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Luminosity of accretion disks in compact objects with a quadrupole

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The presentation is based on our recent paper:

Boshkayev K., Konysbayev T., Kurmanov T., Luongo O., Malafarina D. and Quevedo H. PRD 104, 084009 (2021) arXiv:2106.04932

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Outline

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- Review of the Kerr metric
- Orbital parameters of test particles
- Radiative flux and spectral luminosity
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Motivation

The theory of black hole accretion was developed by Novikov & Thorne (1973) and Page & Thorne (1974), and has been successfully applied to astrophysical black hole candidates to explain the features of their observed spectra for many years. However, observations are almost always interpreted under the assumption that the black hole is in a vacuum (i.e. the Kerr metric). It is only in recent years that attempts have been made to study the theoretical properties of accretion discs in a geometry that departs from the Kerr line element (Harko, Kovacs & Lobo 2009a,b; Bambi & Barausse 2011; Bambi & Malafarina 2013).

We consider the circular motion of test particles in the gravitational field of a static and axially symmetric compact object described by the *q* metric. To this end, we calculate orbital parameters of test particles on accretion disks such as angular velocity (Ω), total energy (E), angular momentum (L), and radius of the innermost stable circular orbit as functions of the mass (m) and quadrupole (q) parameters of the source. The radiative flux, differential, and spectral luminosity of the accretion disk, which are quantities that can be experimentally measured, are then explored in detail. The obtained results are compared with the corresponding ones for the Schwarzschild and Kerr black holes in order to establish whether black holes may be distinguished from the q metric via observations of the accretion disk's spectrum.

Review of *q* metric

$$ds^2 = \left(1 - \frac{2m}{r}\right)^{1+q} dt^2 - \left(1 - \frac{2m}{r}\right)^{-q}$$
$$\times \left[\left(1 + \frac{m^2 \sin^2\theta}{r^2 - 2mr}\right)^{-q(2+q)} \left(\frac{dr^2}{1 - \frac{2m}{r}} + r^2 d\theta^2\right)\right.$$
$$\left. + r^2 \sin^2\theta d\phi^2 \right], \tag{1}$$

Orbital parameters of test particles in the field of q metric

$$\Omega^2 = \left(1 - \frac{2m}{r}\right)^{1+2q} \frac{(1+q)m}{r^2(r-(2+q)m)}, \qquad (2)$$

$$E^2 = \left(1 - \frac{2m}{r}\right)^{1+q} \frac{r-(2+q)m}{r-(3+2q)m}, \qquad (3)$$

$$L^2 = \left(1 - \frac{2m}{r}\right)^{-q} \frac{(1+q)mr^2}{r-(3+2q)m}, \qquad (4)$$

Innermost stable circular orbits in the *q* metric

$$r^{\pm}_{\rm ISCO} = m\left(4 + 3q \pm \sqrt{5q^2 + 10q + 4}\right),$$

$$M_T = m(1+q)$$

$$\frac{r_{\rm ISCO}}{M_T} = 3 + \frac{1}{1+q} + \sqrt{5 - \frac{1}{(1+q)^2}},$$

Review of the *Kerr* metric

$$ds^2 = \left(1 - \frac{2Mr}{\Sigma}\right)dt^2 - \frac{\Sigma}{\Delta}dr^2 + \frac{4Mra}{\Sigma}\sin^2\theta d\phi dt$$
$$- \Sigma d\theta^2 - \left(r^2 + a^2 + \frac{2Mra^2}{\Sigma}\sin^2\theta\right)d\phi^2,$$

$$M_T = M$$

$$j = a/M$$

Orbital parameters of test particles in the field of the *Kerr* metric

$$\Omega^2 = \frac{M}{r^3 \pm 2ar^2\sqrt{M/r} + a^2 M},$$

$$E^2 = \frac{(\sqrt{r}(r-2M) \pm a\sqrt{M})^2}{r^2(r \pm 2a\sqrt{M/r} - 3M)},$$

$$L^2 = \frac{M(r^2 \mp 2a\sqrt{M/r} + a^2)^2}{r^2(r \pm 2a\sqrt{M/r} - 3M)},$$

Innermost stable circular orbits in the *Kerr* metric

$$\frac{r^{\pm}_{\mathrm{ISCO}}}{M_T} = \left(3 + Z_2 \mp \sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)}\right),$$

$$M_T = M$$

$$Z_1 \equiv 1 + (1 - j^2)^{\frac{1}{3}}((1 + j)^{\frac{1}{3}} + (1 - j)^{\frac{1}{3}})$$

$$Z_2 \equiv (3j^2 + Z_1^2)^{\frac{1}{2}}.$$

Radiative flux and spectral luminosity

To model the accretion disc, we follow the standard treatment and assume that particles follow circular geodesics in the equatorial plane ($\theta = \pi/2$).

$$\mathcal{F}(r) = -\frac{\dot{m}}{4\pi\sqrt{g}} \frac{\Omega_{,r}}{(E - \Omega L)^2} \int_{r_{\rm ISCO}}^{r} (E - \Omega L) L_{,\tilde{r}} d\tilde{r},$$

$$\frac{d\mathcal{L}_\infty}{d\ln r} = 4\pi r\sqrt{g} E \mathcal{F}(r)$$

$$\mathcal{L}_\infty = (1 - E_{\rm ISCO})\dot{m}$$

is net luminosity that reaches observers at infinity. This quantity represents the amount of rest mass energy of the accreting matter that is converted into radiation, in other words, the efficiency of the source in converting infalling mass into emitted radiation.

Joshi P. S., Malafarina D., Narayan R., 2014, CQG, 31, 015002

Radiative flux and spectral luminosity

For a better presentation of our results, it is convenient to introduce new dimensionless functions by defining $\Omega^*(r) = M_{\rm T}\Omega(r)$, $L^*(r) = L(r)/M_{\rm T}$, where $M_T = m(1+q)$he system with a black hole and dark matter, and M $M_T = M$ or the system without dark matter. Then in our dimensionless units the energy remains unchanged, namely $E^*(r) = E(r)$.

$$\nu\mathcal{L}_{\nu,\infty} = \frac{60}{\pi^3}\int_{r_{\rm ISCO}}^{\infty} \frac{\sqrt{g}E}{M_T^2} \frac{(u^t y)^4}{\exp\left[u^t y/\mathcal{F}^{*1/4}\right] - 1} dr$$

$$u^t(r) = \frac{1}{\sqrt{g_{tt} + 2\Omega g_{t\varphi} + \Omega^2 g_{\varphi\varphi}}}$$

$$y = h\nu/kT_*, \qquad \mathcal{F}^*(r) = M_T^2\mathcal{F}(r).$$

- *Novikov I. D., Thorne K.S., 1973, in DeWitt C., DeWitt B., eds, Black Holes (Les Astres Occlus), Gordon and Breach, N.Y., p. 343*
- *Page D. N., Thorne K. S., 1974, ApJ, 191, 499*
- *Joshi P. S., Malafarina D., Narayan R., 2014, CQG, 31, 015002*

Numerical results

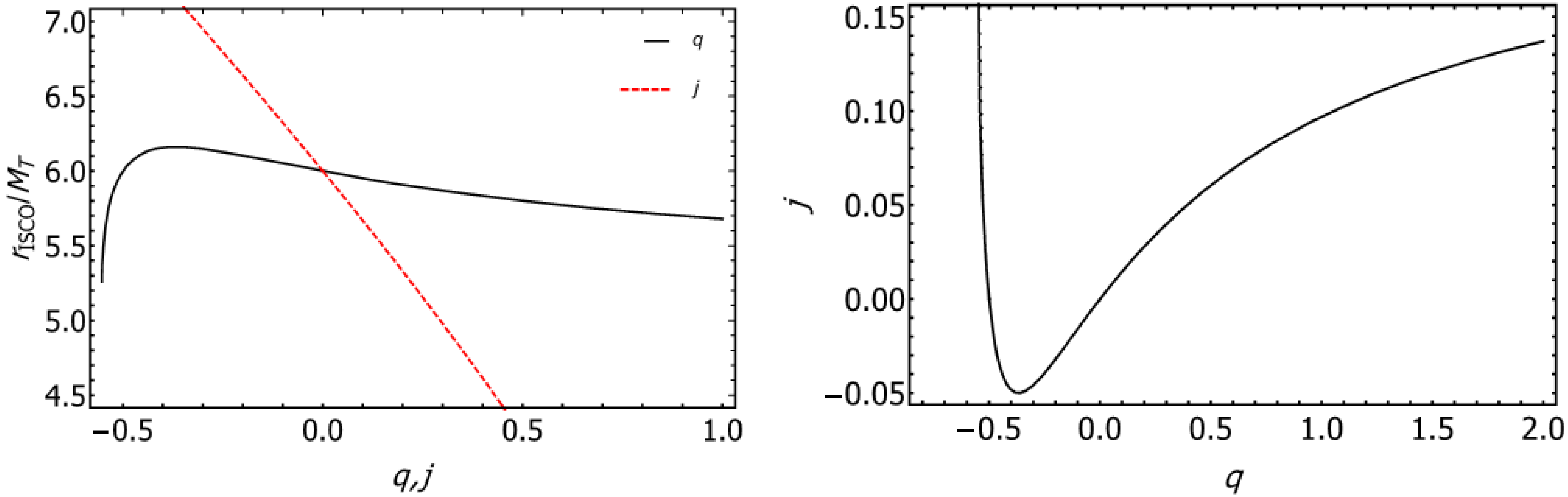

FIG. 1. Left panel: the ISCO radii for the q metric in units of total mass M_T as a function of the quadrupole parameter q as compared to the ISCO for the Kerr space-time as a function of the dimensionless angular momentum j. Right panel: degeneracy for the value of the ISCO between the q metric and the Kerr space-time. Namely for each allowed value of q there is a corresponding value of j for which the two sources with the same active mass M_T produce the same ISCO.

$j = a/M$

Numerical results

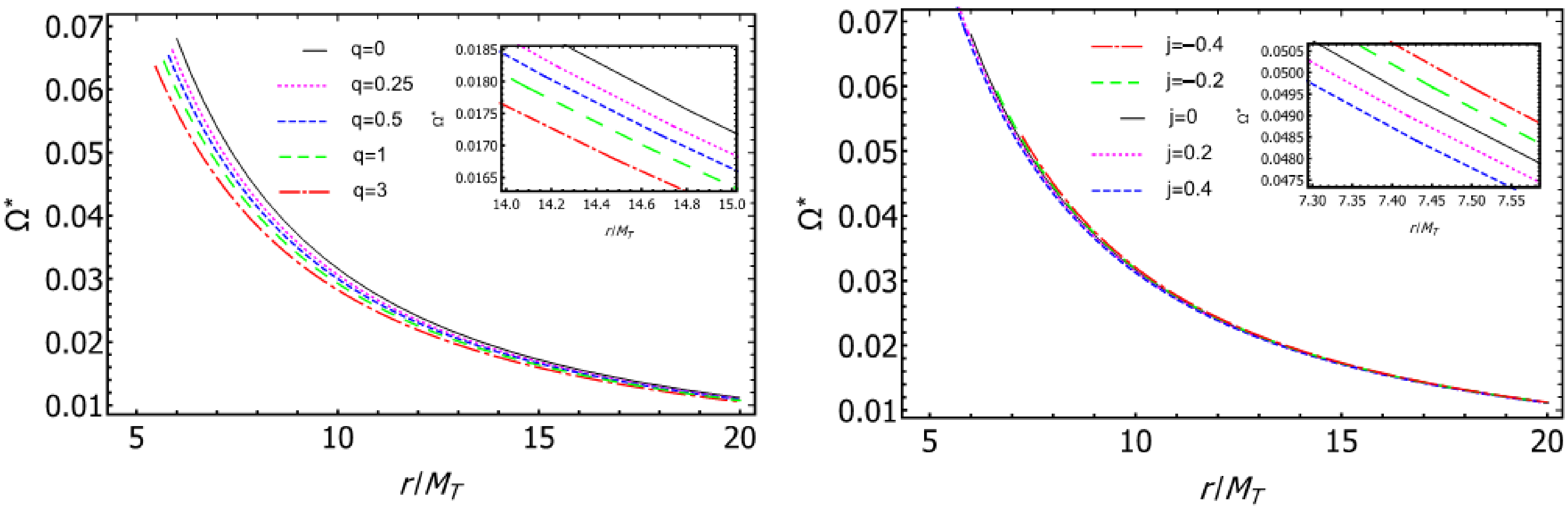

FIG. 2. Left panel: angular velocity of test particles versus radial distance r normalized in units of total mass M_T in the oblate q metric. Right panel: angular velocity of test particles versus radial distance r normalized in units of total mass M_T in the Kerr space-time.

$j = a/M$

Numerical results

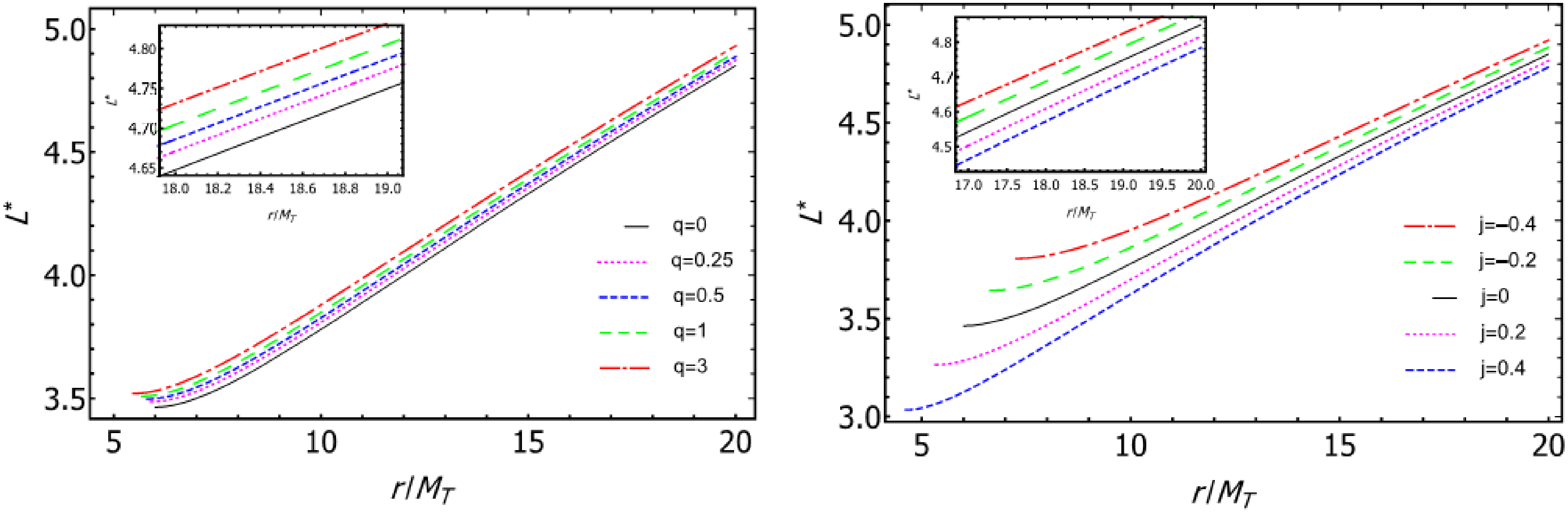

FIG. 3. Angular momentum L^* of test particles versus radial distance r normalized in units of total mass M_T. Left panel: L^* in the q metric for oblate sources ($q > 0$). Right panel: in the Kerr space-time.

$j = a/M$

Numerical results

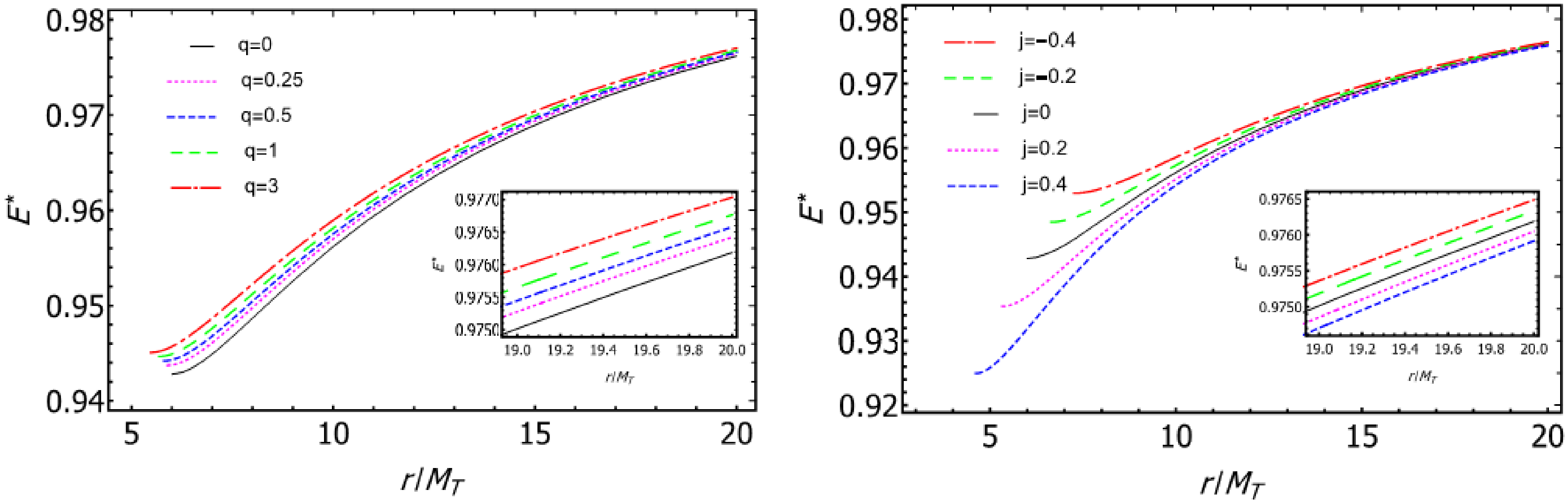

FIG. 4. Energy E^* of test particles versus radial distance r normalized in units of total mass M_T. Left panel: E^* in the oblate q metric. Right panel: E^* in the Kerr space-time.

$j = a/M$

Numerical results

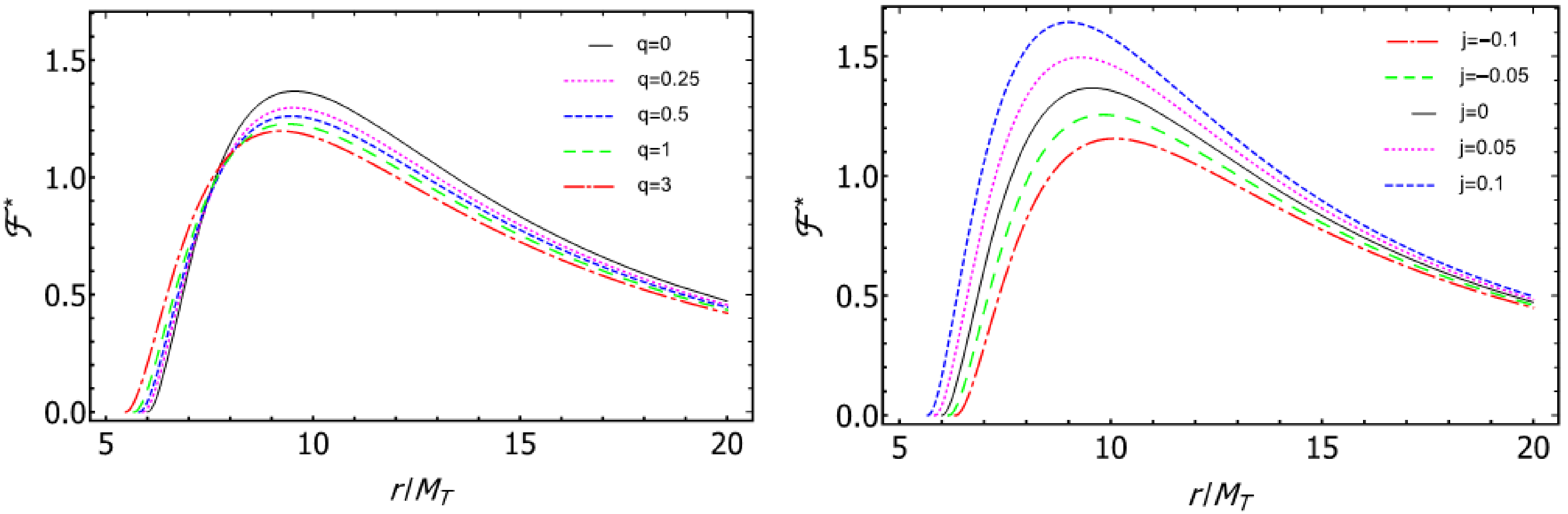

FIG. 5. Radiative flux $\mathcal{F}^*$ multiplied by 10^5 of the accretion disk versus radial distance r normalized in units of total mass M_T. Left panel: $\mathcal{F}^*$ in the oblate q metric. Right panel: $\mathcal{F}^*$ in the Kerr space-time. Notice that for the Kerr metric the change in j causes the flux to increase or decrease everywhere, while for the oblate q metric the flux increases at small radii and decreases at large radii with respect to Schwarzschild.

$j = a/M$

Numerical results

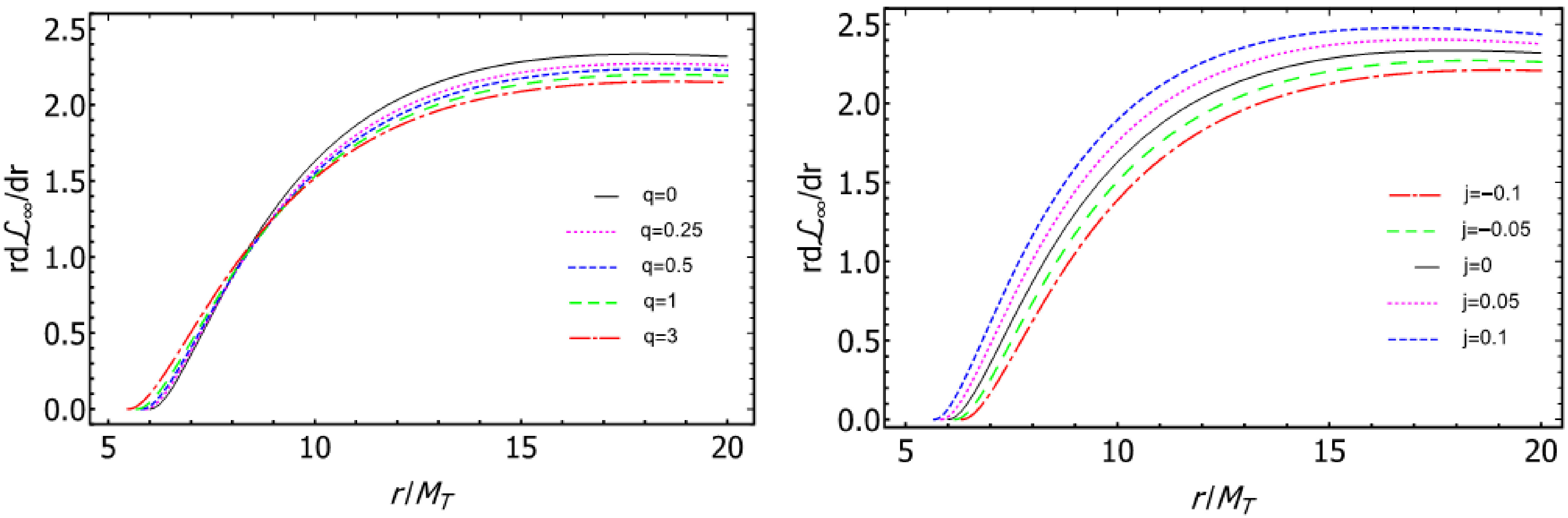

FIG. 6. Differential luminosity multiplied by 10^2 of the accretion disk versus radial distance r normalized in units of total mass M_T. Left panel: differential luminosity in the oblate q metric. Right panel: differential luminosity in the Kerr space-time.

$j = a/M$

Numerical results

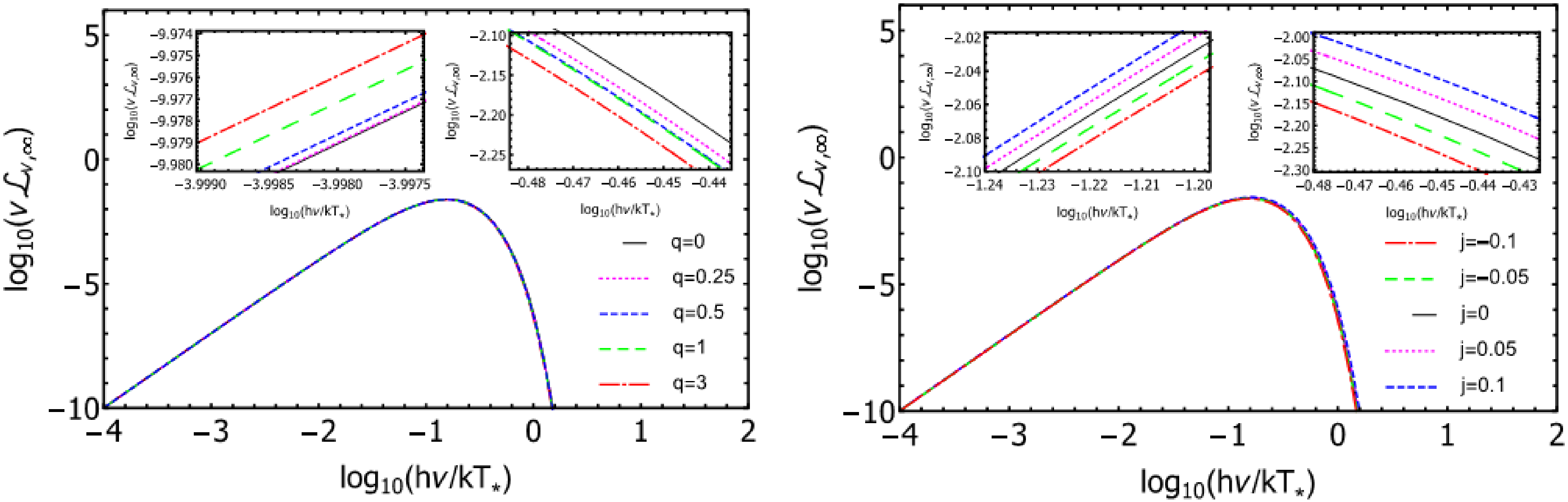

FIG. 7. Spectral luminosity versus frequency of the emitted radiation for blackbody emission of the accretion disk. Left panel: Spectral luminosity in the oblate q metric. Right panel: spectral luminosity in the Kerr space-time.

$j = a/M$

Numerical results

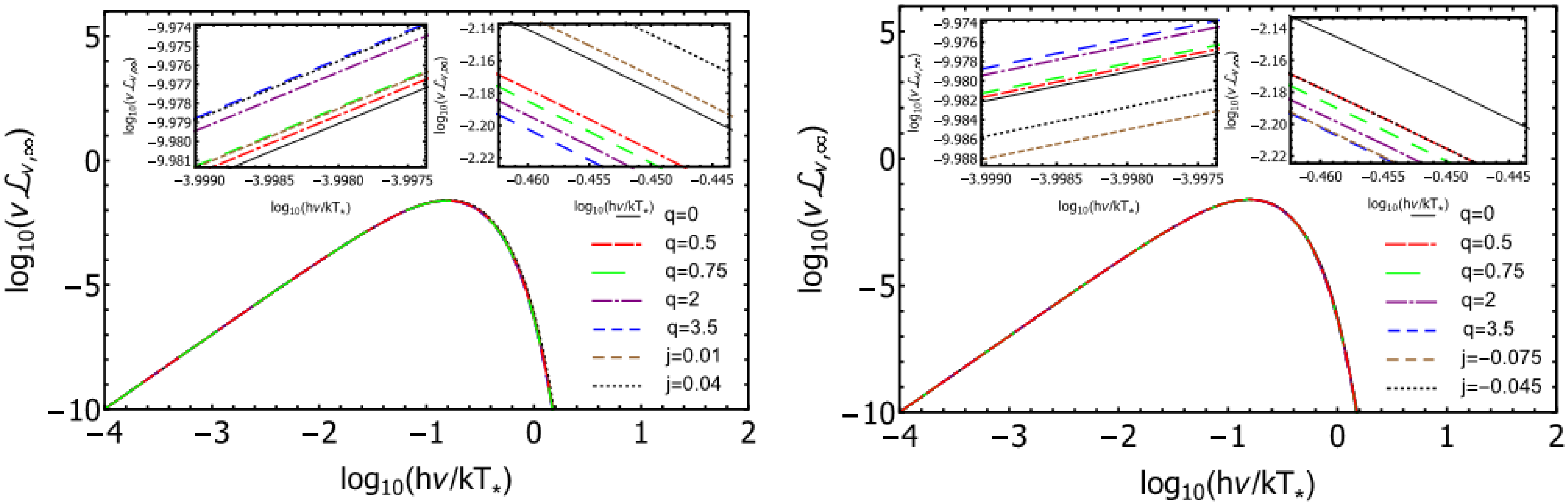

FIG. 8. Comparison of the spectral luminosities of the accretion disks for the oblate q metric with Kerr black holes. Left panel: various values of $q > 0$ are compared with corotating disks $j > 0$. Right panel: the same values of $q > 0$ are compared with counterrotating disks $j < 0$.

$j = a/M$

Numerical results

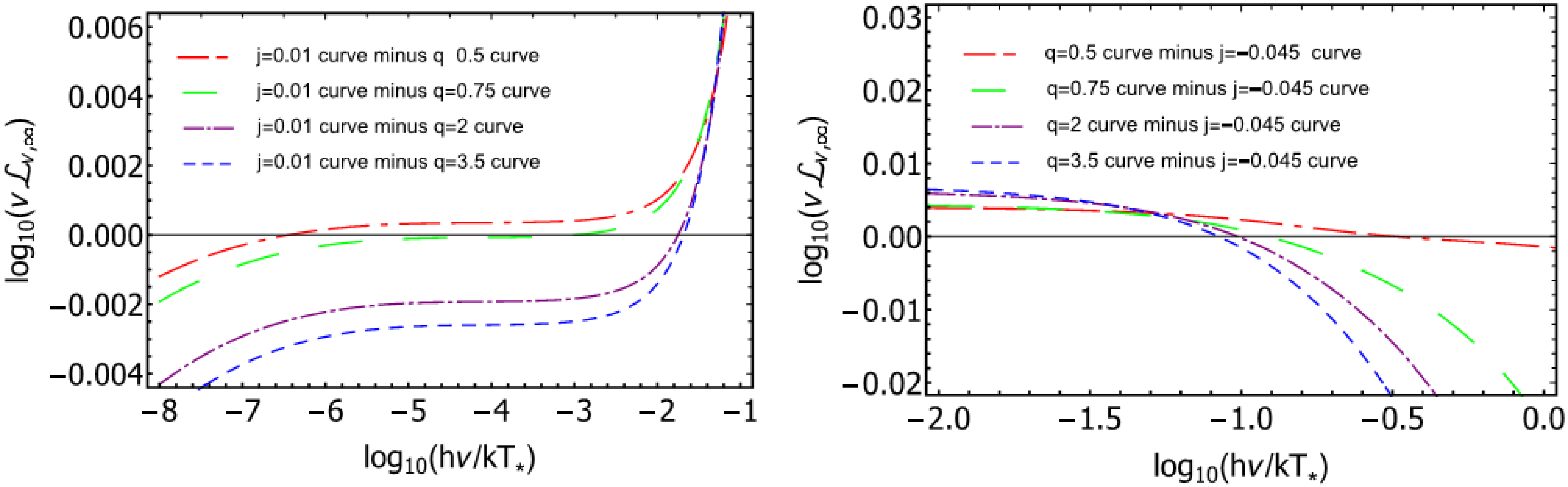

FIG. 9. Left panel: difference between spectral luminosities for accretion disks in the q metric with $q > 0$ and corotating Kerr metric with $j = 0.01$. Right panel: difference between spectral luminosities for accretion disks in the q metric with $q > 0$ and counterrotating Kerr metric with $j = -0.045$.

$j = a/M$

Numerical results

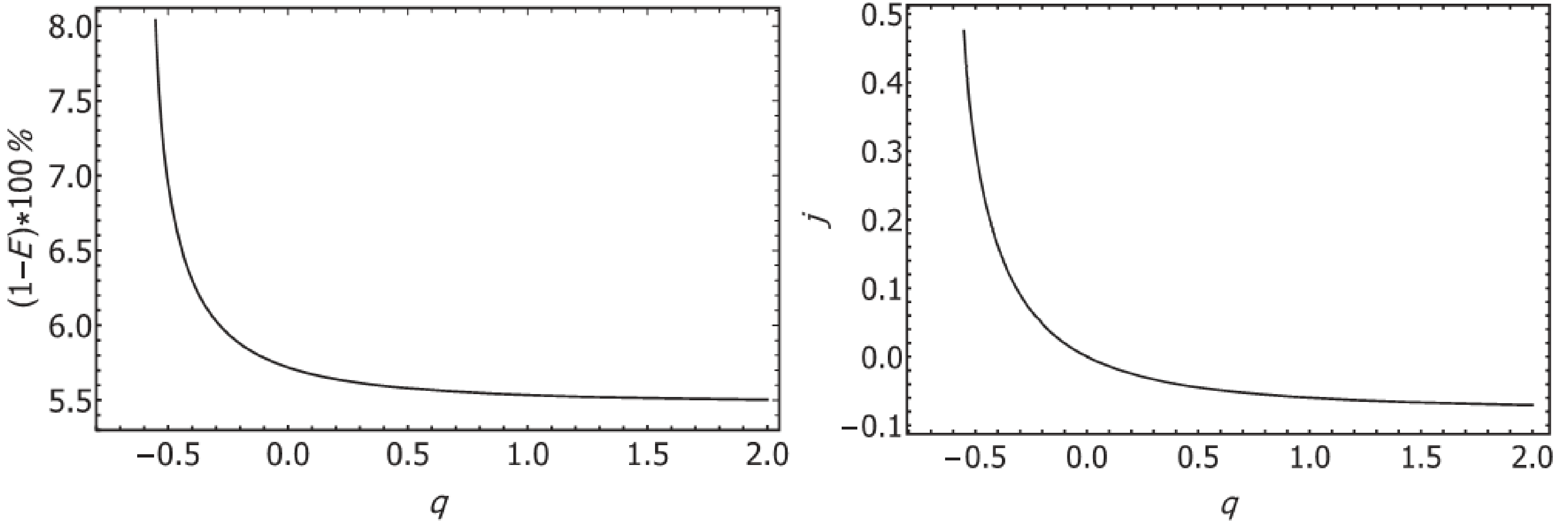

FIG. 10. Left panel: radiative efficiency for the oblate q metric. Notice that the oblate q metric is less efficient in converting accreting mass into radiation with respect to Schwarzschild. Right panel: degeneracy between the radiative efficiency in the q metric and the Kerr metric. For each value of the deformation parameter q there exists a value of the angular momentum j in the Kerr space-time for which the accretion disks in the two geometries have the same efficiency.

$j = a/M$

Conclusion

1. *We considered the q metric, a static and axially symmetric vacuum solution of Einstein's equations, as the possible exterior field of an exotic compact object and considered the eventuality that such a source may be distinguished from a Kerr black hole from the observation of the accretion disk's spectrum.*
2. *Of course the features of the spectra emitted by real accretion disks surrounding compact objects in the Universe are much more complicated than the simple toy models employed here. In fact, modern models include more realistic situations that take into account the effects due to the existence of plasma distributions within thick disks in the presence of magnetic fields and to the interaction with gray body radiation under the assumption of local thermal equilibrium. Additionally, the spectra of the accretion disks of astrophysical compact objects contains extra components such as coronal emission and reflection spectrum which complicate the task of comparing the features of real spectra with the mathematical models.*
3. *The simple models we investigated in this work cannot be used to practically determine the actual geometry in the proximity of astrophysical black hole candidates. However, we believe that the general considerations obtained here do indicate a road towards the possibility in the future of experimentally constraining geometric quantities, such as the mass quadrupole moment of compact sources and thus answer the question whether astrophysical black holes are indeed described by the Kerr metric.*

Thanks for your attention!