Quarks and Nucleons:

A General Relativistic Perspective on Strong Forces

Unifying the gravitational and strong nuclear forces at the femtometer scale by demonstrating how GR-curved spacetime confines spin-1/2 Dirac quarks.

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A thump rule to begin with:

Quarks inside nucleons are not point particles.

To comply with Heisenberg, they have to be waves.

They have to have a wavelength λ of order *fm* or smaller.

A wave of about 0.65 *fm* gives the energy

$$\hbar\omega \approx \hbar \mathrm{kc} = \hbar \mathrm{c}/\lambda \approx 300\ MeV$$

For 3 quarks $3\hbar\omega = 900\ MeV$, almost emergent mass of nucleons.

The foundational questions addressed in this study:

1. There is no established wave function for hadrons or constituent quarks that parallels standard wave equations (e.g., Dirac waves for electrons).
2. No definitive understanding of the exact physical nature of the strong forces exists, except that they are strong and confining.

We assume:

a. Quarks as spin-½ particles are Dirac spinors.
b. Quarks as massive entities curve their surroundings.

Core Research Thesis

We demonstrate that the general relativistic gravitation of quarks is sufficient to confine them within hadrons, while simultaneously allowing them to remain asymptotically free.

Main Findings:

We demonstrate at femtometer scales, strong and general relativistic gravitational forces are one and the same.

Definition of the problem

From Miguel Alcubierre:

1. **Spacetime metric** $ds^2 = -\alpha^2(r)dt^2 + a^2(r)dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$

2. **Drac spinor for spin ±1/2:**

$$u_\uparrow(\mathbf{x}) \text{ or } d_\uparrow(\mathbf{x}) = \frac{e^{+i\varphi/2}}{\sqrt{4\pi}} \begin{bmatrix} f\sin\theta/2 \\ if\cos\theta/2 \\ ig\sin\theta/2 \\ g\cos\theta/2 \end{bmatrix},$$

$$u_\downarrow(\mathbf{x}) \text{ or } d_\downarrow(\mathbf{x}) = \frac{e^{-i\varphi/2}}{\sqrt{4\pi}} \begin{bmatrix} f\cos\theta/2 \\ -if\sin\theta/2 \\ ig\cos\theta/2 \\ -g\sin\theta/2 \end{bmatrix},$$

3. Differential equations to satisfy:

$$f' = -(\omega + m)g, \qquad g' = -\frac{2}{r}g + (\omega - m)f, \tag{1}$$

$$r^2 g'' + 2rg' + \left(k^2 r^2 - 2\right) g = 0, \quad k^2 = (\omega^2 - m^2), \tag{2}$$

$$\alpha' = 4\pi r S_r^r = 2r(fg' - f'g), \tag{3}$$

$$a' = 4\pi r S_t^t = -2\omega r(f^2 + g^2). \tag{4}$$

k propagation vector, ω frequency, m quark rest mass.

Eq.(2) is spherical Bessel function of order 1, is not square integrable

Yukawa amelioration can impose square integrability by complexifying the propagation vector k

$$K_\pm = k \pm i\kappa, \quad K^2 = K_+ K_- = (k^2 + \kappa^2),$$

$$z = \frac{K_+}{K} r, \quad \frac{\partial}{\partial z} = \frac{K_-}{K} \frac{\partial}{\partial r}, \quad r = \frac{K_-}{K} z, \quad \frac{\partial}{\partial r} = \frac{K_+}{K} \frac{\partial}{\partial z},$$

$$z^2 \partial_z^2 g + 2z\partial_z g + (K^2 z^2 - 2) g = 0. \quad (5)$$

(5) Is modified spherical Bessel equation function of order 1

Solutions of (5):

$$f_i = \frac{\sin Kz}{Kz} = \left(1 - \frac{1}{6}K^2z^2 + \cdots\right),$$

$$g_i = -\frac{K_-}{K}\left(\frac{\cos Kz}{Kz} - \frac{\sin Kz}{K^2z^2}\right)$$

$$= -\frac{K_-}{3K}Kz\left(1 - \frac{1}{10}k^2z^2 + \cdots\right),$$

$$f_e = \frac{e^{iKz}}{Kz},$$

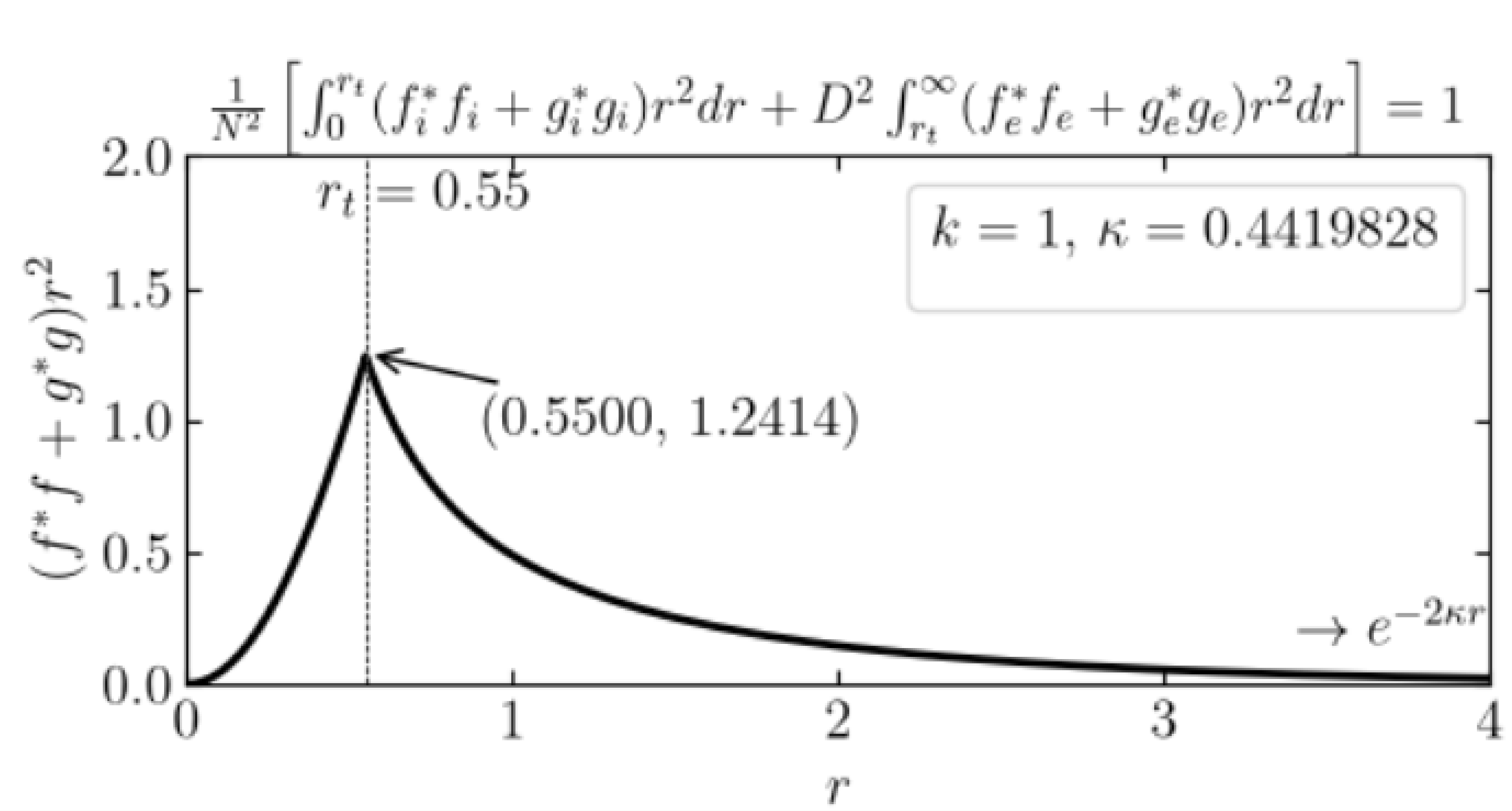
$$g_e = -\frac{K_-}{K}e^{iKz}\left(\frac{i}{Kz} - \frac{1}{K^2z^2}\right).$$

Note asymptotic behaviors:

f_i and $g_i \to$ finite as $r \to 0$, and $\to \infty$ as $r \to \infty$.

f_e and $g_e \to \infty$, as $r \to 0$, and $\to 0$ as $r \to \infty$.

Probability density of a quark: (f∗f + g∗g)r2



Metric coefficients:

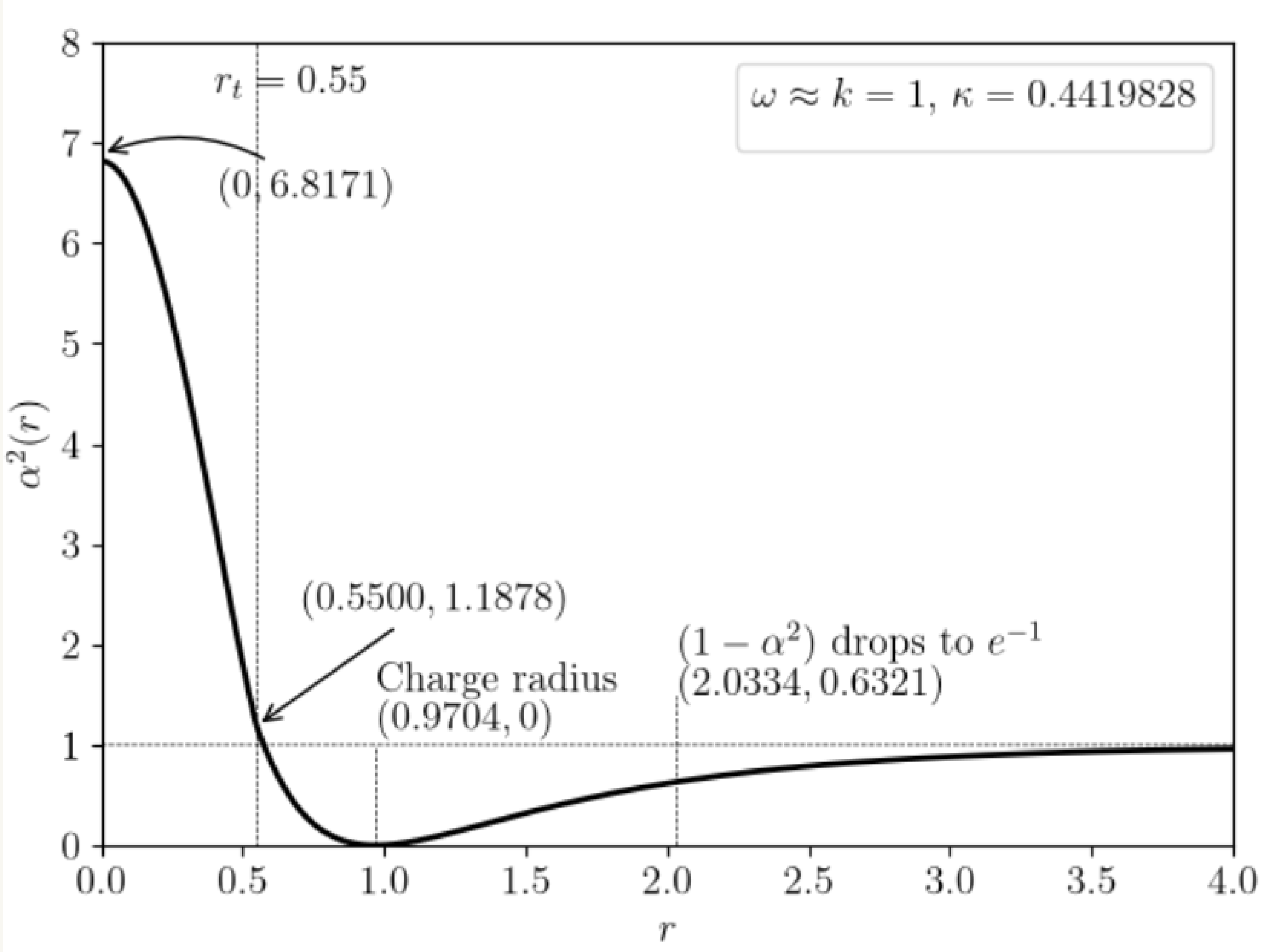
$$\alpha_i(r) = C_\alpha + \frac{3}{N^2}\left(\frac{1}{K^4 r}\left(k(k^2-5\kappa^2)\sin 2kr + \kappa(5k^2-\kappa^2)\sinh 2\kappa r\right) + \frac{(k^2-\kappa^2)}{2K^4r^2}(-\cos 2kr + \cosh 2\kappa r) + 2\frac{(k^4-4k^2\kappa^2+\kappa^4)}{K^4}\int(-\cos 2kr+\cosh 2\kappa r)\frac{dr}{r}\right), \quad r \le r_t.$$

$$a_i(r) = C_a - \frac{1}{N^2}\left(\frac{6\omega}{2K^3 r}(k\sin 2kr + \kappa\sinh 2\kappa r) + \frac{6\omega}{4K^3r^2}(-\cos 2kr + \cosh 2\kappa r) - \frac{6\omega k^2}{K^3}\int(-cos2kr + \cosh 2\kappa r)\frac{dr}{r}\right). \quad r \le r_t.$$

$$\alpha_e(r) = 1 + \frac{3D^2}{N^2}\left(e^{-2\kappa r}\left(\frac{-2\kappa(5k^2-\kappa^2)}{K^4 r} + \frac{(k^2-\kappa^2)}{K^4r^2}\right) r \ge r_t, - \frac{4(k^4+\kappa^4-4k^2\kappa^2)}{K^4}\int e^{-2\kappa r}\frac{dr}{r}\right), \quad r \ge r_t$$

$$a_e(r) = 1 + \frac{6\omega D^2}{N^2}\left(e^{-2\kappa r}\left(\frac{\kappa}{K^3 r} + \frac{1}{2K^3r^2}\right) + \frac{2(k^2-2\kappa^2)}{K^3}\int e^{-2\kappa r}\frac{dr}{r}\right), \quad r \ge r_t.$$

Metric coefficient $\alpha 2$:



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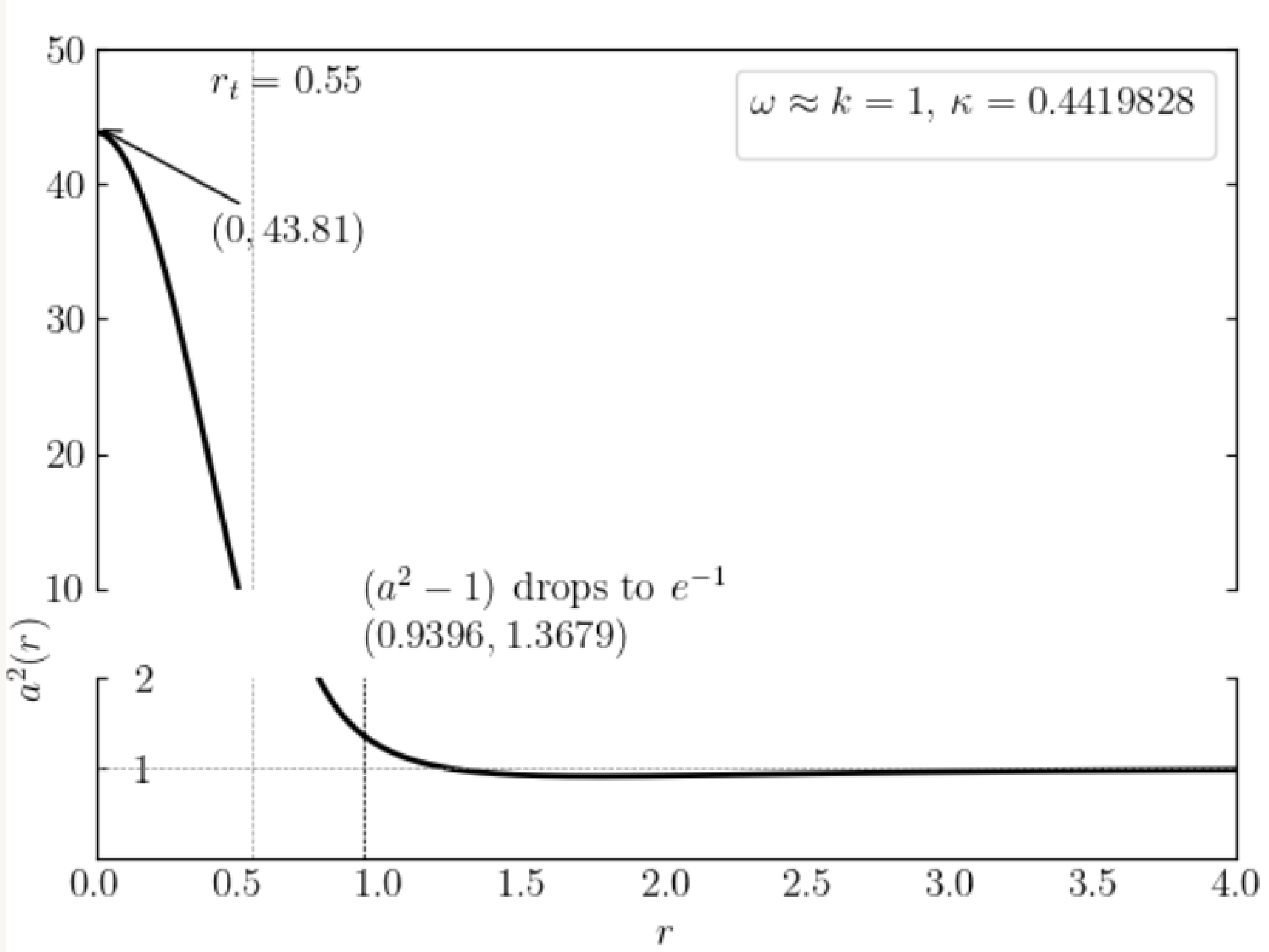


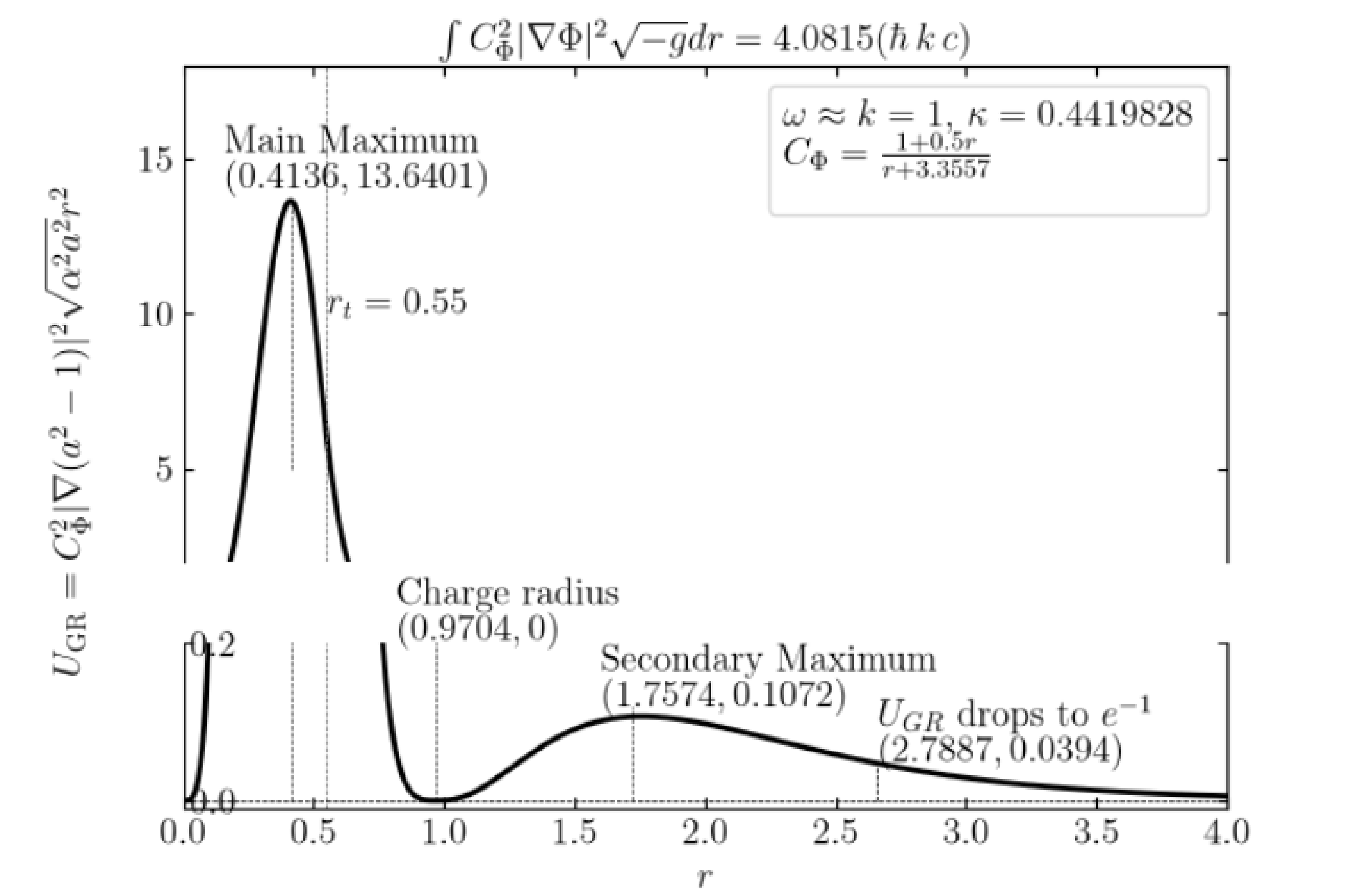
Origin unstable equilibrium. r=0.97, charge radius, stable equilibrium. Quarks are driven to charge radius and manifest enhanced electromagnetic properties.

$(\alpha^2 - 1)/2$ is gravitational potential. Its descending slope is outward force on quarks. Its ascending slope is inward force. The two forces should balance each other (Von Laue stability condition).

No force on quarks in the vicinity of charge radius (asymptotic freedom).

No scape to infinity because of exponential decay to flat spacetime (confinement).

General Relativistic energy inside nucleons - Grav force, $F = \nabla\alpha^2$, Grav potential energy density= $1/2|\nabla\alpha^2|$



Distribution of the (emergent) GR energy inside a nucleon, derived from general relativistic and quantum considerations. It is zero at the center, reaches maximum at r = 0.4136, drops to zero at r_c = 0.9704, is followed by a secondary maximum at 1.7574, and eventually decays exponentially to zero at infinity.

Conclusions:

Quarks are normalizable Dirac spinors. They are massive and curve the surrounding spacetime.

To ensure square-integrability of the spinors, we introduce a complex-valued wave vector a la Yukawa.

Assuming a static, spherically symmetric spacetime metric, we find:

- Thanks to the wave nature of spinors, there is no singularity in the resulting spacetime.
- Inside the nucleon, spacetime is flat at the origin. Outside the nucleon the spacetime is asymptotically flat.
- There are two distinct equilibrium points to the spacetime, the unstable origin and the stable charge radius (r_c).
- Quarks occupy the force-free environment of the stable charge radius.
- While asymptotically free they are confined by the exponential decay of their wave density outside this zone.

Conclusions (continued):

- Eventually, we have demonstrated a direct structural correspondence between our general relativistic model of nucleons and the experimental mass profiles published by the J/Ψ - 007 collaboration.
- Far out a nucleon, there are r^{-1}, r^{-2}, r^{-3} forces. The first one may be of interest to those who may fancy to explain the Tully-Fisher relation, the flat rotation curves of spiral galaxies.
- We acknowledge that treating $(\alpha^2 - 1)/2$ as an effective Newtonian potential is a simplifying approximation that may draw scrutiny from general relativity purists.
- Nonetheless, this work demonstrates that the general relativistic forces at the femtometer scale is remarkably strong and capable of replicating the known features of the strong force.
- This suggests that the fundamental interactions of nature might ultimately be classified into just two primary categories: the electroweak and the gravostrong forces.