# Black Holes in Alternative Theories of Gravity

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The Sixth Zeldovich Meeting

# Outline

1. Introduction
2. EdGB BHs
3. EsGB BHs
4. EsGB+R BHs
5. Conclusions

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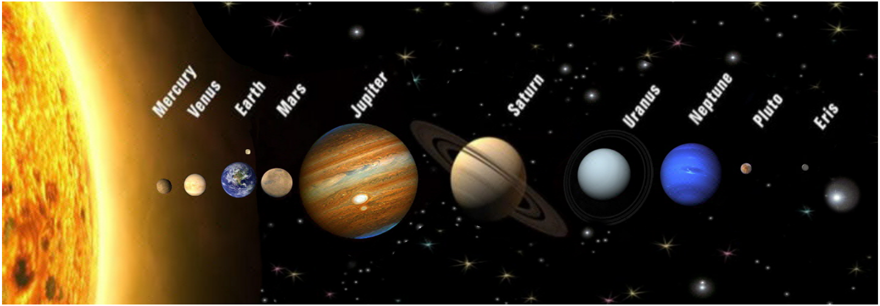
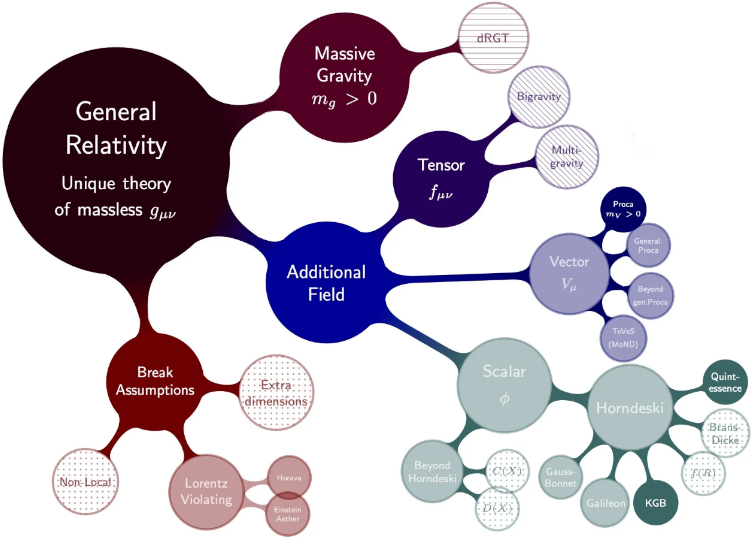
# Alternative Theories of Gravity




- Compatible with all solar system tests!
- Strong gravity?
  - Black holes
  - Neutron stars
  - Exotic compact objects
- Cosmology?

# Alternative Theories of Gravity



# Einstein-scalar-Gauss-Bonnet Theories

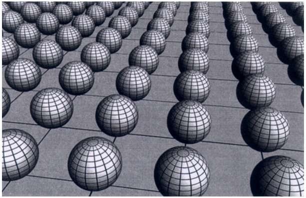
EsGB action

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial_\mu\varphi)^2 + f(\varphi)R^2_{\rm GB}\right]$$

Gauss-Bonnet term: quadratic in the curvature

$$R^2_{\rm GB} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

coupling function $f(\varphi)$



The resulting set of equations of motion are of second order (Horndeski).

# Einstein-scalar-Gauss-Bonnet Theories

generalized Einstein equations

$$\begin{aligned} G_{\mu\nu} &= -\frac{1}{4} g_{\mu\nu} \partial_\rho \varphi \partial^\rho \varphi + \frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi \\ &- \frac{1}{2} \left( g_{\rho\mu} g_{\lambda\nu} + g_{\lambda\mu} g_{\rho\nu} \right) \eta^{\kappa\lambda\alpha\beta} \tilde{R}^{\rho\gamma}{}_{\alpha\beta} \nabla_\gamma \partial_\kappa f(\varphi) \end{aligned}$$

scalar equation

$$\nabla_\mu \nabla^\mu \varphi + \frac{df}{d\varphi} R^2_{\mathrm{GB}} = 0$$

crucial: choice of coupling function $f(\varphi)$

- GR black hole solutions do not remain solutions
  $\Longrightarrow$ only hairy black holes result
- GR black hole solutions do remain solutions
  $\Longrightarrow$ in addition scalarized black holes emerge

# Outline

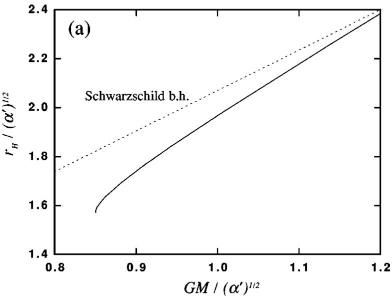


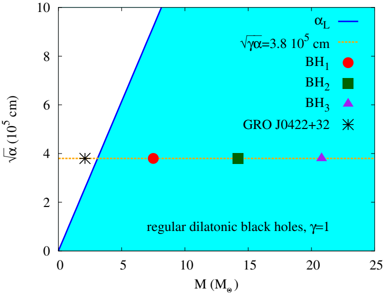
# EdGB black holes: properties

Kanti et al. 9511071, Torii et al. 9606034, Blázquez-Salcedo et al. 1706.03262

dilatonic coupling function $f(\varphi) = \frac{\alpha}{4} e^{-\gamma\varphi}$

static black holes



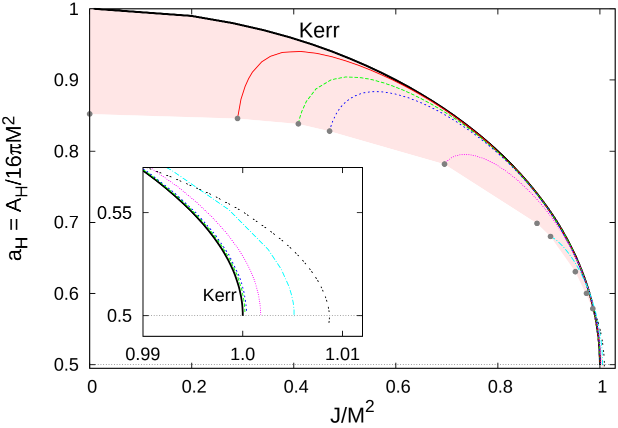

critical black holes:
lower bound on $M$

upper bound on the coupling

# EdGB black holes: properties

Kleihaus et al. 1101.2868

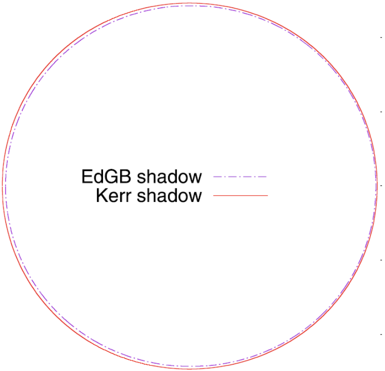
horizon area versus angular momentum



# EdGB black holes: properties

Cunha et al. arXiv:1701.00079

shadow

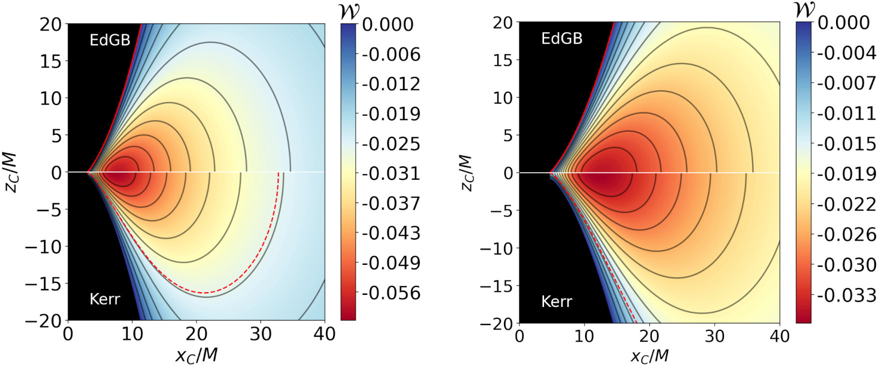



$$\alpha/M^2 = 0.172, \;\; J/M^2 = 0.41$$

# EdGB black holes: properties

Gjorgjieski et al. in preparation

Polish doughnuts: EdGB (upper) vs Kerr (lower)




prograde

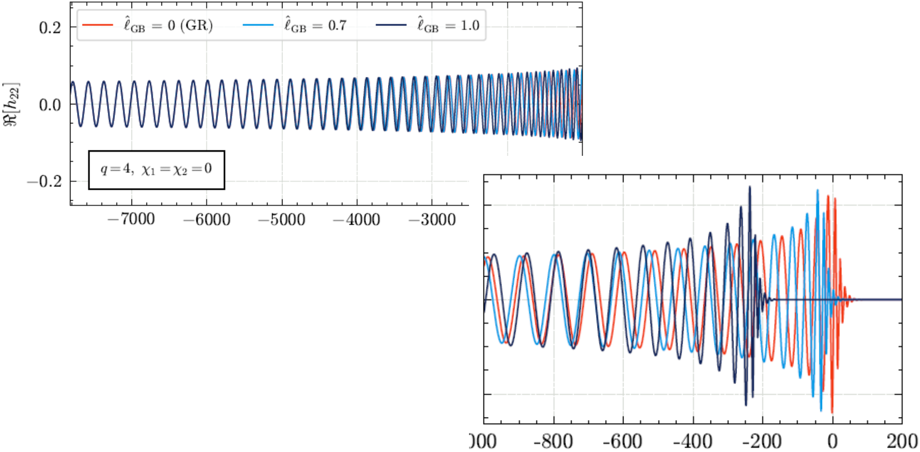
retrograde

$j = 0.4,\ \alpha = 0.2$

# EdGB black holes: effective-one-body formalism

Julié et al. 2406.13654

Inspiral-merger-ringdown waveforms in EdGB



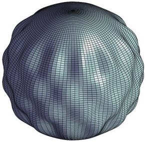
# EdGB black holes: QNMs of BHs

perturbation theory: damped oscillations
metric

$$g_{\mu\nu} = g^{(0)}_{\mu\nu}(r) + \epsilon h_{\mu\nu}(t, r, \theta, \phi)$$

scalar

$$\phi = \varphi_0(r) + \epsilon \delta\varphi(t, r, \theta, \phi)$$



polar(-led) modes: (leading) even-parity perturbations
axial(-led) modes: (leading) odd-parity perturbations

eigenvalue equation: static ODEs, stationary PDEs
eigenvalue $\omega$

$$\omega = \omega_R + i\omega_I$$

frequency: $\omega_R$ decay time: $\tau = 1/\omega_I$

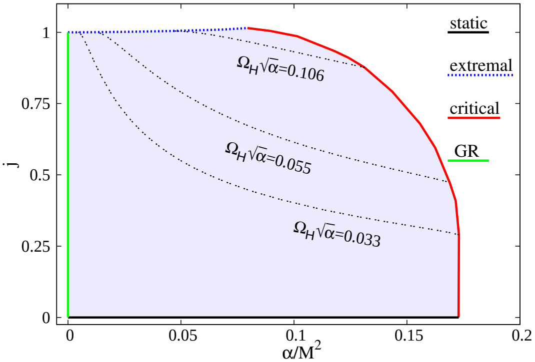
# EdGB black holes: QNMs of rotating BHs

Blazquez-Salcedo et al. arXiv: 2407.20760

numerical methods for rapid rotation

- spectral decomposition

  $r$ in Chebyshev polynomials, $\theta$ in Legendre functions



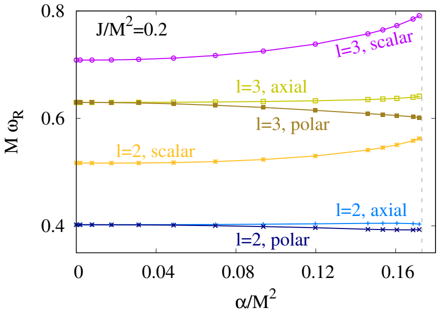

limited domain of existence of background black hole solutions

# EdGB black holes: QNMs of rotating BHs

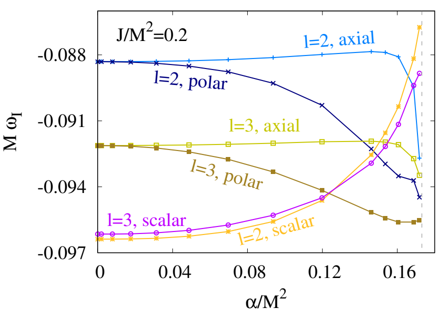
Blázquez-Salcedo et al. 2407.20760, 2412.17073

polar-led, axial-led, and scalar-led QNMs versus coupling constant $\alpha/M^2$

$$J/M^2 = 0.2$$




real part



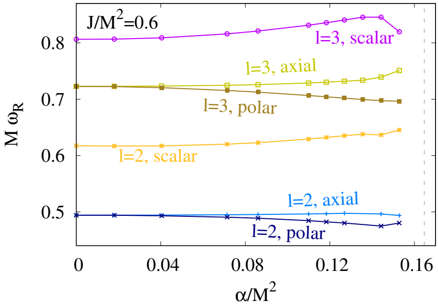

imaginary part

# EdGB black holes: QNMs of rotating BHs

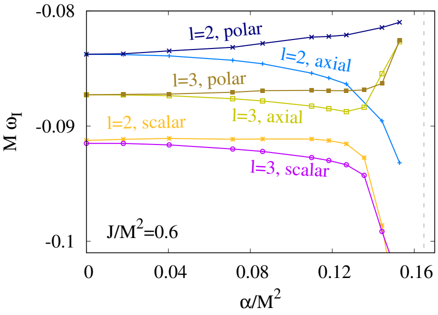
Blazquez-Salcedo et al., 2407.20760, 2412.17073

polar-led, axial-led, and scalar-led QNMs versus coupling constant $\alpha/M^2$

$$J/M^2 = 0.6$$




real part




imaginary part

# Outline



# Spontaneously scalarized black holes

Doneva et al. 1711.01187, Silva et al. 1711.02080, Antoniou et al. 1711.03390
Cunha et al. 1904.09997, Collodel et al. 1912.05382, Dima et al. 2006.03095

scalar equation

$$\nabla_\mu \nabla^\mu \varphi + \frac{df}{d\varphi} R^2_{\rm GB} = 0$$

Gauss-Bonnet: Kerr

$$R^2_{\rm GB} = \frac{48M^2}{(r^2+\chi^2)^6}\left(r^6 - 15r^4\chi^2 + 15r^2\chi^4 - \chi^6\right) \ , \quad \chi = a\cos\theta$$

effective mass

$$m^2_{\rm eff}(r) = -\eta R^2_{\rm GB} < 0$$

- $\eta > 0$

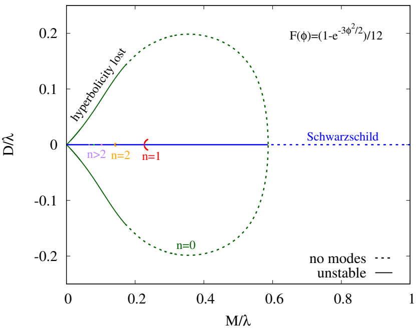
  $\Longrightarrow$ spin suppresses scalarization

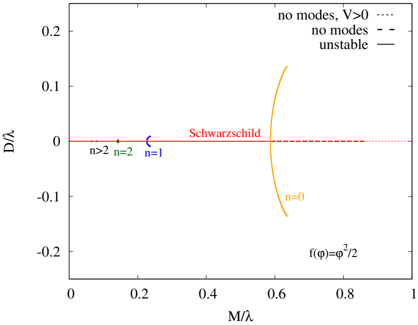
- $\eta < 0$

  $\Longrightarrow$ spin induces scalarization

# Spontaneously scalarized black holes

Blazquez-Salcedo et al. 1805.05755




coupling function

$$f(\varphi) = \frac{1}{12}\left(1 - e^{-6\varphi^2}\right)$$

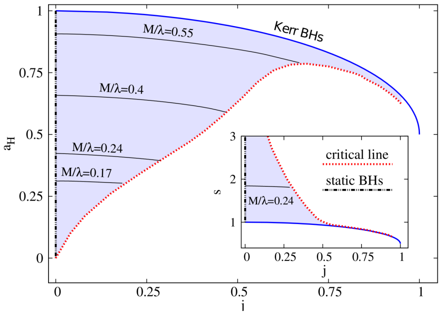
coupling function

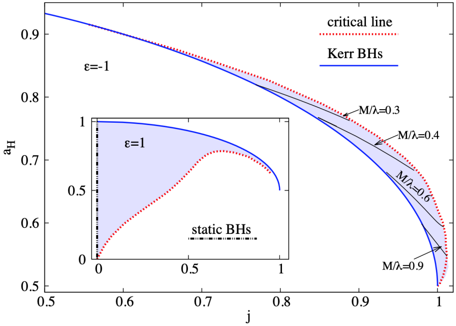
$$f(\phi) = \frac{\phi^2}{2}$$

# Spontaneously scalarized black holes

Cunha et al. arXiv:1904.09997, Herdeiro et al. arXiv:2009.03904

coupling function
$$f_1(\varphi) = \frac{1}{12}\left(1 - e^{-6\varphi^2}\right)$$



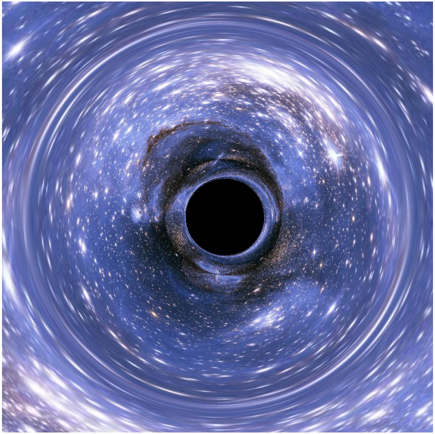

area/entropy vs angular momentum $\alpha > 0$

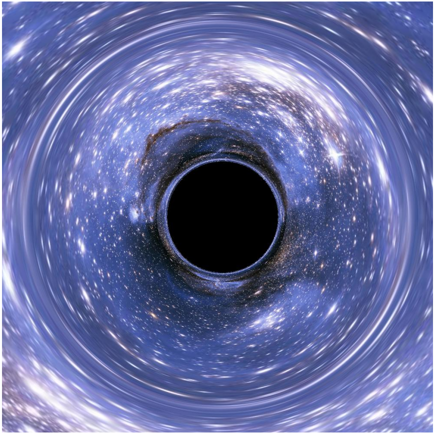
area vs angular momentum $\alpha < 0$

QNMs: arXiv:2411.19069?

# Spontaneously scalarized black holes

Cunha et al. arXiv:1904.09997





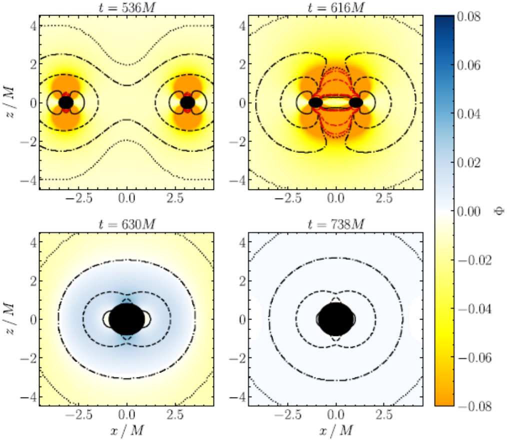
EsGB

Kerr

$$M/\lambda = 0.237 (j = 0.24)$$

# Dynamical descalarization of scalarized black holes

Elley et al. arXiv:2205.06240




spin-induced dynamical scalarization and descalarization

# Outline



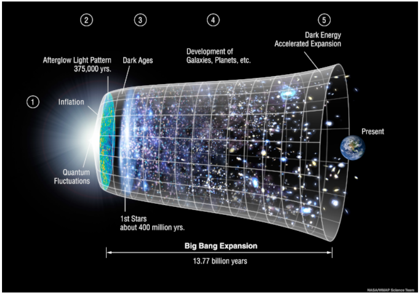
# Einstein-scalar-Gauss-Bonnet with Ricci coupling

Antoniou et al. 2004.14985, Antoniou et al. 2105.04479, ...
Compact object scalarization with general relativity as a cosmic attractor

EsGB with Ricci action

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial_\mu\varphi)^2 + \frac{\varphi^2}{2}\left(\alpha R^2_{\rm GB} - \frac{\beta}{2}R\right)\right]$$

coupling function $f(\varphi) = \frac{\varphi^2}{2}$, tachyonic instability: independent of $\beta$ $(R = 0)$



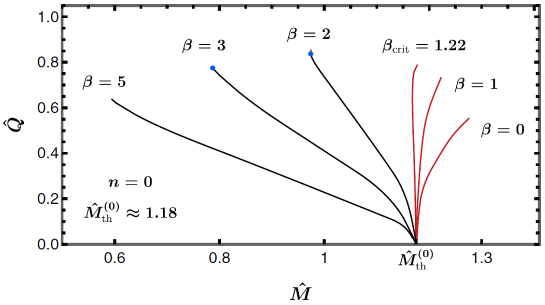
# Static curvature induced scalarized black holes

Antoniou et al. 2105.04479

domain of existence of spontaneously scalarized static black holes

$$m_{\rm eff}^2 = \frac{\beta}{2} R - \alpha R_{\rm GB}^2 < 0$$

tachyonic instability: independent of $\beta$ ($R = 0$)

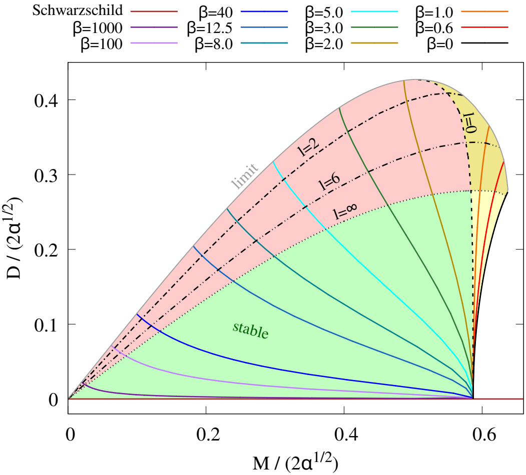



scaled scalar charge vs scaled mass for varying Ricci coupling $\beta$

# Non-radial instabilities of static scalarized black holes

Blázquez-Salcedo et al. 2607.07208

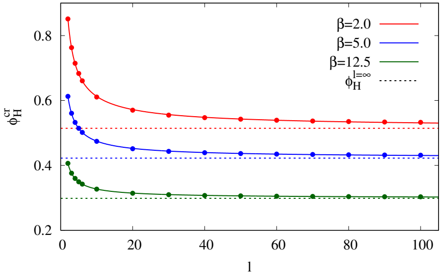
Ricci coupling




hierarchy of angular instabilities: $l = 2$, $l = 3$, $l = 4$, $l = 5$,...

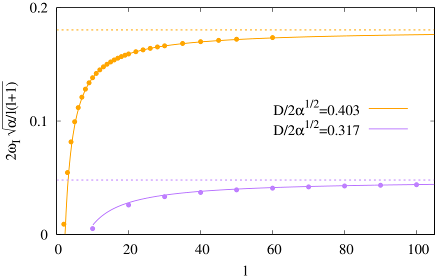
# Non-radial instabilities of static scalarized black holes

Blázquez-Salcedo et al. 2607.07208




onset values $\phi_H^{\rm cr}(l)$ for 3 values of $\beta$
the curves are fits to

$$\phi_H^{\rm cr}(l) = \phi_H^{l=\infty} + A\, l^{-p}$$

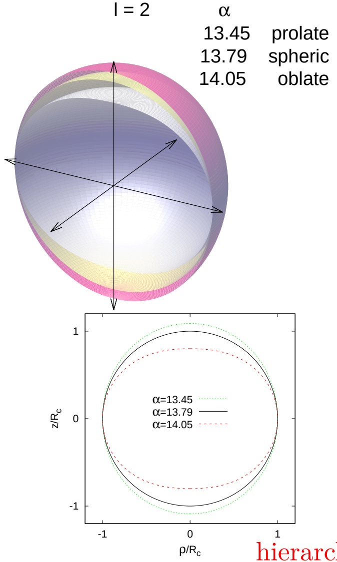


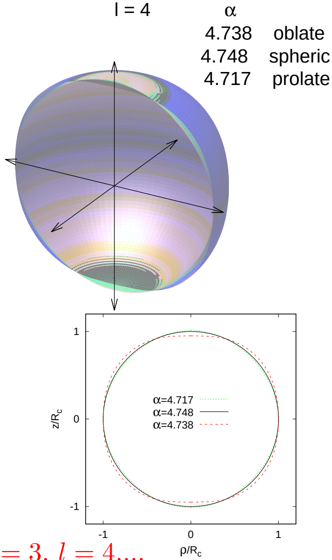

eikonal scaling of unstable modes
2 solutions on $\beta = 2$ branch

$$\frac{\omega_I(l)}{\sqrt{l(l+1)}} = a + \frac{b}{l}$$

# Non-radial instabilities of static scalarized black holes

Kleihaus et al. 2303.04107




hierarchy: $l = 2$, $l = 3$, $l = 4$,...

# Outline

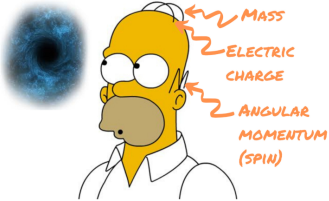


# Conclusions

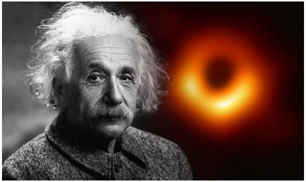
GR versus generalized gravity theories

## GR black holes

- Kerr: no hair
- gravitational waves
- shadow
- ...




## black holes beyond GR: hair



- EdGB
  - GW: QNMs, EOB approach
  - perturbatively stable
  - shadow
- EsGB, EsGB+Ricci
  - GW: static
  - stability?
  - ...

# THANKS