

Induced Wald Charge on Magnetized Kerr Black Hole



ICRANet

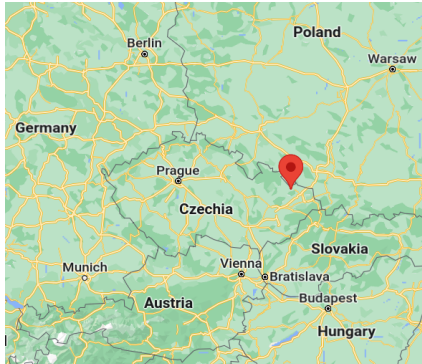
Martin Kološ, Farukh Abdulkhamidov,
Arman Tursunov, Jorge Rueda, ...

The 6th Zeldovich meeting, Jul 13-17, 2026 ICRANet, Pescara

Institute of Physics, Silesian University in Opava (CZ)

- Institute of Physics: ~ 50 people (Z.Stuchlík, R.Konoplya, A.Zhidenko, G.Török ...)
- Research centres: Theoretical Physics and Astrophysics; Computational Physics and Data Processing; Educational Centre
- **Annual relativistic conference RAGtime28: Nov 2-6, 2026 – you are welcome!**

<https://ragtime.physics.cz/>



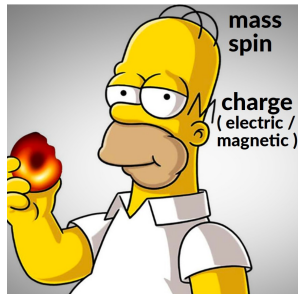
Black hole magnetosphere

pulsar magnetosphere = rotating dipole; black hole = ?

A) Black hole is alone – BH's own EM field

- BH no-hair theorem – black holes have only three hairs: mass, spin, monopole (electric/magnetic) charge

⇒ there can be induced monopole electric charge on rotating black hole in magnetic field



B) Plasma presence around BH

– electromagnetic field around the BH generated by moving plasma

0) Maxwell Equations

linear eqn; J^μ

1) Force-Free Electrodynamics

$B^2 \gg \rho c^2$

2) Magnetohydrodynamics

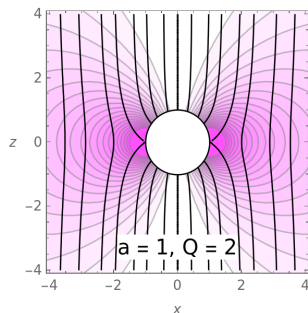
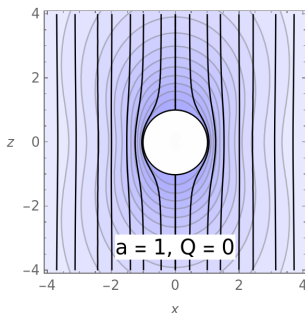
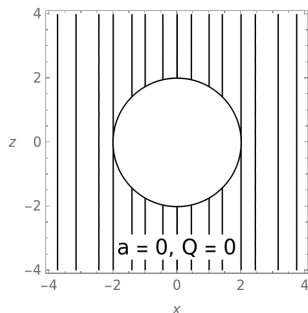
fluid description

3) Kinetic Approach: Particle-In-Cell

charged particles

Rotating BH immersed in uniform magnetic field

⇒ the induced electric field cannot be screened by any choice of el. charge Q on BH;
el.potential along z-axis $\varphi_H - \varphi_\infty = (Q - Q_W)/2M$ where $Q_W = 2aMB$ is Wald charge



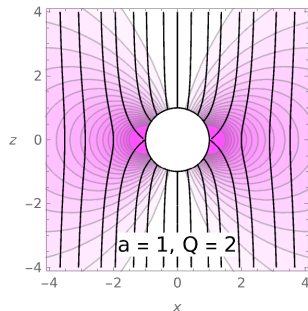
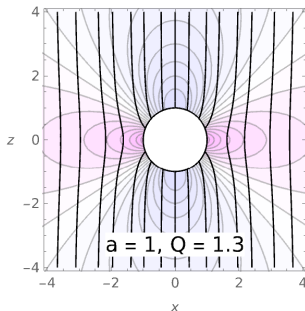
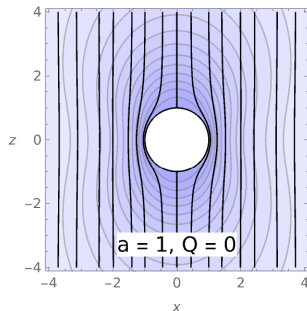
electrostatic potential with respect to infinity



- Robert M. Wald: Black hole in a uniform magnetic field, Physical Review D, 10 (1974)
- Serguei S. Komissarov: Electrically-charged black holes and the Blandford-Znajek mechanism, MNRAS, 512/2 [arxiv.org/abs/2108.08161] (2022)

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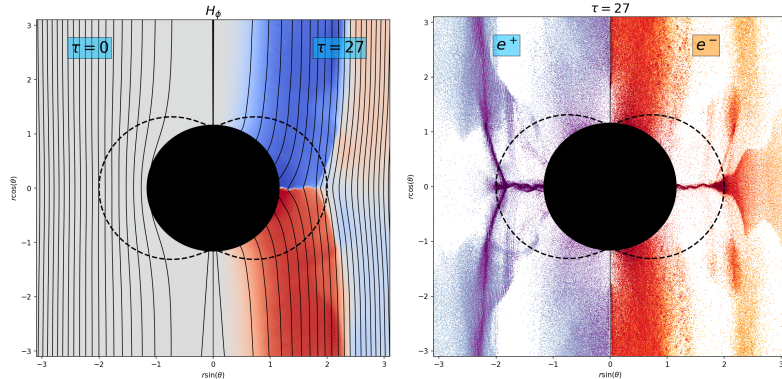


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General Relativistic Particle-in-Cell (GRPIC) setup

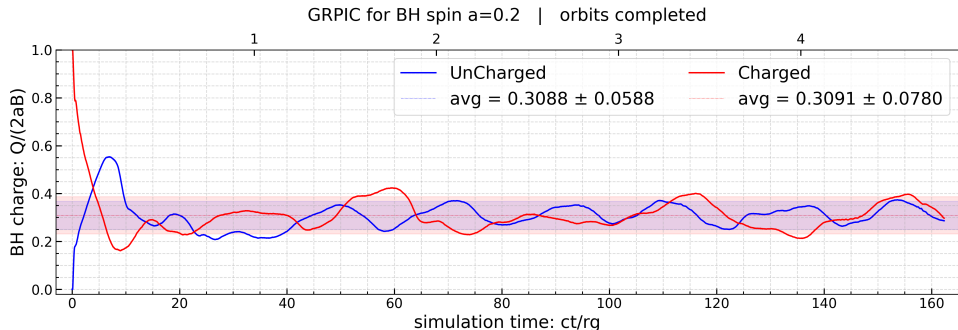
The GRPIC simulation is modeling plasma by tracking many individual charged "macro-particles" as they move through a spatial grid. At each step, the particles charges and currents are deposited onto the grid, the EM fields are solved, and then interpolated back to push the particle forward

- Benjamin Crinquad: Particle acceleration in Kerr black-hole magnetospheres, PhD thesis (2016)



Our setup: uniform external magnetic field, fixed on the outer boundary. Particle pairs e^-/e^+ created that fill the space around the BH and change the structure of the initial EM field.

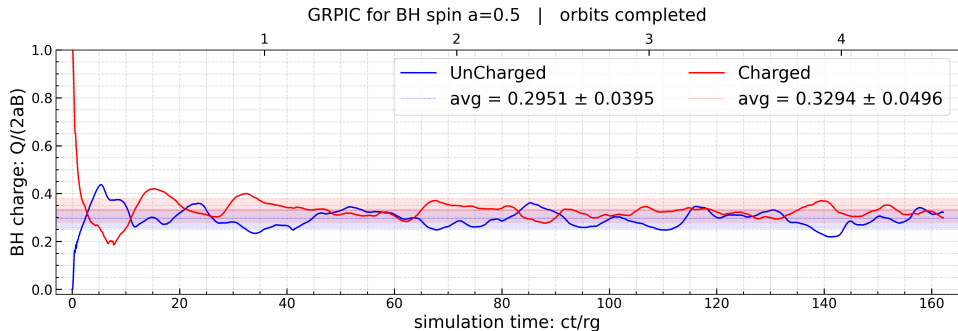
Induced charge on the BH using GRPIC simulation



- We started with two initial configurations:
 - ▶ uncharged BH ($Q = 0$, blue)
 - ▶ charged BH ($Q = Q_W$, red)
- in both cases dynamical equilibrium is quickly reached
- for BH spin $a = 0.2$ we got charge $Q \sim 0.3 Q_W$; for spin $a = 0.9$ charge $Q \sim 0.2 Q_W$

Simulations done by Farukh Abdulkhmidov using GRZeltron GRRPIC code (Benoit Cerutti).

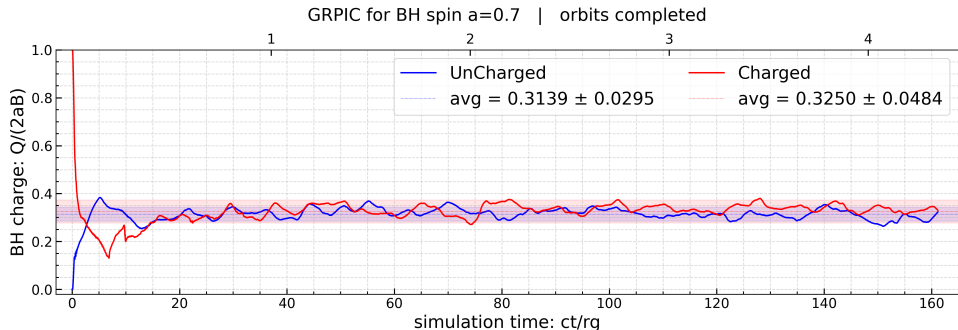
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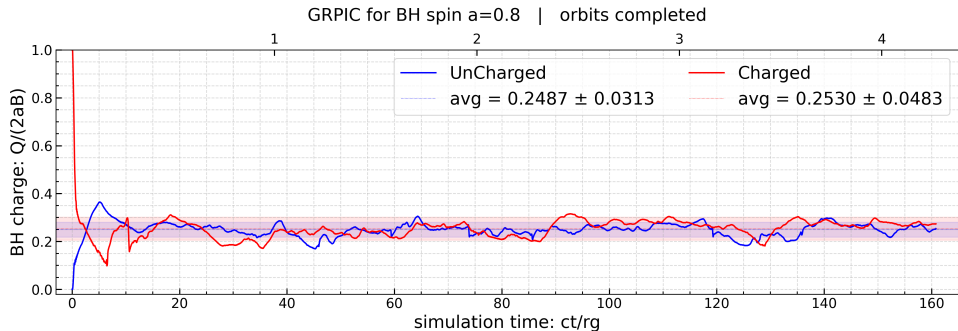
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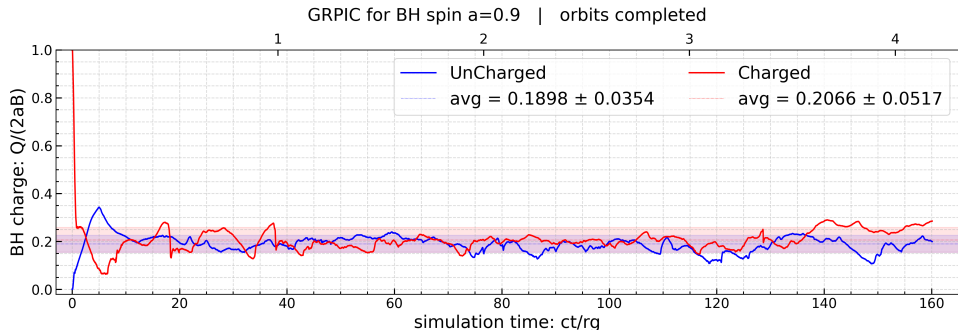
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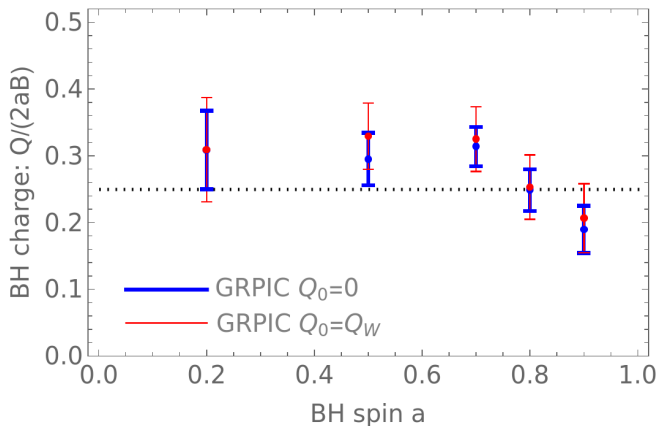
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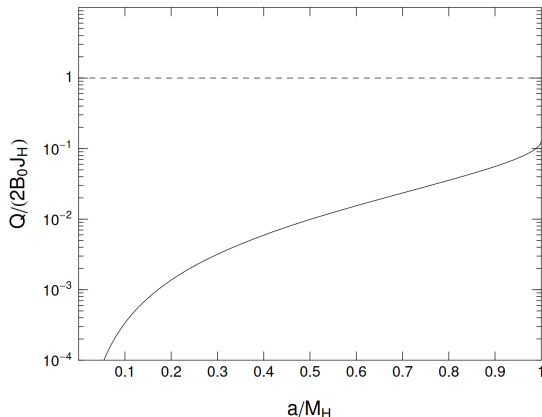
Simulations done by Farukh Abdulkhmidov using GRZeltron GRRPIC code (Benoit Cerutti).

Black hole charge Q as a function of BH spin a



- The charge induced on BH in GRPIC simulation is changing with BH spin.
- BH charge can be approximated as $Q \sim 0.25 Q_{\text{Wald}} = 0.25(2aB) = 0.5aB$.
- BH charge drops down for larger BH spins.

Explaining data: configuration with the lowest energy



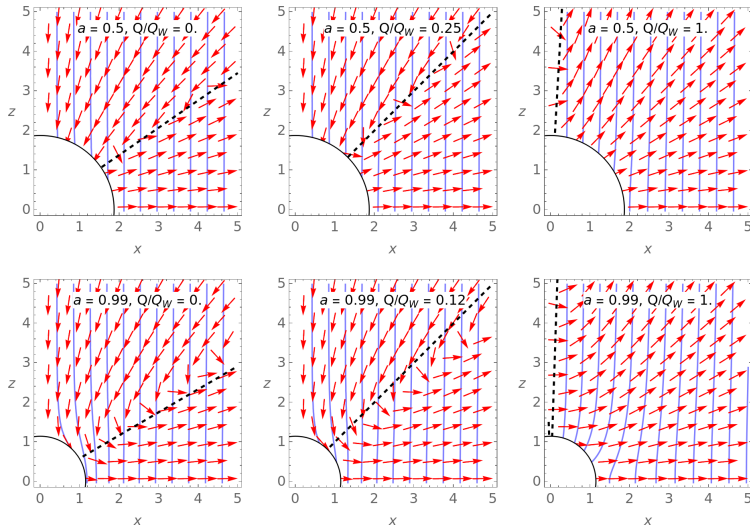
Total energy of EM field around BH should be the lowest:

(!) Wald charge is increasing with BH spin;

(!) gives too low value - max $Q = 0.13 Q_W$ for $a \rightarrow 1$

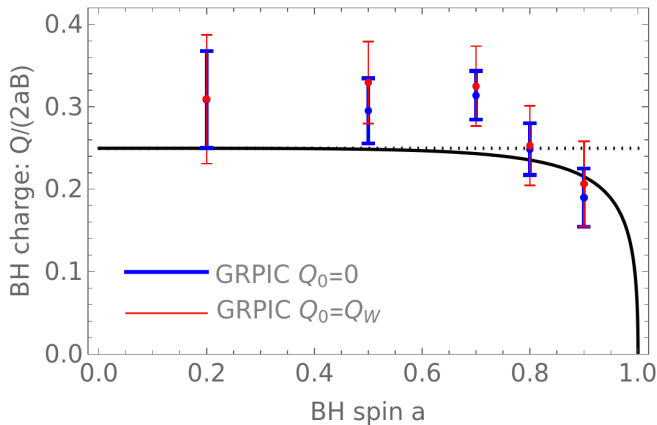
• Li-Xin Li. Electromagnetic energy for a charged Kerr black hole in a uniform magnetic field.
Phys. Rev. D, 61(8):084033 (2000)

We assume selective accretion for charged particles



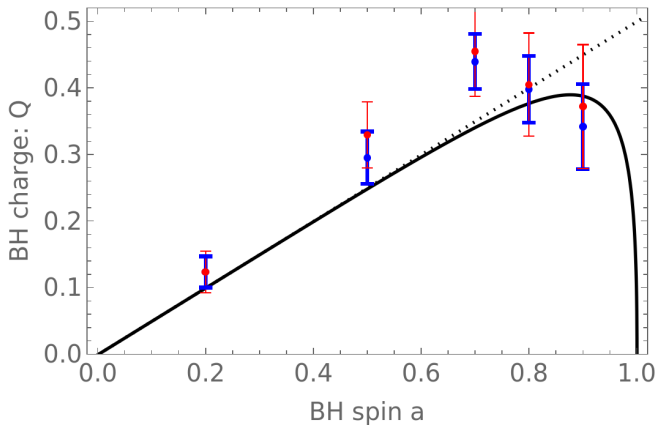
• Jorge A. Rueda and Remo Ruffini. Kerr black hole energy extraction, irreducible mass feedback, and the effect of captured particles charge. *Eur. Phys. J. C*, 84(11):1166 (2024)

Selective accretion - preliminary 45° formula



- To fit GRPIC data we assume the same flux of electric field in/from BH (45° formula).
- BH spin is increasing linearly as $Q = 0.25 Q_{\text{Wald}} = 0.5aB$ (dotted) and then drop down for larger BH spins (thick black curve): $Q(a) = \frac{-a^4 - 3\sqrt{1-a^2}a^2 - 7a^2 + 8\sqrt{1-a^2} + 8}{-12\sqrt{1-a^2}a^2 - 28a^2 + 32\sqrt{1-a^2} + 32}$.

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Summary, problems & future plans

- The charged induced on BH in GRPIC simulation is changing with BH spin; $Q \sim 0.25 Q_{\text{Wald}}$ and then gets down for larger BH spins.
- Problem with axisymmetric 2.5D and full 3D GRPIC simulation; problem with proper analytic formula explaining the data
- GRPIC simulations for large BH spins are still in progress; more computation time.

Thank you for your attention and do not hesitate to contact us for discussion!

