Black hole Hyperaccretion Disks and Gamma-ray Bursts

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Mehr 1400

Introduction to the Brightest Bursts (GRBs)

The Brightest Bursts

- Gamma-ray bursts are sudden release of about $10^{50-54} erg$ energy in a volume with a radius of less than $100Km$, which lasts from $0.01s$ till $100s$.
- The rate of its occurrence is about $\frac{10^{-5}-10^{-6}}{yr.galaxy}$ for both LGRB and SGRB. Currently, orbiting satellites detect on average one GRB per day.
- *Remarkable results from observations:*
  1. No two gamma-ray bursts light curves are identical. Large variation observed in almost every property: *there is wide variation both in time-structure and duration.*
  2. However, the observed duration suggests the existence of two separate populations: *SGRB (less than 2sec)* and *LGRB (more than 2sec).*
  3. *Redshift distributions:*

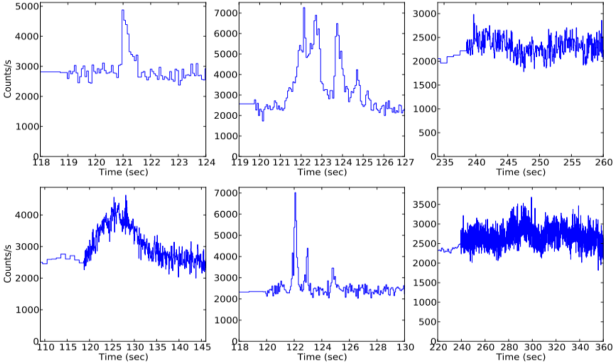
     LGRBs: Most of LGRB's have been detected in younger star forming galaxies with redshift up to $25-30$. A result in agreement with collapsar model!

     SGRBs: They seem to be less regular. However, their redshift distributions are more similar to the old objects population. A result consistent with merger model!

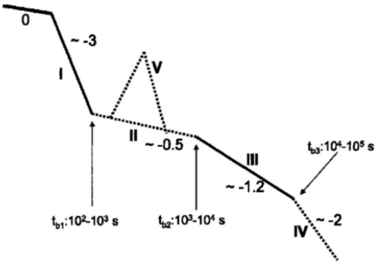
GRB's Specteral Features

Prompt emission

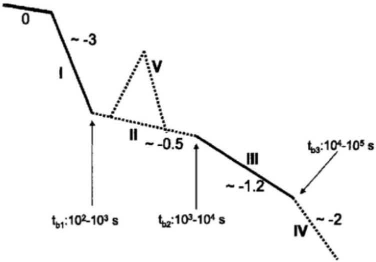
- It consists of soft gamma rays ($E > 50keV$) followed by hard X-ray emissions ($E \sim (5-50)keV$) called X-ray Flashes (XRF).
- The time duration (T_{90}) is $(0.01-100)s$. Long Gamma-Ray Bursts (LGRBs) are of $T_{90} > 2s$, and Short Gamma-Ray Bursts (SGRBs) have $T_{90} < 2s$.



- *Afterglow*
  - This part of GRB's spectrum is considered to be low energy emissions in X, radio and opitical wavelengths bandpass, that will be produced after prompt emission lasting up to several hours or days.
  - ***X-ray afterglows*** Thier light-curves consist of five main parts:
    1. *Steep decay phase:* Being of a steep slope of about t^{-10}, this phase is mostly considered as an emission tail of prompt phase.



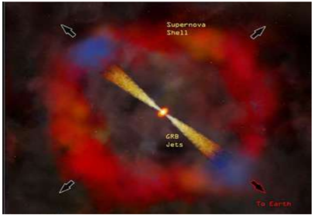
2. *Shallow decay phase:* Being considered as the most important feature of the early afterglows, this phase has a smooth slope of between 0 and -0.5, and lasts for less than $10^4 s$. It can be described via "*standard external shock model*".
3. *Normal decay phase:* The decay index of this section is of about -1, in agreement with the perediction of "*standard external shock model*".
4. *Post jet break phase:* This late time phase is of a slope of about -2. "*Jet break*" mechanism proposes that if the jet lorentz factor gets less than $1/\theta_j$, the flux detected by the earth observer will decrease by coefficient $\Gamma^2\theta_j^2$.
5. *X-ray flashes:* These flares share many properties with prompt emission pulses. They can be regarded as fast ups and downs in X-ray light curves, with luminosities between $1\% - 10\%$ of those in prompt phase. It is widely accepted that they are powered by late central engine activities. So they can be considered as promising hints to the central engines features.



- ***Optical afterglows*** These low energy emissions happen in hours after the prompt phase. They decay power index is of about -1, and some times, they include another section with power index -2, as well.
- ***Radio afterglows*** Being difficult to be detected, around 30% of the observed GRBs include radio afterglows. Their light curves begin to grow at $8GHz$ and get to a maximum in 3-6 weeks.

GRB's Progenitors

- *LGRB's progenitor*
  - Most of the LGRBs host galaxies are star forming dwarf galaxies which might reflect the relation between LGRBs and massive stars death (the collapsar model).
  - *Woosly 1993:* Rapidly rotating massive stars $\longrightarrow$ Iron core collapse
    - If $M > 30M_{\odot}$ $\longrightarrow$ BH formation + Fallback supernova
    - $\Longrightarrow$ Formation of continuously feeding BH accretion disk $(0.1 - 5M_{sun})$
    - $\Longrightarrow$ LGRB + Weak SN



- *SGRB's progenitor*
  - Most of the SGRB's host galaxies are elliptical ones with low star formation rate. This might strengthen the "NS-NS" or "NS-BH" merger model as SGRBs' progenitor.

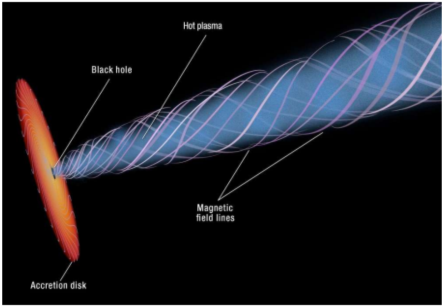
GRB's central engine

- Being the same for two kinds of GRBs (LGRB and SGRB), central engines of GRBs should meet some physical characteristics:
  1. Capable to produce jet of a high luminosity.
  2. The produced jet should be clean, namely the energy per baryon must be much more than $m_p c^2$, so that the ultra relativistic velocities (or $\Gamma > 10^2$) could be met.
  3. Capable to get reactivated in a periodic and irregular manner.
  4. Be able to stay activated in long term, to create late time less energitic bursts.
- ***Hyper accreting black holes*** and ***magnetars*** are two promising candidates that fulfill these features.

Hyper accreting black holes

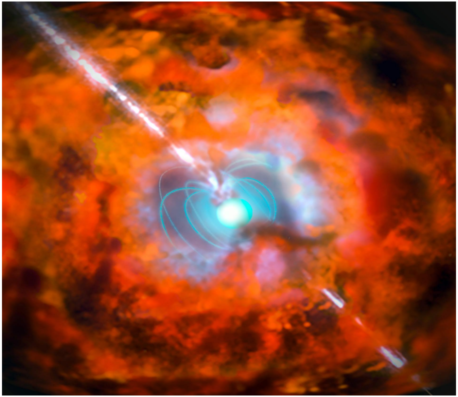
- Hyper accretion disks of a high accretion rate, $(0.01 - 10)M_{\odot}/s$, are able to produce GRBs.
- High accretion rate $\Longrightarrow$ Hot plasma with trapped photons $\Longrightarrow$ Efficient neutrino cooling
- Plausible mechanisms that might cause jet production:
  1. Neutrino-antineutrino anihilation
  2. Blandford-Znajek mechanism (BZ)
  3. Blandford-Payn mechanism (BP)

- *BZ mechanism* Four decades ago Blandford and Znajek (1977) proposed a process (BZ) in which rotational energy of black hole can be efficiently extracted. If there are sufficient charge distributions around the black hole to provide the force-free condition, then the magnetic field lines exert no force and corotate rigidly with the rotating black hole. The torque exerted by external magnetic field lines on plasma causes magnetic braking of the black-hole rotation through jet production.



Fast magnetars

- Magnetars are those of rapidly rotating highly magnetized ($\sim 10^{15} G$) neutron stars with a radius of about 20 Km and a mass of around $((2-3)M_{\odot})$. Regarding instabilities related to magnetic field, magnetars account for central engines of GRBs.



Non-electromagnetic radiations

- *Ultra high energy neutrinos contemporary with gamma-rays*
- *Precursor neutrinos*
- *Gravitational waves*
  - The gamma-rays and the afterglows of GRB are thought to be produced at distances from the central engine where the plasma has become optically thin, $r \geq 10^{13} cm$, which is much larger than the Schwarzschild radius of a stellar mass black hole (or of a neutron star). Hence we have only very indirect information about the inner parts of the central engine where the energy is generated. However, in any stellar progenitor model of GRB (NS-NS or BH-NS mergers) one expects that gravitational waves should be emitted from the immediate neighborhood of the central engine, and their observation should give valuable information about its identity. Therefore, it is of interest to study the gravitational wave emission from GRB associated with specific progenitors.

Neutrino Dominated Accretion Flows (NDAFs)

Neutrino Physics

- *Neutrino emission*
  - Neutrinos are generated through neutronization and thermal emission.
  - The most important thermal emissions are as follows:
    - *Electron-positron pair annihilation:* Considering "i" as different neutrino flavors, the process is

$$e^- + e^+ \to \nu_i + \bar{\nu_i}$$

with the cooling rate

$$q^-_{\nu_i,\bar{\nu_i}} \approx 5 \times 10^{33} T_{11}^9 ergs\ cm^{-3}\ s^{-1}$$

where $T_{11} = T/10^{11} K$.

    - *Nucleons bremsstrahlung emission:* $n + n \to n + n + \nu_i + \bar{\nu_i}$
    - *Plasmon decay:* $\tilde{\gamma} \to \nu_i + \bar{\nu_i}$

- The most significant neutronization is considered to be electron-positron pair capture on nuclei (i.e. URCA Process)

$$p + e^- \rightarrow n + \nu_e;$$
$$n + e^+ \rightarrow p + \bar{\nu_e}$$

- The cooling rate per volume is

$$q^-_{eN} = q^-_{e^+n} + q^-_{e^-p} \approx 9 \times 10^{33} \rho_{10} T_{11}^6 X_{nuc} \; ergs \; cm^{-3} s^{-1}$$

with mass fraction of free nucleons

$$X_{nuc} = min\{1, 295.5 \rho_{10}^{-3/4} T_{11}^{9/8} exp(-0.8209/T_{11})\}$$

Neutrino opacity

- The inverse processe of each neutrino emission mechanism can be accounted for neutrino absorption, besides the possible scattering processes.
- Electron-positron pair production and the inverse process of neutronization are the two dominant absorption mechanism with optical depth for the first one as

$$\tau_{a,\nu_i\bar{\nu_i}} \approx \frac{q^{-}_{\nu_i,\bar{\nu_i}} H}{4(7/8)\sigma T^4} \approx 2.5 \times 10^{-7} T_{11}^5 H$$

and the one for the latter is considered as

$$\tau_{a,\nu_e} \approx \frac{q^{-}_{eN} H}{4(7/8)\sigma T^4} \approx 2.5 \times 10^{-7} T_{11}^2 X_{nuc} \rho_{10} H$$

- Being of less importance with respect to absorption, neutrino scattering optical depth is

$$\tau_{s,\nu_i} = 2.7 \times 10^{-7} T_{11}^2 \rho_{10} H$$

NDAFs Thermodynamics

- The equation of state in NDAFs is

$$p = p_{gas} + p_{rad} + p_{deg} + p_{\nu}$$

where

$$p_{gas} = \sum_j n_j k_B T = \frac{\rho k_B T}{m_p} \frac{1 + 3X_{nuc}}{4}$$

$$p_{rad} = \frac{1}{3} a T^4$$

$$p_{deg} = \frac{2\pi hc}{3} \left(\frac{3\rho}{16\pi m_u}\right)^{4/3}$$

$$P_{\nu} = \frac{u_{\nu}}{3}$$

with neutrino energy density

$$u_{\nu} = \sum_i \frac{(7/8) a T^4 (\tau_{\nu_i}/2 + 1/\sqrt{3})}{\tau_{\nu_i}/2 + 1/\sqrt{3} + 1/(3\tau_{a,\nu_i})}$$

- The energy balance equation is considered as

$$Q_{vis} = Q_{adv} + Q_{\nu}^{-}$$

where

$$Q_{vis} = \frac{1}{2\pi}\dot{M}\Omega_K^2 fg, \;\; f = 1 - j/\Omega_K R^2, \;\; g = -d\ln\Omega_K/d\ln R$$

$$Q_{adv} = \frac{1}{2\pi}\frac{\xi\dot{M}c_s^2}{R^2}$$

$$Q_{\nu} = \sum_i \frac{(7/8)\sigma T^4}{(3/4)[\frac{\tau_{\nu_i}}{2} + \frac{1}{\sqrt{3}} + \frac{1}{3\tau_{a,\nu_i}}]}$$

Gravitational Instabilities in NDAFs

- Gravitational Instability (GI) in the **outer disk** leads to

  EITHER

  → Inspiral structure with dense fragments inside → Time variable accretion → Jet layars of different lorentz factors → Episodic emissions (Perna et al. (2006))

  OR

  → Outer disk fragmentation → Time variable accretion → Jet layars of different lorentz factors→ X-ray flares (Dall'Osso et al. (2016); Shahamat et al. (2020)))

- GI in the **inner disk** may also be possible:

  Masada et al. (2007): Neutrino trapping and MRI reduction in the inner disk → Baryonic mass accumulation → GI in the inner disk → Fast accretion restart → Episodic prompt emissions

  Shahamat et al. (currently): { Neutrino viscosity (proposed by Masada et. al. (2007)) + self-gravity + turbulence owing to GI in the inner disk's neutrino opaque region } → Inner disk's fragmentation → Fast accretion restart due to the fragments' viscose spreading → Highly variable prompt emissions

Self-gravity in Magnetized Neutrino-dominated Accretion Disks

Physical Model

Basic Formalism

- Our system is considered to be a steady and axisymmetric magnetized NDAF ($\partial/\partial t = 0,\ \partial/\partial \phi = 0$) in which vertical self-gravity has been taken into account. Regarding magnetic field influence on both large scale (magnetic braking mechanism) and small scale (viscous dissipation effects), through which the disk's rotational energy extraction and angular momentum transfer happen, we have the following relations for ***continuity*** and ***angular momentum*** equations

$$\dot{M} = -2\pi R \Sigma v_R = constant$$

$$\dot{M} = \frac{2\pi\alpha R^2 \Pi}{\Omega_k R^2 - j} + \frac{B_\phi B_z R^2}{\frac{\partial}{\partial R}(R^2\Omega_k)}$$

- The second term reflects the magnetic braking effect, in which

$$\Sigma = 2\int_0^H \rho dz$$

$$\Pi = 2\int_0^H p dz$$

$$\Omega_k = (GM/R)^{1/2}/(R - R_g) \longrightarrow \text{Keplerian angular velocity}$$

$$j = 1.8cR_g \longrightarrow \text{Angular momentum in ISCO}$$

Self-gravity in Magnetized NDAF

- ***Energy balance*** equation has components as pointed out previously, in addition to a magntic term to take the magnetic braking effect into consideration

$$Q_{vis} = Q_{adv} + Q_{\nu}^{-} + Q_{B}^{-}$$

where

$$Q_{B}^{-} = 2R\Omega(B_{\phi}B_{z}/4\pi)$$

- We consider the polytropic EOS, $p = K\rho^{4/3}$, in vertical direction.
- The hydrostatic balance equation in the vertical direction reads

$$4\pi G\Sigma_{z} + \frac{\partial\Psi}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{1}{8\pi\rho}\left(\frac{\partial B_{\varphi}^{2}}{\partial z} + \frac{\partial B_{R}^{2}}{\partial z}\right) - \frac{1}{4\pi\rho}B_{R}\frac{\partial B_{z}}{\partial R} = 0$$

where $\Psi = \frac{-GM}{\sqrt{R^{2}+z^{2}} - R_{g}}$ is the pseudo-Newtonian potential.

Toomre Parameter

- Disk is gravitationally unstable if

$$Q = \frac{c_s \Omega_k}{\pi G \Sigma_z} < 1$$

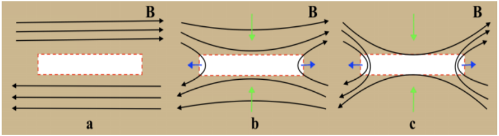
- In magnetized case this criterian reads

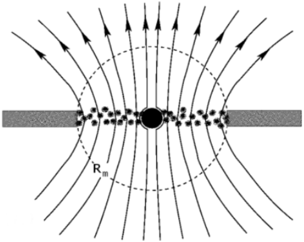
$$Q_M = \frac{\sqrt{c_s^2 + v_A^2}\Omega}{\pi \Sigma G} < 1$$

- where $v_A = \frac{B}{\sqrt{4\pi\rho}}$ is the Alfven velocity.

Magnetic Barrier Mechanism

- It is proposed that the accretion rate decline (the case for the late time activity of NDAFs) can result in X-ray flares in GRBs.
- Being inspired by Magnetically Arrested Disks (MADs), the magnetic field accumulation in the inner regions might halt the accretion flow, at magnetospheric radius R_m, intermittently.
- For $R > R_m$ the flow is axisymmetric while for $R < R_m$ the flow breaks up into blobs or streams, of a velocity much less than the free fall velocity ($v_r = \epsilon v_{ff}$ *with* $\epsilon < 1$), that fight its way towards the BH through magnetic reconnection.





- *Estimation of R_m:*

$$F_m\Big|_{R_m} = F_g\Big|_{R_m} \longrightarrow 2B_R B_z/4\pi\Big|_{R_m} = (GM_{BH}\Sigma/R^2)\Big|_{R_m}$$

$$\Sigma = \dot{M}/(2\pi R\epsilon v_{ff}) \; with \; v_{ff} = (2GM_{BH}/R)^{1/2}$$

With $B_R \approx B_z = B$, we then have

$$\Phi \approx \pi R^2 B(r) = 5\times 10^{28}\epsilon_{-3}^{-1/2}(R/R_g)^{3/4}\dot{M}_1^{1/2}M_3 \; cm^2 G$$

where $\epsilon_{-3} = 10^3\epsilon$, $\dot{M}_1 = \dot{M}/1M_\odot \; s^{-1}$ and $M_3 = M_{BH}/3M_\odot$.

$$\Longrightarrow r_m \approx 60\epsilon_{-3}^{2/3}\dot{M}_1^{-2/3}M_3^{-4/3}\Phi_{30}^{4/3}$$

where $\Phi_{30} \equiv \Phi/(10^{30} \; cm^2 G)$.

- Magnetic barrier happens if

$$t_{dif} \approx \frac{H}{v_A} > t_\nu = \int_{3R_g}^{R} \frac{1}{v_R} dR$$

Fragmentation

- In case of having $Q_M < 1$, fragmentation happens if

$$t_{cool} \approx (H/R)^2 t_\nu < t_{cirt} \approx 3\Omega^{-1}$$

Neutrino and Blandford-Payne Luminositis

- *Neutrino luminosity:* After the neutrino cooling rate Q_ν is calculated, we are able to measure the neutrino luminosity, L_ν, which is expressed as

$$L_\nu = 2\pi \int_{R_{in}}^{R_{out}} Q_\nu R dR$$

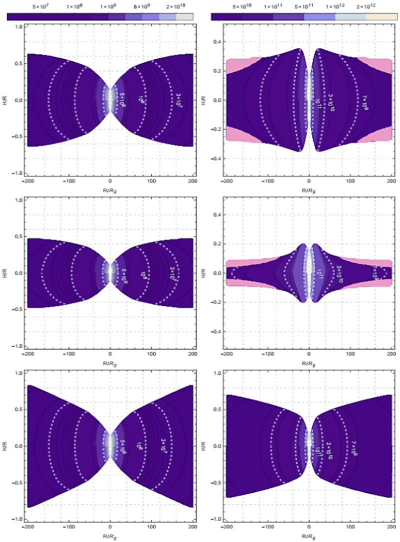
where $R_{in} = R_g$ and $R_{out} = 200 R_g$.

- *Blandford-Payne (BP) luminosity:* An outflow of matter can be driven centrifugally by large-scale magnetic fields anchored at the surface of the disk (Blandford & Payne 1982). Hence, the risk of baryonic pollution is much larger than what might happen through BZ mechanism, as the wind originates from high-density regions.

- The BP power output from a disk is equal to the power of disk magnetic braking and can be calculated as

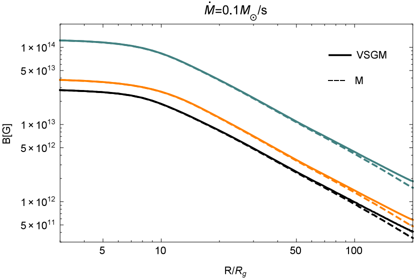
$$L_{BP} = 2\pi \int_{R_{in}}^{R_{out}} Q_B R dR.$$

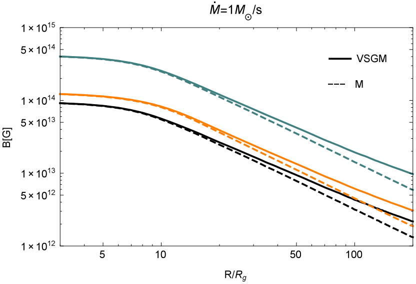
Results

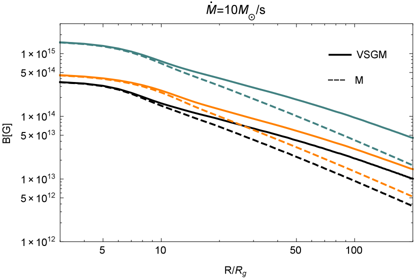


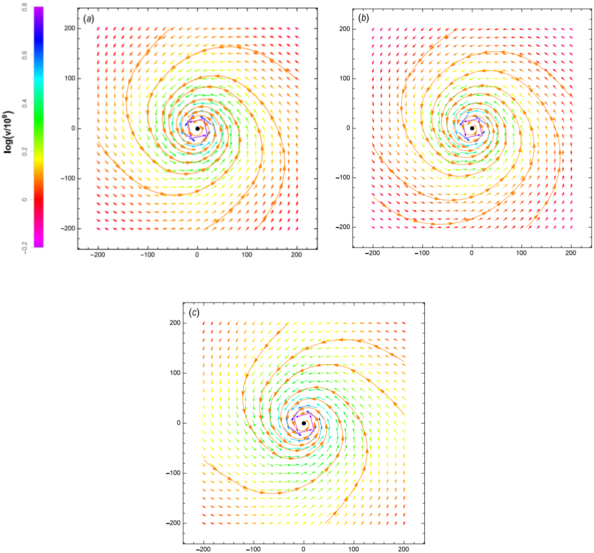
5×10^7 1×10^8 1×10^9 8×10^9 2×10^10
5×10^10 1×10^11 5×10^11 1×10^12 2×10^12
H/R
R/R_g

Self-gravity in Magnetized NDAF

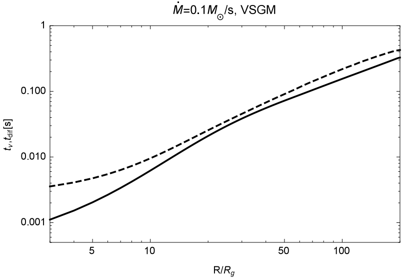


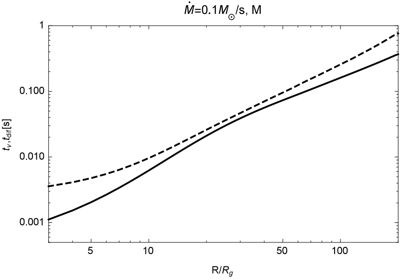


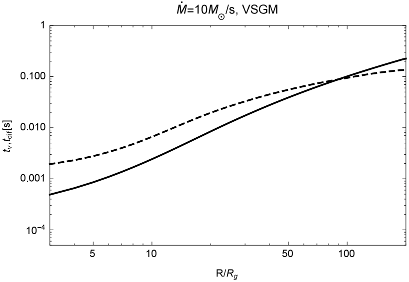


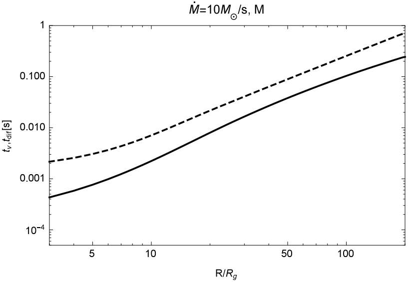

log(v/10^9)
(a)
(b)
(c)

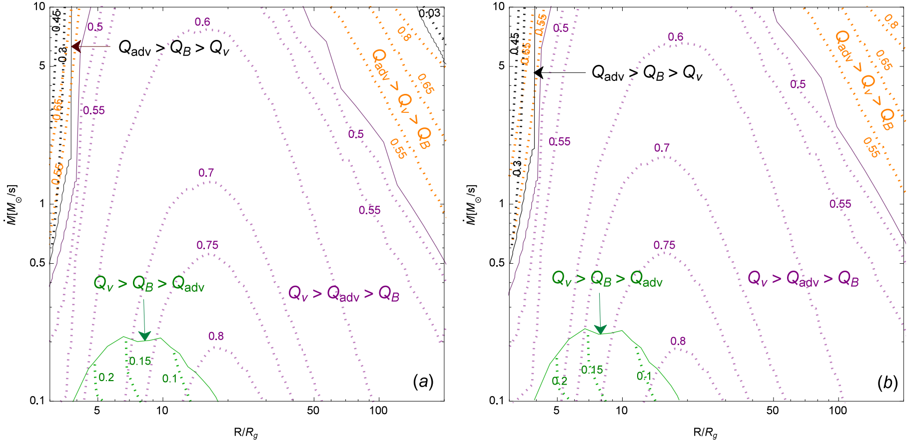
Self-gravity in Magnetized NDAF





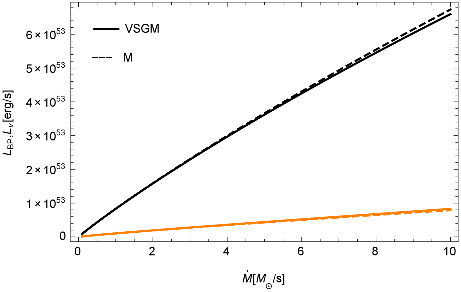


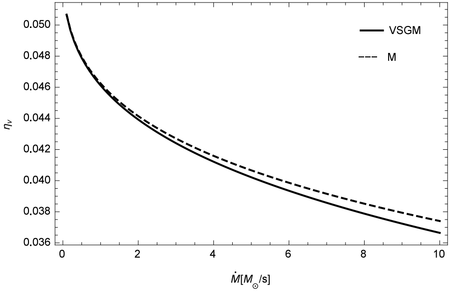




Qadv > QB > Qv
Qadv > Qv > QB
Qv > QB > Qadv
Qv > Qadv > QB
(a)
(b)
Ṁ[M⊙/s]
R/Rg

Self-gravity in Magnetized NDAF





Estimates and Observational Evidences

- To estimate the conditions needed to restart accretion, the accretion energetics, and related timescales, we ask what is the mass of a disk with Σ high enough to reduce r_m from its own magnitude in the late-time evolution to 3 or so. Before that, we should make an estimation for r_m in the late-time disk's activity.

$$\dot{M} = 10 M_\odot s^{-1} \Longrightarrow \Phi_{30}\Big|_{r=3R_g} \simeq 0.02$$

$$\text{If } \dot{M} = 10^{-3} M_\odot s^{-1},\ \Phi_{30} \simeq 0.02,\ \epsilon = 10^{-3} \Longrightarrow r_m = 33 R_g$$

- For $3R_g < r < 33R_g$ disk mass is of about $0.25 M_\odot$, and the time it takes to accrete is then

$$100s < \Delta t_{acc} = 250s < 1000s \checkmark$$

GRB variabilities and the following gravitational waves induced by gravitational instability in NDAFs

Early time fluctuations in the GRB's prompt emission

- *Di Matteo et al. (2002)*: For $\dot{M} \gtrsim 1\ M_\odot\ \mathrm{s}^{-1}$, neutrinos are sufficiently trapped in the inner disk, so that flow is advection dominated in this realm instead of being neutrino dominated.

- Masada et al (2007) investigated this region to probe the effectiveness of the magnetorotational instability (MRI) and found that the energy and momentum transport by neutrinos (i.e., neutrino viscosity) could suppress the growth of the MRI significantly, when the magnetic field strength $B \lesssim 10^{14}$ G is considered. Having said that, MRI can drive active magnetohydrodynamic turbulence in the outer neutrino-transparent region regardless of the field strength. This gives birth to an accumulation of the baryon matter into the inner dead zone, where the MRI grows inactively and the accretion of matter is suppressed. When the dead zone achieves a large amount of mass and becomes gravitationally unstable, the intense mass accretion onto the BH is triggered through the gravitational torque, episodically. This process can account for the short-term variability in the prompt emission of GRBs.

- Being inspired by these ideas, we consider this inner region to be self-gravitating besides the consideration of the neutrino viscosity as the most effective factor in transportion of the energy and momentum.

Physical Model

- The neutrino viscosity is described by

$$\nu = \frac{4}{15}\frac{U_\nu}{\rho c} <\lambda> \approx 5.2 \times 10^{12} T^2 \rho^{-2}$$

where $U_\nu = 4\sigma_B T^4/c$ is the neutrino energy density. The avaraged mean free path of the neutrino introduced as $<\lambda> = H/\tau_{tot}$, in which the total neutrino optical depth reads $\tau_{\rm tot} \approx 1.3 \times 10^{-38} T^2 \rho H$.

- Concerning the disk structure, we assume a quasisteady axisymmetric configuration for the inner neutrino opaque regions of our self-gravitating disk. Therefore, we have

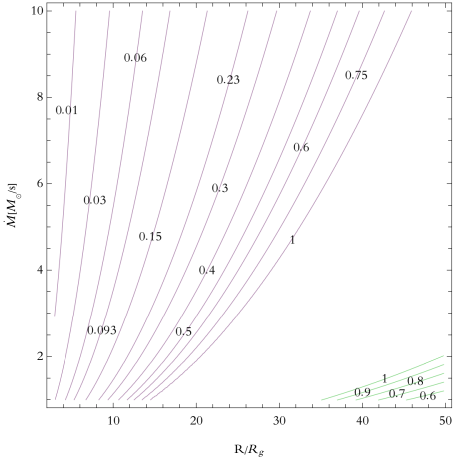
$$\nu\Sigma = -\frac{\dot{M}}{2\pi R}\Omega(\frac{\partial\Omega}{\partial R})^{-1}$$

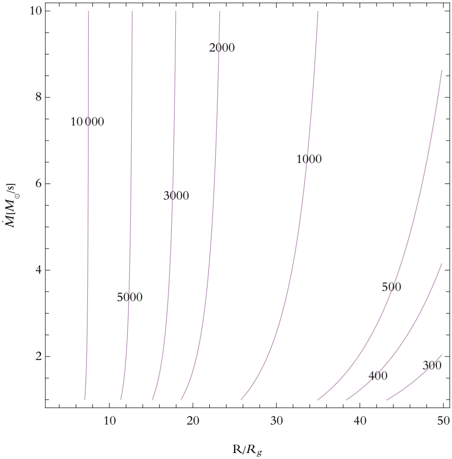
- The vertical structure of this self-gravitating disk can be accounted for by the hydrostatic balance equation

$$2\pi G\Sigma + \Omega^2 H - \frac{c_s^2}{H} = 0$$

- If we take the advection mechanism as the dominante cooling factor into account, and approximate the pressure by the gass pressure, one can solve the two last equations introduced above.

Gravitational Instability and Fragmentation (Neutrino viscosity case)



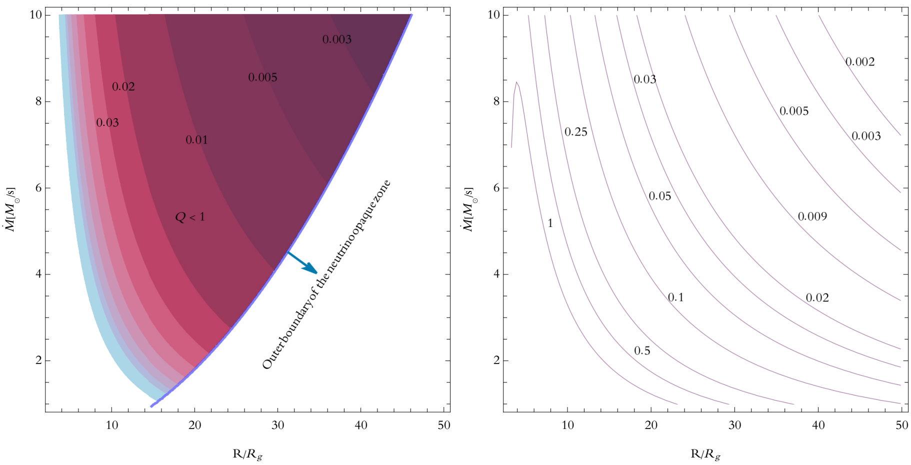


Gravitational Instability and Fragmentation (Beta-Neutrino Viscosity Prescription)

- It is argued that in such a self-gravitating case with the GI is possible, the disk may settle into a quasi-steady state of self-gravitating turbulence, in which an outward transport of angular momentum via gravitational torques is expected (Lynden-Bell & Kalnajs (1972); Lodato & Rice (2004), (2005); Lin & Kratter (2016)).

- Assuming the GI as a source of turbulence, and consequently viscosity, we consider Duschel (2006) approach to take this type of viscosity in the same form as that of the β parameterizations, i.e., $\nu = \beta R v_\phi$. We then have

$$\beta R^2 \Omega \Sigma^2 + \Sigma \frac{\dot{M}}{2\pi R} \Omega (\frac{\partial \Omega}{\partial R})^{-1} + 20.8 \times 10^{12} T^2 H^2 = 0.$$



10
8
6
4
2
10
20
30
40
50
$\dot{M}[M_\odot/s]$
R/R_g
0.003
0.005
0.02
0.03
0.01
$Q < 1$
Outerboundaryof the neutrinoopaquezone
0.002
0.03
0.005
0.25
0.003
0.05
0.009
1
0.1
0.02
0.5

Accretion of the Fragments and Observational Evaluation

- In order for a fragment to start accretion due to central BH's gravitational torque, as well as disk's viscous torque, the following condition should be met by the shear force per unit length f_ν, tidal force f_T, and self-gravity force of the fragment f_{SG},

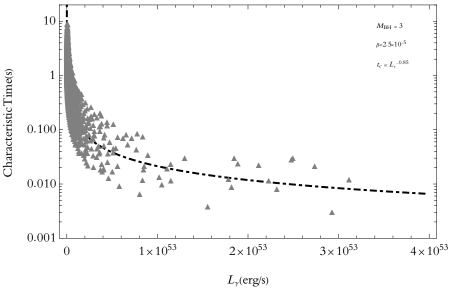
$$f_\nu + f_T > f_{SG}.$$

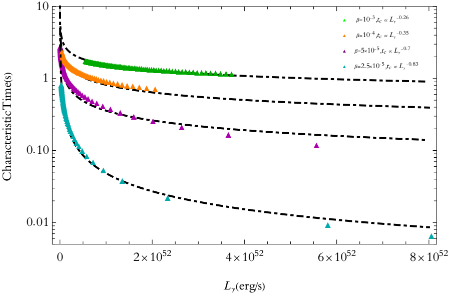
- One may estimate $f_T = \frac{GM_{\rm BH}}{R^2}(\frac{l_f}{R})$, $f_{SG} = \frac{3GM_f}{5l_f^2}$, and $f_\nu = \frac{l_f \nu \Sigma R}{M_f} \mid \frac{{\rm d}\Omega}{{\rm d}R} \mid$, in which M_f and l_f are considered to be the fragment's mass and size.
- The consequent gamma-ray luminosity owing to the fragments' accretion can be considered as

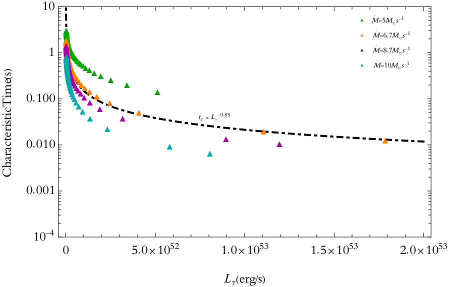
$$L_\gamma = f\eta \dot{M}_f c^2$$

where η is the energy conversion efficiency from the rest-mass energy to the gravitational one, and f is taken as the conversion factor from the gravitational energy to the radiative one.

Early time fluctuations in the GRB's prompt emission







Fragments' Migration and GWs

- Imagine a system composed of a single fragment in a circular orbit around the BH. The GW strain amplitude for a source at the distance of $D = 100D_{100}$ Mpc approximately reads the following relation

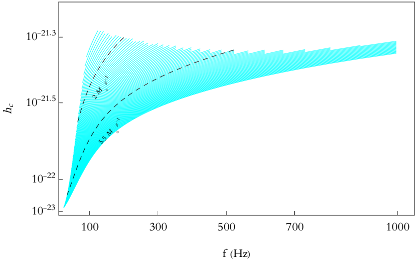
$$h_0 = 6.4 \times 10^{-24} \Theta M_f (\frac{M_{BH}^{2/5}}{M_\odot})^{5/3} f_{100}^{2/3} D_{100}^{-1}$$

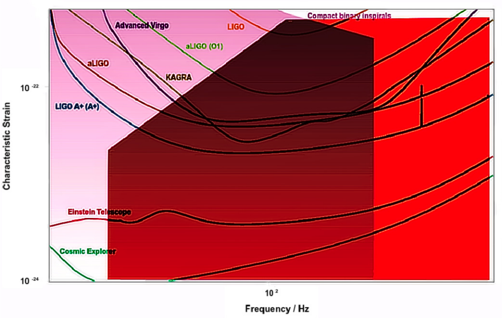
where $\Theta = 0 - 4$ is introduced in terms of orientation of the source and the antenna pattern of the detector, and $f = 100f_{100}$ Hz is the frequency.

- The characteristic strain is then introduced as

$$h_c = h_0 n_{cyc}^{1/2}$$

where $n_{cyc} \equiv f^2/\dot{f}$ is the number of cycles spent within a bandwidth $\Delta f \sim f$ centered on f. Regarding $f = \Omega/\pi$ and taking the Keplerian angular velocity $\Omega = \sqrt{\frac{GM_{BH}}{R^3}}$ into account, this can be rewritten as $n_{cyc} = 2fR/3\dot{R}$, with $\dot{R}/R$ is the total inward migration rate. One can approximate this factor as $|\dot{R}/R| = t_\nu^{-1} + t_g^{-1}$.


$10^{-21.3}$
$10^{-21.5}$
10^{-22}
10^{-23}
h_c
100
300
500
700
1000
f (Hz)


Compact binary inspirals
Advanced Virgo
LIGO
aLIGO (O1)
aLIGO
KAGRA
LIGO A+ (A+)
Einstein Telescope
Cosmic Explorer
10^{-22}
10^{-24}
Characteristic Strain
10^2
Frequency / Hz

Viscous Evolution of Magnetized Clumps: a Source for X-ray Flares in Gamma-ray Bursts

Late time activity of central engine and X-ray flares

- Being detected in both long and short GRBs, X-ray flares appear mostly in $10^{2-5}s$ time window, hence overlapping with the afterglow time-scale.
- The leading external shock scenario failed to justify the temporal properties of X-ray flares as a consequence of inhomogeneities.
- A variety of efforts have been conducted to attribute the flares to central engine evolutions:
    - *King et al. (2005):* Fragmentation of a collapsing star and its subsequent accretion
    - *Perna et al. (2006):* Ring-like fragmentation of the outer regions in a hyperaccreting disk
    - *Proga and Zhang (2006):* Late time episodic accretion caused by a magnetic flux accumulation in the inner disc regimes, i.e. magnetic barrier
- Having proposed by Perna et al. (2006), Dall'Osso et al. (2017) developed the idea of the disc fragmentation in a more quantitative manner. They suggest that various flares with different arrival times (including both early and late time ones) might be attributed to the viscous spreading of different clumps created during the early or late time phase of central engine evolution.

Ring-like Fragmentation and Their Viscous Evolution

- After getting gravitationally unstable, fragmetation occurs in the outer disk if

$$t_{cool} < t_{\mathrm{cirt}} \approx 3\Omega^{-1}$$

- Fragments merge and/or accrete until their tidal influence gets strong enough to open a gap in the disc. This happens when the mass of the clump has increased to

$$M_{frag} \simeq (\frac{H}{R})^2 \alpha^{1/2} M_{BH}$$

- In what follows, regarding the model proposed by Perna et al. (2006), we suppose such a ring-like clump has been created in the outer regions as a sharp accumulated mass, more specifically, a delta function, at radius R_0. Computing the viscous evolution of the clump, we also consider the standard equation for axisymmetric accretion discs

$$\frac{\partial}{\partial t}\Sigma(R,t) = \frac{1}{R}\frac{\partial}{\partial R}[R^{1/2}\frac{\partial}{\partial R}(3\nu\Sigma R^{1/2})]$$

- The small scale effect of magnetic field can be regarded through magnetic viscosity term, i.e. ν. To consider the large scale effect of magnetic field we add a term corresponding to the torque exerted by the Lorentz force

$$\frac{\partial}{\partial t}\Sigma(R,t) = -\frac{1}{R}\frac{\partial}{\partial R}\left[\frac{\frac{\partial}{\partial R}(R^3\nu\Sigma\frac{d\Omega}{dr}) + \frac{R^2 B_\phi B_z}{2\pi}}{\frac{d}{dR}(R^2\Omega)}\right]$$

- Considering $B_z \approx B_R$, we can make use of the magnetic viscosity equation

$$\frac{B_R B_\phi}{4\pi} = \frac{3}{2}\alpha P$$

- On the other hand, we know that

$$P = -\frac{1}{\alpha}\rho\nu R\frac{d\Omega}{dR},$$

and consider $H\rho \approx \Sigma$, as a vertically averaged approximation. Finally, we have

$$\frac{\partial}{\partial t}\Sigma(R,t) = -\frac{1}{R}\frac{\partial}{\partial R}\left[\frac{\frac{\partial}{\partial R}(R^3\nu\Sigma\frac{d\Omega}{dr}) + 3\frac{d\Omega}{dR}\frac{\Sigma}{h}\nu R^2}{\frac{d}{dR}(R^2\Omega)}\right]$$

Accreting Clumps: an X-Ray Flare Source

- In general, the viscosity ν depends on the surface density and this equation is non-linear. If, however, ν is only a function of radius, then the equation is linear and much more amenable to analytic methods. Therefore, we adopt the viscosity to have a radial power law, $\nu \propto R^n$, to achieve an exact solution for $\Sigma(R,t)$ using a Green's function G,

$$\Sigma(R,t) = \int_{R_{in}}^{\infty} G(R,R',t)\Sigma(R',t=0)dR',$$

in which $\Sigma(R,t=0)$ is a given arbitrary profile at $t=0$. Having $\Sigma(R,t)$, L_{acc} reads

$$L_{acc} \simeq \int_{R_{in}}^{\infty} \frac{9}{4}\Sigma(R,t)\nu(R)\Omega^2(R)2\pi R dR.$$

- To compute Green's function we consider two different boundary conditions (BCs):

1. *Zero central torque boundary condition*, which is of astrophysical interest especially in case of accretion on to a black hole or a slowly rotating star, at radii larger than the radius of innermost circular orbit or the stellar surface, respectively.
2. *Zero mass flux boundary condition* at inner radious. Having a strong central source of angular momentum, the accretion gas will be prevented to flow in, and instead accumulate near the center. Such solutions can describe accretion discs around a compact binary, and compact objects with strong central magnetic fields (such as the case for magnetically arrested discs (MADs) and the consequent magnetic barrier (Narayan et al. (2003); Shahamat and Abbassi (2017))).

- Obtaining the appropriate Green's function, in case of zero torque boundary condition, the surface density integral takes the following form

$$\frac{\Sigma}{\Sigma_0} = \int_0^\infty \frac{R}{R_0}^{-n+(1-b_1)/2} (\frac{R_0}{R_{in}})^{2-n} (1-n/2) e^{-2\kappa^2(1-n/2)^2 t/t_{\nu in}} [Y_l(\kappa) J_l(\kappa x_0)$$

$$-Y_l(\kappa x_0) J_l(\kappa)] \frac{[Y_l(\kappa) J_l(\kappa x) - Y_l(\kappa x) J_l(\kappa)]}{[Y_l^2(\kappa) + J_l^2(\kappa)]} \kappa d\kappa$$

- In case of zero mass flux boundary condition one may achieve

$$\frac{\Sigma}{\Sigma_0} = \int_0^\infty \frac{R}{R_0}^{-n+(1-b_1)/2} (\frac{R_0}{R_{in}})^{2-n} (1-n/2) e^{-2\kappa^2(1-n/2)^2 t/t_{\nu in}} [Y_{l-1}(\kappa) J_l(\kappa x_0)$$

$$-Y_l(\kappa x_0) J_{l-1}(\kappa)] \frac{[Y_{l-1}(\kappa) J_l(\kappa x) - Y_l(\kappa x) J_{l-1}(\kappa)]}{[Y_{l-1}^2(\kappa) + J_{l-1}^2(\kappa)]} \kappa d\kappa$$

Note that we considered $\Sigma(R, t=0) = \Sigma_0 R_0 \delta(R - R_0)$.

Determination of Σ_0 *and* R_0

- One can assume the ring-like clump as a sharp concentration at radius R_0. This radius can be regarded the same as the gravitational instability radius. Therefore, we approximated Σ_0 with the expression

$$\Sigma_0 \simeq \frac{M_{frag}}{2\pi R_0 l_{cl}} \approx \frac{M_{frag}}{2\pi R_{ins} l_{cl}}$$

where l_{cl} denotes the clump size, that can be estimated as the local Jeans length (λ_j).

$$M_{frag} \simeq (\frac{H}{R})^2 \alpha^{1/2} M_{BH}$$

- We estimate R_0 to be the same as R_{ins}, which can be achieved by Toomre criterion

$$Q_{mag} = \frac{\sqrt{c_s^2 + v_A^2}\,\Omega}{\pi \Sigma G} < 1$$

Accreting Clumps: an X-Ray Flare Source

- Through some mathematical considerations we applied previousely, the following expression for B^2

$$B^2 = -\frac{0.3}{\alpha}\rho\nu R\frac{d\Omega}{dR}$$

a long with the application of $\dot{M} = 3\pi\Sigma\nu$ (valid for a steady accretion), and $n = 1/2$ (an acceptable value for advection-dominated discs, results in

$$R > (3h^2\alpha\frac{M}{\dot{M}}\sqrt{1.03h^2GM})^{2/3} = R_{ins}.$$

- To estimate l_{cl}, we need local Jeans length that reads

$$\lambda_{jmag} = \lambda_j\sqrt{1+\frac{v_A^2}{c_s^2}} = c_s\sqrt{\frac{\pi}{G\rho}}\sqrt{1+\frac{v_A^2}{c_s^2}} = \sqrt{\frac{3.1h^5\pi^2\alpha M\sqrt{GMR_{ins}}}{\dot{M}}}.$$

Now, we have our approximately estimated values for Σ_0 and R_0.

What about R_{in}?

- In case of zero torque boundary condition, we choose R_{in} to be equal to the Innermost Stable Circular Orbit (ISCO), while in zero mass flux boundary condition, in which the magnetic barrier can be taken into account, we considered the magnetospheric radius as the inner radius.

$$R_m \approx 60\epsilon_{-3}^{2/3}\dot{M}_1^{-2/3}M_3^{-4/3}\phi_{30}^{4/3}$$

where $\phi_{30} \equiv \phi/(10^{30}\ cm^2 G)$, with ϕ is taken to be of about 10^{-2}, which is a typical value for GRB's central engine activity. Also $\epsilon_{-3} = 10^3\epsilon$, and we adopted a value of 10^{-3} for ϵ.

- Metzger et al. (2008) reported that a neutrino cooled accretion disc, in its late time viscous evolution, experiences a decreasing mass accretion rate with a self similar behavior $\dot{M} \propto t^{-4/3}$. Hence, we approximate the late time accretion rate to be

$$\dot{M} = \dot{M}_0(\frac{t}{t_0})^{-4/3}$$

where $\dot{M}_0$ is the accretion rate related to a given time, t_0, specified (arbitrarily) during the late activity phase. We have fixed $\dot{M}_0$ to be of the value of $0.04M_\odot/s$ at $t_0 = 50s$ after the prompt phase (that is supposed to have an accretion rate of about $0.1 - 10M_\odot/s$, typically).

What about R_{out}?

- In the context of the outer boundary, we consider R_{out} as a radius at which the viscous evolution of clump sets in. To estimate such a radius, we take shear, self-gravity and tidal forces per unit mass, exerted on the clump. We then add Lorentz force in order to account for the large scale effects of magnetic field.

- A clump of linear size l_{cl} and mass M_{cl} will be affected by the shear force per unit mass

$$F_\nu = \frac{l_{cl} \dot{M} \Omega}{2\pi M_{cl}}$$

- Given the self-gravity force per unit mass of the clump, $F_{SG} = \frac{GM_{cl}}{l_{cl}^2}$, the tidal force due to the central object, $F_T = \frac{GM_{BH}}{r^2}(\frac{l_{cl}}{r})$, and Lorenz force per unit mass, $F_B = \frac{B_r B_\phi}{4\pi\Sigma}$, viscous spreading of the clump will start roughly when

$$F_\nu + F_T + F_B > F_{SG}$$

- Considering magnetic viscousity equation and $\dot{M} = 3\pi\Sigma\nu$, Lorentz force can be rewritten in the form of $\frac{9GM\alpha h}{4r^2}$. After all, the condition for being viscously evolved reads

$$\frac{GMl_{cl}}{r^3} + \frac{l_{cl}\dot{M}}{2\pi M_{cl}}\sqrt{\frac{GM}{r^3}} + \frac{9GM\alpha h}{4r^2} - \frac{GM_{cl}}{l_{cl}^2} > 0$$

Bolometric and X-ray luminosities

- the bolometric luminosity of the flare is

$$L_{bol} = f_{rad} L_{acc}$$

  where f_{rad} is the conversion efficiency of energy from mass accretion into radiation which is considered to be constant, for the sake of simplicity. In case of zero torque BC we adopt $f_{rad} \approx 0.5$, while it can be regarded more radiatively efficient in case of zero mass flux BC, considering MADs, i.e. ~ 1.0.

- An efficiency coefficient is required to consider the X-ray energy band. Here, we approximate X-ray flare luminosity as a constant coefficient of bolometric luminosity, $L_X = f_X L_{bol}$, where f_X (X-ray flare efficiency) is estimated as ~ 0.1.

Results

Model Parameters and Spectral Properties Correlation

- *Model parameters:* Our model consists of some parameters such as M_{BH}, h, and α.
- *Specteral parameters:* We interoduce two specteral parameters which describe light curve properties:
  1. The ratio of the width $\Delta t = t_2 - t_1$, to the peak time, i.e. $w = \Delta t / t_p$. t_1 and t_2 refer to the times with fluxes of around $\frac{1}{e} L_p$ (the former during the rise time interval and the latter during the decay one).
  2. The asymmetry parameter which is regarded to be the ratio t_d/t_r, with rise time $t_r = t_p - t_1$ and decay time $t_d = t_2 - t_p$.
- Regarding $0.5 \leqslant h \leqslant 1$ (valid for the late time advection dominated phase of GRB's central engine, and $0.01 \leqslant \alpha \leqslant 0.2$, some of our outcome can be considered as follows.

Table 1. Spectral and model parameters for zero torque and zero mass flux boundary condition.

M_{BH} ($M_\odot$)	t_{off} (s)	h	α	M_{cl} ($M_\odot$)	t_p (s)	L_p ($10^{48} erg/s$)	Δt (s)	w	k
10	100.0	0.6	0.01	0.36	110.0	1.4	16.0	0.15	2.5
10	200.0	0.6	0.01	0.36	220.5	0.6	32.0	0.15	2.3
10	100.0	0.5	0.01	0.25	105.0	1.6	7.5	0.07	2.7
10	200.0	0.5	0.01	0.25	210.5	0.7	15.0	0.07	2.0
5	50.0	0.9	0.01	0.4	61.5	2.12	21.5	0.3	2.5
5	100.0	0.9	0.01	0.4	122.5	0.73	43.0	0.3	2.7
3	50.0	0.9	0.01	0.2	57.0	2.1	12.5	0.2	2.51
3	100.0	0.9	0.01	0.2	113.5	0.7	26.0	0.22	2.7
3	50.0	0.9	0.1	0.76	57.0	2.7	12.5	0.2	2.5
3	100.0	0.9	0.1	0.76	114.5	1.15	26.5	0.23	2.7
2.5	1000.0	0.6	0.01	0.09	1094.5	0.023	148.0	0.14	2.4
2.5	1000.0	0.5	0.01	0.06	1048.5	0.03	69.0	0.07	2.2
2.5*	200.0	0.8	0.15	0.6	215.0	6.0	26.0	0.12	2.7
2.5*	300.0	0.8	0.15	0.6	340.5	1.12	70.5	0.2	2.7
3*	200.0	0.8	0.13	0.7	218.0	8.0	31.5	0.15	2.7
3*	300.0	0.8	0.13	0.7	349.0	1.5	85.0	0.24	2.6

*Cases with zero flux boundary condition.

- First of all, as h grows, the clump mass increases, and this leads to a decline in the peak luminosity, and an increase in width parameter as well as peak time, i.e. the light-curve gets wider with less maximum radiated flux.
- Secondly, our data demonstrate that any increase in α parameter might enhance remarkably the clump mass, while the width parameter might not be affected considerably, so that the peak luminosity grows. The fact that has been also inferred by Dall'Osso et al. (2017).
- Finally, we came into conclusion that a black hole mass growth may lead to a heavier clump with a rather higher luminosity that happens in a longer duration time scale.
- Over all, such correlations between model parameters and spectral quantities are inferable from our data, however, in agreement with Dall'Osso et al. (2017) and observational analysis, the shape parameters w and k are not strongly affected by our model parameters, regardless of their somehow scattered behavior.

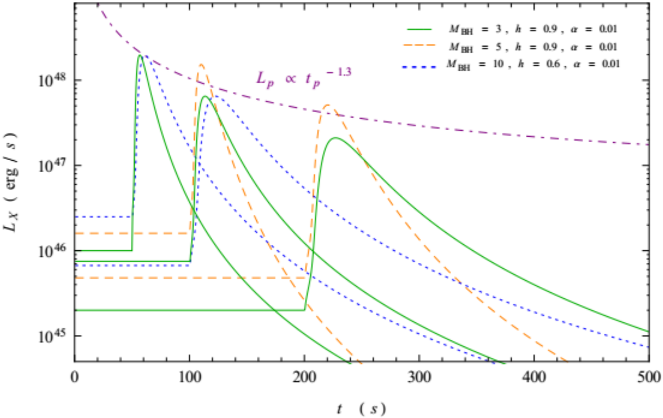
Observations and model predictions

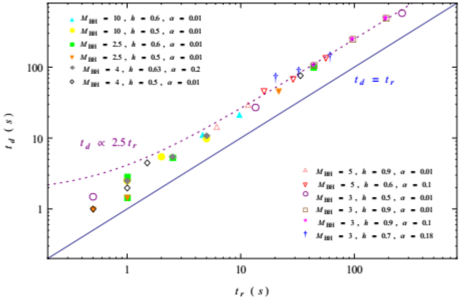
- some observational characteristics of the X-ray light curves can be clarified as follows:

1. The rise over decay time ratio is constant, implying that both timescales grow by the same factor, so that $t_d \approx 2t_r$. Consequently, flares are self-similar in time.
2. The width linearly evolves with the peak time: $w \approx 0.2$.

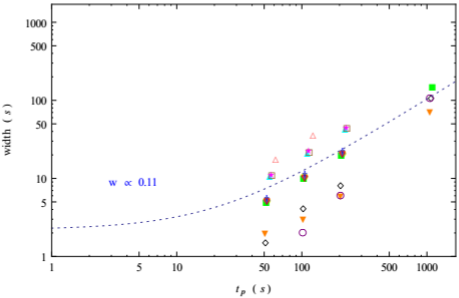
   These two points are the key features that strongly distinguish the flare emission from the prompt phase.
3. The peak luminosity decreases with the peak time, following a power-law behavior $L_p \propto t_p^{-1.27}$.

Accreting Clumps: an X-Ray Flare Source

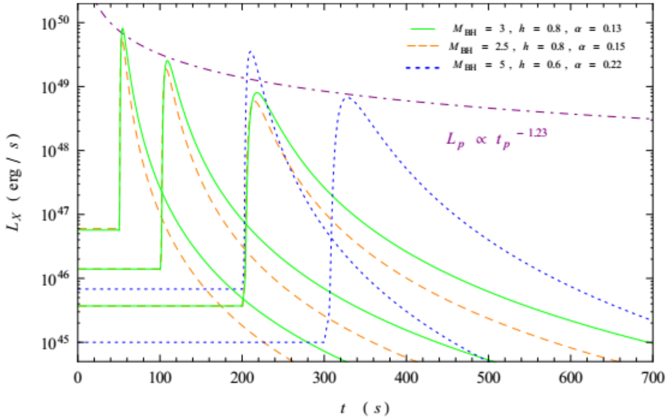


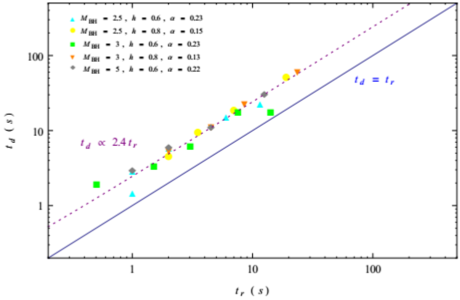


M_BH = 10 , h = 0.6 , α = 0.01
M_BH = 10 , h = 0.5 , α = 0.01
M_BH = 2.5 , h = 0.6 , α = 0.01
M_BH = 2.5 , h = 0.5 , α = 0.01
M_BH = 4 , h = 0.63 , α = 0.2
M_BH = 4 , h = 0.5 , α = 0.01
M_BH = 5 , h = 0.9 , α = 0.01
M_BH = 5 , h = 0.6 , α = 0.1
M_BH = 3 , h = 0.5 , α = 0.01
M_BH = 3 , h = 0.9 , α = 0.01
M_BH = 3 , h = 0.9 , α = 0.1
M_BH = 3 , h = 0.7 , α = 0.18
t_d = t_r
t_d ∝ 2.5 t_r
t_d (s)
t_r (s)

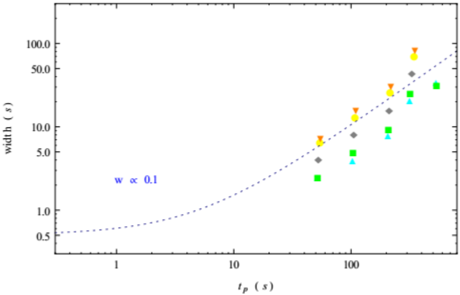


w ∝ 0.11
width (s)
t_p (s)


M_{BH} = 3 , h = 0.8 , α = 0.13
M_{BH} = 2.5 , h = 0.8 , α = 0.15
M_{BH} = 5 , h = 0.6 , α = 0.22
$L_p \propto t_p^{-1.23}$
L_X (erg / s)
10^{50}
10^{49}
10^{48}
10^{47}
10^{46}
10^{45}
0
100
200
300
400
500
600
700
t (s)



M_{BH} = 2.5 , h = 0.6 , α = 0.23
M_{BH} = 2.5 , h = 0.8 , α = 0.15
M_{BH} = 3 , h = 0.6 , α = 0.23
M_{BH} = 3 , h = 0.8 , α = 0.13
M_{BH} = 5 , h = 0.6 , α = 0.22
$t_d = t_r$
$t_d \propto 2.4 t_r$
t_d (s)
t_r (s)



w ∝ 0.1
width (s)
t_p (s)


THANK
YOU!