



Production of Thermal QCD Axions in the Early Universe

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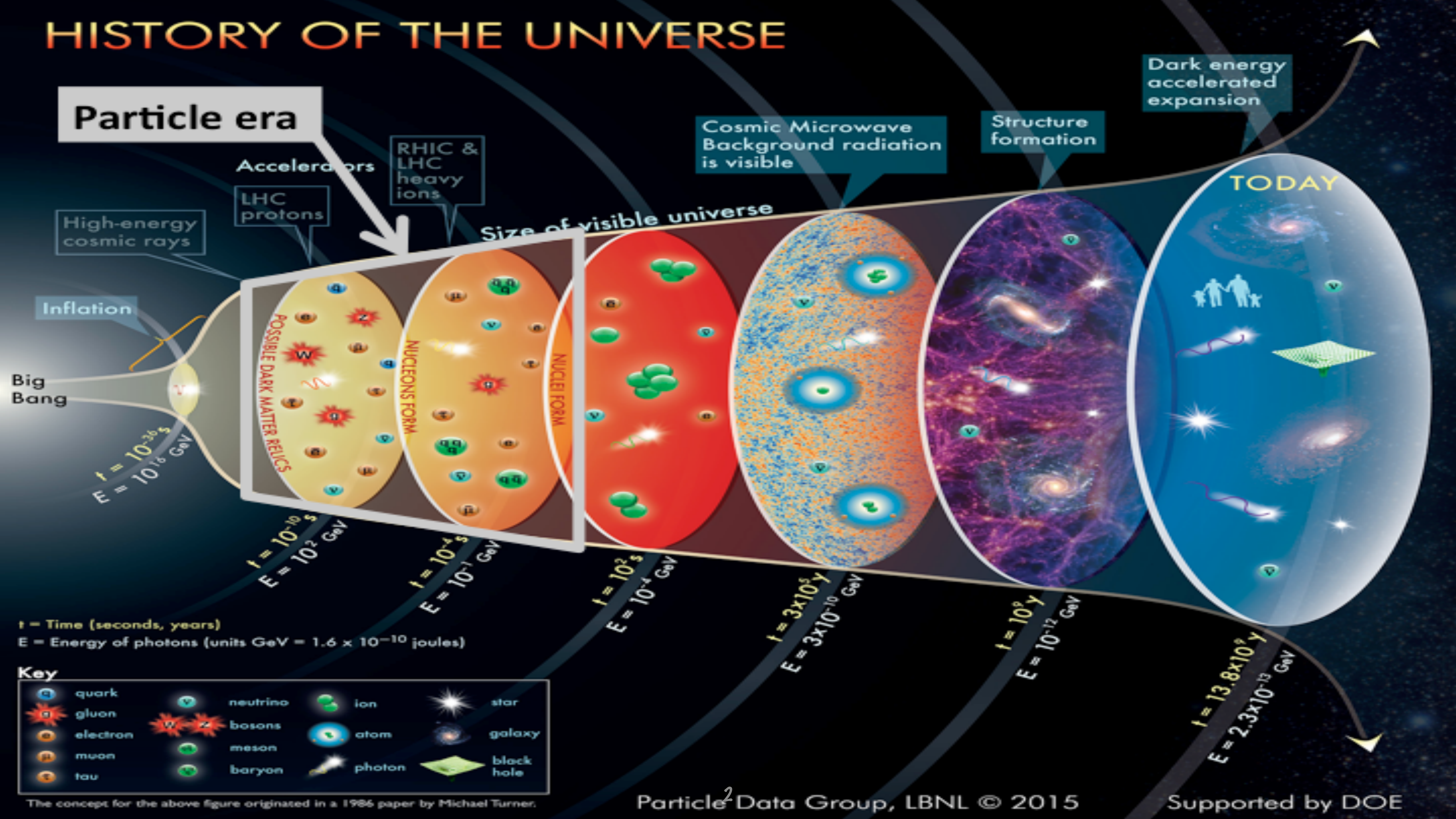
Based on [arXiv:2108.04259](https://arxiv.org/abs/2108.04259) and [arXiv:2108.05371](https://arxiv.org/abs/2108.05371) (JHEP)

in collaboration with

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HISTORY OF THE UNIVERSE



The concept for the above figure originated in a 1986 paper by Michael Turner.

Dark Radiation

- * There is a discrepancy (ΔN_{eff}) from the theoretical value of number of effective neutrinos: $N_{\text{eff}} = 3.044$ and the observed value from cosmic microwave background (CMB) and big bang nucleosynthesis (BBN).
- * Extra degrees of freedom from dark sector can be responsible for dark radiation (DR) or light species.
- * Beyond the standard model physics have some proposals for DR like sterile neutrinos, **axion**, etc. Moreover, gravitational wave background may have a contribution to ΔN_{eff} .
- * Future CMB experiments can put stronger bound on light relics and ΔN_{eff} .
- * Relativistic degrees of freedom depending on their nature can decouple at different temperatures. They may be connected to the new physics!
- * To estimate the accurate amount of dark radiation contributed to the CMB from a typical DR candidate we require to consider the precise thermal background and all the possible production channels of DR in the early universe.

Thermal Background of SM Particles

We need precise equation of state for thermal bath of SM at different temperatures. Energy and entropy density of thermal bath:

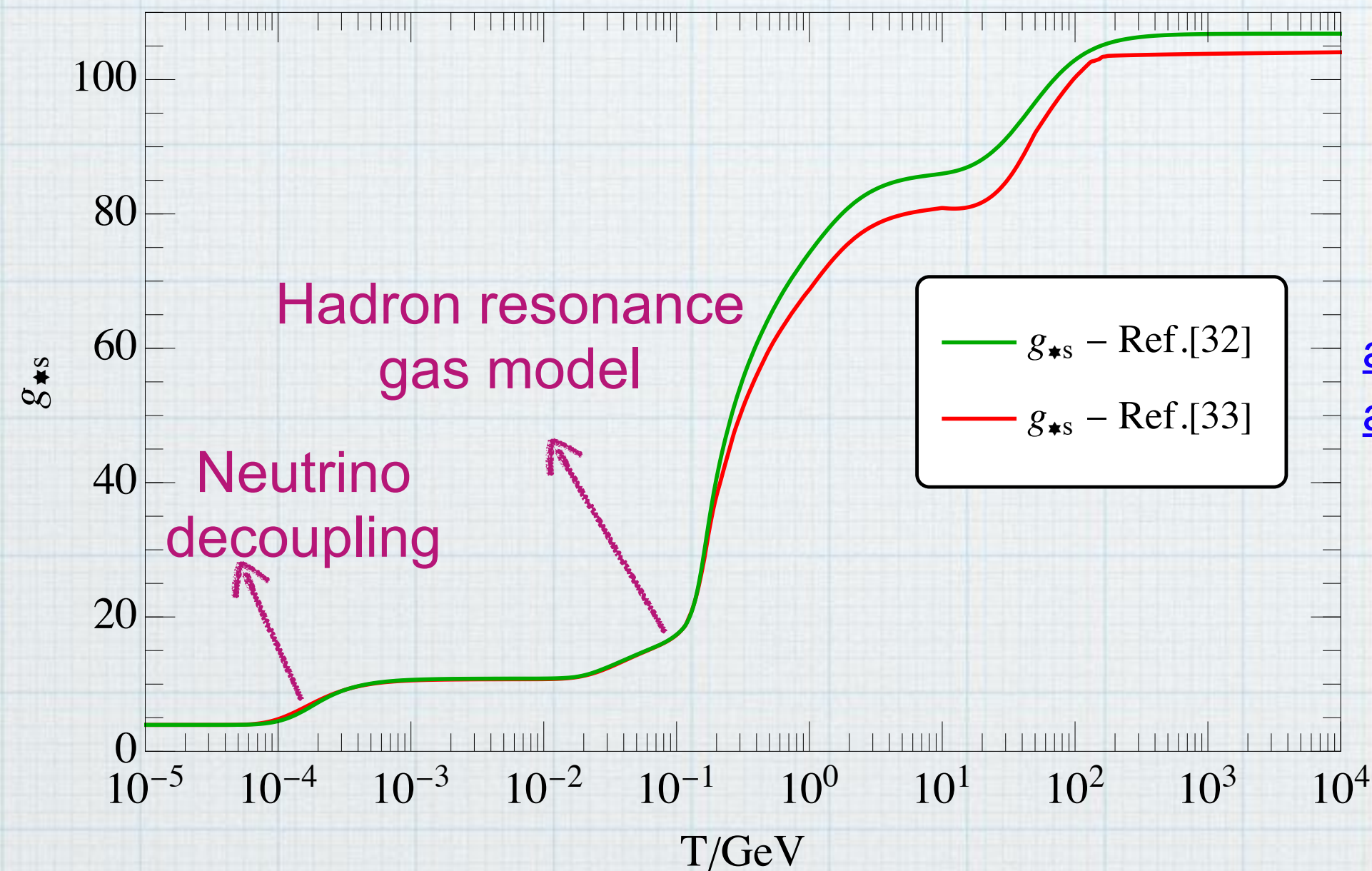
$$s = \frac{2\pi^2}{45} g_{*s}(T) T^3$$

Degrees of freedom

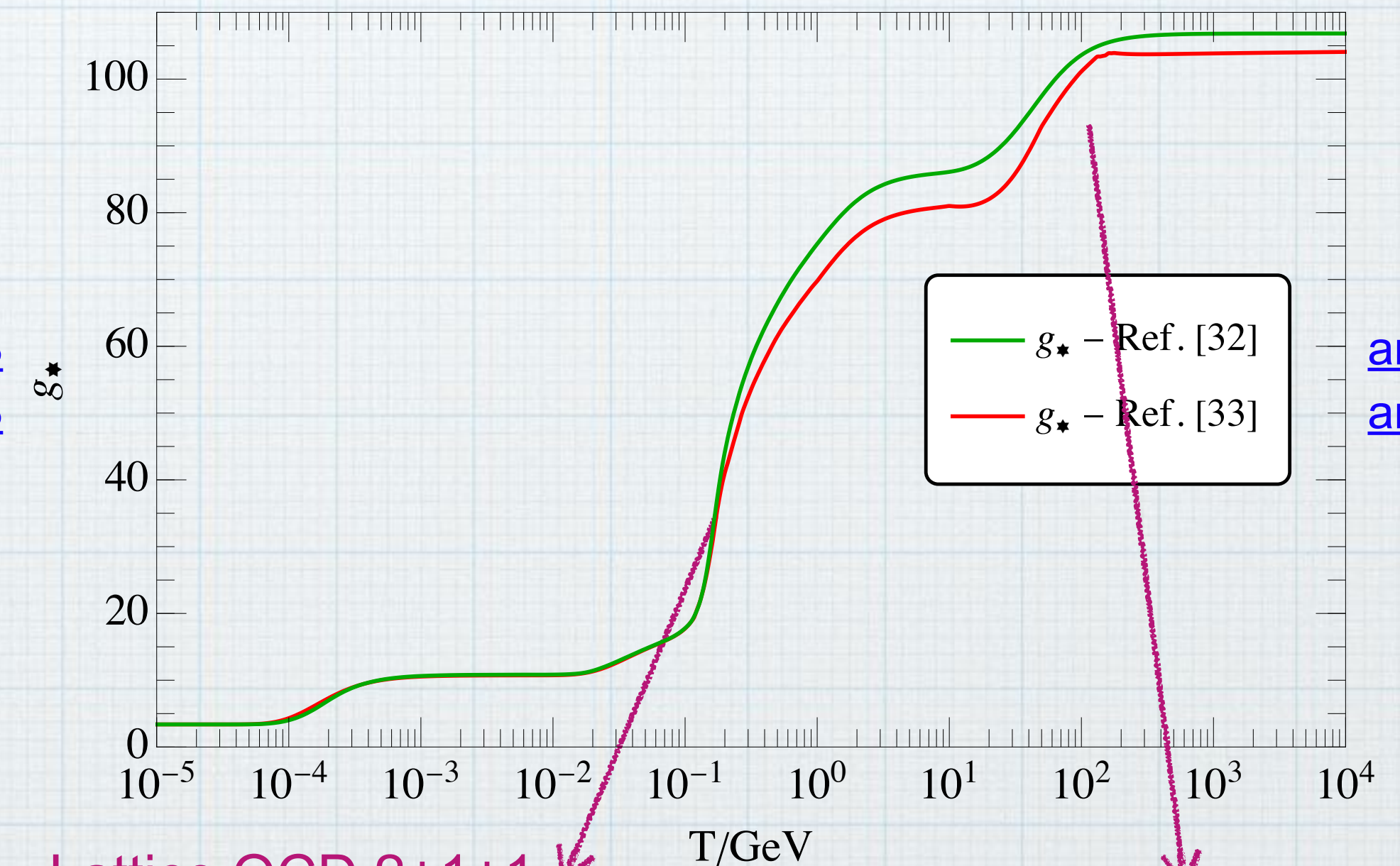
$$\rho = \frac{\pi^2}{30} g_*(T) T^4$$

Entropy density degrees of freedom

Energy density degrees of freedom



[arXiv:1503.03513](https://arxiv.org/abs/1503.03513)
[arXiv:1803.01038](https://arxiv.org/abs/1803.01038)



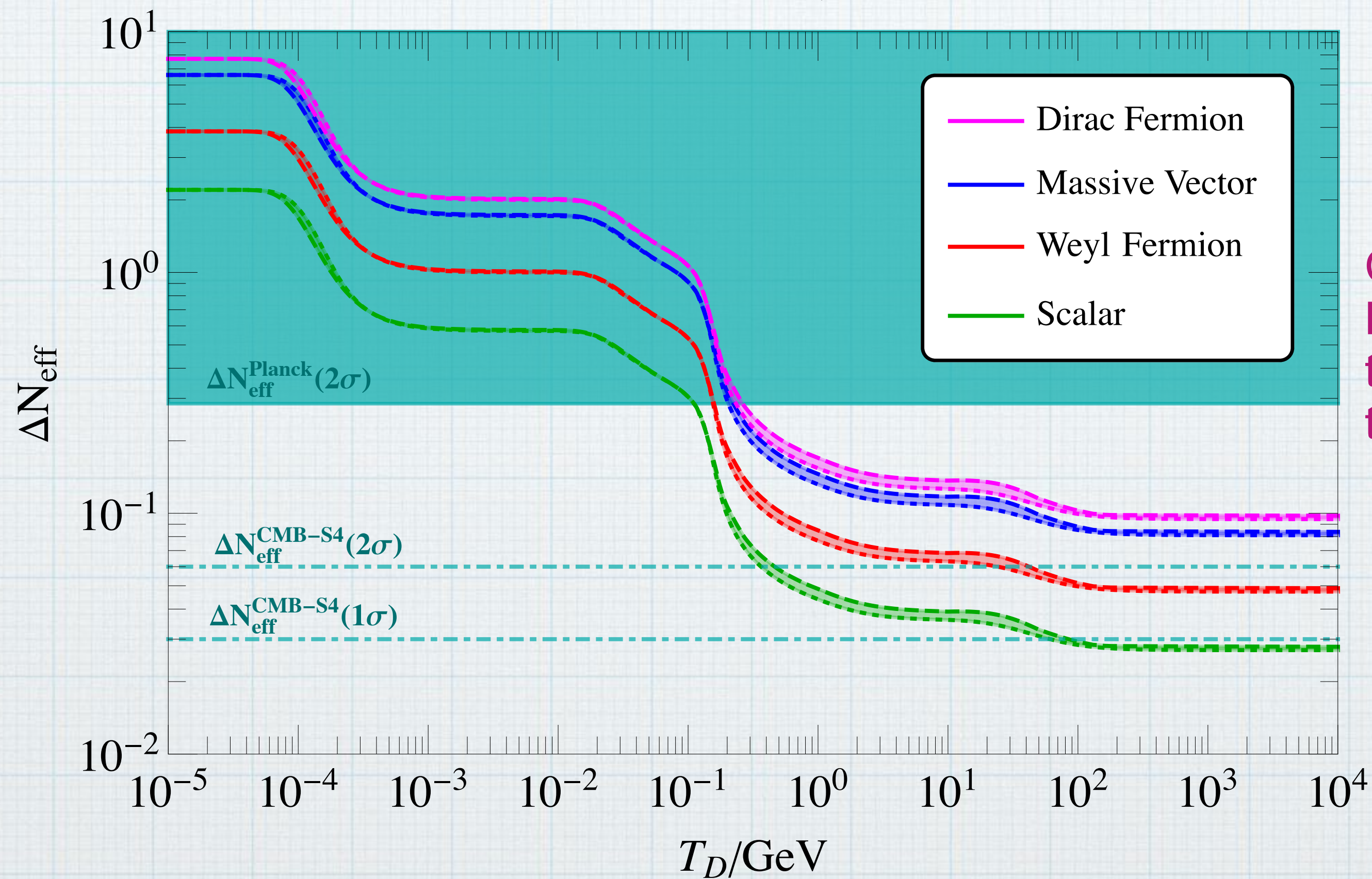
[arXiv:1503.03513](https://arxiv.org/abs/1503.03513)
[arXiv:1803.01038](https://arxiv.org/abs/1803.01038)

The rest of SM particles considered free!

Different Types of Dark Radiation

Number of effective neutrinos ΔN_{eff} versus decoupling temperature T_D of a typical DR:

$$\Delta N_{\text{eff}} \equiv N_{\text{eff}} - N_{\text{eff}}^{\text{SM}} = \frac{8}{7} \left(\frac{11}{4} \right)^{4/3} \frac{\rho_{\text{DR}}}{\rho_\gamma} \simeq g_{*\Phi} \times 13.69 \times g_{*s}^{\text{SM}}(T_D)^{-4/3}$$



CMB-S4 experiment can probe DR candidates decoupled around the QCD transition and above that!

QCD Axion and Strong CP problem

- * QCD axion is a solution for the strong CP problem. It can explain the tiny value of neutron electric dipole moment!
- * Connection to the UV completion physics through Peccei-Quinn symmetry restoration/breaking at high scales → giving mass to axions as pseudo Nambu-Goldstone bosons
- * Possible contribution to the dark matter or dark radiation ($m_a \ll 1 \text{ eV}$) depending on the production mechanisms. The relation between axion mass and axion decay constant is:

$$m_a \simeq 5.7 \times 10^{-6} \text{ eV} \left(\frac{10^{12} \text{ GeV}}{f_a} \right)$$

- * There are some constraints from laboratory, astrophysical and cosmological experiments. The supernova constraint puts a lower bound on axion decay constant f_a . The upper bound on that comes from the condition on the overclosure of the universe. Axion constraints can be model dependent ...

Computation of Axion Yield

We need precise equation of state for thermal bath of the SM at different temperatures. Entropy and energy density of thermal bath are:

[arXiv:1503.03513](https://arxiv.org/abs/1503.03513)
[arXiv:1803.01038](https://arxiv.org/abs/1803.01038)

$$s = \frac{2\pi^2}{45} g_{*s}(T) T^3 \quad \text{Degrees of freedom} \quad \rho = \frac{\pi^2}{30} g_*(T) T^4$$

Boltzmann equation for the evolution of the number density of axions: $\frac{d}{dt} n_a + 3Hn_a = \gamma_a \left(1 - \frac{n_a}{n_a^{\text{eq}}} \right)$

Number density $\leftarrow n_i^{\text{eq}} = \frac{m_i^2}{2\pi^2} T K_2 \left(\frac{m_i}{T} \right) \propto T^3 \rightarrow$ For relativistic case

$$s H x \frac{dY_a}{dx} = \left(1 - \frac{1}{3} \frac{\ln g_{*s}}{\ln x} \right) \gamma_a(x) \left(1 - \frac{Y_a}{Y_a^{\text{eq}}} \right), \quad x = \frac{m}{T}, \quad Y_a = \frac{n_a}{s}, \quad \gamma_a \equiv n_a^{\text{eq}} \Gamma_a$$

Axion yield

Axion production rate from two body scattering of SM particles in a general case:

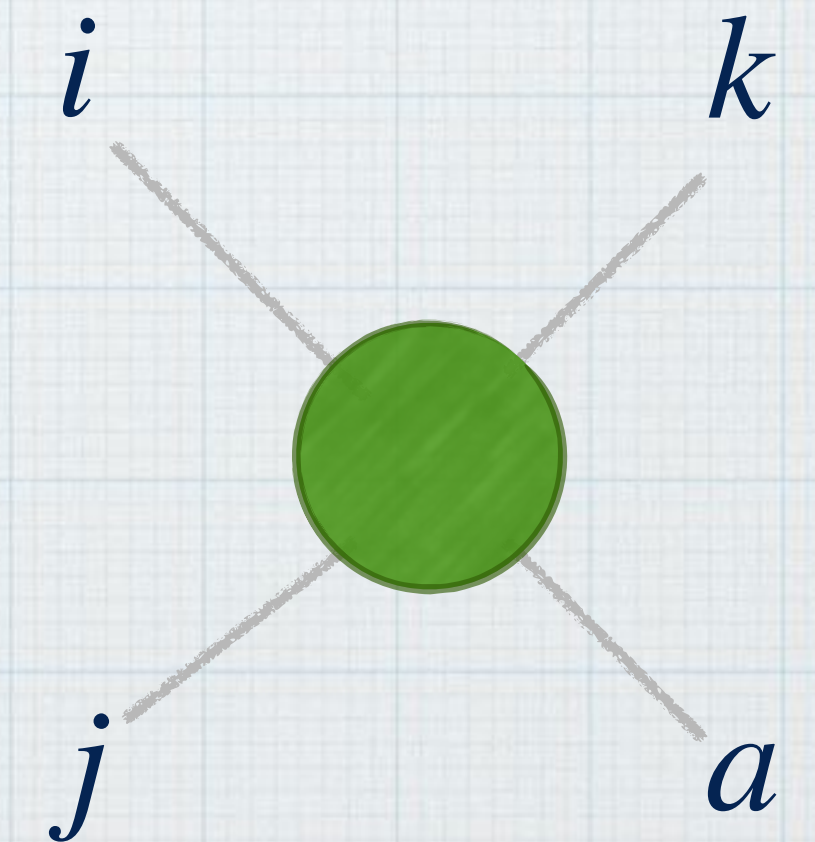
Degrees of freedom of incoming particles $\leftarrow \Gamma_{a,S} = \frac{g_i g_j}{32\pi^4 n_a^{\text{eq}}} T \int_{s_{\text{min}}}^{\infty} ds \frac{\lambda(s, m_i, m_j)}{\sqrt{s}} \sigma_{ij \rightarrow ka}(s) K_1 \left(\frac{\sqrt{s}}{T} \right)$

Center of mass energy $\leftarrow s_{\text{min}} = \text{Max} \left[(m_i + m_j)^2, m_k^2 \right]$

$$\lambda(x, y, z) \equiv [x - (y + z)^2] [x - (y - z)^2]$$

[arxiv:2012.04736](https://arxiv.org/abs/2012.04736)

[arXiv:2003.01100](https://arxiv.org/abs/2003.01100)



Axion Interactions with the SM Particles

A general Lagrangian for effective axion models:

Invariant under shift symmetry \leftarrow
 $a \rightarrow a + \text{const}$

$$\mathcal{L}^{(a)} = \mathcal{L}_{\text{gauge}}^{(a)} + \mathcal{L}_{\text{matter}}^{(a)}$$

Axion field Fermion coupling

Axion decay constant \leftarrow

$$\mathcal{L}_{\text{int}} \supset \frac{1}{f_a} \left[a c_X \frac{\alpha_X}{8\pi} X^{a\mu\nu} \widetilde{X}_{\mu\nu}^a + \partial_\mu a c_\psi \bar{\psi} \gamma^\mu \psi \right]$$

Gauge boson coupling

SM gauge bosons:

$$X = \{G, W, B (A)\}$$

Quarks and leptons:

$$\psi = \{Q_L, u_R, d_R, L_L, e_R\}$$

Below the QCD confinement scale (150 MeV) chiral perturbation theory for hadrons should be considered. For example axion-pions interaction $(1/f_a) \partial_\mu a \pi \partial^\mu \pi$.

Axion Production in the Early Universe

Axions can have different couplings with the SM particles depending on the axion model.

Lagrangian of KSVZ axion model:

$$\mathcal{L}_{aG} = \frac{\alpha_s}{8\pi} \frac{a}{f_a} G_{\mu\nu}^A \widetilde{G}^{A\mu\nu}$$

$$\mathcal{L}_{a\pi} = \frac{\partial_\mu a}{f_a} \frac{c_{a\pi\pi\pi}}{f_\pi} \left[\pi^0 \pi^+ \partial^\mu \pi^- + \pi^0 \pi^- \partial^\mu \pi^+ - 2\pi^+ \pi^- \partial^\mu \pi^0 \right]$$

Coupling depends on light quark masses

Pion decay constant

Main interactions for the KSVZ axion case above and below the QCD transition:

$$T \gtrsim \Lambda_N : q + \bar{q} \rightarrow g + a, \quad q/\bar{q} + g \rightarrow q/\bar{q} + a, \quad g + g \rightarrow g + a$$

$$T \lesssim T_{\text{QCD}} : \pi^+ + \pi^0 \rightarrow \pi^+ + a, \quad \pi^- + \pi^0 \rightarrow \pi^- + a, \quad \pi^+ + \pi^- \rightarrow \pi^0 + a$$

The production rate from rest of hadrons e.g. kaons, neutrons, protons, etc. contribute up to 10 % at $T_{\text{QCD}} \simeq 150$ MeV. It can safely be ignored!

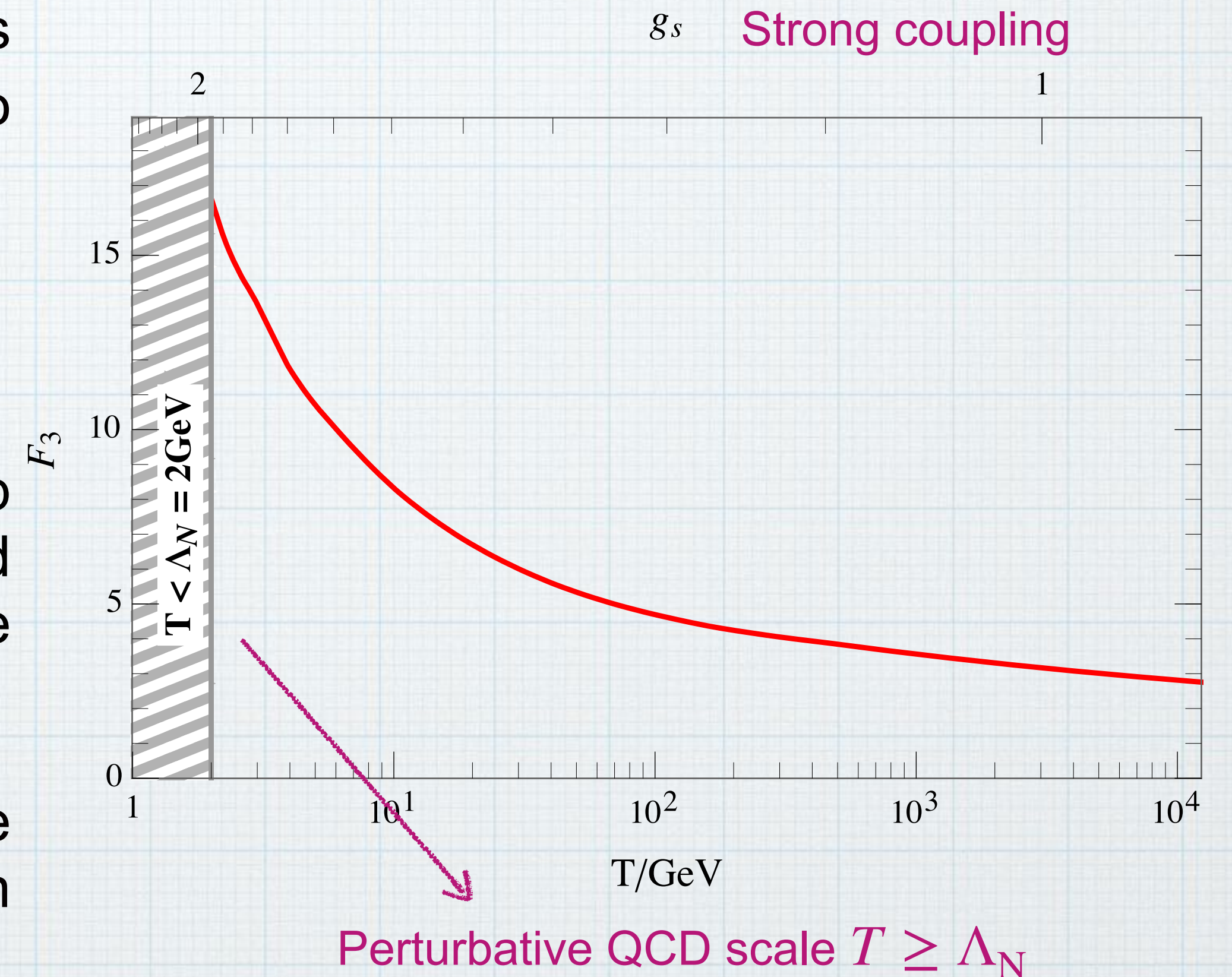
Thermal Effects on Gluon and Quark Scatterings

We consider the running of strong coupling g_s up to four loop orders for axion production from gluon and quark scatterings using **RunDec** package. Taking care of IR divergences due to massless gluons:

$$\gamma_{gg} = \frac{2\zeta(3)d_g}{\pi^3} \left(\frac{\alpha_s}{8\pi f_a} \right)^2 F_3(T) T^6$$

The only diagram contributing to the rate is the one-loop axion two-point function with virtual gluons exchanged, and with the tree-level gluon propagator replaced with the resummed thermal one.

Using continuum and pole approximations outside and inside the light cone for transverse and longitudinal parts of gluon propagator we can numerically calculate the function $F_3(T)$.



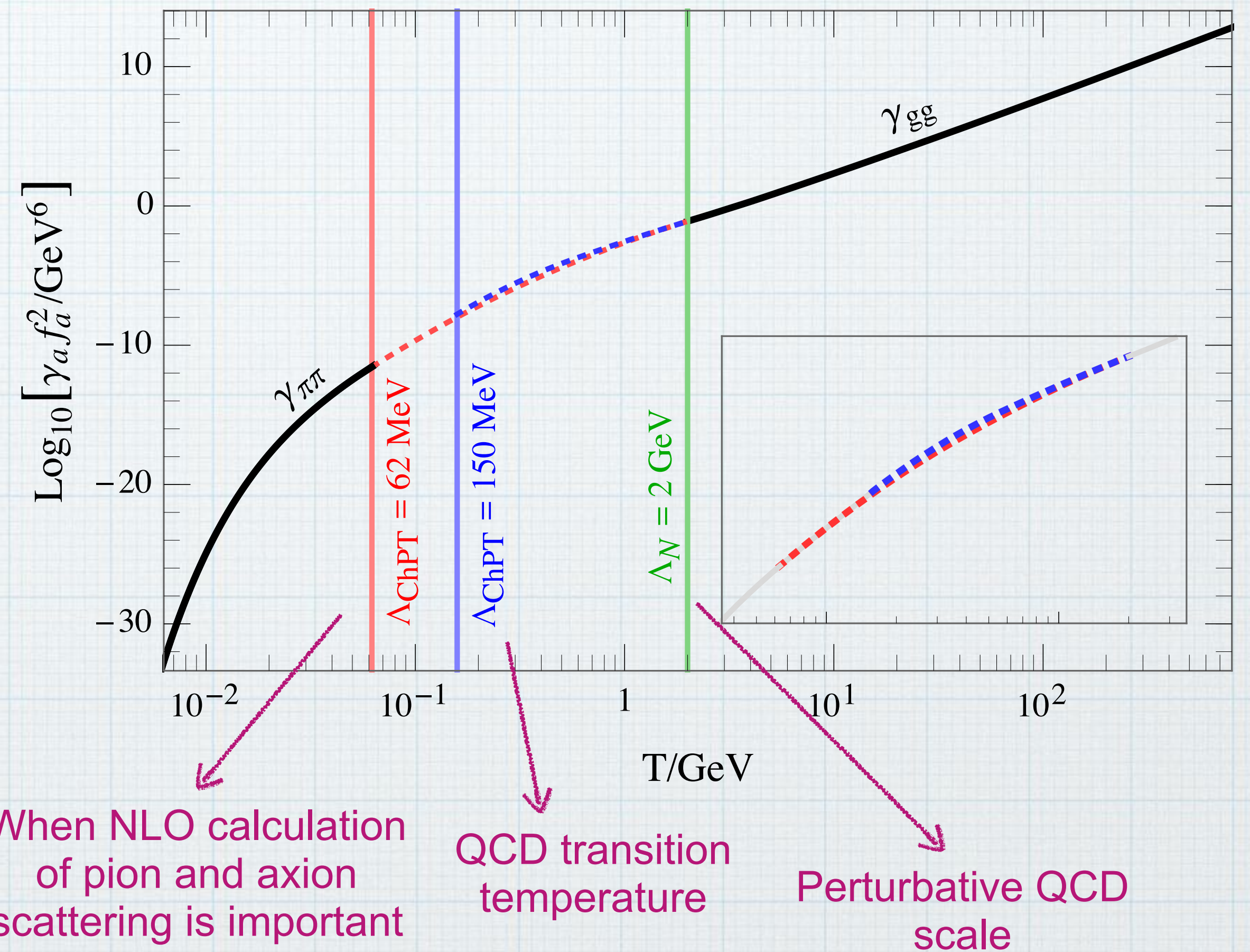
This calculation is also important for the DFSZ model and contributes to the rate!

Total Rate of KSVZ Axion

Calculating the total rate of KSVZ axion at low temperatures:

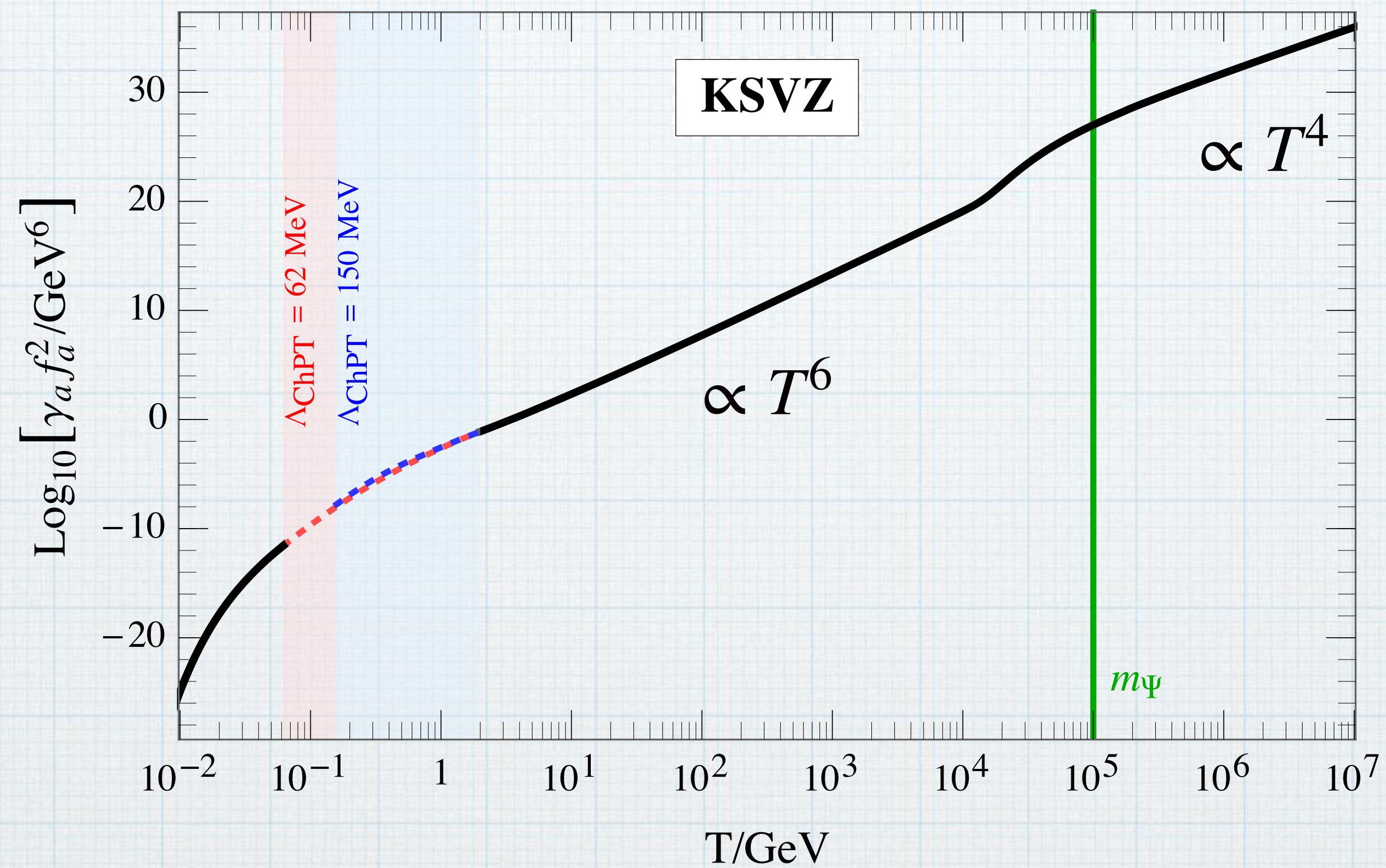
Due to crossover nature of QCD transition in the early universe at vanishing baryon asymmetry we can interpolate between the two regimes that gives a reasonable rate in the intermediate QCD confinement region.

There is a small change in the rate due to different interpolation regime either choosing $\Lambda_{\text{ChPT}} = 62$ or 150 MeV!



KSVZ Axion Production Rate

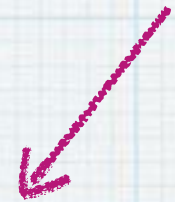
Total axion production rate for the KSVZ model at different temperatures:



Effective Number of Relativistic Axion ΔN_{eff}

By calculating all axion interactions for any given model the number of effective extra degrees of freedom dominated by axions can be evaluated:

$$\Delta N_{\text{eff}} \equiv N_{\text{eff}} - N_{\text{eff}}^{\text{SM}} = \frac{8}{7} \left(\frac{11}{4} \right)^{4/3} \frac{\rho_a}{\rho_\gamma}$$

$$\Delta N_{\text{eff}} \simeq 75.6 Y_a^{4/3}$$


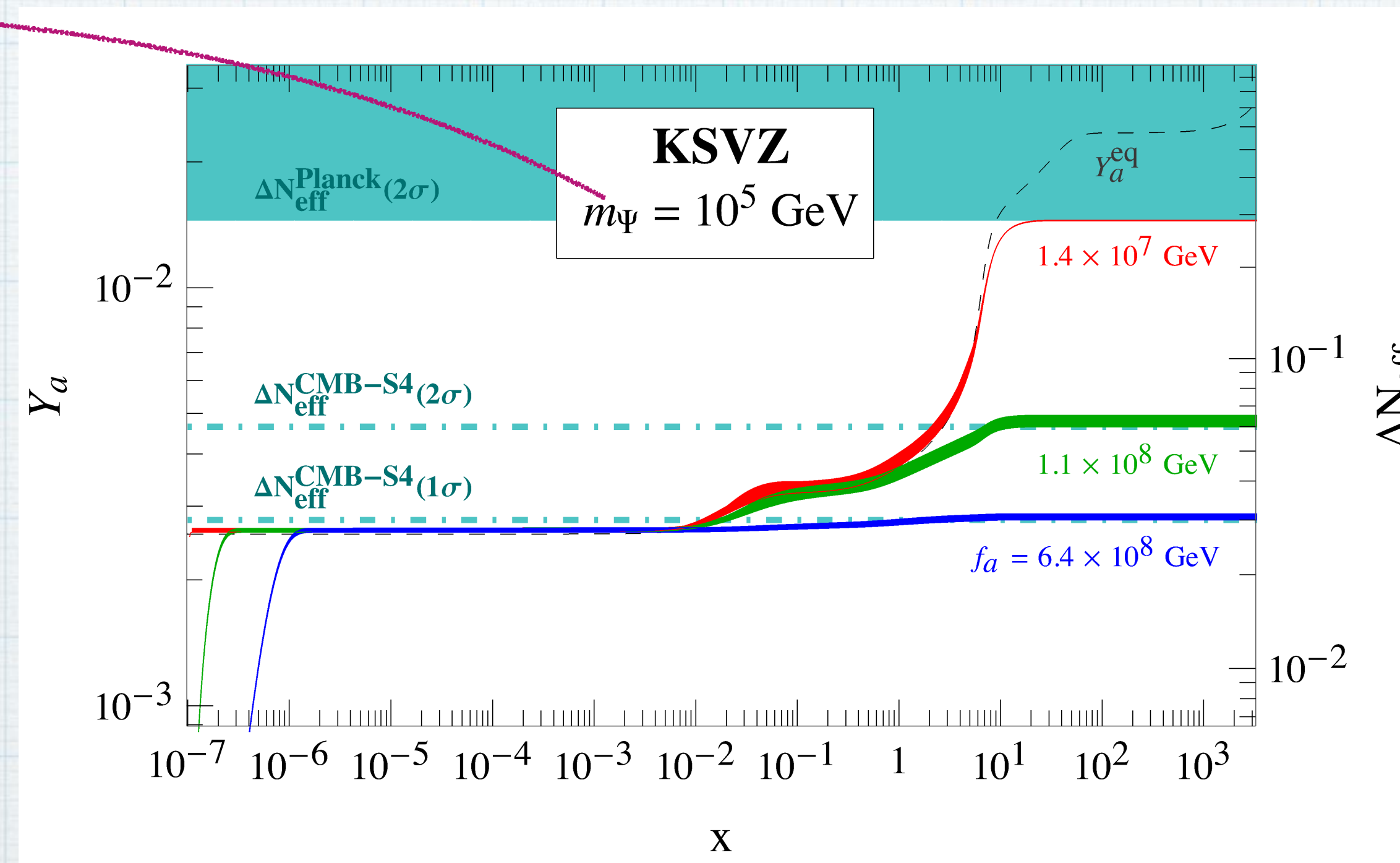
It can be constrained by CMB experiments which can falsify different axion models!

When axions thermalise ($\Gamma_a \gtrsim H$) the value of ΔN_{eff} gets its maximum. If axions do not reach the thermal equilibrium the final value of ΔN_{eff} depends on the initial abundance of them!

Axion Yield in KSVZ Model

Axion yield evolution versus time or temperature in the KSVZ model:

Mass of Peccei-Quinn fermion becomes important at high temperatures or high values of f_a !



Similar calculation for DFSZ model!

Detectability Perspective for Different Axion Decay Constants

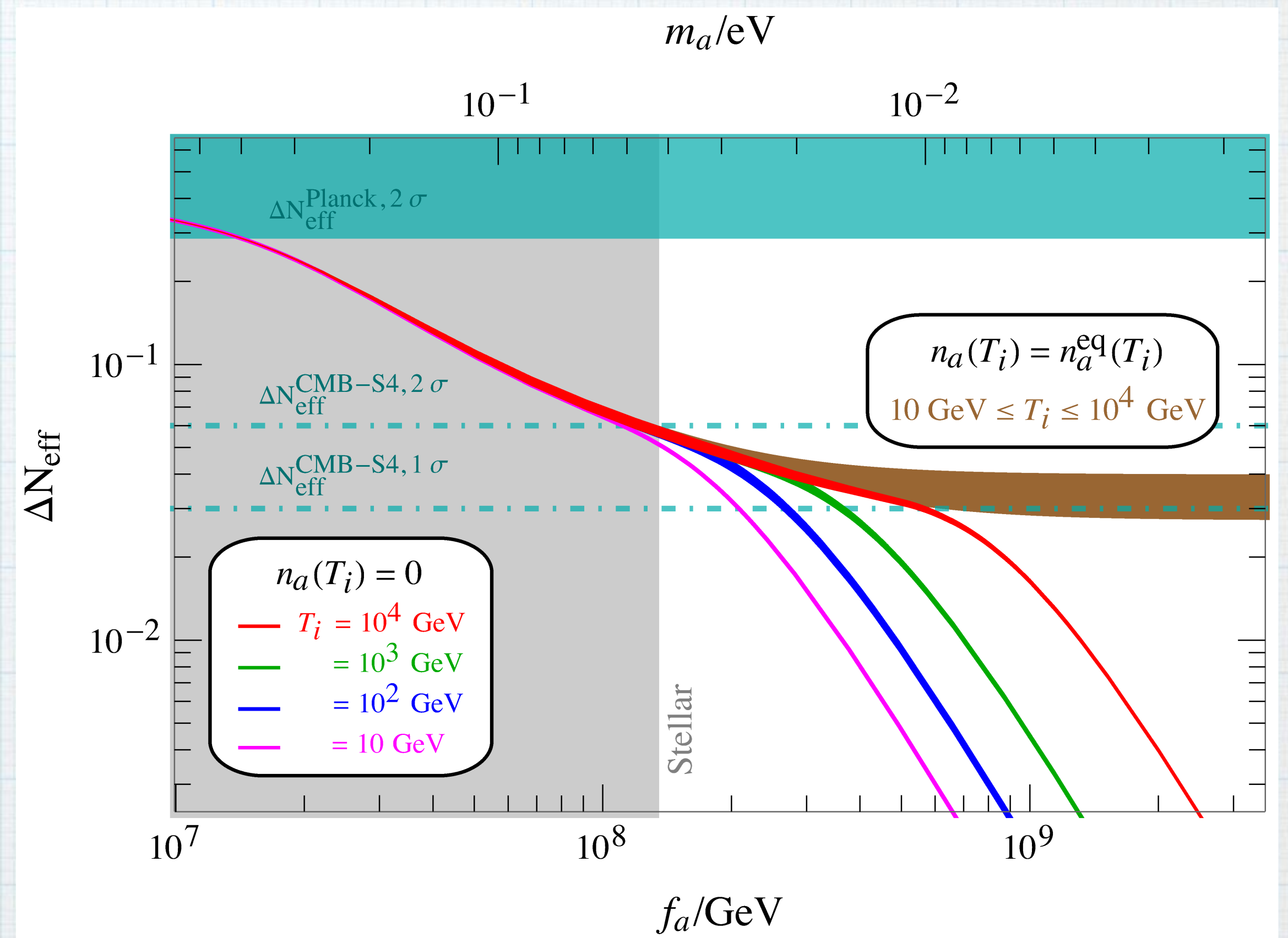
Number of effective neutrinos ΔN_{eff} versus axion decay constant f_a :

Planck is not sensitive enough for large values of f_a that is not constrained by Supernova bound!

We consider different initial temperatures and initial axion densities.

CMB-S4 experiment can probe a large portion of this parameter space in case $\Delta N_{\text{eff}} \gtrsim 0.03$ for different f_a 's!

Similar prediction is done for the DFSZ model with the relevant constraints!



Stellar bound from supernova SN1987A
 $f_a \gtrsim 1.4 \times 10^8 \text{ GeV}!$

DFSZ Model

Different parts of Lagrangian of DFSZ axion interactions after Peccei-Quinn symmetry breaking and field redefinitions:

Coupling depends on light quark masses and Higgs vev's

$$\mathcal{L}_{aG} = \frac{\alpha_s}{8\pi} \frac{a}{f_a} G_{\mu\nu}^A \widetilde{G}^{A\mu\nu}$$

Pion decay constant

$$\mathcal{L}_{a\pi} = \frac{\partial_\mu a}{f_a} \frac{c_{a\pi\pi\pi}}{f_\pi} \left[\pi^0 \pi^+ \partial^\mu \pi^- + \pi^0 \pi^- \partial^\mu \pi^+ - 2\pi^+ \pi^- \partial^\mu \pi^0 \right]$$

Strong interactions for DFSZ axion case above and below the QCD transition:

$$T \gtrsim \Lambda_N : q + \bar{q} \rightarrow g + a, q/\bar{q} + g \rightarrow q/\bar{q} + a, g + g \rightarrow g + a$$

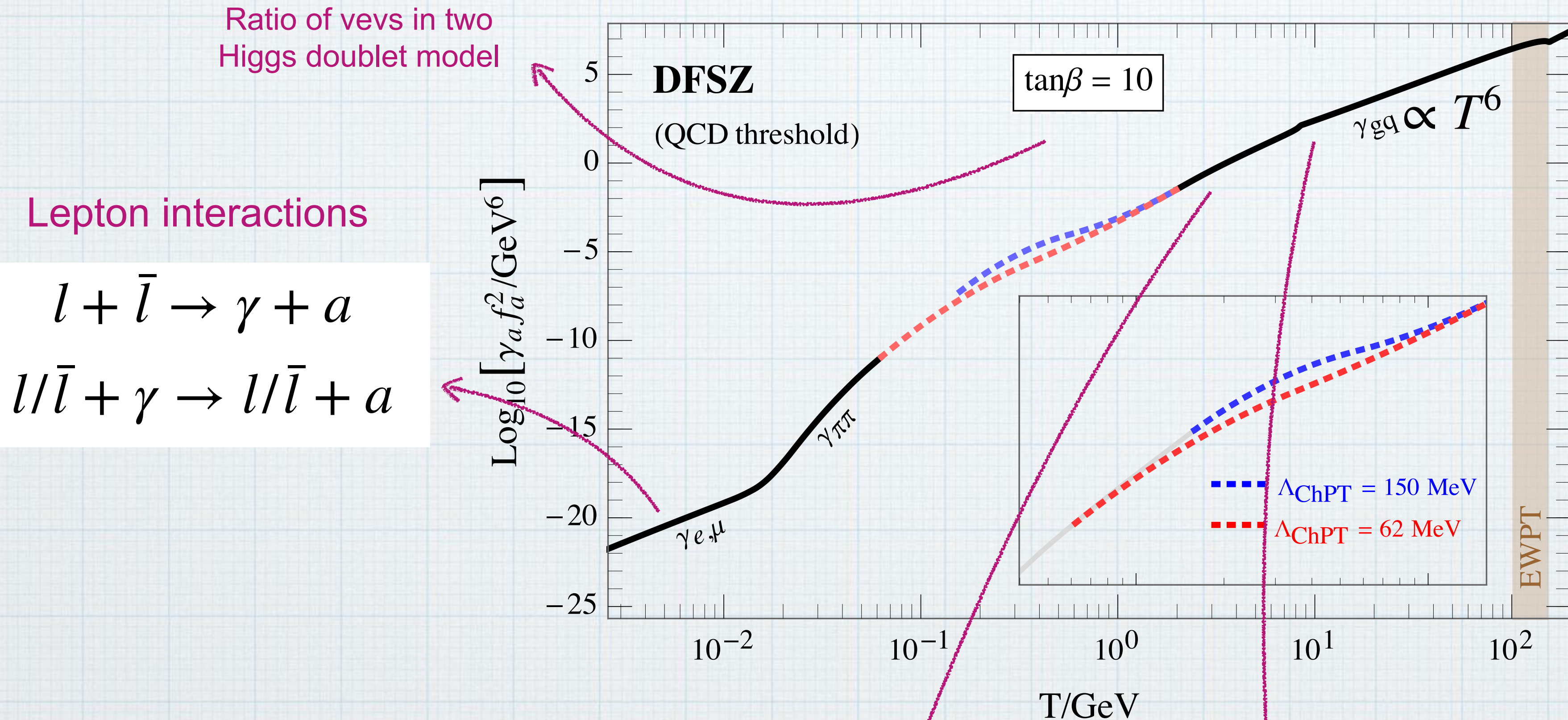
$$T \lesssim T_{\text{QCD}} : \pi^+ + \pi^0 \rightarrow \pi^+ + a, \pi^- + \pi^0 \rightarrow \pi^- + a, \pi^+ + \pi^- \rightarrow \pi^0 + a$$

In the DFSZ model axion also couples to leptons, quarks and weak gauge bosons and we have two Higgs doublets. Coupling of axion with leptons leads to leptons and photons interactions:

$$\mathcal{L}_{al} = \frac{\partial_\mu a}{f_a} \sum_f c_f \bar{f} \gamma^\mu f$$

Axion Production Rate in DFSZ Model

Axion production rate for DFSZ model at low temperatures below the EWPT:



Ratio of vevs in two Higgs doublet model

Lepton interactions

$$l + \bar{l} \rightarrow \gamma + a$$

$$l/\bar{l} + \gamma \rightarrow l/\bar{l} + a$$

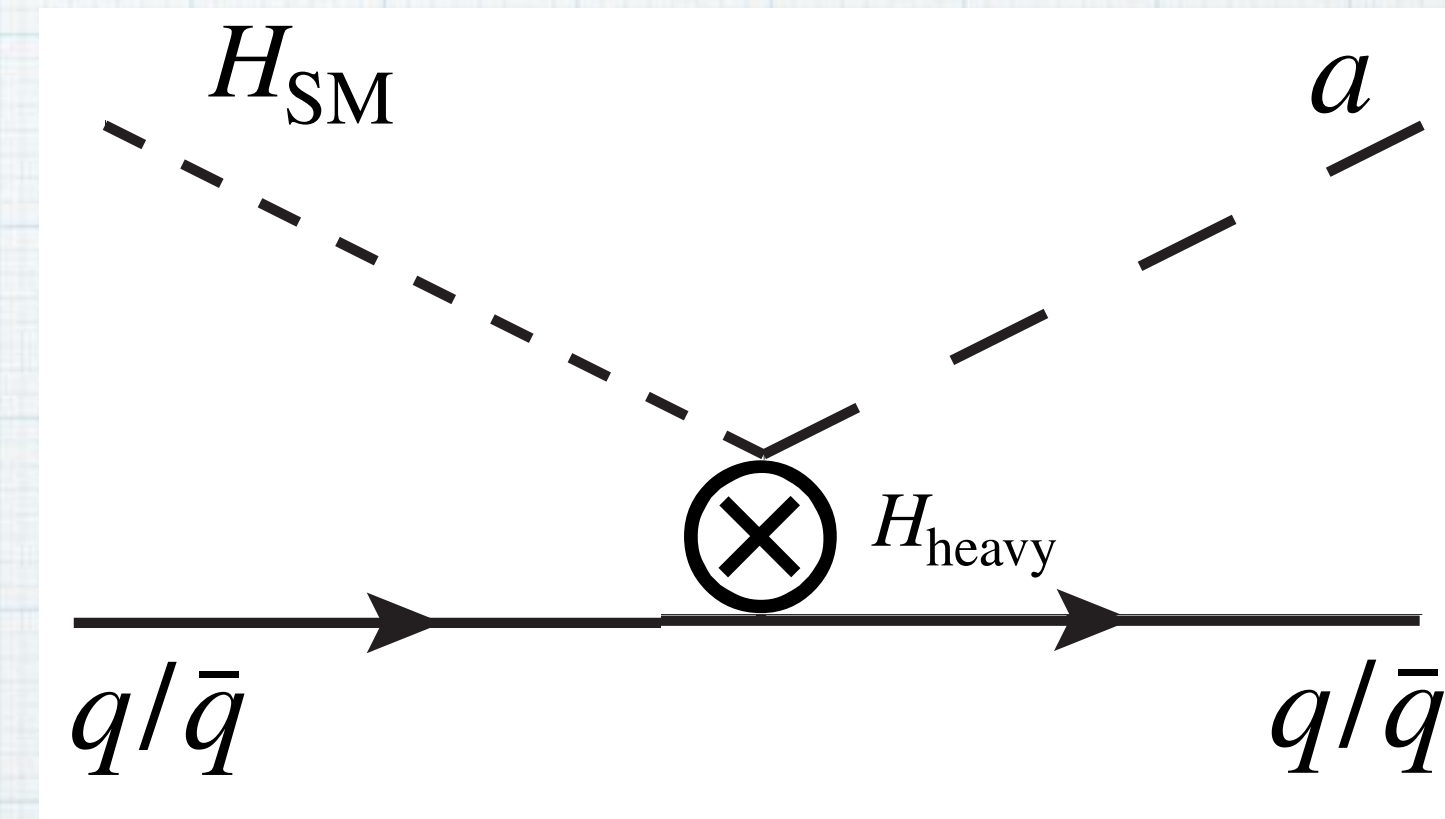
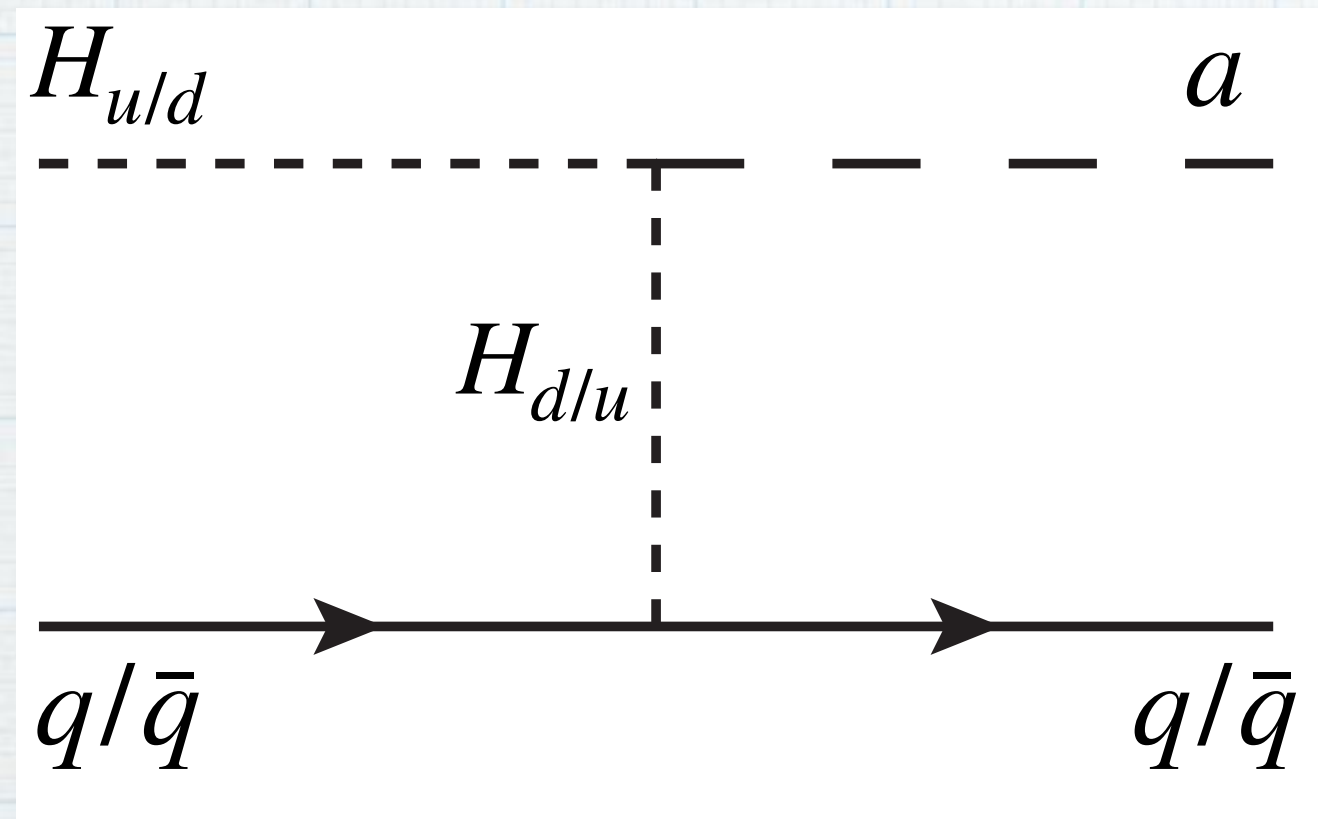
Quark interactions with the EW sector 17

1-PI correction for the SM quarks

DFSZ Model

Electroweak sector interaction for axion production assuming the decoupling limit for the two Higgs doublet model:

$$\mathcal{L}_{\text{DFSZ}}^{\text{linear}} \supset i \frac{a}{f_a} \frac{B}{3} (H_u^T i \sigma^2 H_d) + \text{h.c.}$$



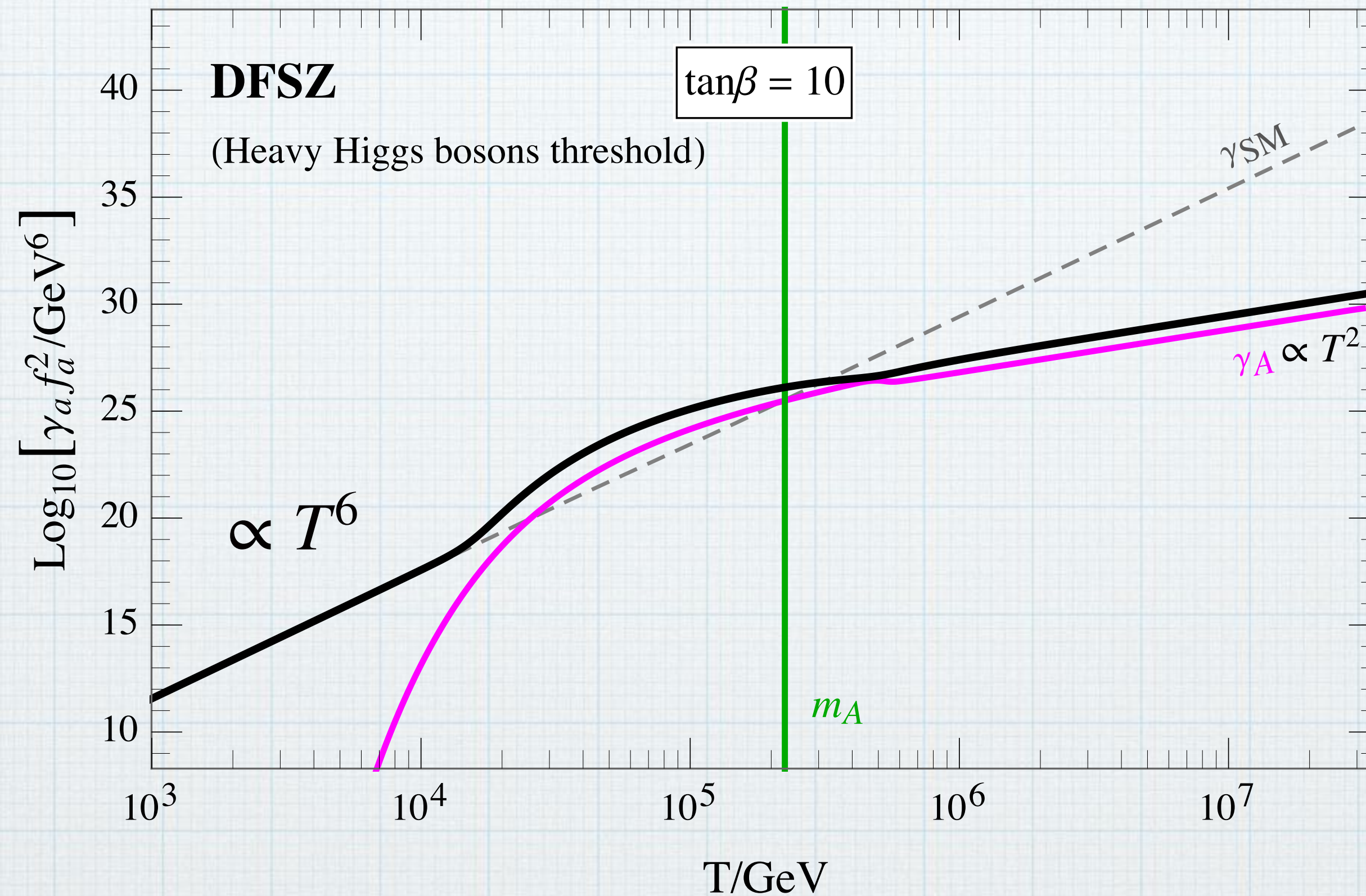
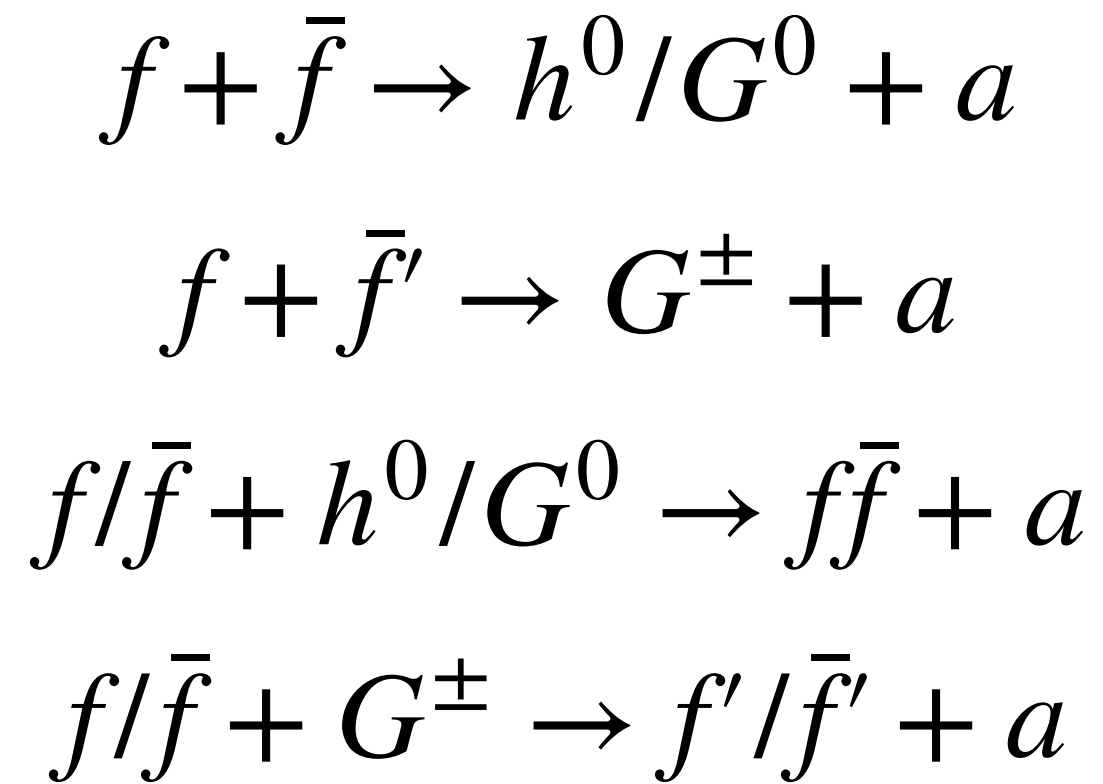
Below the pseudo-scalar mass scale one can integrate out the heavy Higgses:

$$\mathcal{L}_{\text{DFSZ}}^{\text{linear}} \Big|_{T < m_A} = i \frac{a}{f_a} \left(-\frac{\cos \alpha \cos \beta}{3} \bar{Q}_L \widetilde{H}_{\text{SM}} Y^{(u)} u_R - \frac{\sin \alpha \sin \beta}{3} \bar{Q}_L H_{\text{SM}} Y^{(d)} d_R - \frac{\sin \alpha \sin \beta}{3} \bar{L}_L H_{\text{SM}} Y^{(e)} d_R \right) + \text{h.c.}$$

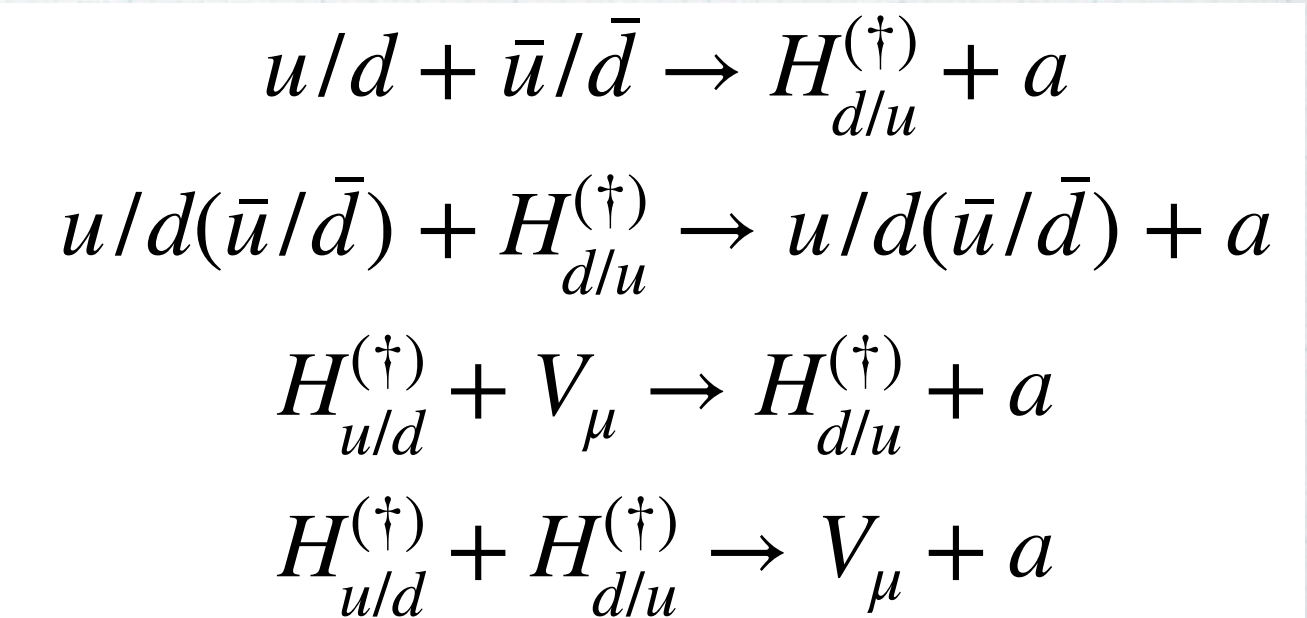
Axion Production Rate in DFSZ Model

Axion production rate for DFSZ model around the psuedo-scalar mass and above the EW scale:

Fermion interaction with the EW sector

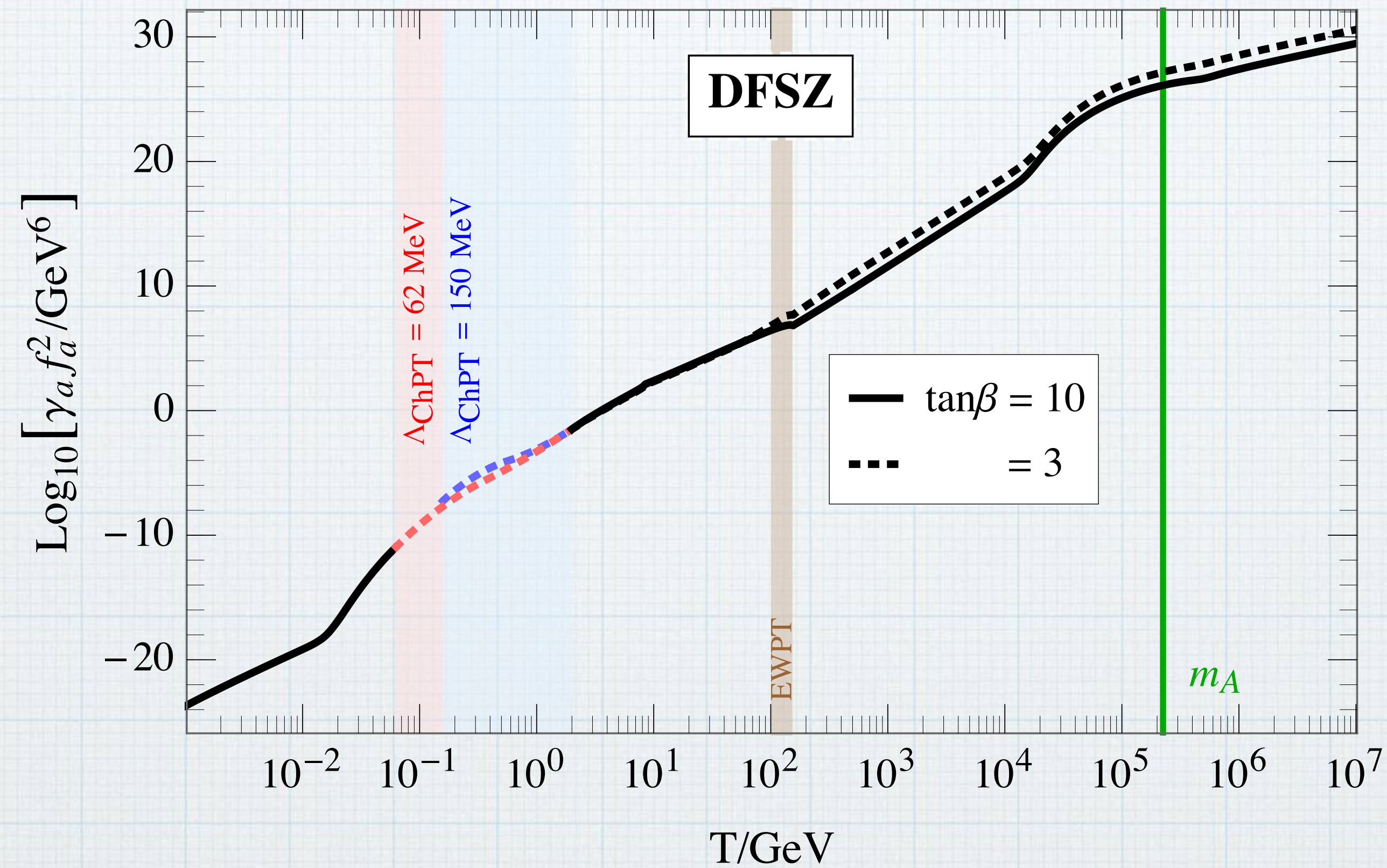


SM fermion interactions with the EW sector at high temperatures



Axion Production Rate in DFSZ Model

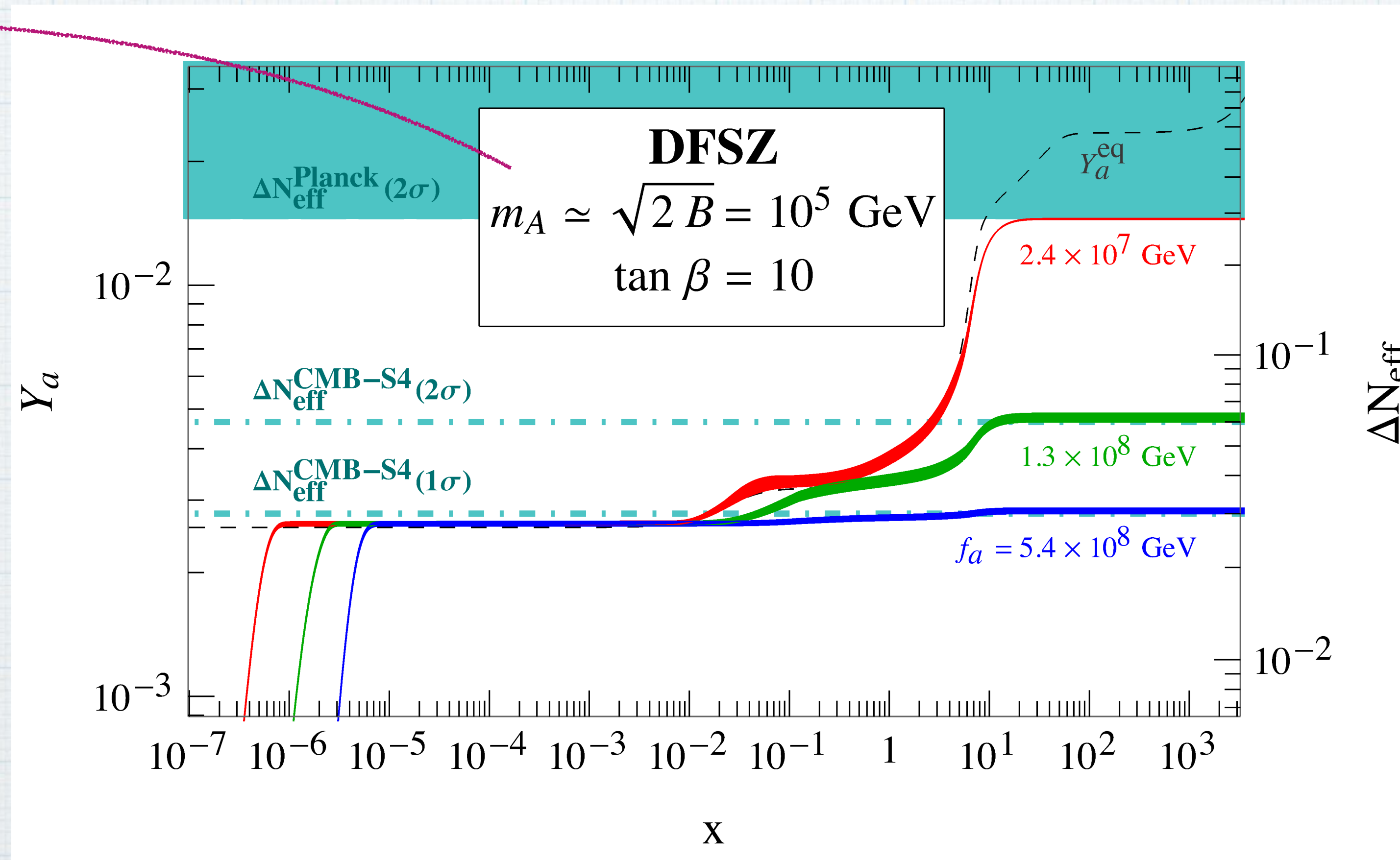
Total axion production rate for the DFSZ model at different temperatures:



Axion Yield in DFSZ Model

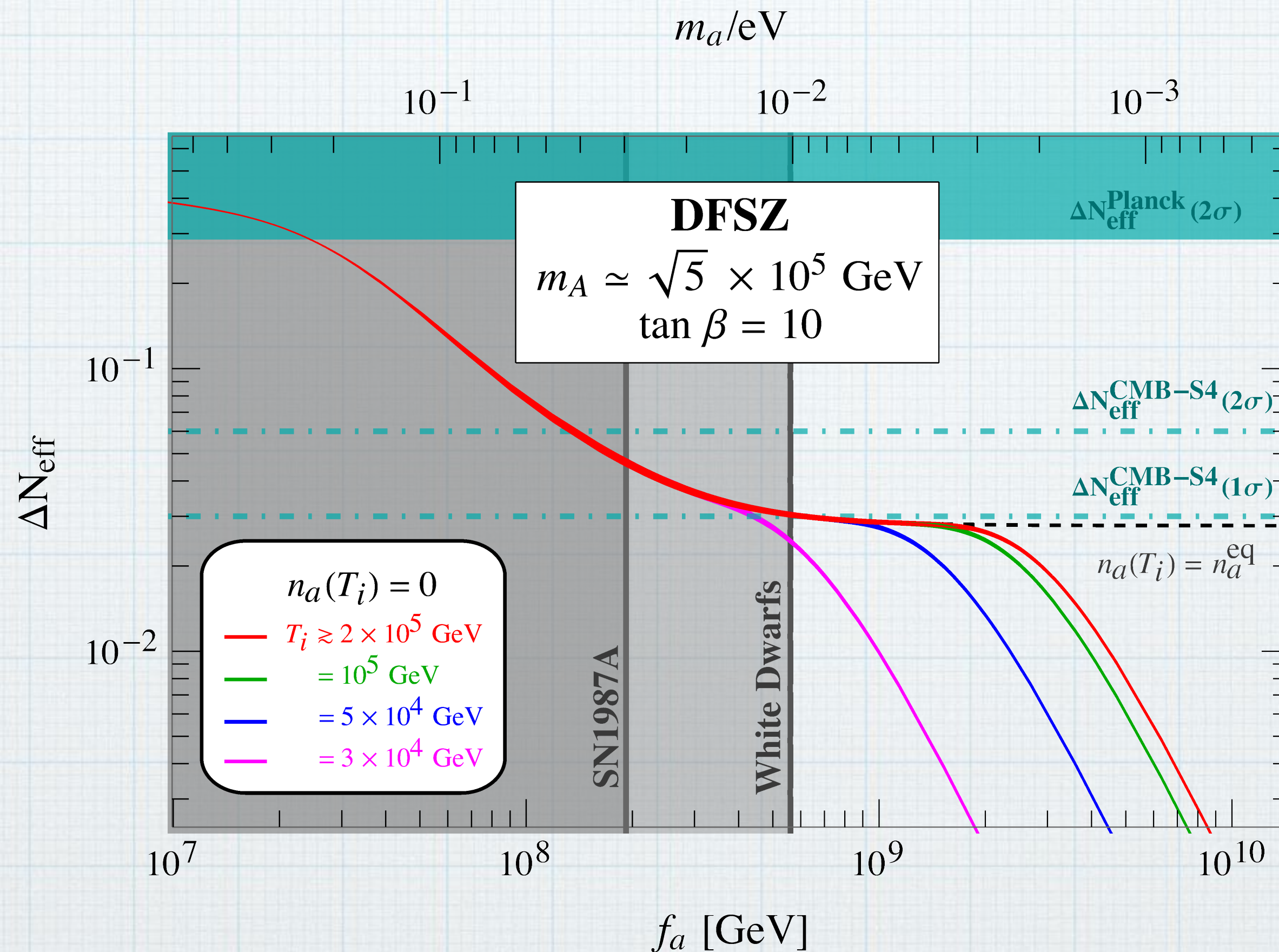
Axion yield evolution versus time or temperature:

Pseudo-scalar mass



Detectability Perspective For Different Axion Decay Constants

Number of effective neutrinos ΔN_{eff} versus axion decay constant f_a for the DFSZ model:



Summary and Conclusions

- * Computing the axion production rate below and above the QCD crossover transition and matching the two limits with the interpolation that is physically meaningful. Also we did calculation across the electroweak transition and above that...
- * Considering precise interaction rates of axion including thermal effects at different scales that improves the theoretical prediction for dark radiation from KSVZ and DFSZ axion models.
- * Using accurate treatment of thermal bath at different temperatures to have a correct estimation of axion abundance.
- * Predicting the precise value of ΔN_{eff} for different axion models is theoretically motivated for future probes by experiments like CMB-S4.

Thank you for your attention!

