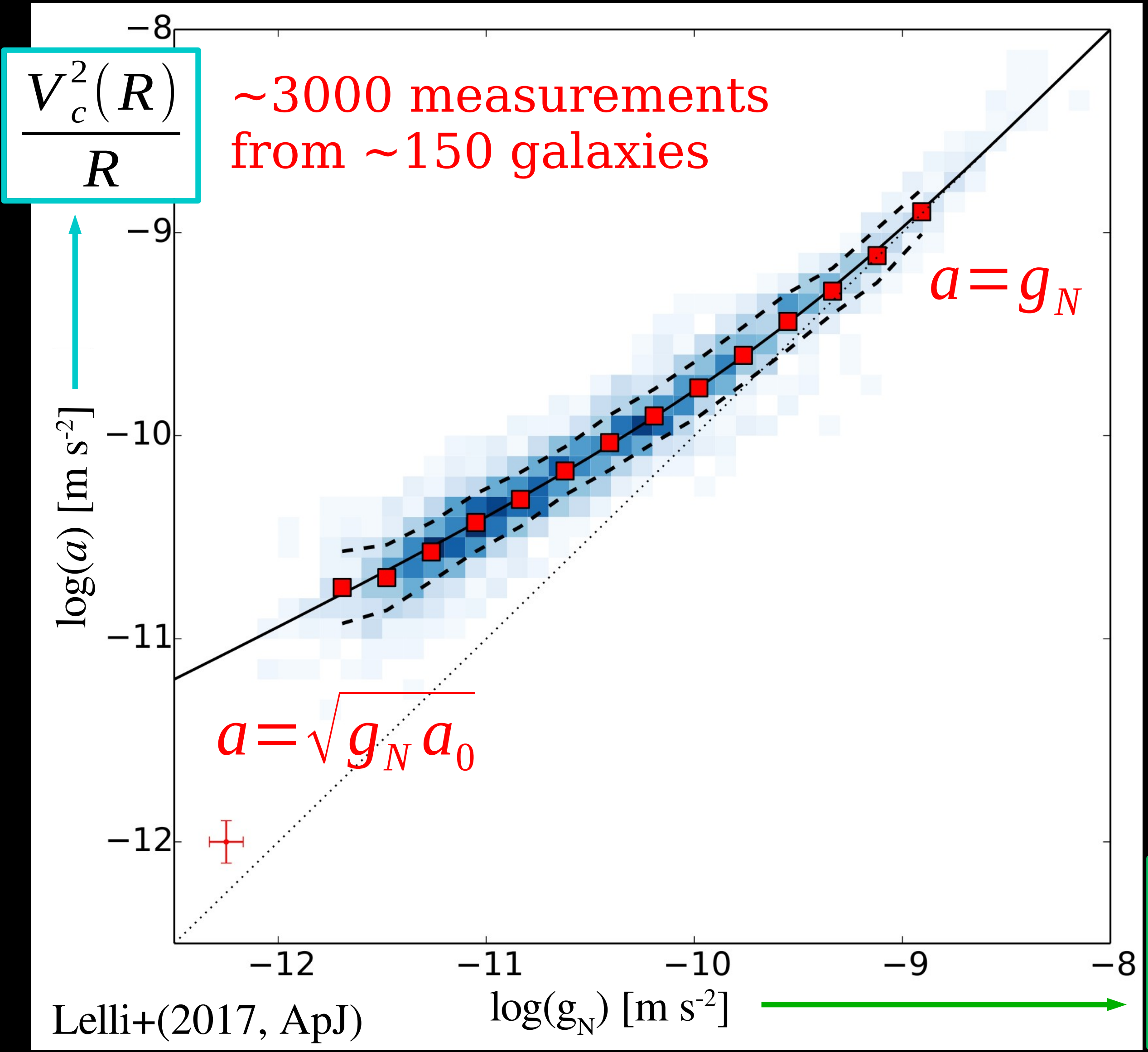

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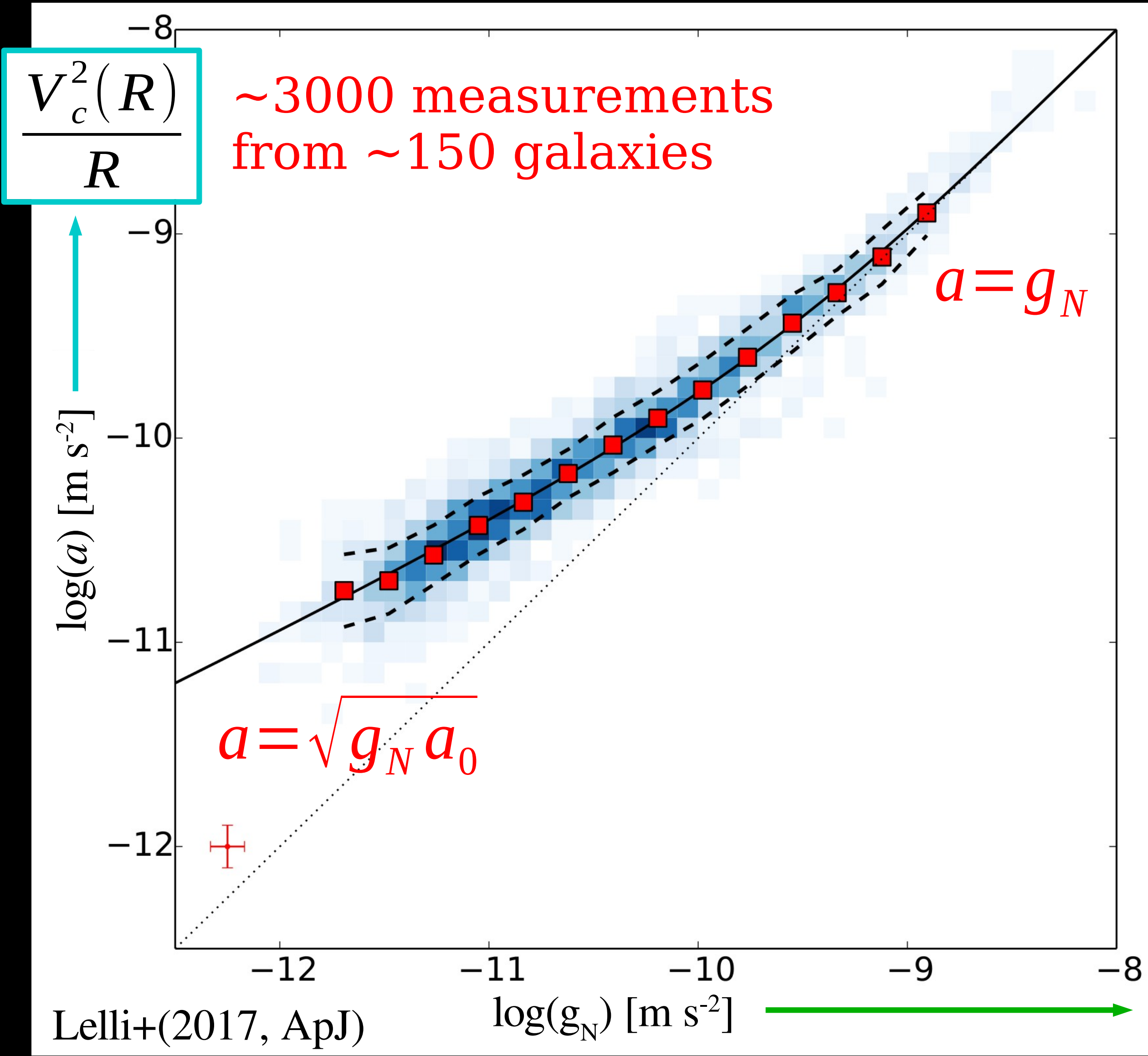

Lelli+(2017, ApJ)

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- ✔ Acceleration scale $a_0 \simeq 10^{-10}$ m/s^2
- ✔ MOND asymptotic limits
- ✔ Negligible scatter (~ uncertainties)

→ MOND interpolation function ν

$$a = \nu(g_N/a_0)\, g_N = \left(1 - e^{-\sqrt{g_N/a_0}}\right)^{-1} g_N$$

We can now assume $\nu(g_N/a_0)$ and predict rotation curves given ρ_b (within the errors)

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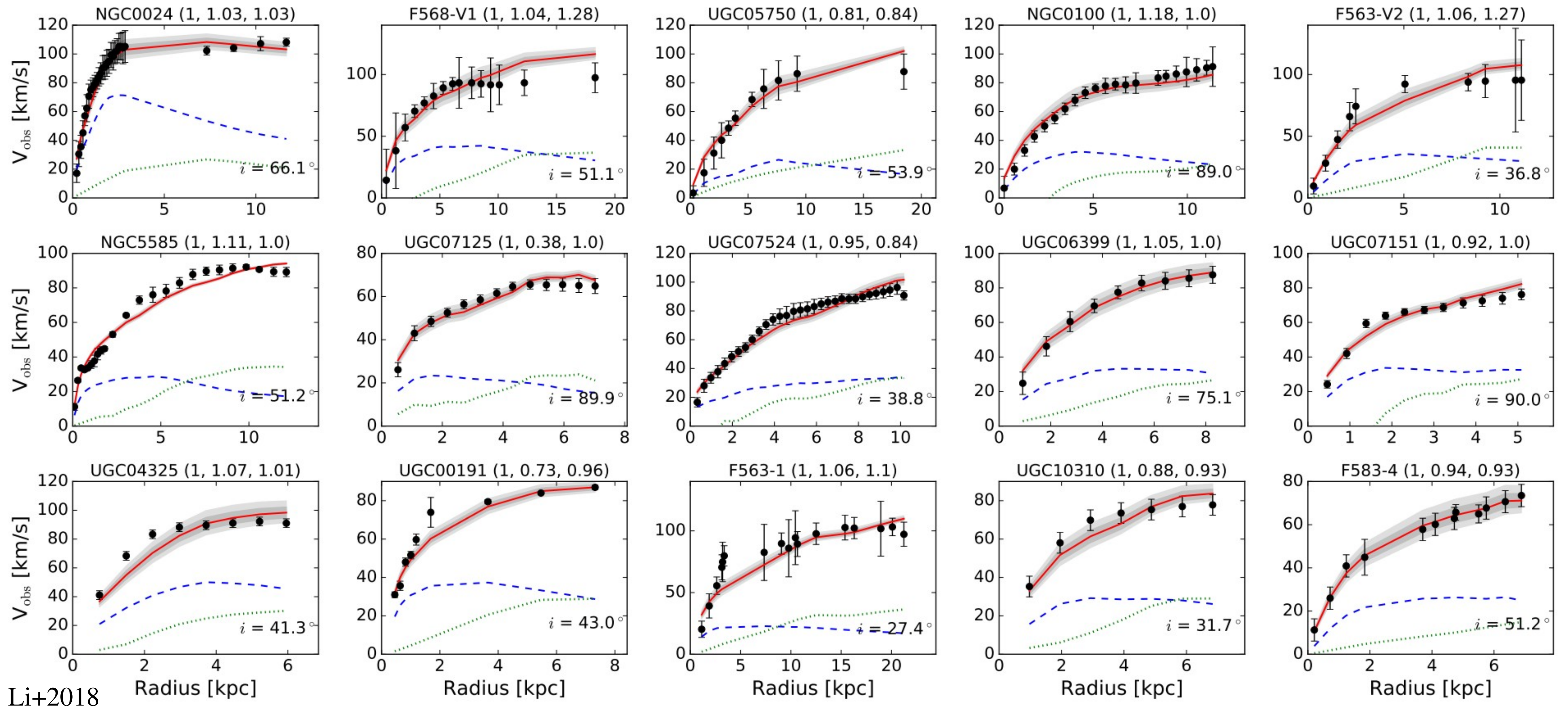

Li+2018

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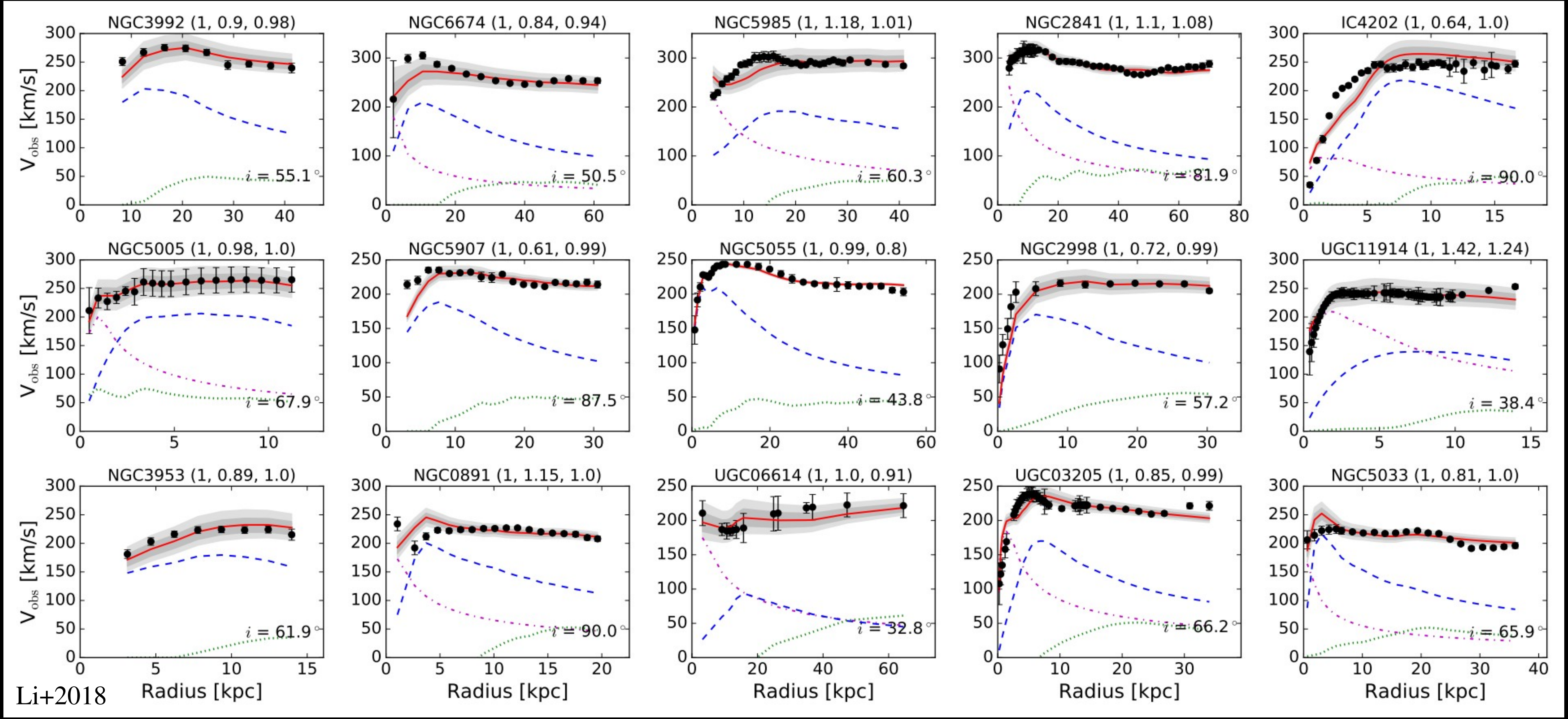

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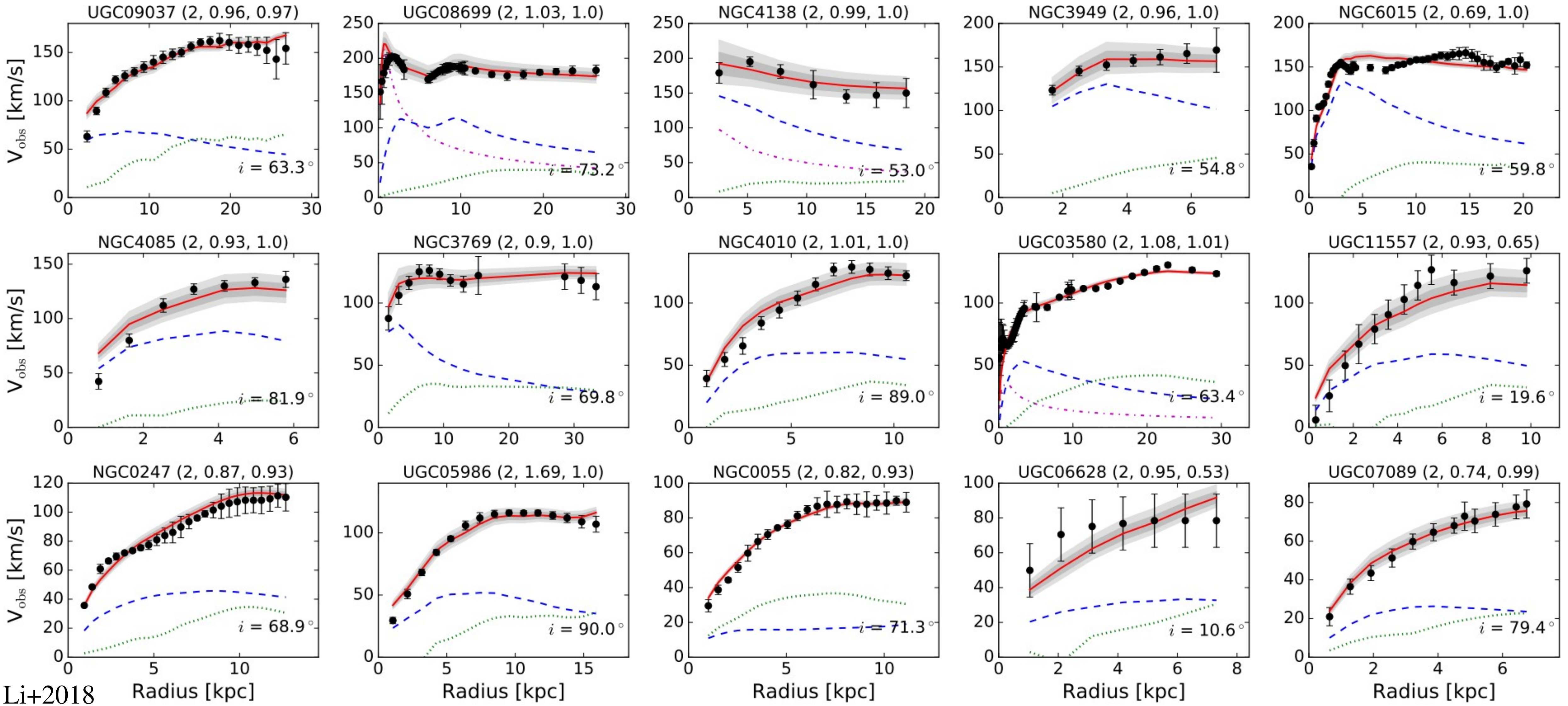

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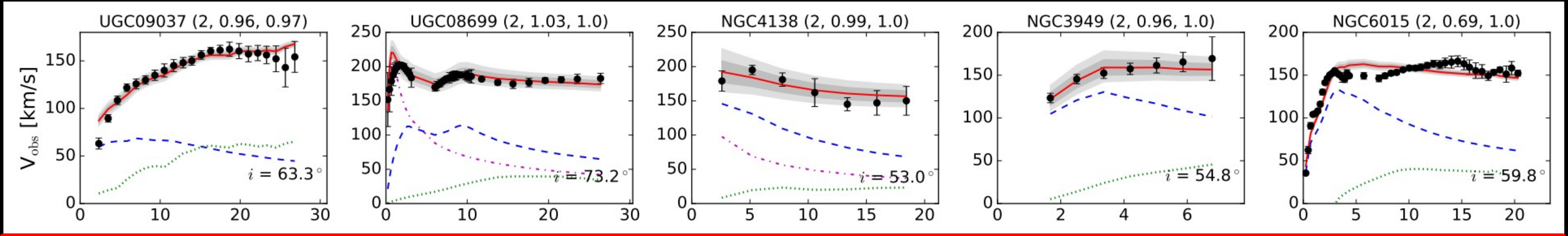
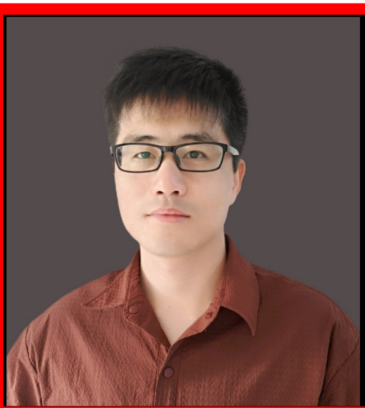
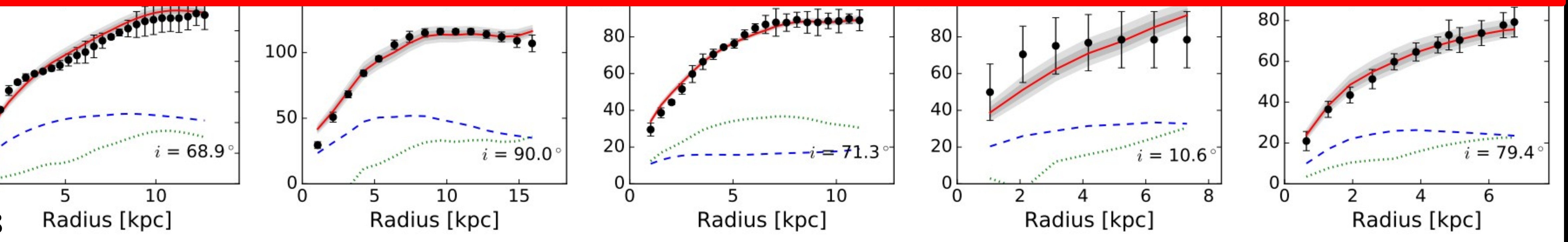




Former PhD student Pengfei Li → now Prof. at Nanjing University!

For the full sample of 175 galaxies, see Li, Lelli+(2018, A&A)

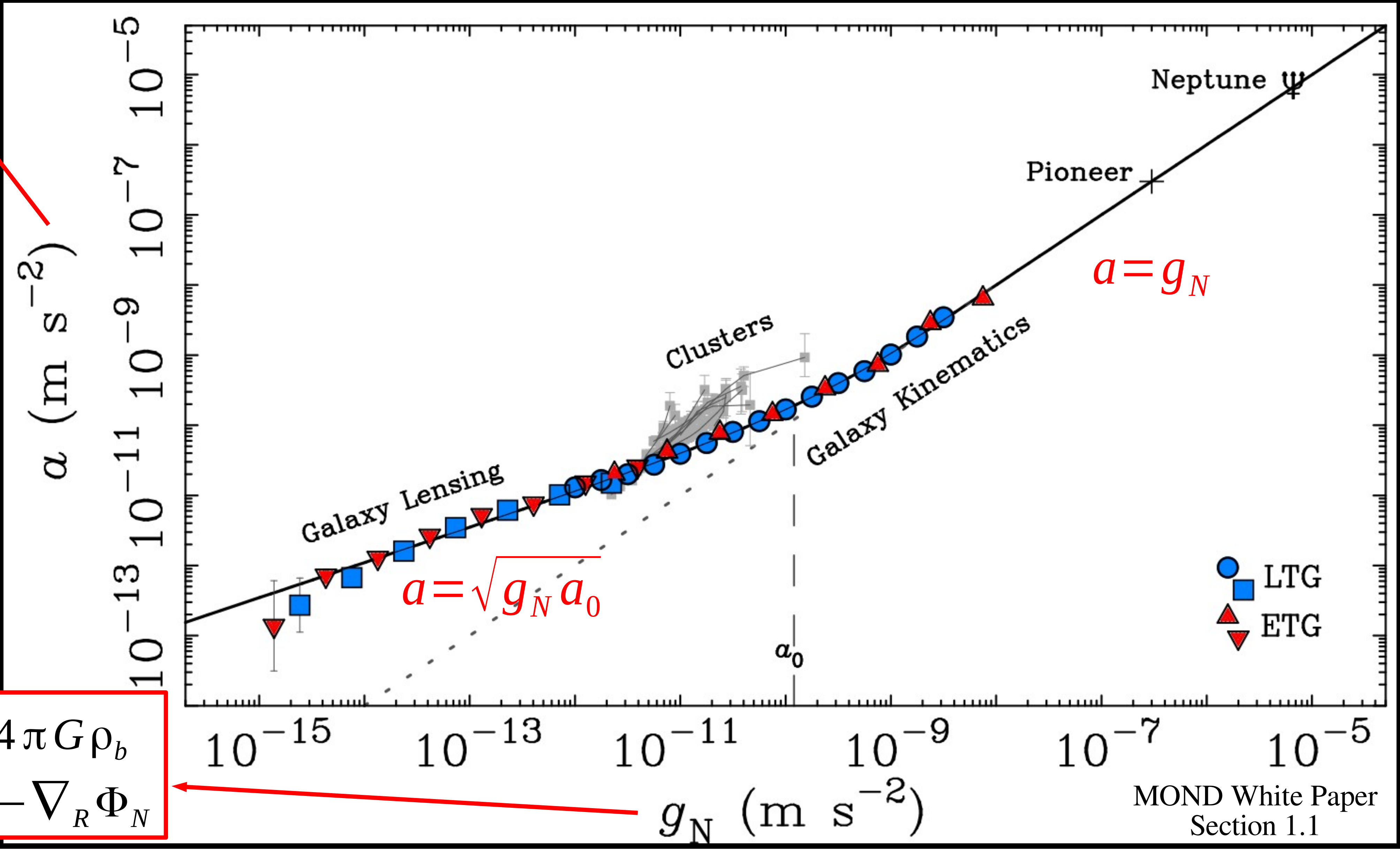
or the SPARC database (http://astroweb.cwru.edu/SPARC/)



Li+2018

Radial Acceleration Relation extended by Weak Lensing

$$\frac{V_c^2(R)}{R}$$

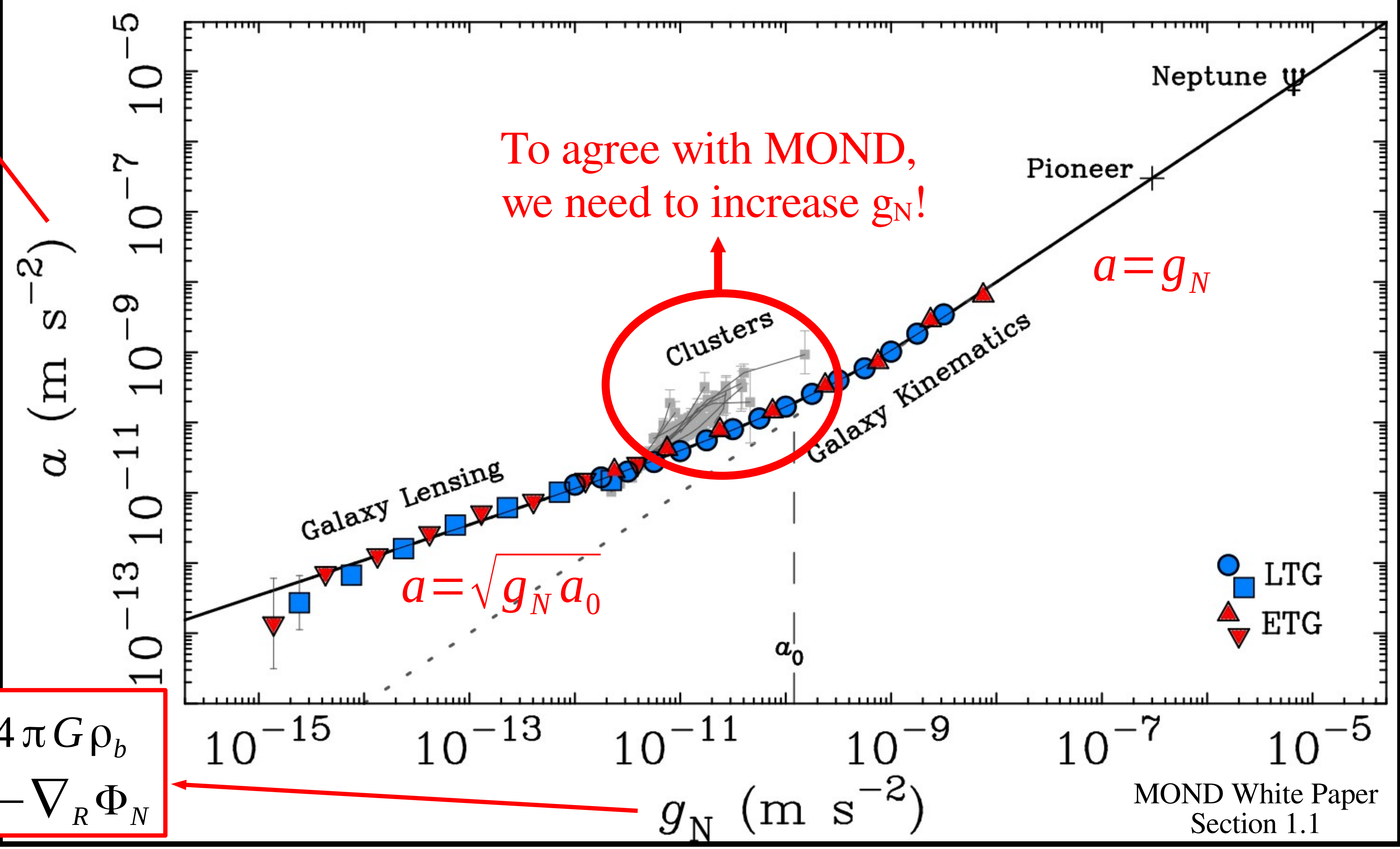


MOND White Paper
Section 1.1

$$\nabla^2 \Phi_N = 4\pi G \rho_b$$
$$g_N(R) = -\nabla_R \Phi_N$$

Radial Acceleration Relation extended by Weak Lensing

$$\frac{V_c^2(R)}{R}$$



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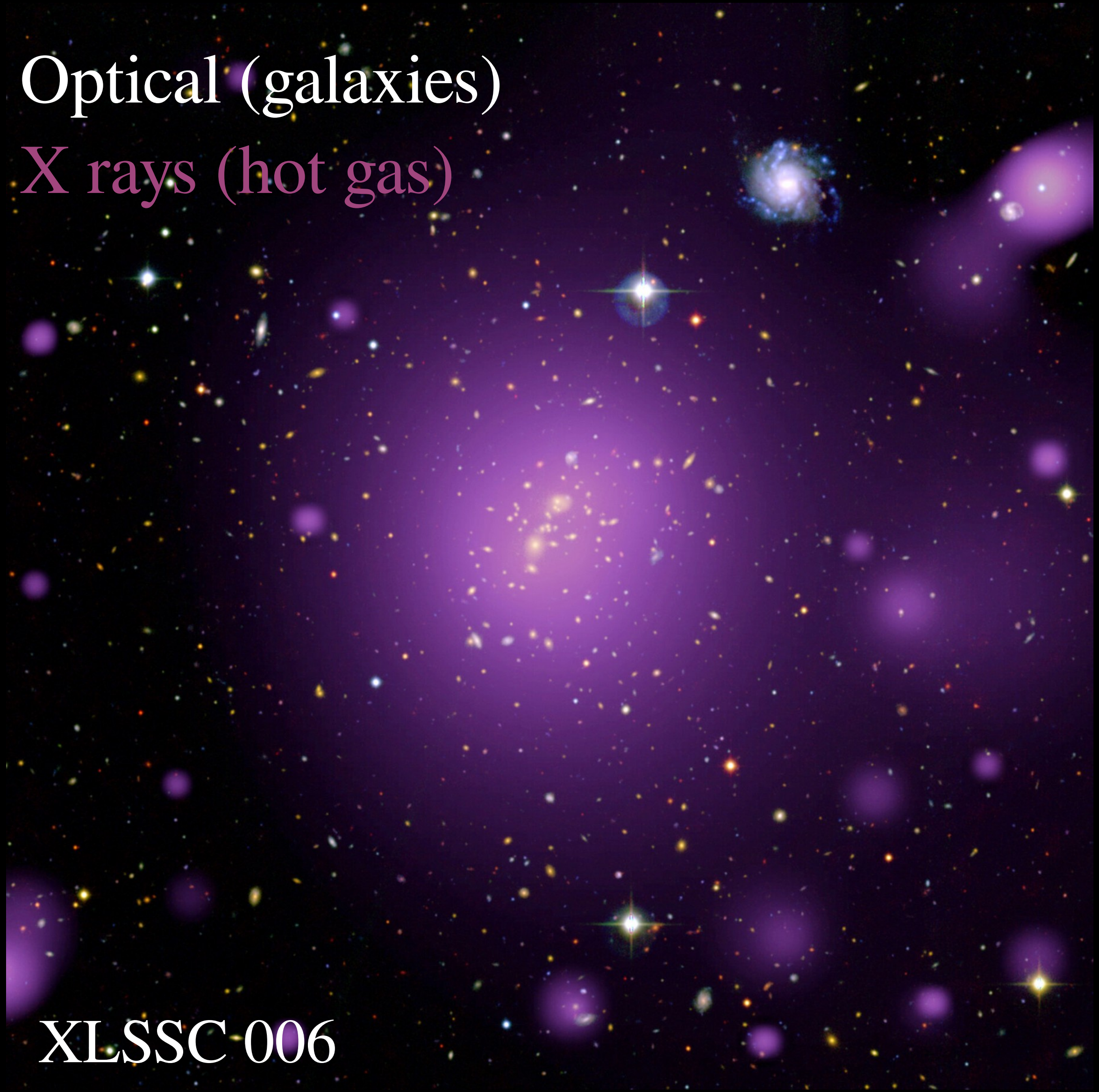
MOND White Paper
Section 1.1

Tests in Galaxy Clusters

Galaxy Clusters: hot gas (~0.9 M_b) plus some galaxies (~0.1 M_b)



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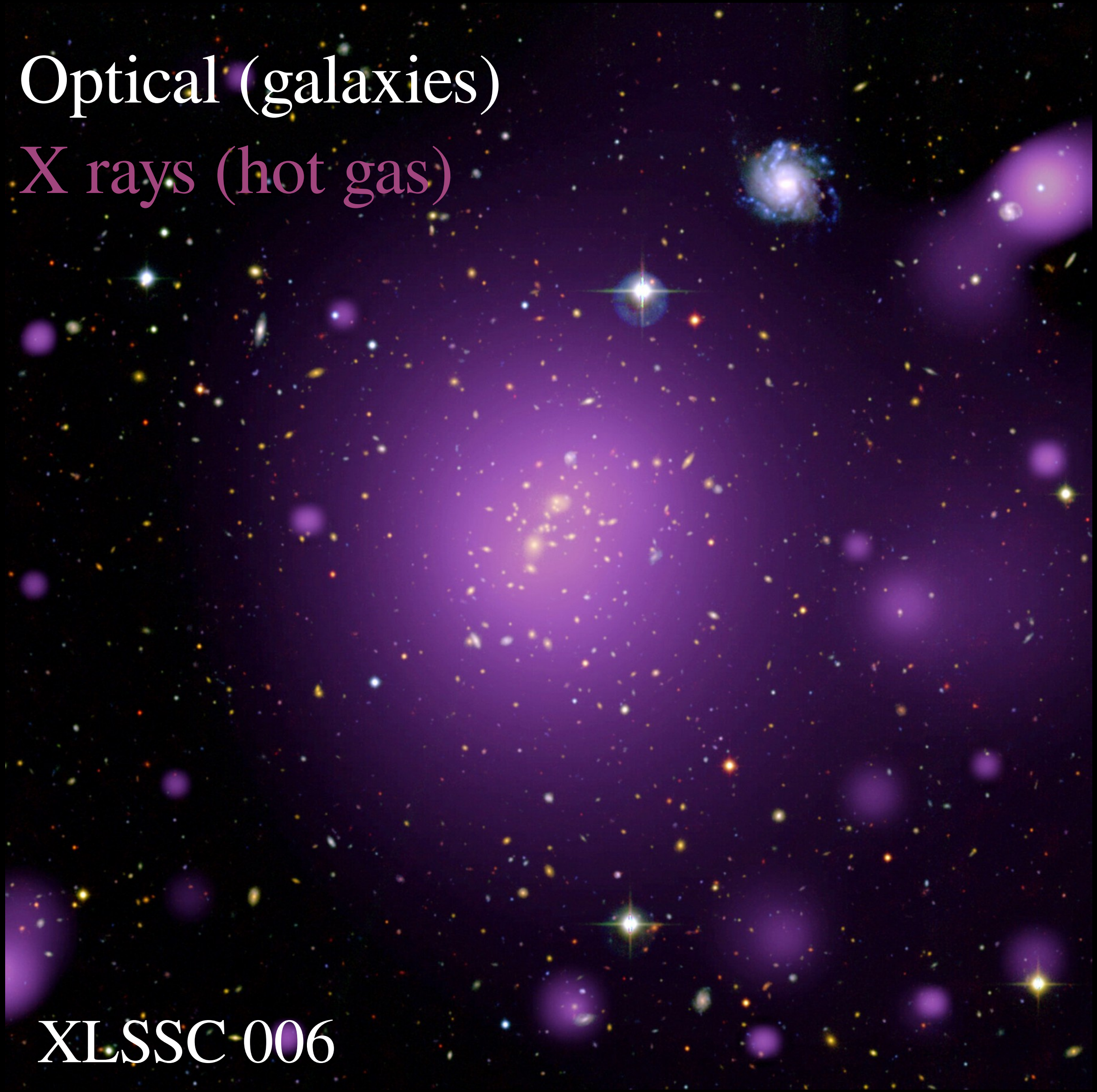

Sphere in Hydrostatic Equilibrium

$$g(r) = -\frac{1}{\rho_{gas}(r)} \frac{\partial P_{gas}(r)}{\partial r}$$

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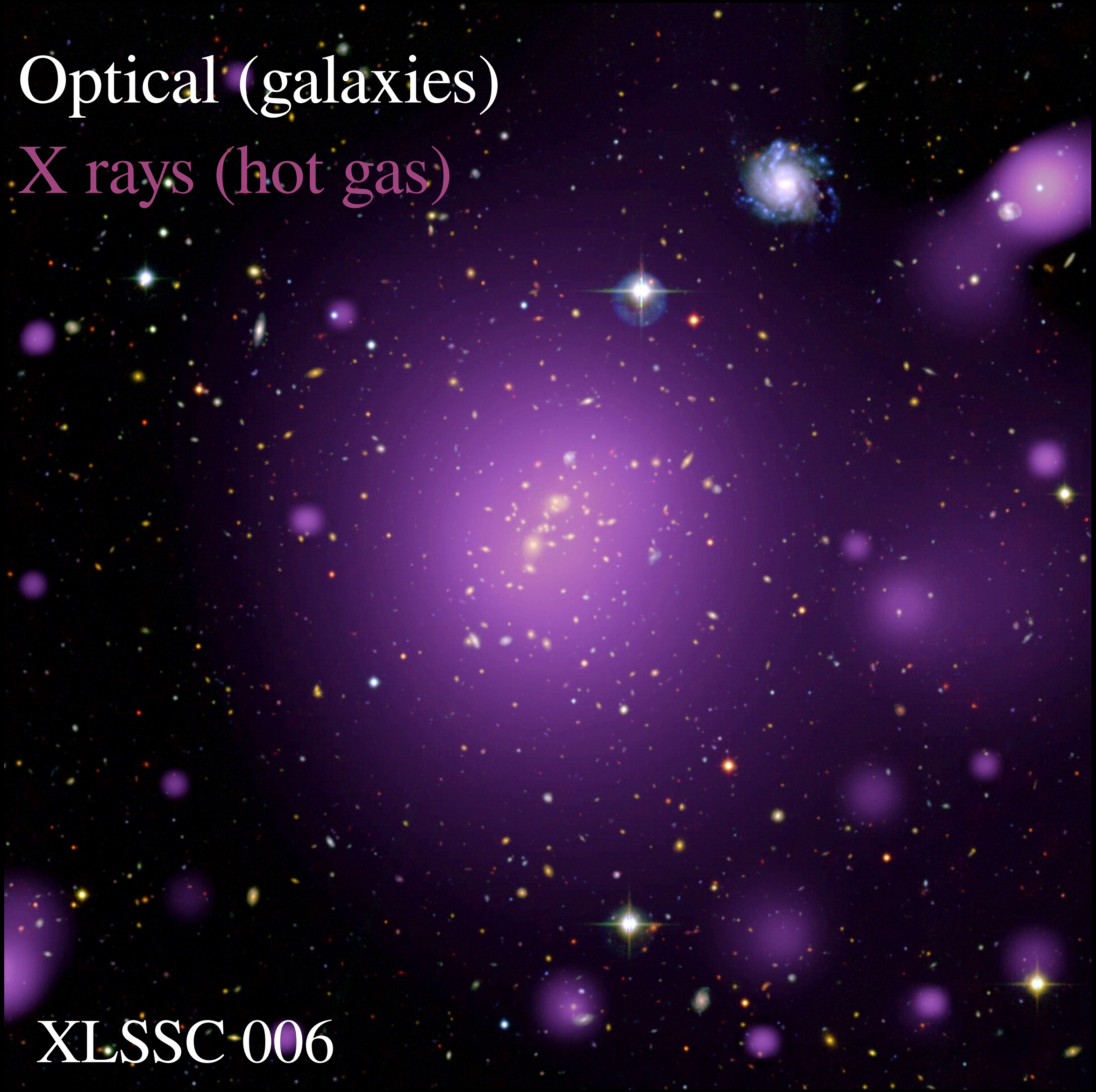

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$$P_{gas} = \frac{k_B}{w\, m_p} \rho_{gas} T_{gas}$$

⇨ From X-ray observations

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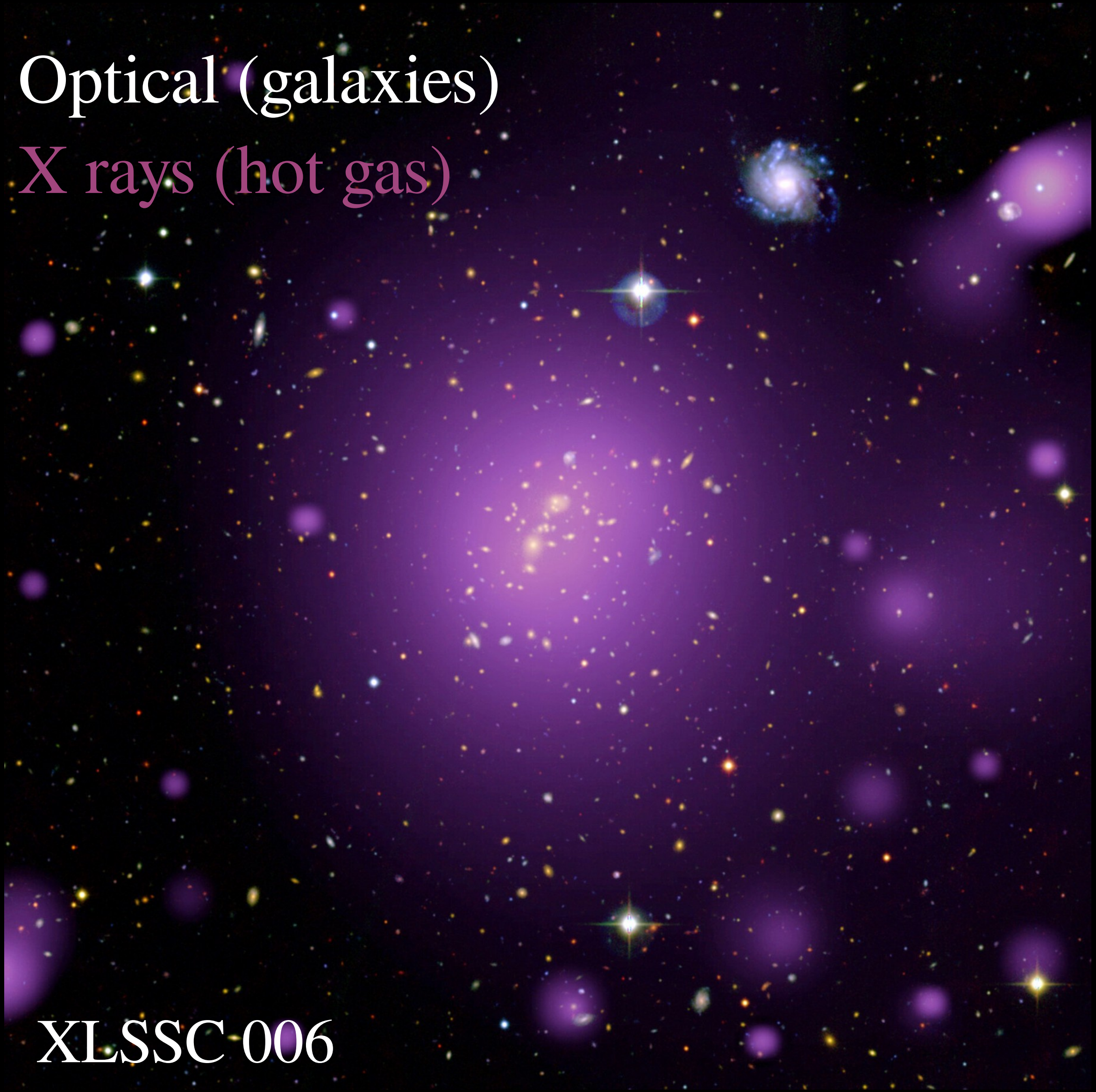

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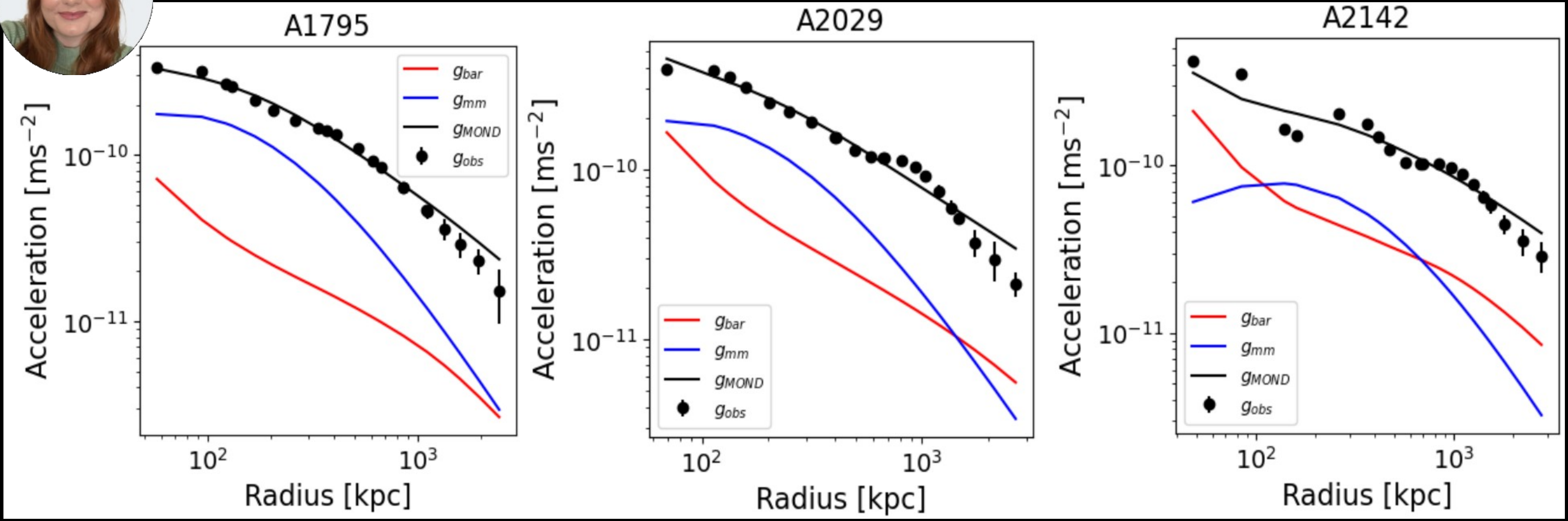
$$P_{gas} = \frac{k_B}{w\, m_p} \rho_{gas} T_{gas}$$ ⇒ From X-ray observations

$$y = \frac{\sigma_T}{m_e c^2} \int_z P_{gas}\, dz$$ ⇒ S-Z effect (Planck data)

S-Z effect gives g at large r ≃ 1-3 Mpc!

MOND fits with an extra component (Kelleher & Lelli 2024, A&A)



Density profile of missing mass:

$$\rho_{mm}(r) = \frac{\rho_0}{(1 + r/r_s)^4}$$

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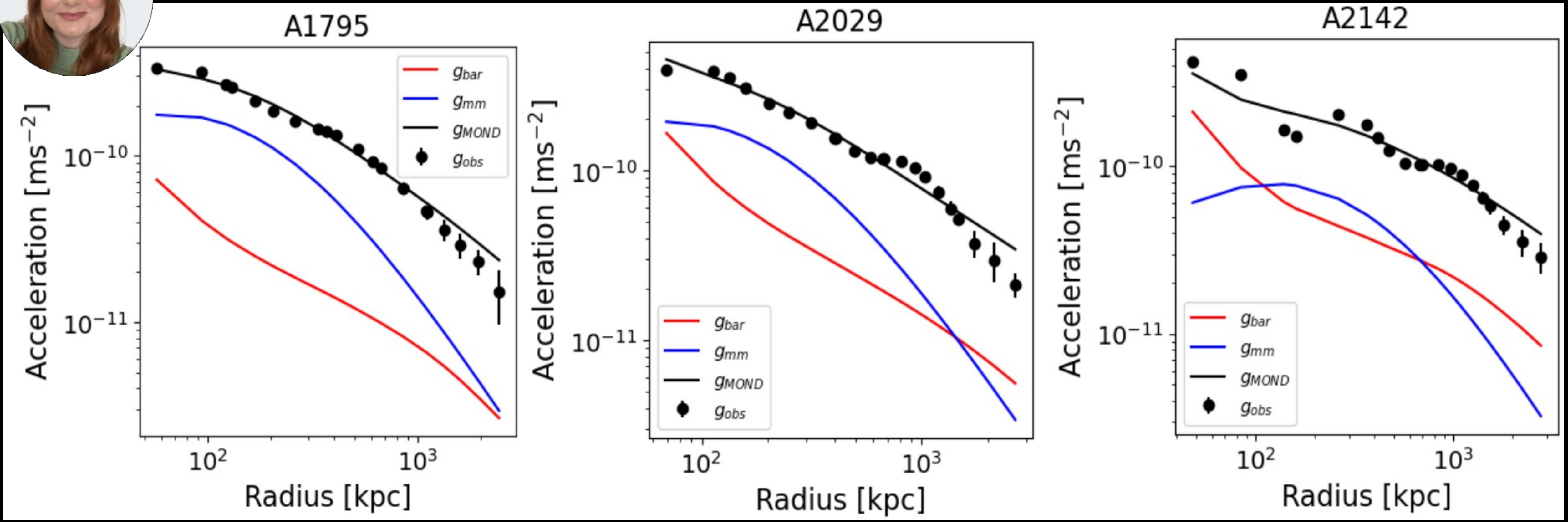


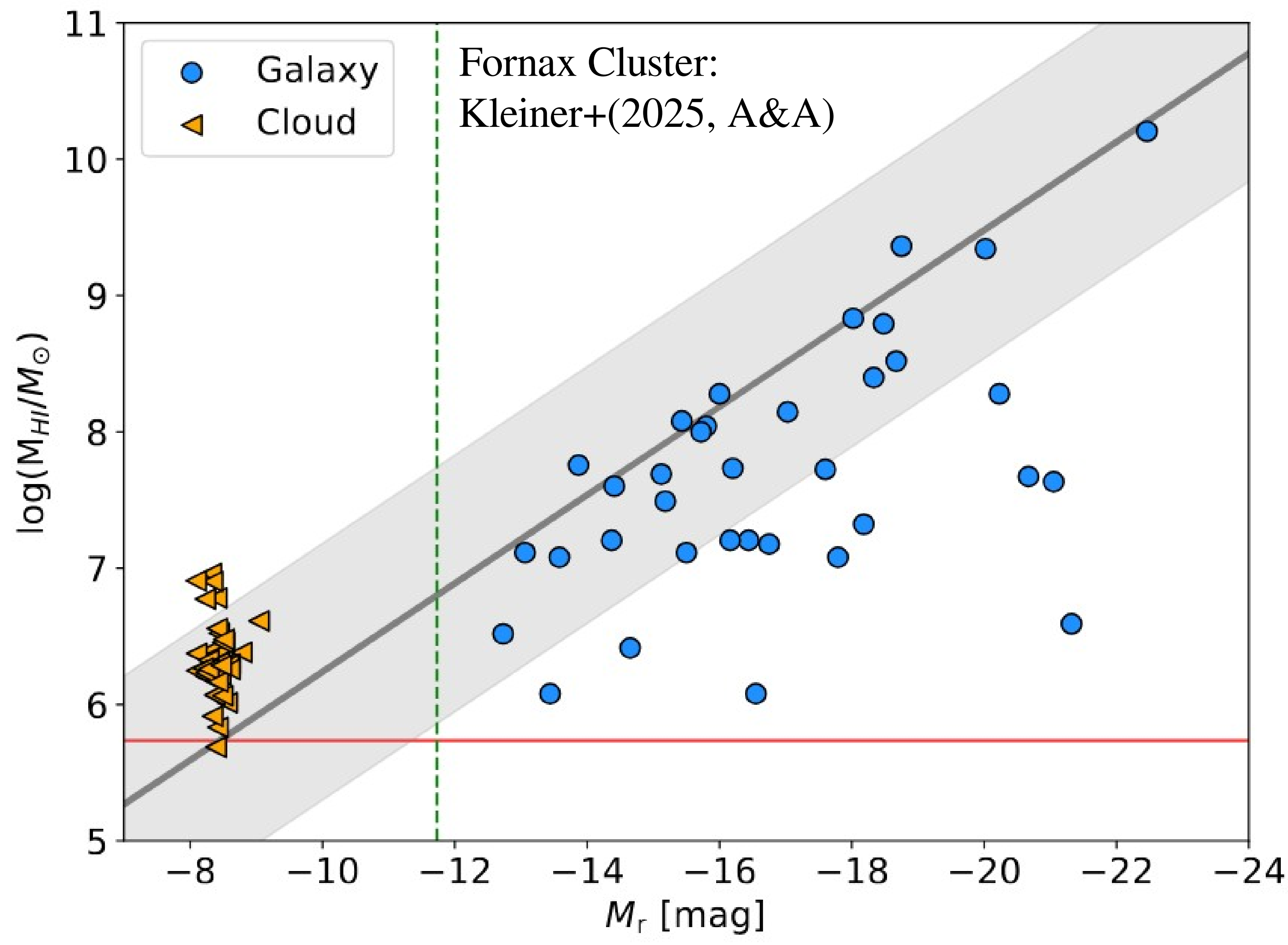
Density profile of missing mass:

$$\rho_{mm}(r) = \frac{\rho_0}{(1 + r/r_s)^4}$$

Converged (physical) total mass:

$$\Rightarrow \quad M_{mm} = \int_0^{\infty} \rho_{mm}(r)\, 4\pi r^2\, dr = \frac{4\pi}{3} \rho_0 r_s^3$$

Missing mass in clusters: compact clouds of cold gas?



Deepest HI surveys of the nearest galaxy clusters (Virgo, Fornax): $M_{HI} \gtrsim 10^6\, M_\odot$

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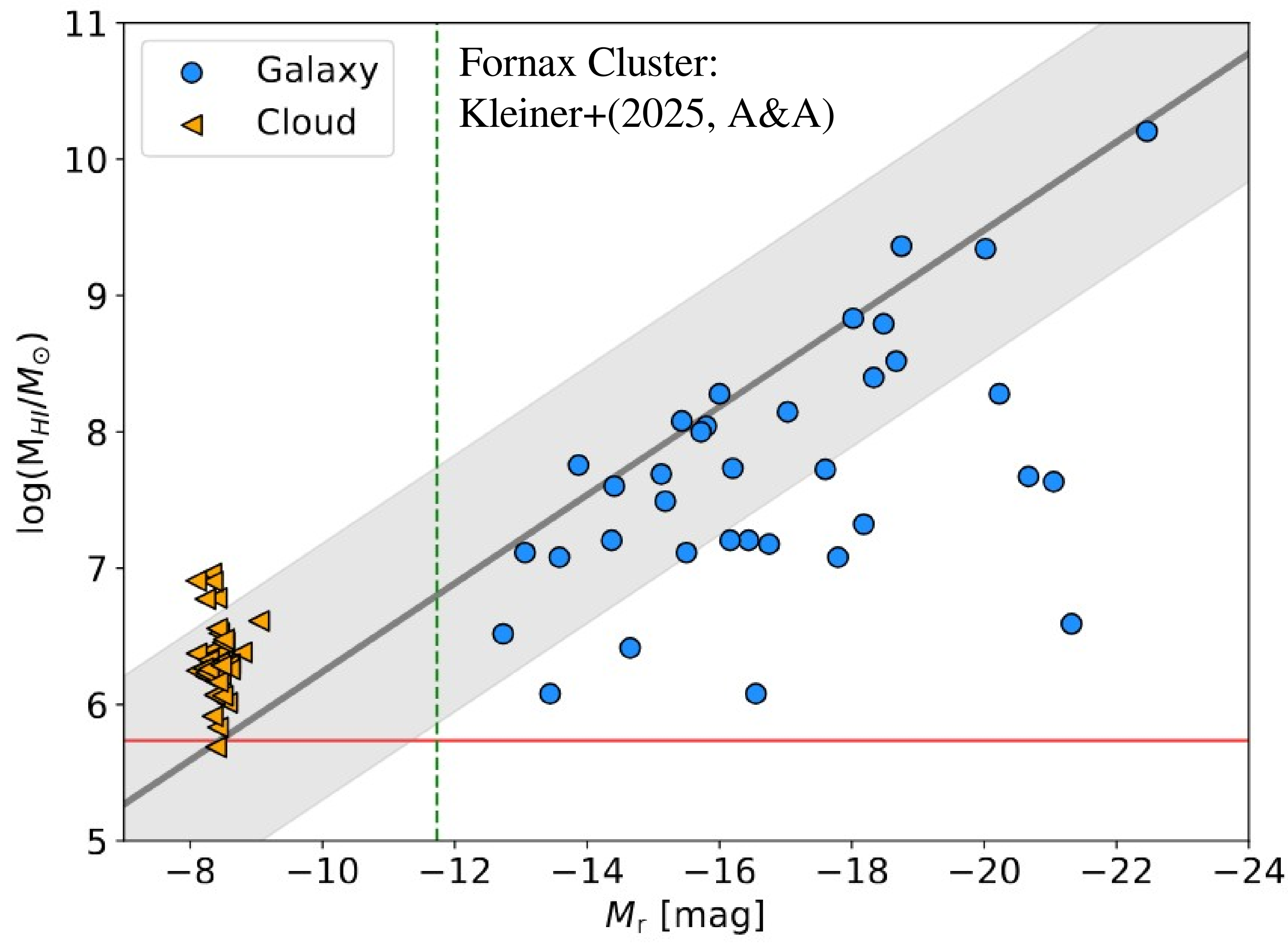


Deepest HI surveys of the nearest galaxy clusters (Virgo, Fornax): $M_{HI} \gtrsim 10^6\, M_\odot$

To solve the problem:

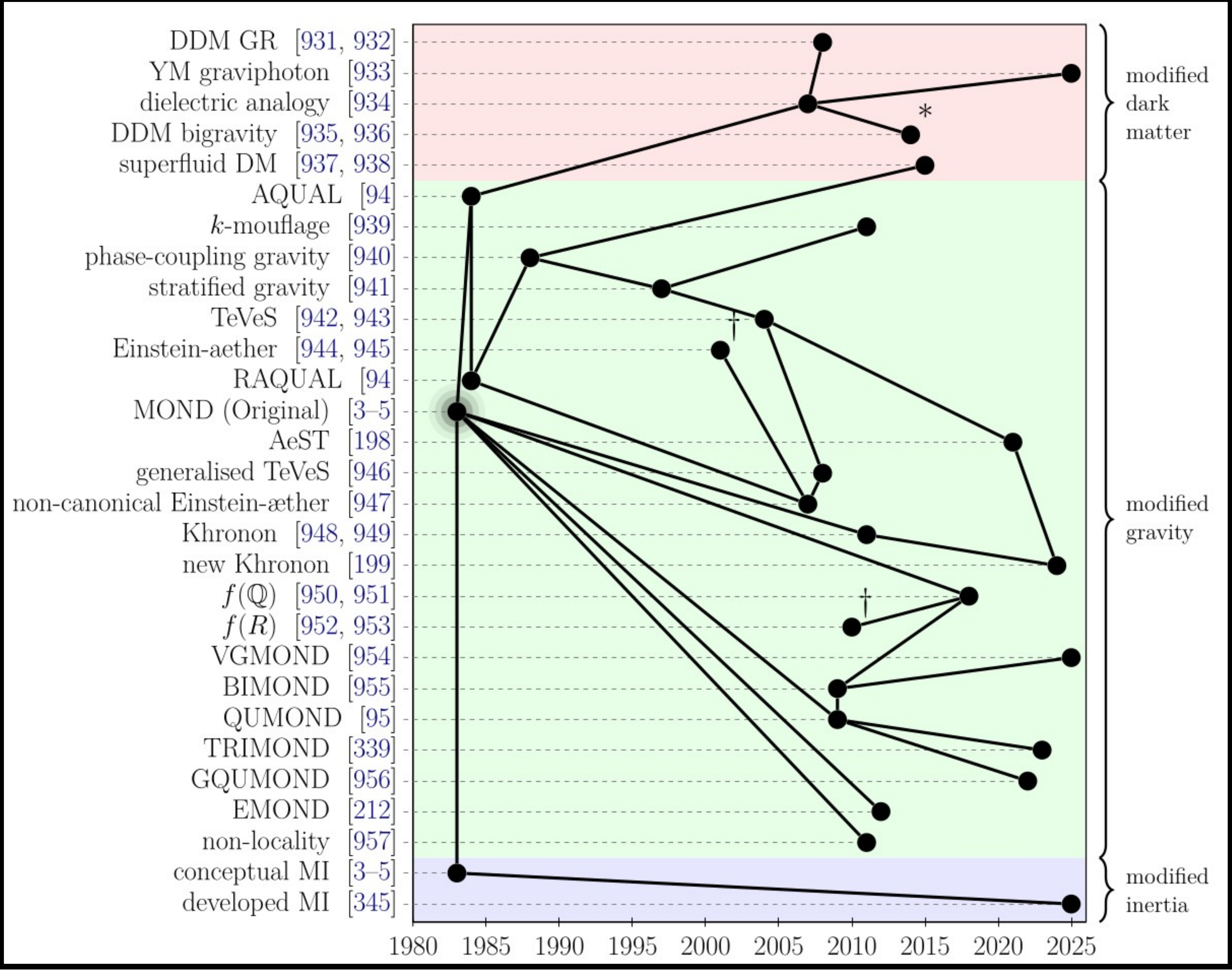
$M_{mm} \simeq M_{ICM} \simeq 10^{13\text{-}14}\, M_\odot$

Many ($\sim 10^{10}$) tiny clouds with $M_{cloud} \simeq 10^3\text{-}10^4$?

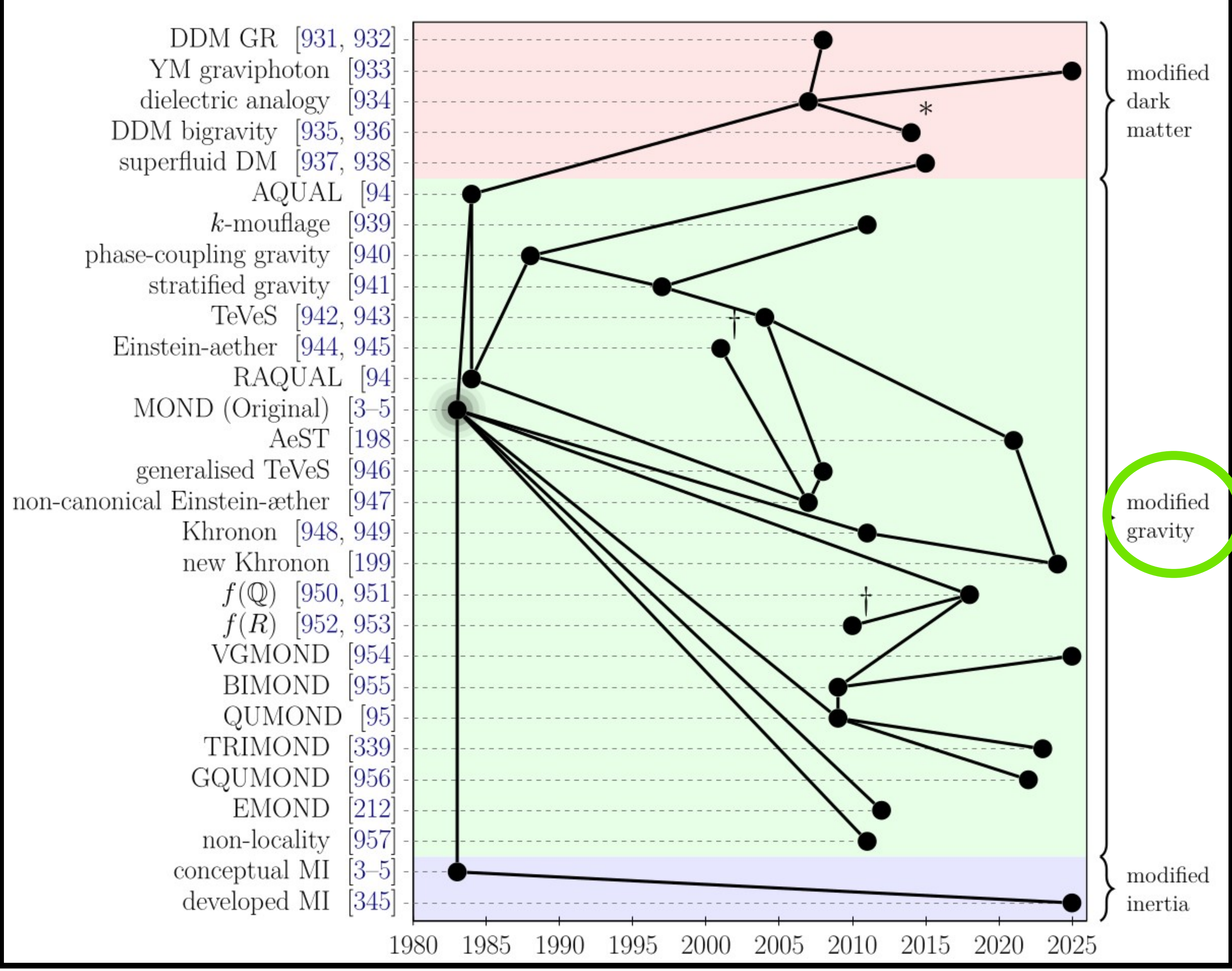
Or even more clouds with smaller masses...

Relativistic Theories & Cosmology

Taxonomy of MOND theories

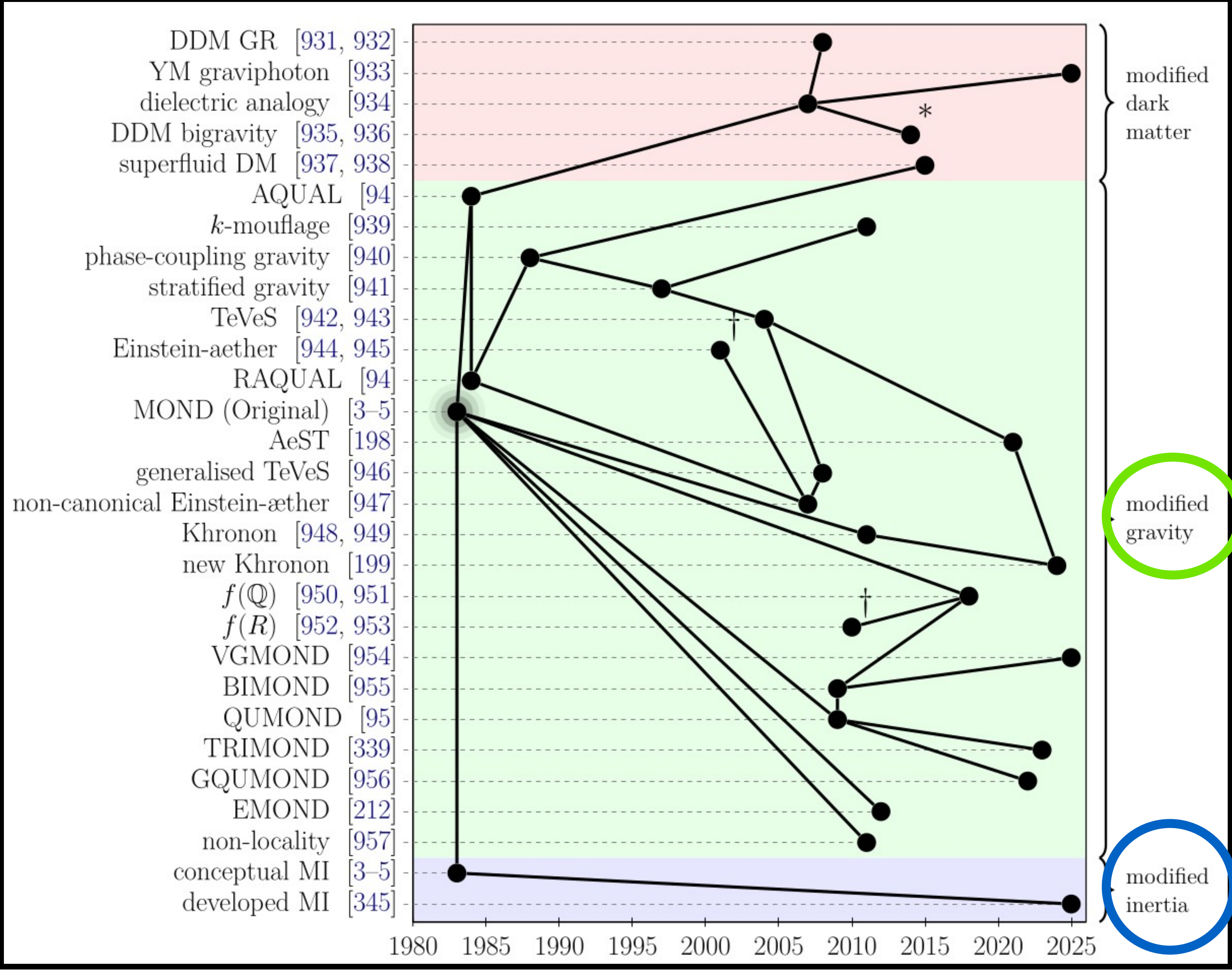


Taxonomy of MOND theories



Modify E-H gravitational action of GR.
In the stationary non-relativistic limit, they give a modified Poisson's equation.

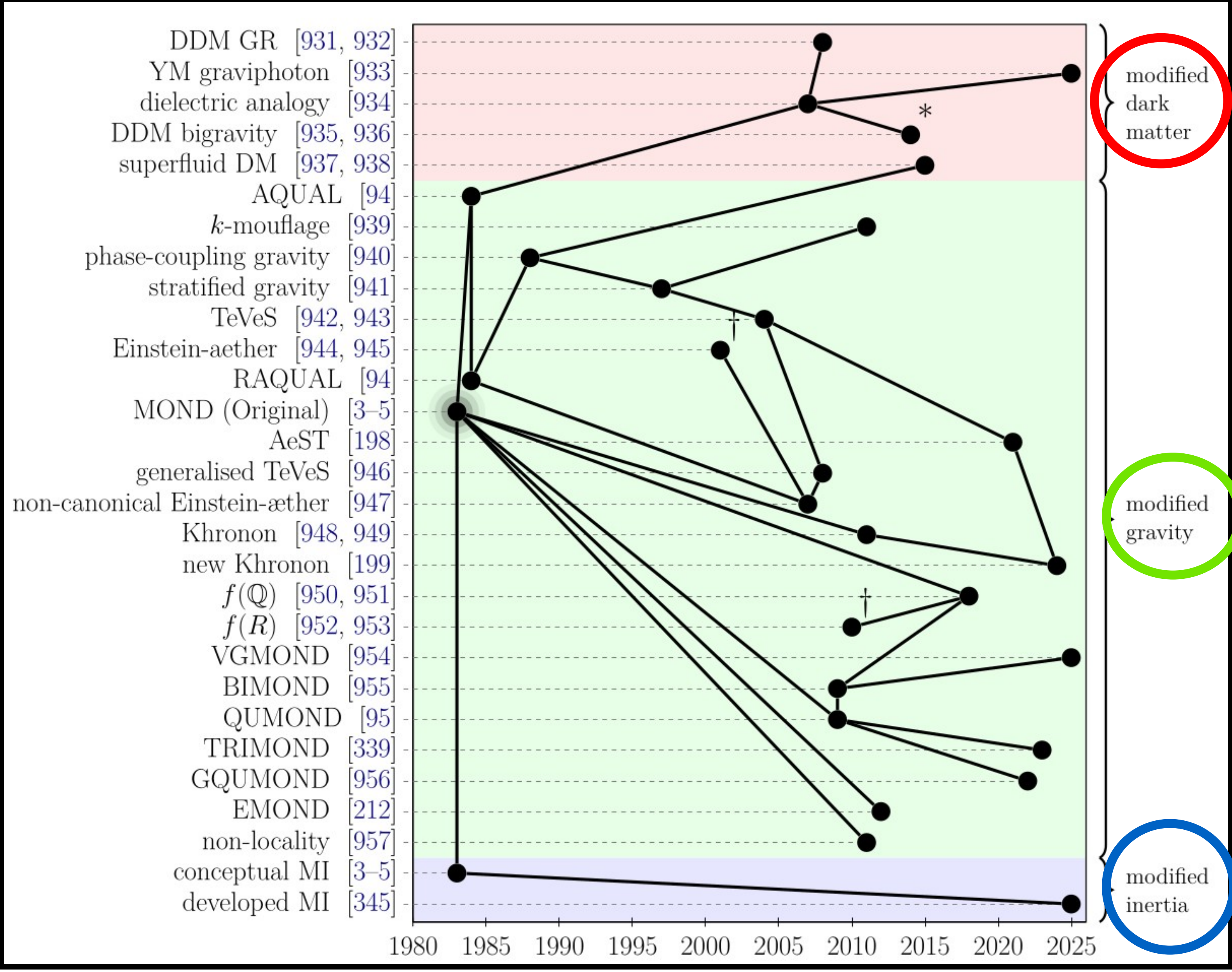
Taxonomy of MOND theories



Modify E-H gravitational action of GR.
In the stationary non-relativistic limit,
they give a modified Poisson's equation.

Modify 2[nd] law of Newton (F=ma)
in the non-relativistic limit.
No proper relativistic version yet.

Taxonomy of MOND theories



Hybrid approach: non-baryonic DM particles that interact with baryons to give a MOND-like behaviour on galaxy scales, while preserving ΛCDM successes on large cosmological scales.

Modify E-H gravitational action of GR. In the stationary non-relativistic limit, they give a modified Poisson's equation.

Modify 2[nd] law of Newton (F=ma) in the non-relativistic limit. No proper relativistic version yet.

Lovelock-Grigore Theorem

GR (+Λ) is the only theory that satisfy these assumptions:

1) Geometry is Reimannian

2) Action depends only on $g_{\mu\upsilon}$

3) It is diffeomorphism invariant

4) It is local

5) It leads to 2nd order field equations

Lovelock-Grigore Theorem

GR (+Λ) is the only theory that satisfy these assumptions:

1) Geometry is Reimannian

2) ~~Action depends only on~~ $\cancel{g_{\mu\upsilon}}$

3) It is diffeomorphism invariant

4) It is local

5) It leads to 2nd order field equations

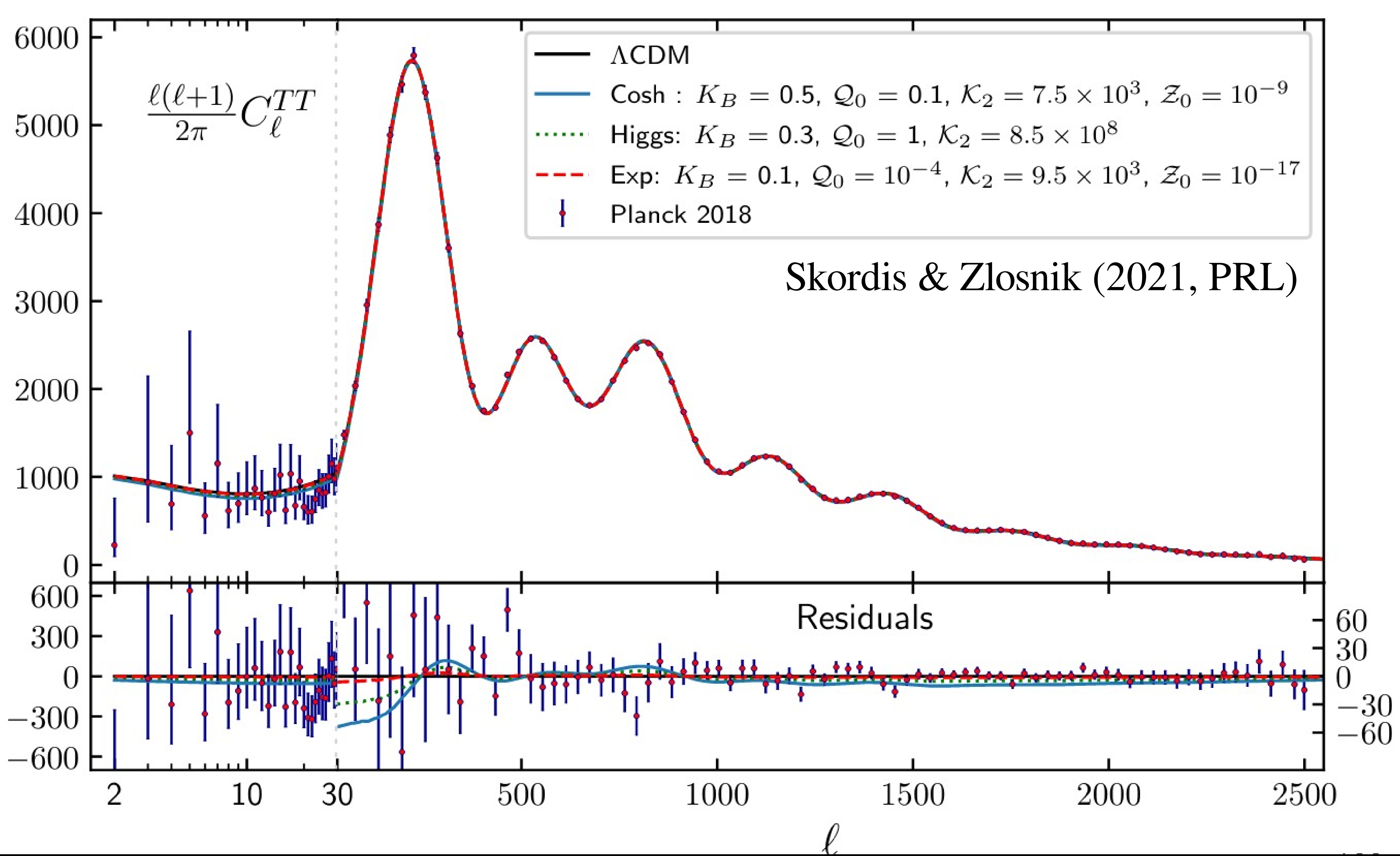
Add gravitational DoFs:

Scalar field or vector field or both

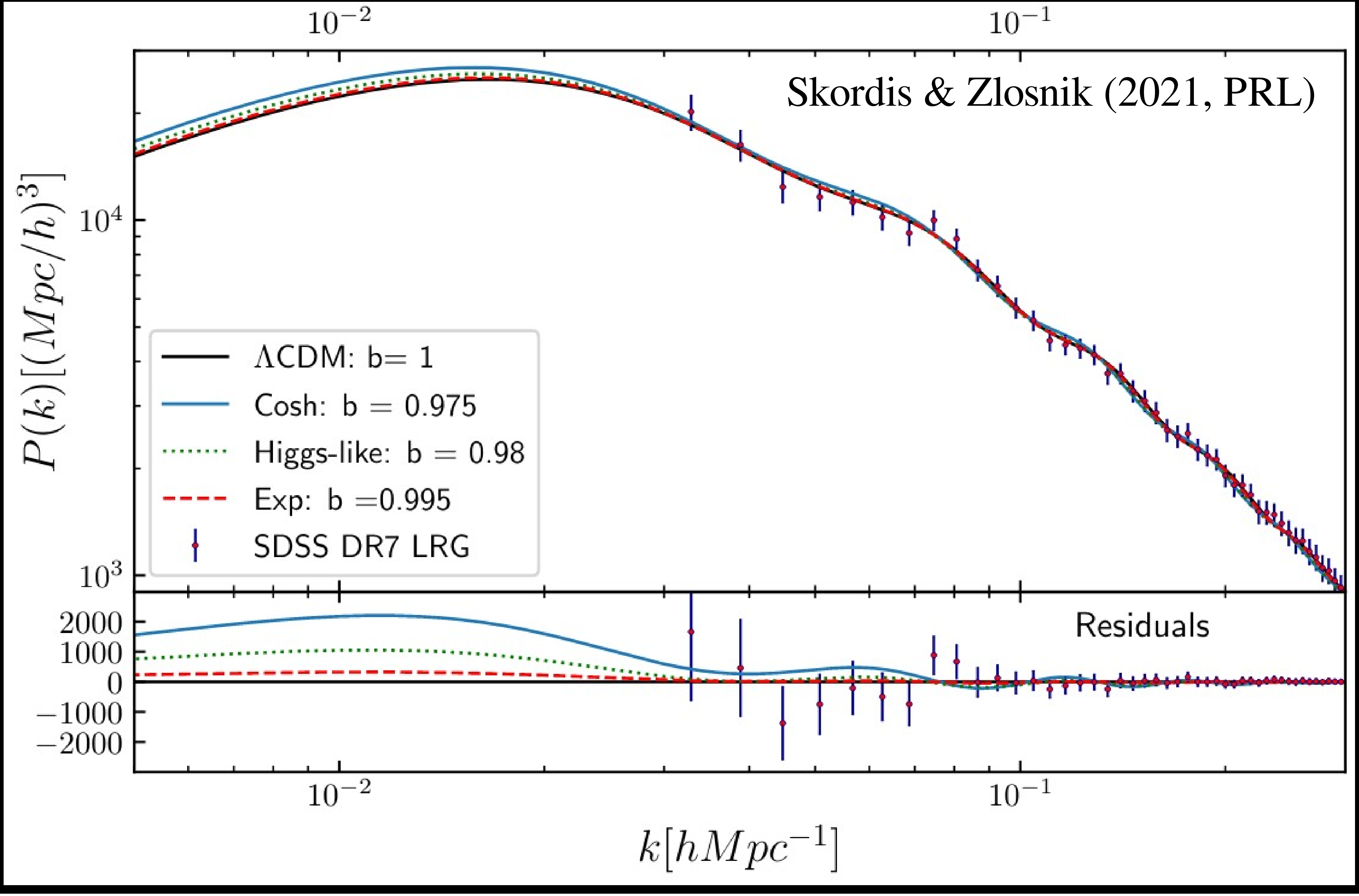
AeST (Skordis & Zlosnik 2021)

Khronon (Blanchet & Skordis 2024)

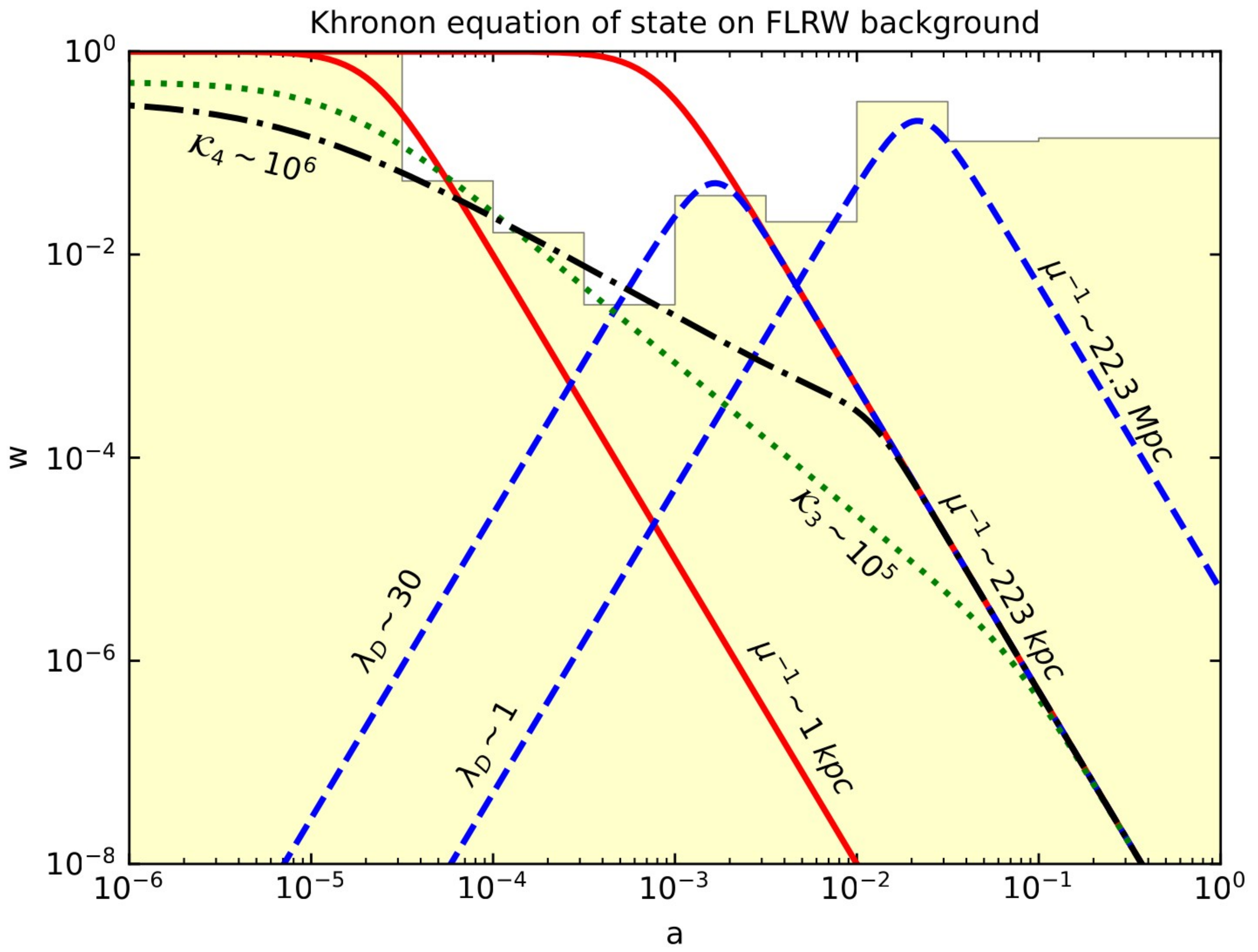
CMB Power Spectrum (from AeST or Khronon)



Linear Matter Power Spectrum (from AeST or Khronon)



CMB constraints on the cosmic evolution of $P_K = \omega_K \rho_K$

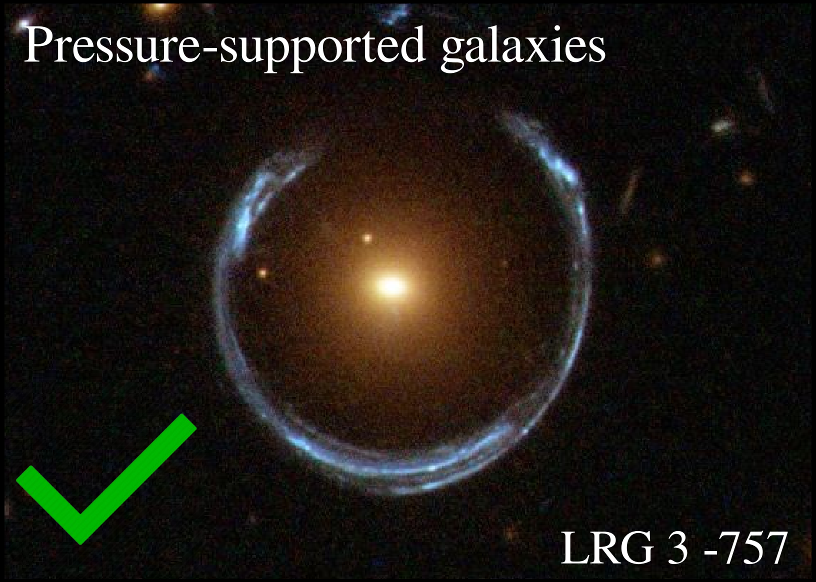


Dust-like solutions from shift-symmetric k-essence (like a ghost condensate)

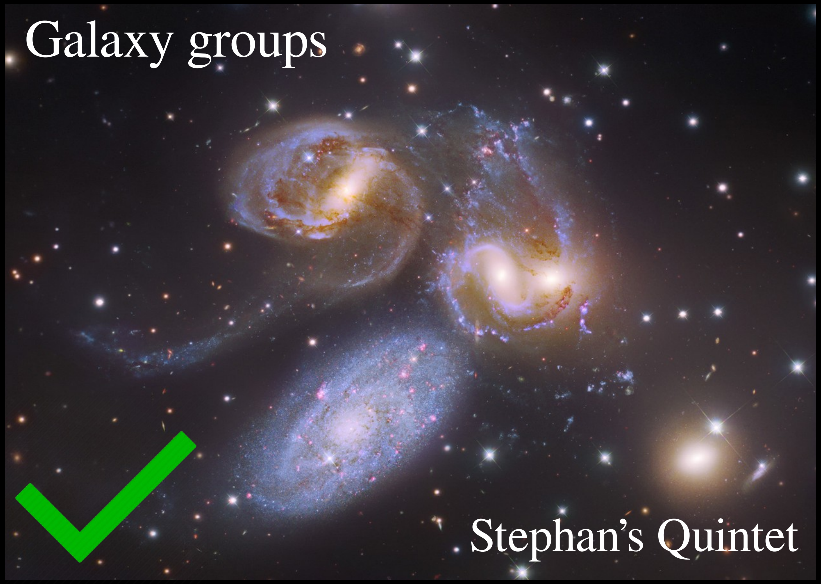
MOND White Paper Section 5

Summary: Status of MOND at Various Scales

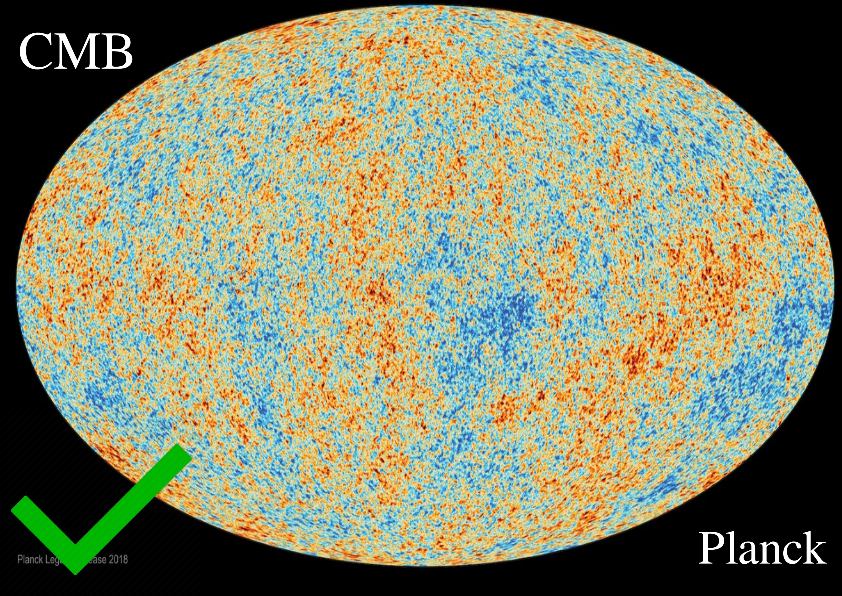
Galaxy Scales (~1-100 kpc)

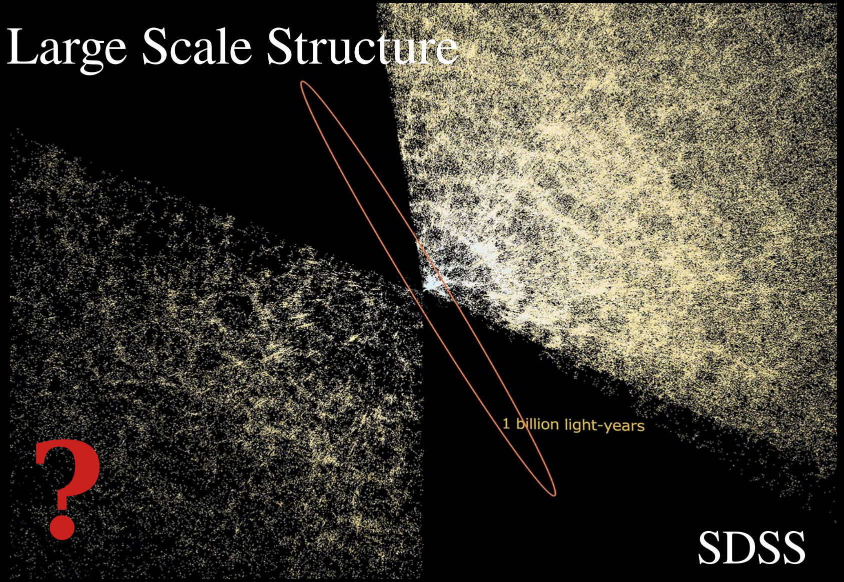


Groups/Clusters Scales (~1-3 Mpc)



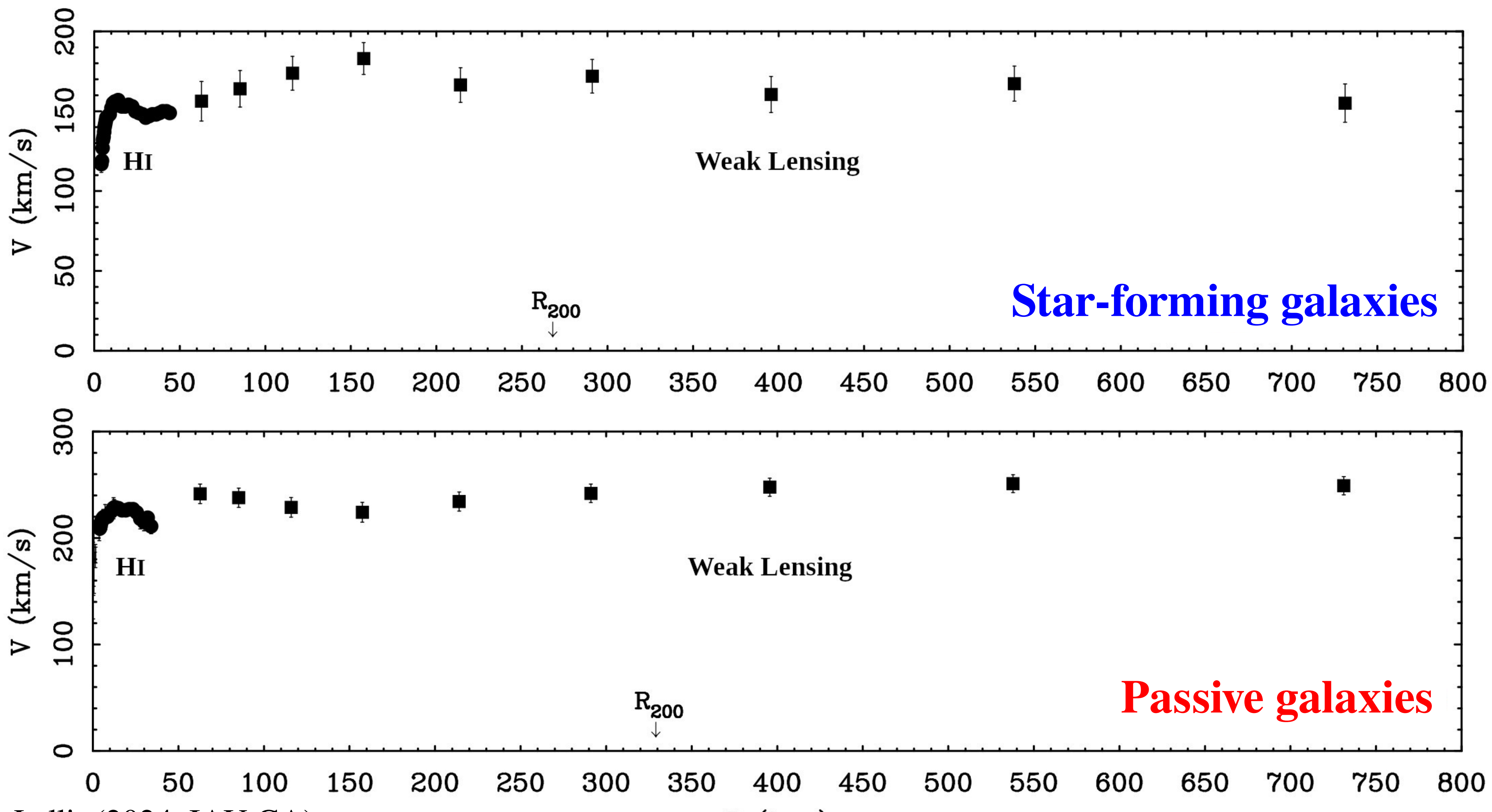
Cosmological Scales (>100 Mpc)





More Slides

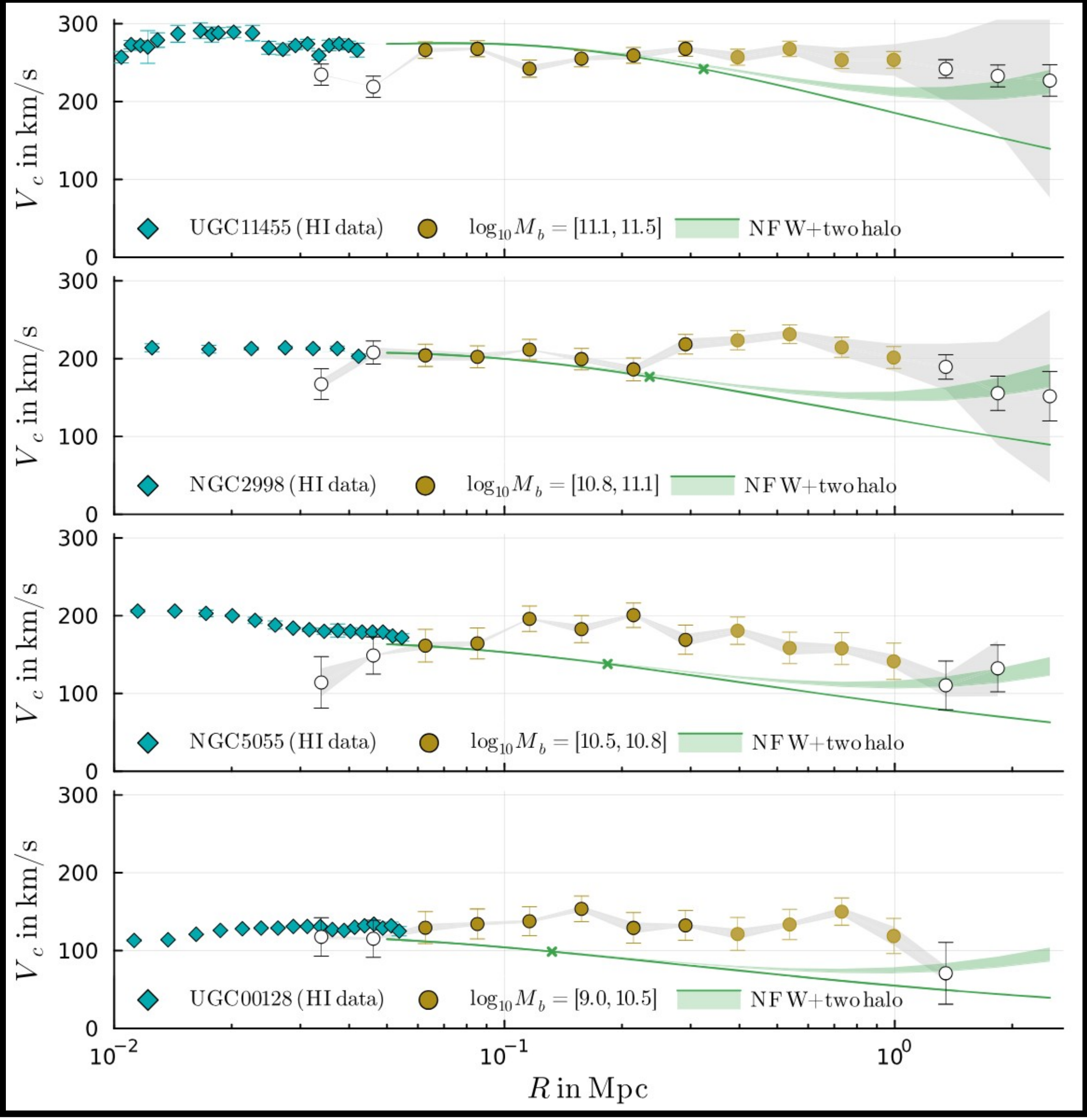
(1) $V_c(R)$ = const at large R for isolated objects → Weak Lensing



Lelli+(2024, IAU GA)

Tobias Mistele

Circular-velocity curves from Weak Lensing vs ΛCDM predictions



Tobias Mistele

Binning by baryonic mass:

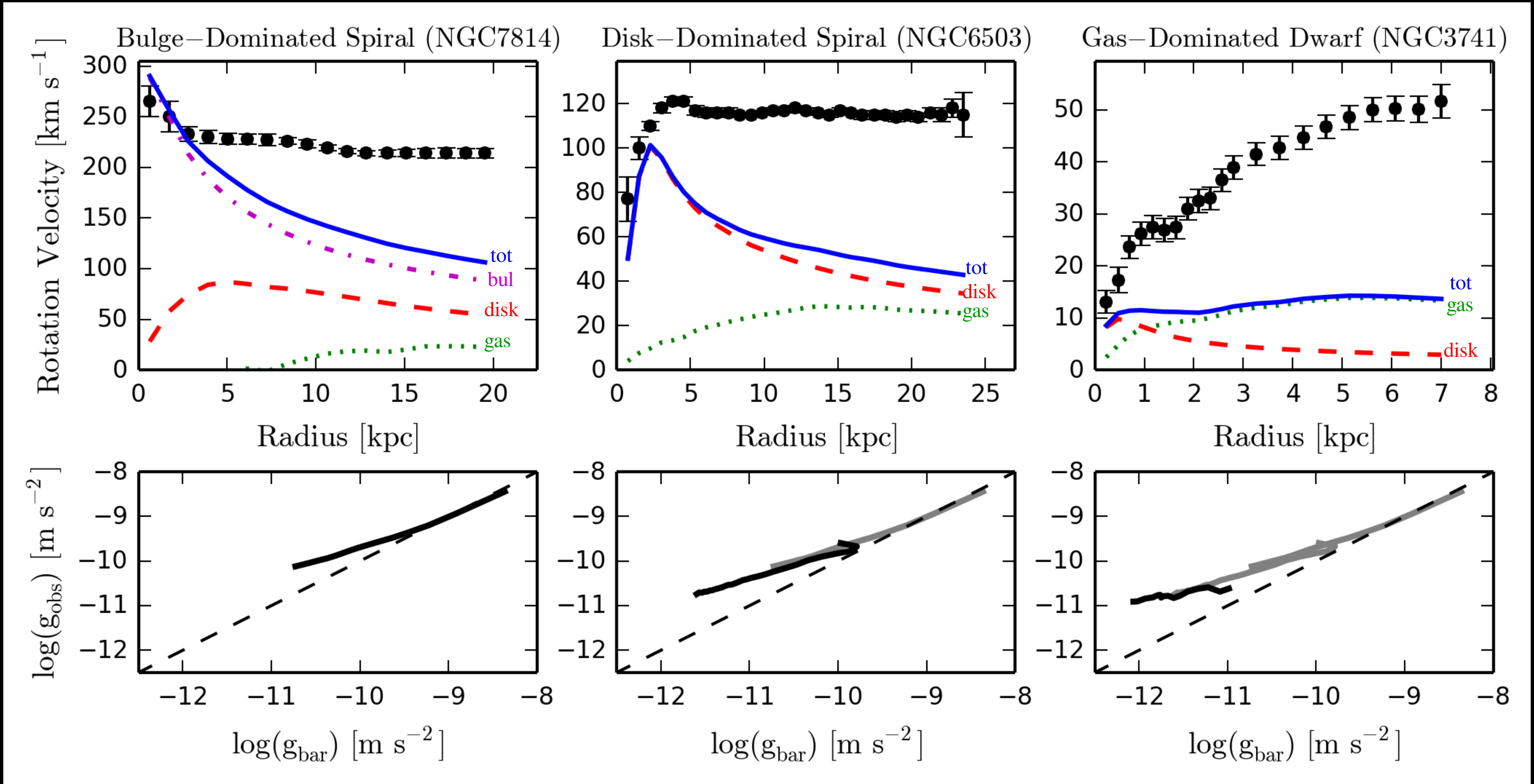
Circular-velocity curve stays flat beyond the “virial radius” of the DM halo:

→ In contradiction with ΛCDM predictions

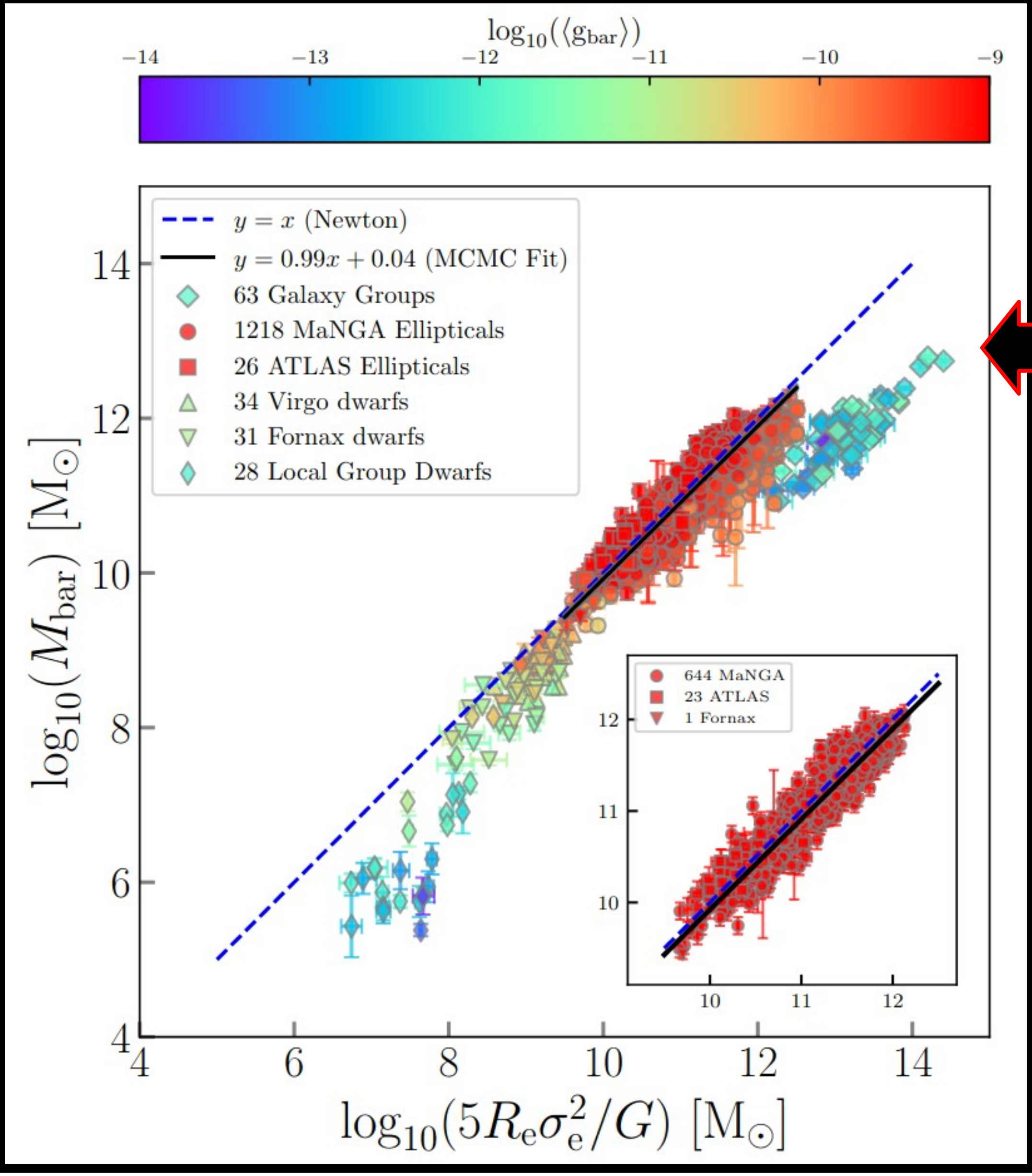
→ In agreement with MOND predictions

Mistele+(2024b, ApJ)

Galaxies lie on the same RAR *despite* their diversity



Two Virial theorems for different acceleration regimes:

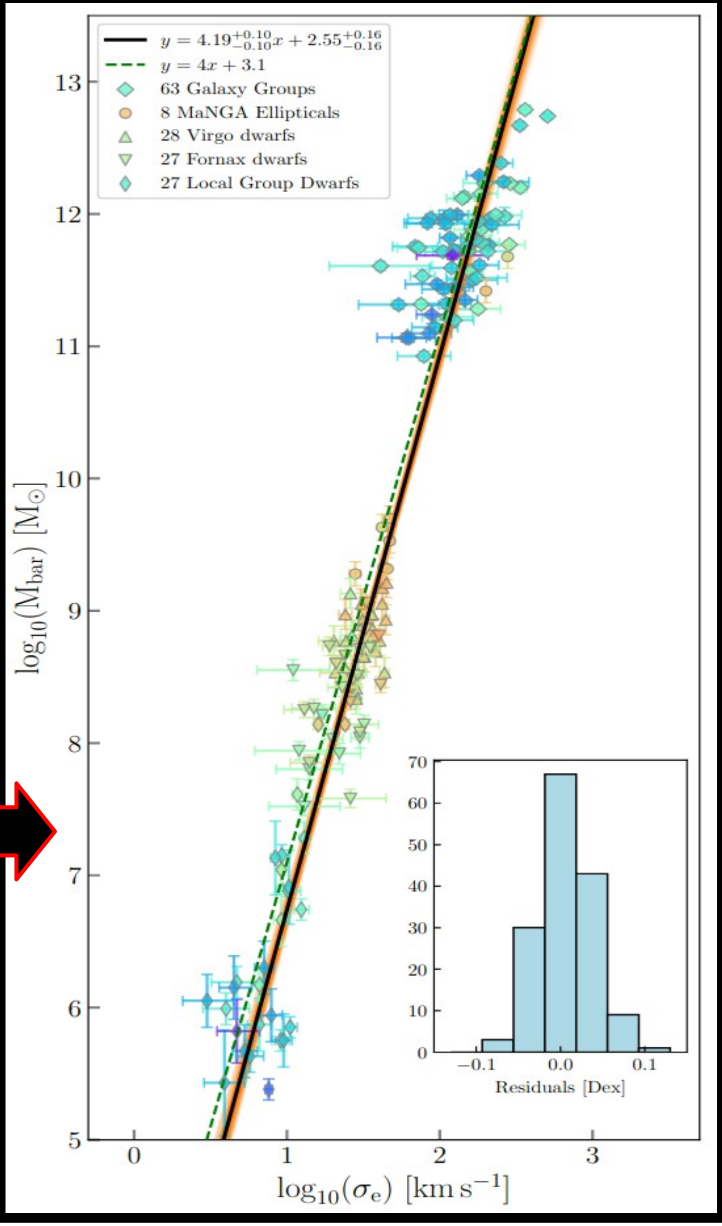


For $g_b \gg a_0$: Newton

$$\sigma_V^2 = \frac{G\,M_b}{5\,R_e}$$

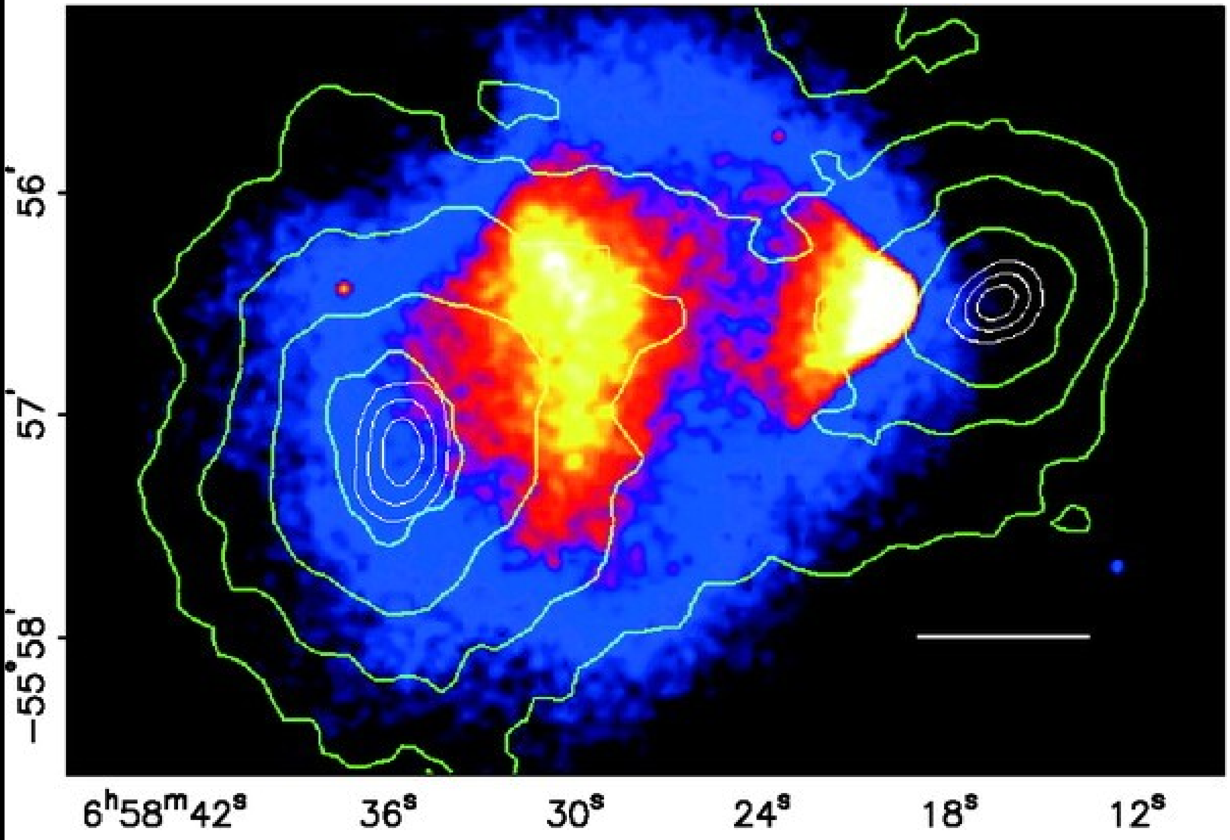
For $g_b \ll a_0$: MOND

$$\sigma_V^4 = \frac{a_0 G}{c} M_b$$



The bullet cluster in MOND: missing mass must be collisionless

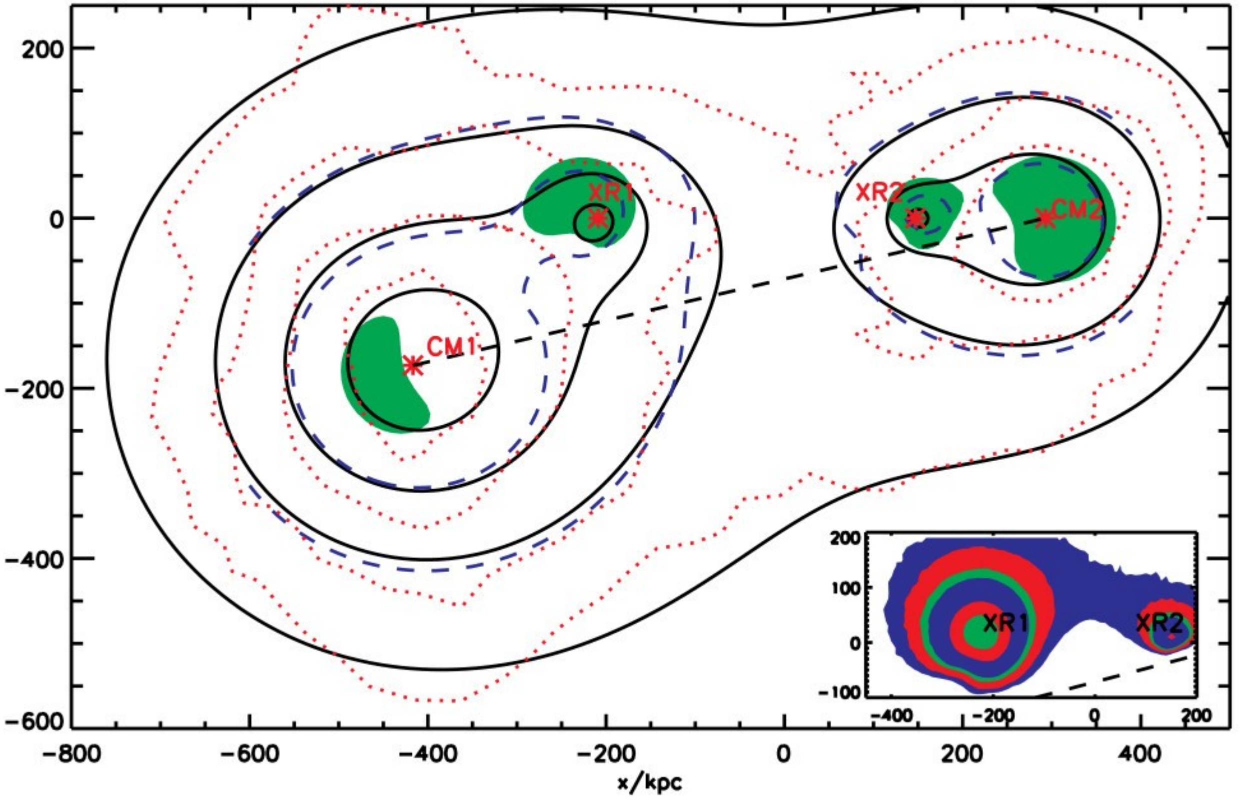
OBSERVATIONS (Clowe+2004, ApJ)



Green: Observed lensing map (total mass)

Blue/**Red**/**Yellow**: X-ray emission (hot gas)

MOND (Angus+2006, MNRAS; Angus+2007, ApJ)

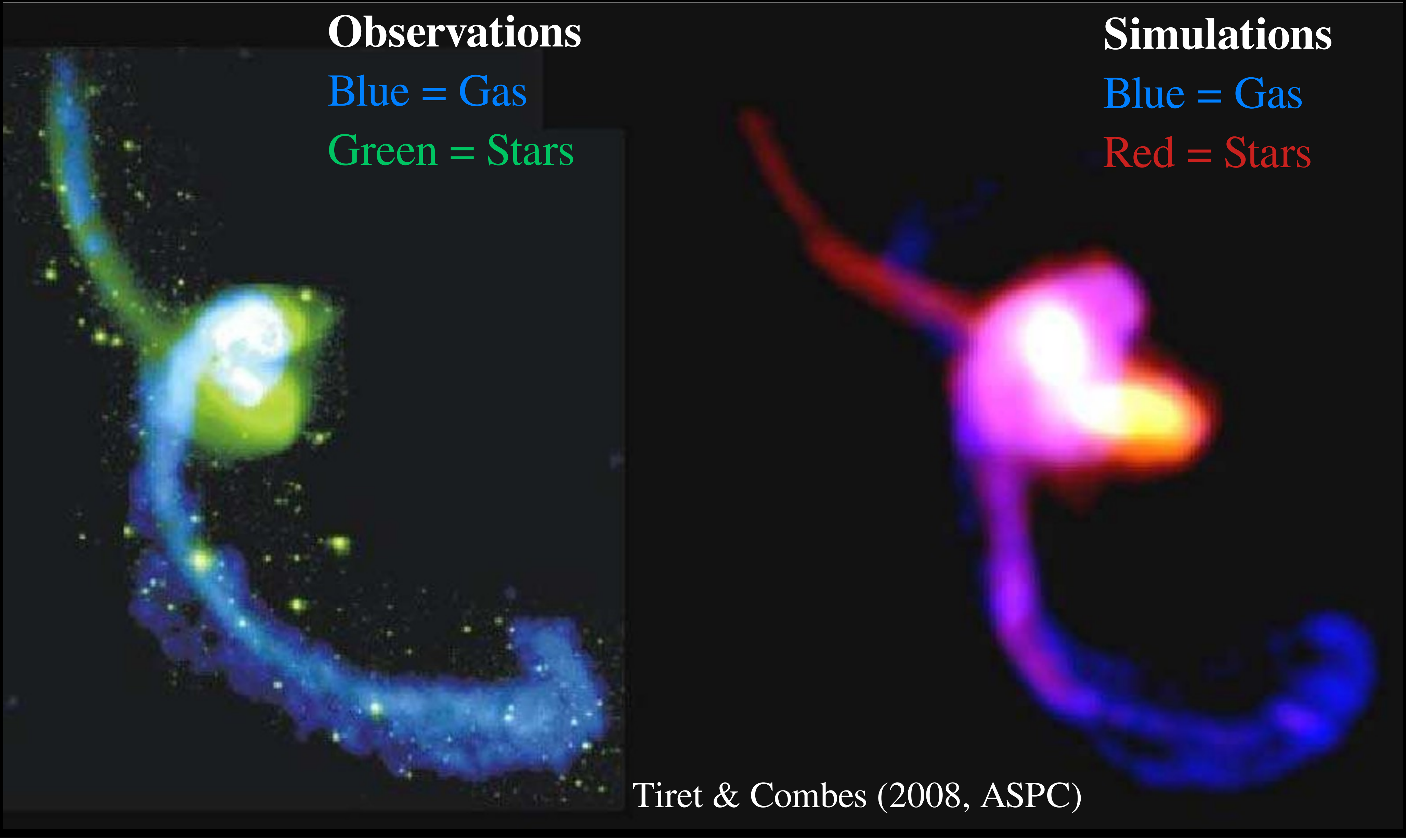


Red: Observed lensing convergence map

Black: TeVeS model with 2eV neutrinos

Blue: total surface densities (baryons+υ)

MOND modified gravity: simulations of interacting galaxies



Cosmology with modified Newtonian dynamics (MOND)

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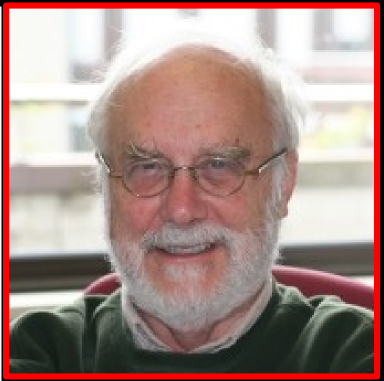
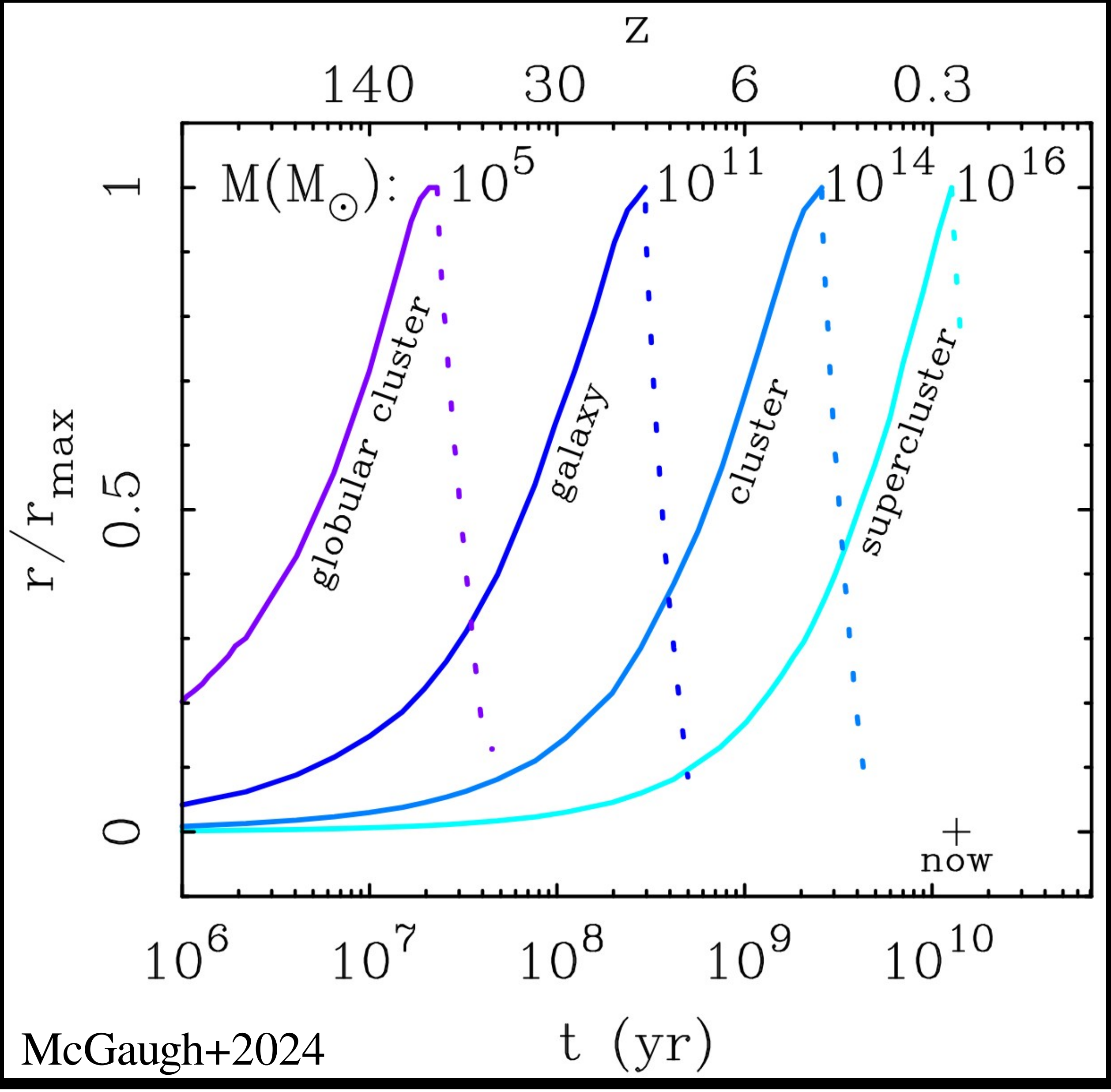


Sanders (1998, MNRAS, 296)

ABSTRACT

It is well known that the application of Newtonian dynamics to an expanding spherical region leads to the correct relativistic expression (the Friedmann equation) for the evolution of the cosmic scalefactor. Here, the cosmological implications of Milgrom's modified Newtonian dynamics (MOND) are considered by means of a similar procedure. Earlier work by Felten demonstrated that in a region dominated by modified dynamics the expansion cannot be uniform (separations cannot be expressed in terms of a scalefactor) and that any such region will eventually recollapse regardless of the initial expansion velocity and mean density. Here I show that, because of the acceleration threshold for the MOND phenomenology, a region dominated by MOND will have a finite size which, in the earlier Universe ($z > 3$), is smaller than the horizon scale. Therefore, uniform expansion and homogeneity on the horizon scale are consistent with MOND-dominated non-uniform expansion and the development of inhomogeneities on smaller scales. In the radiation-dominated era, the amplitude of MOND-induced inhomogeneities is much smaller than that implied by observations of the cosmic background radiation, and the thermal and dynamical history of the Universe is identical to that of the standard big bang model. In particular, the standard results for primordial nucleosynthesis are retained. When matter first dominates the energy density of the Universe, the cosmology diverges from that of the standard model. Objects of galaxy mass are the first virialized objects to form (by $z = 10$), and larger structure develops rapidly. At present, the Universe would be inhomogeneous out to a substantial fraction of the Hubble radius.

MOND prediction, following Sanders (1998)



Evolution of a MOND spherical region in an expanding Universe (Felten 1984):

$$V^2(t) = V_i^2 - \sqrt{\frac{16\pi}{3} a_0 G \rho_b r_0^3} \ln\left(\frac{r(t)}{r_i}\right)$$

Any spherical region will initially expand, then unavoidably recollape due to gravity:

- Structures form faster than in ΛCDM
- Late-time Universe is anisotropic (or homogeneity scale is very large)

MOND – Cosmology Connection?

Two numerical coincidences (Milgrom 1983a, ApJ; Milgrom 1999, PhLA):

$$a_0 \simeq \frac{H_0 \cdot c}{2\pi}$$

H_0 = Hubble constant → maybe a_0(t) ~ H(t) ?

$$a_0 \simeq \frac{c^2 \sqrt{\Lambda/3}}{2\pi}$$

Λ = Cosmological constant → relation to Dark Energy?

IF this numerology has some deeper, fundamental meaning:

Either the state of the Universe at large enters in local dynamics, or

the same parameters enters both Cosmology (Λ) and local dynamics (a_0).

Let's start from the Newtonian non-relativistic Action

$$S=\int dt\,L=\int dt\left(L_{matter}+L_{gravity}+L_{coupling}\right)=\int dt\,d^3x\left(\rho\frac{V^2}{2}-\frac{|\vec{\nabla}\Phi|^2}{8\pi G}-\rho\,\Phi\right)$$

Change this for modified inertia

Change this for modified gravity

Changing this modify both

Principle of Least Action:

$$\frac{\delta S}{\delta\Phi}=0 \rightarrow \nabla^2\Phi=4\pi G\rho$$ Poisson's equation

$$\frac{\delta S}{\delta\vec{x}}=0 \rightarrow \vec{a}=-\vec{\nabla}\Phi$$ Newton's 2nd Law

MOND as Modified Inertia (Milgrom 1994, 1999, 2022)

$\vec{A}[\vec{x}(t); a_0] = -\vec{\nabla}\Phi_N$

$\bar{A}$ is a functional of the full trajectory $\bar{x}(t)$

For $a \gg a_0$, $A \rightarrow a = d^2x/dt^2$ (Newton's 2nd law)

No full theory yet, but two general results:

(1) Imposing Newtonian and MOND limits + Galilei Invariance $\vec{x}(t) \rightarrow \vec{x}(t) + \vec{v}_0 t$

Theory is time non-local: $\vec{A}[\vec{x}(t), a_0] \neq F\left(\frac{d^i \vec{x}}{dt^i}; i = 1, 2, \ldots N\right)$

Accelerations at $(\bar{x}, t)$ depend on the full orbital history!

(2) For purely circular orbits: $\vec{a}\,\mu\left(\frac{a}{a_0}\right) = \vec{g}_N$ holds exactly (RAR for disk galaxies)

The interpolation function is a derived concept valid for circular orbits!

Khronon = Tensor – Scalar Theory (Blanchet & Skordis 2024)

Tensor $g_{\mu\nu} \rightarrow$ Einstein's metric (universally coupled to matter fields Ψ)

Scalar $\tau \rightarrow$ Khronon field (units of time; drive a foliation in 3D space-like hypersurfaces)

$$n_\mu = -\frac{c}{Q} \nabla_\mu \tau \qquad Q = c\sqrt{-g^{\mu\nu} \nabla_\mu \tau \nabla_\nu \tau}$$

Dimensionless unit timelike ($n^\mu n_\mu = -1$) vector, pointing in the future ($n^0 > 0$).

$$A_\mu = c^2 n^\nu \nabla_\nu n_\mu$$

Covariant spacelike acceleration

$$Y = \frac{A_\mu A^\mu}{c^4}$$

Squared "acceleration" with units of $1/L^2$

$$S = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g} \left[R - 2I(Y) + 2K(Q) \right] + S_m[\Psi, g]$$

Ricci Scalar (as in GR)

Free function to get MOND

Free function to get cosmology

Matter Action

Khronon Field Equations (Blanchet & Skordis 2024)

$$G^{\mu\nu} + (I - K)\, g^{\mu\nu} - \frac{2}{c^4}\frac{dI}{dY} A^{\mu} A^{\nu} + \left[\frac{2}{c^4}\nabla_{\rho}\left(\frac{dI}{dY} A^{\rho}\right) - Q\frac{dK}{dQ}\right] n^{\mu} n^{\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

$$T_{\tau}^{\mu\nu} \stackrel{\text{def}}{=} \frac{c^4}{8\pi G}\left(-(I - K)\, g^{\mu\nu} + \frac{2}{c^4}\frac{dI}{dY} A^{\mu} A^{\nu} - \left[\frac{2}{c^4}\nabla_{\rho}\left(\frac{dI}{dY} A^{\rho}\right) - Q\frac{dK}{dQ}\right] n^{\mu} n^{\nu}\right)$$

The Khronon stress-energy tensor contains $g_{\mu\nu}$ explicitly and implicitly in Y and Q.

This is very different from adding DM to GR!

Khronon = Tensor – Scalar Theory (Blanchet & Skordis 2024)

$$S=\frac{c^3}{16\pi G}\int d^4x\sqrt{-g}\left[R-2I(Y)+2K(Q)\right]+S_m[\Psi,g]$$

- Cosmology: 3D space is homogeneous & isotropic → Y = 0 & Q = dτ/dt

$$K(Q)=\mu_Q^2(Q-1)^2+\ \dots\ \Longrightarrow$$

It gives dust-like behaviour: $\rho_K \propto a(t)^{-3}$ like CDM

$\mu_Q =$ constant with dimension of inverse length

- Weak-field stationary limit: no time dependence → $A_0 \sim 0$; $A_i \sim \nabla_i\Phi$

$$\nabla\cdot\left[\underbrace{\left(1+\frac{dI}{dY}\right)}_{\mu(y)}\vec{\nabla}\Phi\right]+\mu_Q^2\Phi=4\pi G\rho_m \Longrightarrow \text{AQUAL} + \mu_Q^2\Phi \quad \mu_Q^{-1}\gg 1\text{ Mpc}$$

$\mu(y)=\mu(|\nabla\Phi|/a_0)$

$y=|\nabla\Phi|/a_0$

$for\ y\ll 1\quad \mu(y)=y \quad\Rightarrow\quad I(Y)=\Lambda-Y+\frac{2c^2}{3a_0}Y^{3/2}+O\left(\frac{Y^4}{a_0^2}\right)$

$for\ y\gg 1\quad \mu(y)=1 \quad\Rightarrow\quad I(Y)=\Lambda_\infty+O(a_0/y)$

Oscillations in Khronon / AeST ?