Cosmologies with Gravitational Anomalies & Axions: modified profiles of Gravitational Waves and Dark Matter properties

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CA18108 - Quantum gravity phenomenology in the multi-messenger approach

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0. Outline

1. Motivation

2. The model: String-Inspired Gravitational Theory with Torsion & Grav. Anomalies, axions and torsion

3. Primordial Gravitational Waves (GW) induced Condensates of Anomalies,

4. Spontaneous Lorentz and CPT-Violation by axion backgrounds & Running Vacuum Model inflation without external inflatons

5. Enhanced cosmic perturbations & densities of primordial black holes (PBH) & GW → dark matter components: PBH, together with the torsion-induced axions

6. Post Inflationary eras & cosmic evolution of the stringy RVM: Spontaneous Lorentz and CPT-Violation by axion backgrounds & Leptogenesis in radiation era → Baryogenesis – role of sterile right-handed neutrinos

7. Modern-era phenomenology: deviations from ΛCDM and alleviation of cosmological data tensions?

8. Warm Dark Matter in Galaxies: the role of sterile neutrinos and their interactions with axions → current constraints using modified (Ruffini-Arguelles-Rueda) profiles

9. Summary & Outlook

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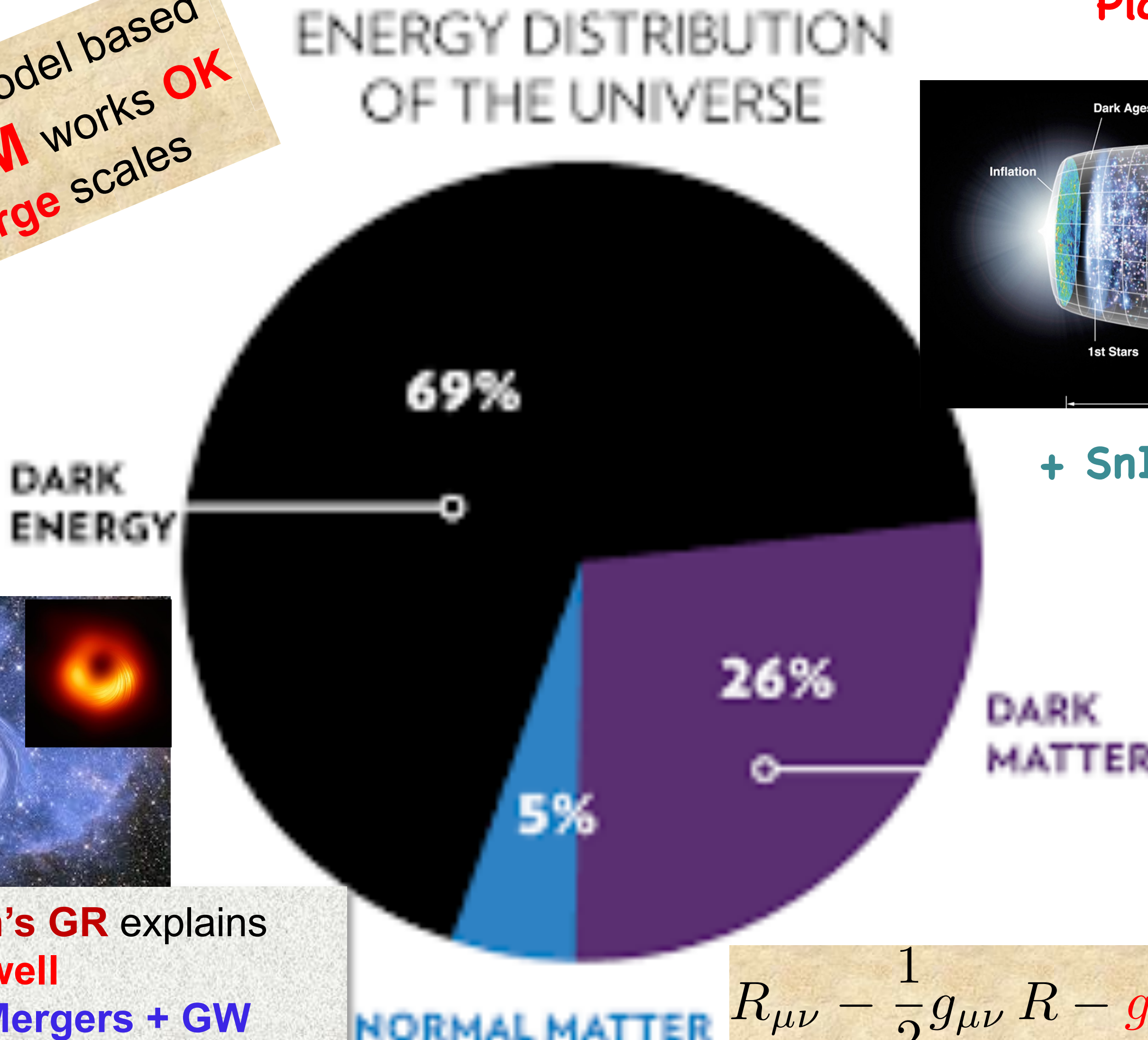
9. Summary & Outlook

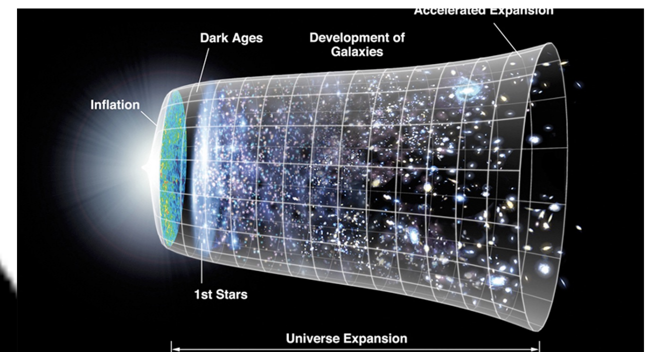
1. Motivation

Important (> last 20 yrs) Discoveries in Cosmology/Astronomy

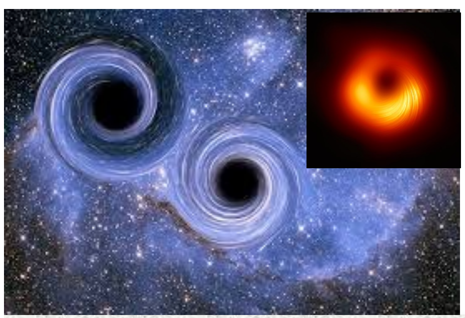
Simplest model based on **ΛCDM** works **OK** for **large** scales

Planck2018 data





+ SnIa, BaO, Lensing



Also **Einstein's GR** explains **sufficiently well**
Black-Hole Mergers + GW (since 2015 LIGO),
Black-Hole `photographs' (EHT),...

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu}\, \Lambda = 8\pi \mathrm{G}\, T_{\mu\nu}$$

$$T_{\mu\nu} \ni \text{Cold Dark Matter}$$

Important (> last 20 yrs) Discoveries in Cosmology/Astronomy

3 data

Sim
on

But....

Need to go Beyond....

What still we do not know/did not observe:

Nature of Dark Energy

Nature of Dark matter

Primordial Gravitational Waves

(through detection of B-mode polarisation in CMB from very early Universe)

Microscopic models of Inflation

(Is it due to fundamental inflatons or dynamical e.g. Starobinsky type? ....)

Lensing

Also
suffic
Black
(since
Black

$8\pi G\, T_{\mu\nu}$

Important (> last 20 yrs) Discoveries in Cosmology/Astronomy

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What still we
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Nature o
Nature o
Primordial G
(through de
polarisation
in CMB from very early Universe)
Microscopic models of Inflation
(Is it due to fundamental inflatons or dynamical e.g. Starobinsky type? ....)

More than one DM species, depending on era?
Role of warm sterile Neutrino DM & axions in Galactic structure?

$8\pi G\, T_{\mu\nu}$

Important (> last 20 yrs) Discoveries in Cosmology/Astronomy

Sim
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But....

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ΛCDM appears to be in tension with local measurements of present-era H_0 & also σ_8 galaxy-growth data ?

Also I
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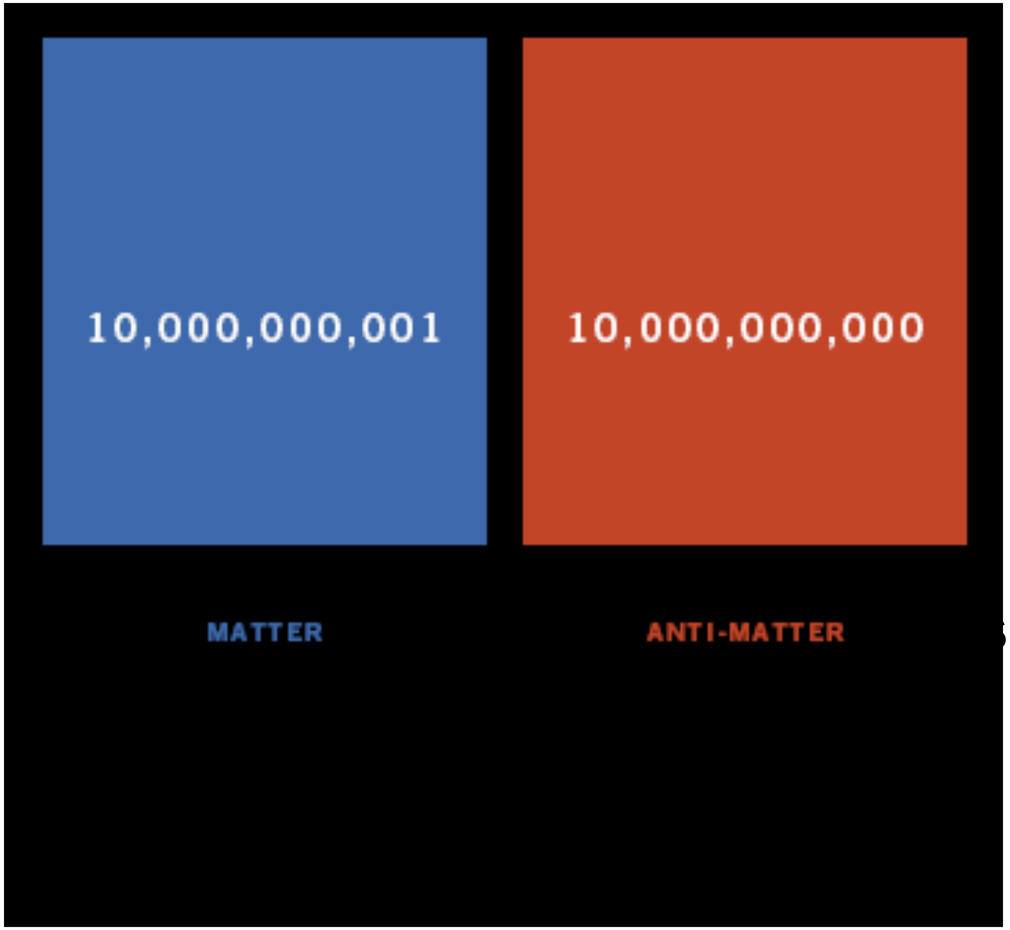
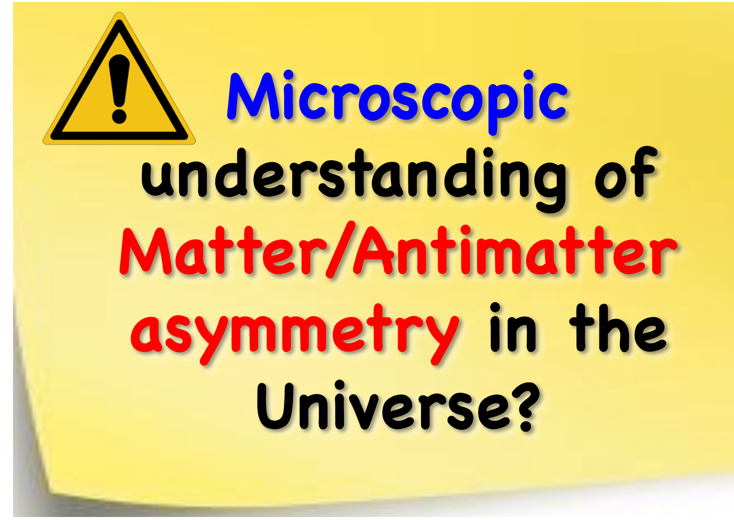
$8\pi G\, T_{\mu\nu}$





The Baryon Asymmetry

$$\frac{n_B - \overline{n}_B}{n_B + \overline{n}_B} \sim \frac{n_B - \overline{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

T > 1 GeV

s = entropy density of Universe

Attempts at Explanation of Baryon Asymmetry – Sakharov 's Conditions

Baryon number violation

C-violation

and CP violation

CPT conservation assumed

Departure from thermodynamic equilibrium (non-stationary system)

Need to go Beyond....

CP |particle> = |anti-particle>

Need new physics beyond the SM → new sources of CP violation?

Attempts at Explanation of Baryon Asymmetry – Sakharov 's Conditions

Baryon number violation

C-violation

and CP violati

What if CPTV geometries in the early Universe ?

Departure from thermodynamic equilibrium (non-stationary system)

Need to go Beyond....

CP |particle> = |anti-particle>

Need new physics beyond the SM → new sources of CP violation?

I will argue that:

Deviations from ΛCDM and alleviation of cosmological-data Tensions in the current era

+

observed matter-antimatter asymmetry

Can be linked with

Microscopic string-inspired models of Cosmology with ANOMALIES, primordial gravitational waves (GW) and induced spontaneous (through gravitational anomaly condensates) Lorentz + CPT Violation

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geometric torsion interpretation of axion Dark matter
Enhanced gravitational pertubations (Primordial Black Boles, GW)

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Role of Right-handed Sterile neutrinos & axions ... Also in galactic structure

Can be linked with

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Enhanced gravitational pertubations (Primordial Black Boles, GW)

Important Ingredients

The anomaly condensate $\langle R_{\mu\nu\rho\sigma} \, \widetilde{R}^{\mu\nu\rho\sigma} \rangle$

The stringy axion fields $b(x) \, , a(x)$

The axions – condensate coupling $(b(x) \, , a(x) \,) \, \langle R_{\mu\nu\rho\sigma} \, \widetilde{R}^{\mu\nu\rho\sigma} \rangle$

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The stringy axion fields $b(x)\,,\, a(x)$

String-model Independent axion

Compactification axion

The axions – condensate coupling $(b(x)\,,\, a(x)\,)\, \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle$

Important Ingredients

The anomaly condensate $\langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle$

The stringy axion fields $b(x)\,,\, a(x)$

Lead to spontaneously violating Lorentz (& CPT) symmetry axion backgrounds

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The stringy axion fields $b(x)\,,a(x)$

Lead to spontaneously violating Lorentz (& CPT) symmetry axion backgrounds
+ leptogenesis if sterile neutrinos present

The axions – condensate coupling $(b(x)\,,a(x)\,)\,\langle R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} \rangle$

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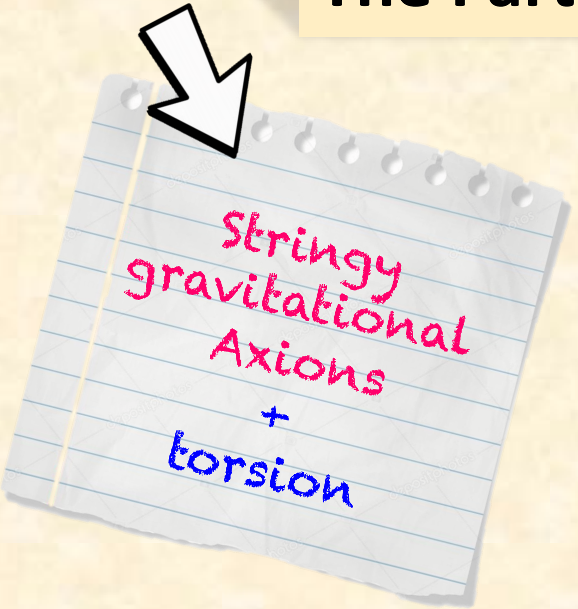
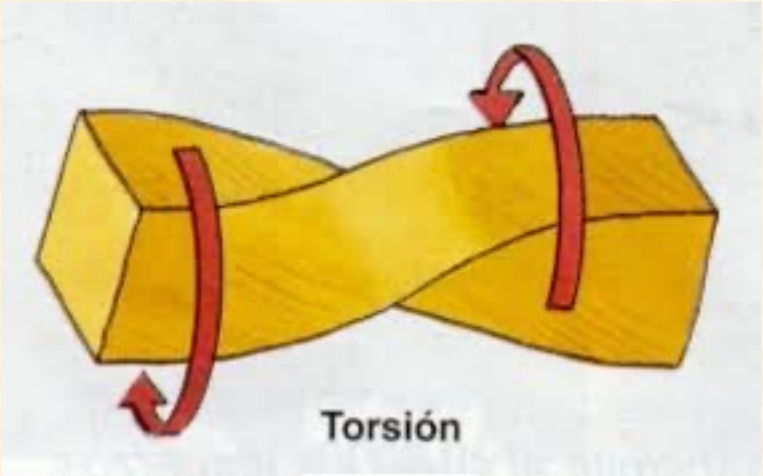
↓

Lead to **Running Vacuum Model** (RVM)
Inflation without external inflaton fields

2. The Model:
String-Inspired Gravitational Theory with Torsion & Grav. Anomalies, axions and torsion

The Parts





Gross and Sloan, Metsaev and Tseytlin

Massless Gravitational multiplet of (closed) strings:

spin 0 scalar (dilaton Φ)

spin 2 traceless symmetric rank 2 tensor (graviton $g_{\mu\nu}$)

spin 1 antisymmetric rank 2 tensor

KALB-RAMOND FIELD $B_{\mu\nu} = -B_{\nu\mu}$

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$$\mathrm{U(1)-symmetry}: B_{\mu\nu} \to B_{\mu\nu} + \partial_{[\mu}\theta(x)_{\nu]}$$

Stringy gravitational Axions + torsion

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4-DIM action

$$S_B = \int d^4x\sqrt{-g}\Big(\frac{1}{2\kappa^2}[-R + 2\,\partial_\mu\Phi\,\partial^\mu\Phi] - \frac{1}{6\kappa^2}e^{-4\Phi}H_{\lambda\mu\nu}H^{\lambda\mu\nu} + \ldots\Big)$$

$\kappa^2 = 8\pi\, G$

$$H_{\mu\nu\rho} = \partial_{[\mu}\, B_{\nu\rho]}$$

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Green, Schwarz

String Anomaly Cancellation requires modification in definition of $H_{\mu\nu\rho}$

$$H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]} \quad \Rightarrow \quad \boldsymbol{H} = \mathrm{dB} + \frac{\alpha'}{8\,\kappa}\left(\Omega_{3\mathrm{L}} - \Omega_{3\mathrm{Y}}\right)$$

$$\Omega_{3\mathrm{L}} = \omega^a_{\ c} \wedge \mathbf{d}\omega^c_{\ a} + \frac{2}{3}\omega^a_{\ c} \wedge \omega^c_{\ d} \wedge \omega^d_{\ a}, \qquad \Omega_{3\mathrm{Y}} = \mathbf{A} \wedge \mathbf{dA} + \mathbf{A} \wedge \mathbf{A} \wedge \mathbf{A},$$

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$$\overline{R}(\overline{\Gamma})$$

generalised curvature

Φ = constant throughout

$$\overline{\Gamma}^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + \frac{\kappa}{\sqrt{3}} H^{\mu}_{\nu\rho} \neq \overline{\Gamma}^{\mu}_{\rho\nu}$$

Contorsion

Stringy gravitational Axions + torsion

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Φ ... thr...

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Contorsion

No tree-level Cosmological Constant

(otherwise scattering matrix would not be defined in perturbative strings)

Gross and Sloan, Metsaev and Tseytlin

Stringy gravitational Axions + torsion

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Φ ... thr...

No tree-level Cosmological Constant

(otherwise scattering matrix would not be defined in perturbative strings)

...will generate dark energy dynamically

generalised curvature

$$\overline{\Gamma}^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + \frac{\kappa}{\sqrt{3}} H^{\mu}_{\nu\rho} \neq \overline{\Gamma}^{\mu}_{\rho\nu}$$

Contorsion

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KALB-R... $B_{\nu\mu}$

4-DIM action

$$S_B = \int d^4x \sqrt{-g}\Big(\frac{1}{2\kappa^2}[\ldots H^{\lambda\mu\nu} + \ldots\Big)$$

b(x) = pseudoscalar Lagrange multiplier implementing **Bianchi identity** ...

$$\mathcal{H} = \mathrm{dB} + \frac{\alpha'}{8\,\kappa}\Big(\Omega_{3\mathrm{L}} - \Omega_{3\mathrm{Y}}\Big)$$

$$d \star H \propto c_1 R \wedge \widetilde{R} - F \wedge \widetilde{F}$$

$\ldots \neq \ \overline{\Gamma}^{\mu}_{\rho\nu}$

quantum torsion → gravitational axion b "dual" to H torsion

Inclusion of Fermions

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}b(x)\left(R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\widetilde{F}^{\mu\nu}\right) + \dots\Big]$$

$$+ S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\left(\mathcal{F}_\mu + \frac{\kappa}{2}\sqrt{\frac{3}{2}}\partial_\mu b\right)J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

or Majorana

$$\mathcal{F}^d = \varepsilon^{abcd}e_{b\lambda}\partial_a e^\lambda_c,$$ vielbeins

$$J^{5\mu} = \bar{\psi}_j\gamma^\mu\gamma^5\psi_j$$

Axial Current

All fermion species

KR-axion anomalous CP-Violating interaction

$$\widetilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}R^{\alpha\beta}{}_{\rho\sigma}$$

$$\widetilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$$

torsion

cf. classically in 4 dim:

$$-3\sqrt{2}\partial_\sigma b = \sqrt{-g}\,\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}$$

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vielbeins

Vanishes for Friedmann-Lemaitre-Roberston-Walker backgrounds

4-fermion contact interaction characteristic of (integrating out) torsion

torsion

cf. classically in 4 dim: (duality relationship)

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Kalb-Ramond (KR) or string-model independent (``gravitational") axion

torsion

cf. classically in 4 dim: (duality relationship)

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The Model

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The Model

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+ S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\,\Big(\frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\,J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

or Majorana

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

All fermion species

Fixed axion coupling constant

$1/f_b$,

b = universal axion

The Model

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa_{\sqrt{3}}}b(x)\left(R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\widetilde{F}^{\mu\nu}\right) + \dots\Big]$$

$$+ S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\Big(\quad \frac{\kappa}{2}\sqrt{\frac{3}{2}}\partial_\mu b\Big)J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

or Majorana

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

All fermion species

Compactification axions **a** come with their own coupling constants f_a (**depending on the details of compactification**)
The **a-axions** *also couple to gravitational anomaly terms , with action:*

$$+\; S_a = \int d^4x\sqrt{-g}\Big(\frac{1}{2}\partial_\mu a\,\partial^\mu a + \frac{1}{f_a}a(x)\widetilde{R}_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \dots\Big)$$

The Model

Anomaly terms

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{2\,\alpha'}{6\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots$$

$$+\, S^{Free}_{Dirac} + \int d^4x\sqrt{-g}\,\Big(-\frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\,J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\,J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

or Majorana

$$= -\frac{\kappa}{2}\sqrt{\frac{3}{2}}\,b\,\nabla_\mu J^{5\mu}$$

$$J^{5\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

All fermion species

Compactification axions **a** come with their own coupling constants f_a (**depending on the details of compactification**), *The* ***a-axions*** *also couple to gravitational anomal*

Anomaly terms

$$+\; S_a = \int d^4x\sqrt{-g}\Big(\frac{1}{2}\partial_\mu a\,\partial^\mu a + \frac{1}{f_a}a(x)\widetilde{R}_{\mu\nu\rho\sigma}\,R^{\mu\nu\rho\sigma} + \dots\Big)$$

NB: Anomalies: (CHIRAL)

Classically conserved current $J^{\mu 5}$

AXIAL FERMION CURRENT $J^{\mu 5}$ CEASES to be conserved @ a quantum level

CHIRAL FERMIONS IN LOOP :

$$\nabla_\mu J^{\mu 5} \propto c_1 R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$c_1 \in \mathbb{R}$

$J^{\mu 5} \equiv \bar{\Psi}_j \gamma^\mu \gamma^5 \Psi_j$, $j = 1 \dots N$ SPECIES

chiral fermion $\gamma^5 \Psi_j = \mp \Psi_j$ (LEFT OR RIGHT HANDED)

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma},$$

$$\tilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} R^{\alpha\beta}{}_{\rho\sigma}$$

The Parts

Dark Energy

("running vacuum model (RVM) type")

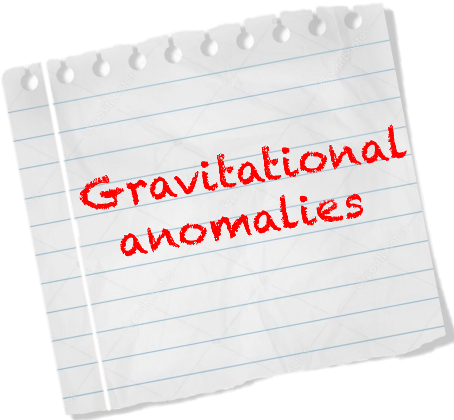
Stringy gravitational Axions

+

torsion

Gravitational anomalies

Gravitational Anomalies & Diffeomorphism Invariance



$$\int d^4x \sqrt{-g}\, b(x) \Big(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\, \widetilde{F}^{\mu\nu} \Big)$$

Spoils conservation of stress tensor (diffeomorphism invariance affected in quantum theory)

Topological, does NOT contribute to stress tensor

$$\delta\Big[\int d^4x \sqrt{-g}\, b\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \Big] = 4 \int d^4x \sqrt{-g}\, \mathcal{C}^{\mu\nu}\, \delta g_{\mu\nu} = -4 \int d^4x \sqrt{-g}\, \mathcal{C}_{\mu\nu}\, \delta g^{\mu\nu}$$

Cotton tensor

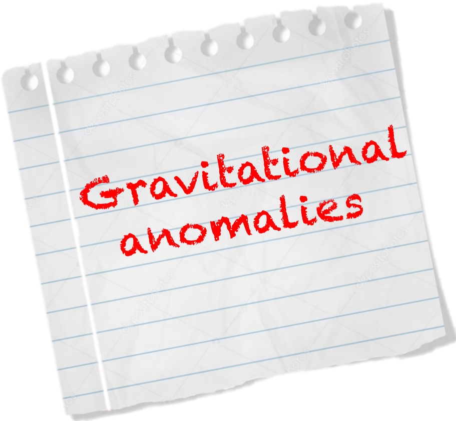
$$\mathcal{C}^{\mu\nu} = -\frac{1}{2}\Big[v_\sigma \Big(\varepsilon^{\sigma\mu\alpha\beta} R^\nu{}_{\beta;\alpha} + \varepsilon^{\sigma\nu\alpha\beta} R^\mu{}_{\beta;\alpha} \Big) + v_{\sigma\tau} \Big(\widetilde{R}^{\tau\mu\sigma\nu} + \widetilde{R}^{\tau\nu\sigma\mu} \Big) \Big] = -\frac{1}{2}\Big[\Big(v_\sigma\, \widetilde{R}^{\lambda\mu\sigma\nu} \Big)_{;\lambda} + (\mu \leftrightarrow \nu) \Big]$$

$$v_\sigma \equiv \partial_\sigma b = b_{;\sigma}, \quad v_{\sigma\tau} \equiv v_{\tau;\sigma} = b_{;\tau;\sigma}$$

Traceless $\qquad g_{\mu\nu}\, \mathcal{C}^{\mu\nu} = 0$

Jackiw, Pi (2003)

Gravitational Anomalies & Diffeomorphism Invariance



$$\int d^4x \, \sqrt{-g} \, b(x) \Big(R_{\mu\nu\rho\sigma} \, \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu} \, \widetilde{F}^{\mu\nu} \Big)$$

Spoils conservation of stress tensor (diffeomorphism invariance affected in quantum theory)

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$$\delta\Big[\int d^4x \sqrt{-g} \, b \, R_{\mu\nu\rho\sigma} \, \widetilde{R}^{\mu\nu\rho\sigma} \Big] = 4 \int d^4x \sqrt{-g} \, \mathcal{C}^{\mu\nu} \, \delta g_{\mu\nu} = -4 \int d^4x \sqrt{-g} \, \mathcal{C}_{\mu\nu} \, \delta g^{\mu\nu}$$

Cotton tensor

$$\mathcal{C}^{\mu\nu} = -\frac{1}{2} \Big[v_\sigma \left(\varepsilon^{\sigma\mu\alpha\beta} R^{\nu}{}_{\beta;\alpha} + \varepsilon^{\sigma\nu\alpha\beta} R^{\mu}{}_{\beta;\alpha} \right) + v_{\sigma\tau} \left(\widetilde{R}^{\tau\mu\sigma\nu} + \widetilde{R}^{\tau\nu\sigma\mu} \right) \Big] = -\frac{1}{2} \Big[\left(v_\sigma \, \widetilde{R}^{\lambda\mu\sigma\nu} \right)_{;\lambda} + (\mu \leftrightarrow \nu) \Big]$$

$$v_\sigma \equiv \partial_\sigma b = b_{;\sigma}, \quad v_{\sigma\tau} \equiv v_{\tau;\sigma} = b_{;\tau;\sigma}$$

not necessarily positive contributions to vacuum energy

Traceless $\quad g_{\mu\nu} \, \mathcal{C}^{\mu\nu} = 0$

Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \mathcal{C}^{\mu\nu} = \kappa^2 T^{\mu\nu}_{\rm matter}$$

$$\mathcal{C}^{\mu\nu}{}_{;\mu} = - \frac{1}{8} v^{\nu} R^{\alpha\beta\gamma\delta} \widetilde{R}_{\alpha\beta\gamma\delta}$$

$$v_\sigma \equiv \partial_\sigma b$$

$$\kappa^2 T^{\mu\nu}_{{\rm matter}\,;\mu} = - \mathcal{C}^{\mu\nu}{}_{;\mu} \neq 0$$

Diffeomoprphism invariance breaking by gravitational anomalies?

Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \mathcal{C}^{\mu\nu} = \kappa^2 T^{\mu\nu}_{\text{matter}}$$

$$\mathcal{C}^{\mu\nu}{}_{;\mu} = - \frac{1}{8} v^{\nu} R^{\alpha\beta\gamma\delta} \widetilde{R}_{\alpha\beta\gamma\delta}$$

$$v_\sigma \equiv \partial_\sigma b$$

$$\kappa^2 T^{\mu\nu}_{\text{matter}\,;\mu} + \mathcal{C}^{\mu\nu}{}_{;\mu} = 0$$

No problem with diffeo

Conserved Modified stress-energy tensor

Gravitational Anomalies & Diffeomorphism Invariance

Einstein's equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \mathcal{C}^{\mu\nu} = \kappa^2 T^{\mu\nu}_{\rm matter}$$

$$\mathcal{C}^{\mu\nu}{}_{;\mu} = - \frac{1}{8} v^{\nu} R^{\alpha\beta\gamma\delta} \widetilde{R}_{\alpha\beta\gamma\delta}$$

Exchange of energy Between axions and Gravitational (anomaly) sector

$$v_{\sigma} \equiv \partial_{\sigma} b$$

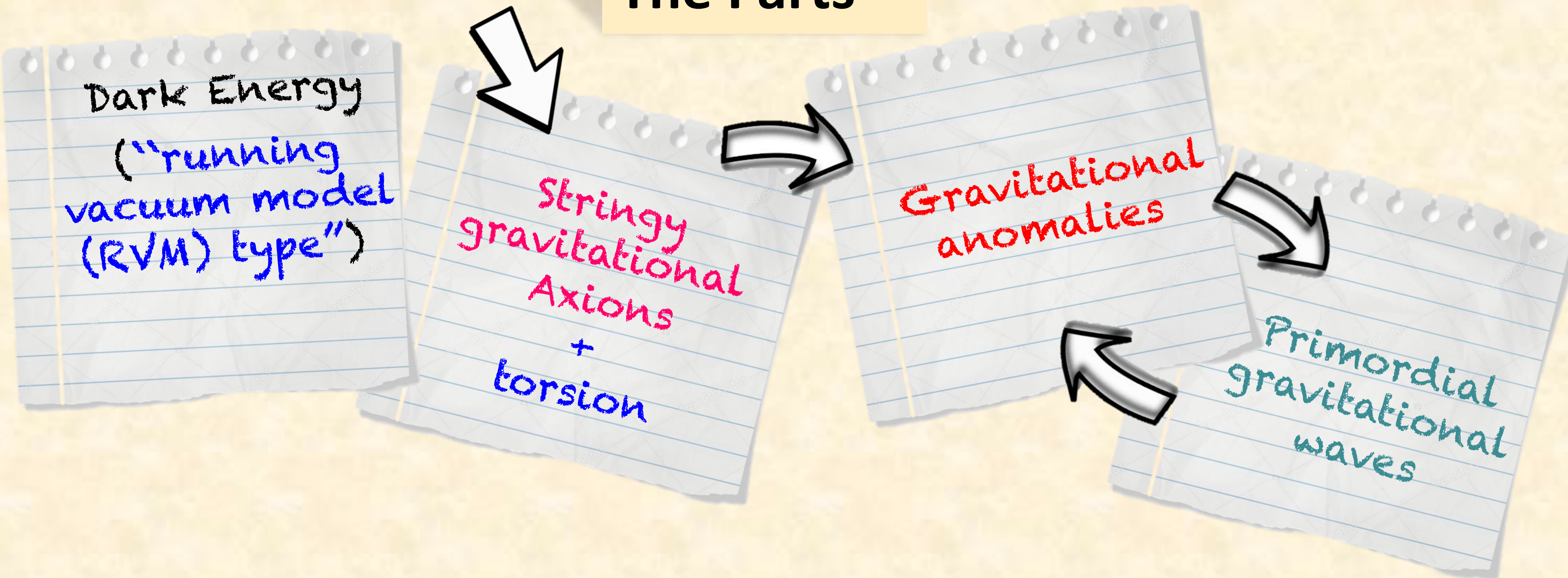
No problem with diffeo

$$\kappa^2 T^{\mu\nu}_{\rm matter\, ;\mu} + \mathcal{C}^{\mu\nu}{}_{;\mu} = 0$$

Conserved Modified stress-energy tensor

3. Primordial Gravitational Waves, Anomaly condensates

The Parts


Dark Energy
("running vacuum model (RVM) type")
Stringy gravitational Axions
+
torsion
Gravitational anomalies
Primordial gravitational waves

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

NB:

absent before formation of GW

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

No potential for KR axion before generation of GW

→ stiff-matter, equation of state **w=+1**

→stiff-axion-matter dominance during very early (pre-inflationary) Universe

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

NB:

absent before formation of GW

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

No potential for KR axion before generation of GW

→ stiff-matter, equation of state **w=+1**

→**stiff-axion-matter dominance during very early (pre-inflationary) Universe**

c.f. Zeldovich but for baryons in his model; cf. also Chavanis

The Model in Early Universe:
only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

Primordial Gravitational Waves
Potential Origins in pre-inflationary era?

NEM, Solà
EPJ-ST
(2020)

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

Basilakos, NEM, Solà (2019-20)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

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One scenario:
Role of (Local) Supersymmetry (SUGRA)

Primordial Gravitational Waves

Potential Origins in pre-inflationary era?

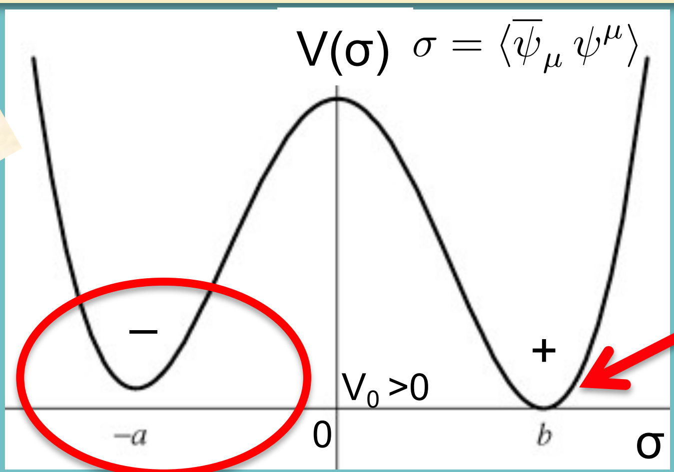
Collapse/collisions of Domain walls formed in theories with (approximate) discrete symmetry breaking, e.g. via bias in double-well potentials of some condensate (gravitino ψ_μ or gaugino)

NEM, Solà EPJ-ST (2020)

The Model in Early Universe:
only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

Basilakos, NEM, Solà (2019-20)

One scenario:
Role of SUGRA



SUGRA broken DYNAMICALLY
gravitino
Condensate σ
stabilised →
RVM GW-induced Inflation

Statistical bias (percolation) in occupation probabilities of the +,- vacua

Lalak, Ovrut, Lola, G. Ross, Thomas

Primordial Gravitational Waves

Potential Origins in pre-inflationary era?

Collapse/collisions of Domain walls formed in theories with (approximate) discrete symmetry breaking, e.g. via bias in double-well potentials of some condensate (gravitino ψ_μ or gaugino)

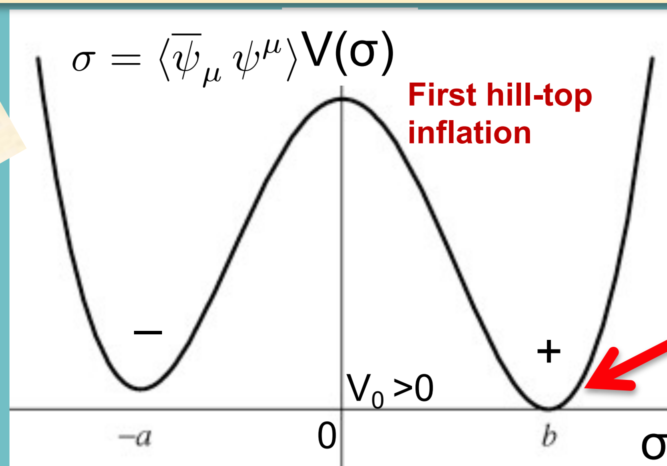
NEM, Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

Basilakos, NEM, Solà (2019-20)

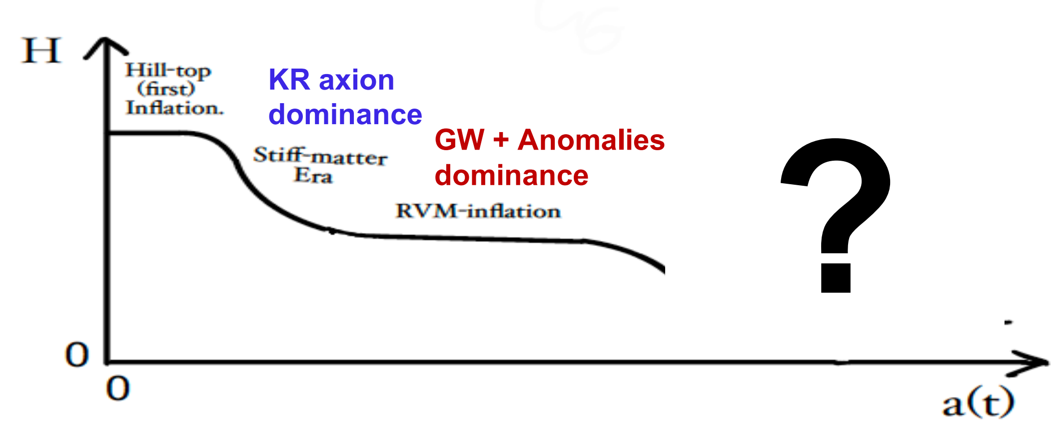
One scenario: Role of SUGRA



SUGRA broken DYNAMICALLY
gravitino
Condensate σ
stabilised →
RVM GW-induced Inflation

Pre-RVM inflationary phase: superstring/supergravity Effective action → *Imaginary parts* → **instabilities**

First Hill-top inflation = finite life –time → System **tunnels** to *RVM inflationary vacuum (GW condense)*



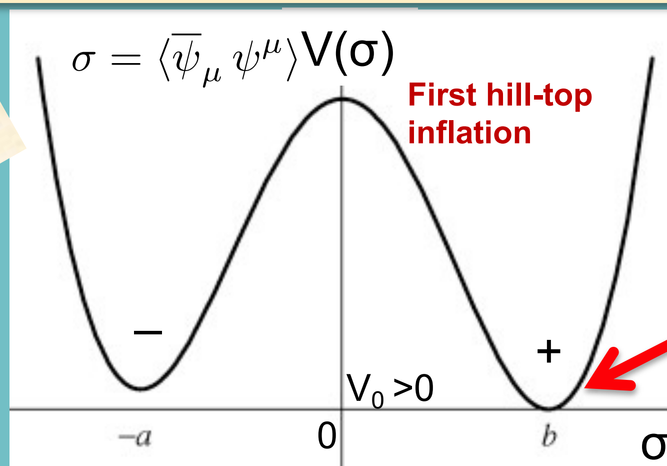
NEM, Solà EPJ-ST (2020)

Ellis, NEM, Alexandre, Houston

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$, ψ_μ)

Basilakos, NEM, Solà (2019-20)

One scenario
Role of SUGRA

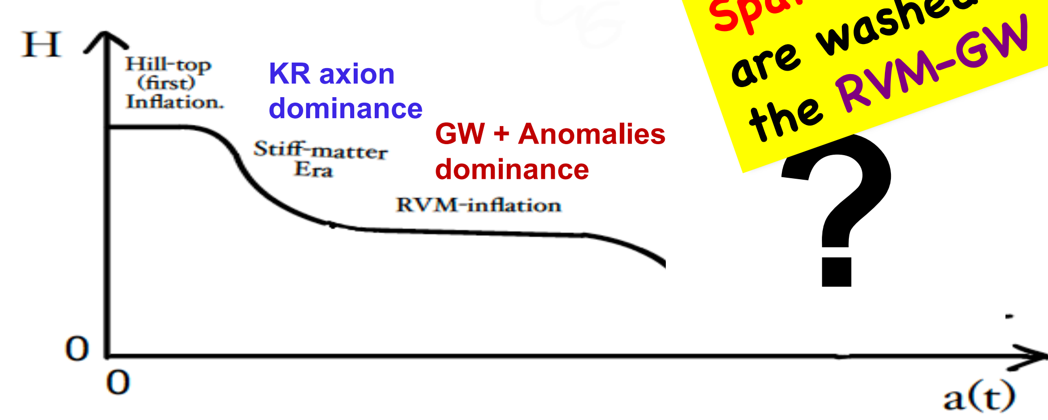


SUGRA broken DYNAMICALLY
gravitino
Condensate σ
stabilised →
RVM GW-induced Inflation

Pre-RVM inflationary phase: superstring/supergravity
Effective action → Imaginary parts → instabilities

First Hill-top inflation = finite life –ti...
System **tunnels** to RVM inflationary va...

First inflation ensures any
Spatial inhomogeneities
are washed out before
the RVM-GW inflation



NEM, Solà
EPJ-ST
(2020)

Ellis, NEM,
Alexandre,
Houston

4. Spontaneous Lorentz & CPT Violation by axion backgrounds and RVM Inflation

The Parts

Dark Energy
("running vacuum model (RVM) type")

Stringy gravitational Axions
+
torsion

Gravitational anomalies

Primordial gravitational waves

Spontaneous Lorentz + CPT Violation
from anomaly condensates

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

Non-trivial if GW present

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

Primordial Gravitational Waves,
&
De Sitter space times &
Spontaneous Lorentz & CPT Violation

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

Gravitational Chern-Simons (gCS)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

Primordial Gravitational Waves →
Condensate < …> of Gravitational Anomalies

$$gCS = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\int d^4x\,\sqrt{-g}\,\Big(\langle\, b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\,\rangle + \,:b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}:\Big)$$

quantum ordered

The Model in Early Universe:
only gravitational d.o.f. ($b, g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

Gravitational Chern-Simons (gCS)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

$$+\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\int d^4x\sqrt{-g}\,\langle b(x)\,R_{\mu\mu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\rangle$$

Condensate < ...> of Gravitational Anomalies

Cosmological-Constant-like

Mild time Dependence (RVM) through H

$$g\mathcal{CS} = \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,\int d^4x\,\sqrt{-g}\,\Big(\langle\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}\,\rangle + \,:b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}:\Big)$$

quantum ordered

The Model in Early Universe: only gravitational d.o.f. (b, $g_{\mu\nu}$)

Basilakos, NEM, Solà (2019-20)

Gravitational Chern-Simons (gCS)

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]$$

$$= \int d^4x\,\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\,\partial_\mu b(x)\,\mathcal{K}^\mu + \dots\Big]\,,$$

$$-\sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\int d^4x\sqrt{-g}\,\langle\partial_\mu b\,\mathcal{K}^\mu\rangle$$

Condensate < ...> of Gravitational Anomalies

Cosmological-"Constant"-like

Mild time Dependence (RVM) through H

Up to boundary terms

$$g\mathcal{CS} = -\sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\int d^4x\sqrt{-g}\,\langle\partial_\mu b\,\mathcal{K}^\mu\rangle + \text{quantum flcts.}$$

Effective action contains **CP violating axion-like coupling**

$$\sqrt{-g}\,\mathcal{K}^\mu(\omega)_{;\mu}$$

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} \qquad '\Big) + \dots\Big]$$

(i) **Assume de Sitter era**, first, to discuss anomaly condensate in the presence of GW perturbation

(ii) **deduce RVM vacuum** behaviour

and

(iii) **Inflation** is **obtained self consistently** from **RVM evolution**

Effective action contains **CP violating axion-like coupling**

Average over inflationary space time in the presence of **primordial Gravitational waves**

n^* = proper number density of sources of GW(assumed of O(1))

b(x)=b(t)

Alexander, Peskin, Sheikh -Jabbari

μ = UV k-momentum Cut-off

$$\frac{d}{dt}\left(\sqrt{-g}\,\mathcal{K}^0(t)\right) = \langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\,\kappa^2 n^* \int^{\mu} \frac{d^3k}{(2\pi)^3}\,\frac{H^2}{2k^3}\,k^4\,\Theta + \mathrm{O}(\Theta^3)$$

Homogeneity & Isotropy

$$\Theta = \sqrt{\frac{2}{3}}\frac{\kappa^3}{12} H\dot{b} \ll 1$$

$$\kappa = M_{\rm Pl}^{-1},$$

$$\dot{b} \equiv db/dt$$

$$a(t) \sim e^{Ht}$$

***H* ≈ const. (inflation)**

Solutions (backgrounds) to the Eqs of Motion

$\alpha' = M_s^{-2}$

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\mathcal{K}^0$$

n* = proper number density of sources of GW(assumed of O(1))

$$\frac{d}{dt}\left(\sqrt{-g}\,\mathcal{K}^0(t)\right) = \langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\kappa^2 n^* \int^{\mu} \frac{d^3k}{(2\pi)^3}\frac{H^2}{2k^3}k^4\,\Theta + \mathrm{O}(\Theta^3)$$

$$\Theta = \sqrt{\frac{2}{3}}\frac{\kappa^3}{12}H\dot{b} \propto \mathcal{K}^0$$

time evolution of Anomaly

μ = UV k-momentum Cut-off

$$\mathcal{K}^0(t) \simeq \mathcal{K}^0_{\rm begin}(0)\exp\left[-3Ht\left(1 - 0.73\times 10^{-4}\left(\frac{H}{M_{\rm Pl}}\right)^2\left(\frac{\mu}{M_s}\right)^4\right)\right]$$

Solutions (backgrounds) to the Eqs of Motion

$\alpha' = M_s^{-2}$

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0$$

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≈ 0

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$$\frac{\mu}{M_s} \simeq 15 \left(\frac{M_{\rm Pl}}{H} \right)^{1/2} \quad \Rightarrow \quad \mathcal{K}^0 = \text{const.}$$

Planck Data $H/M_{\rm Pl} < 10^{-4}$ $\Rightarrow$ **to ensure constant anomaly $\mu / M_s = O(10^3)$**

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathscr{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathscr{K}^0 \sim \text{constant}$$

$$\dot{\bar{b}} \propto \epsilon^{ijk} H_{ijk} = \text{constant}$$

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$$\frac{d}{dt}\left(\sqrt{-g}\,\mathcal{K}^0(t)\right) = \langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\kappa^2 n^* \int^{\mu} \frac{d^3k}{(2\pi)^3}\frac{H^2}{2k^3}k^4\Theta + O(\Theta^3)$$

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Spontaneous LV (+ CPTV) solution ⚠

Planck Data $H/M_{\rm Pl} < 10^{-4}$ ⇒ **to ensure constant anomaly** ***$\mu / M_s = O(10^3)$***

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0 \sim \text{constant}$$

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$$\frac{\mu}{M_s} \simeq 15\left(\frac{M_{\rm Pl}}{H}\right)^{1/2} \quad \Rightarrow \quad \mathcal{K}^0 = \text{const.}$$

No transplanckian modes !

Planck Data $\quad H/M_{\rm Pl} < 10^{-4} \quad \Rightarrow$ **to ensure constant anomaly**

$\mu = O(10^3)\, M_s \;\leq\; M_{planck}$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathscr{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathscr{K}^0 \sim \text{constant}$$

$$\dot{\bar{b}} \sim \varepsilon_{ijk} H^{ijk}$$

≈ constant torsion

Using **slow-roll assumption** *b*

$$\varepsilon = \frac{1}{2}\frac{1}{(HM_{\rm Pl})^2}\dot{\bar{b}}^2 \sim 10^{-2}$$

Planck Data

$$\dot{\bar{b}} \sim \sqrt{2\varepsilon}\, M_{\rm Pl}\, H \sim 0.14\, M_{\rm Pl}\, H$$

$$H = H_{\rm infl} \simeq \text{const.}$$

Solutions (backgrounds) to the Eqs of Motion

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Using **slow-roll assumption** *b*

2.6 x $10^{-5}\, M_{Pl} < M_s \le 10^{-4}\, M_{Pl}$

NEM + Solà (2021)

$$\dot{\bar{b}}\sim\sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H\sim 0.14\,M_{\rm Pl}\,H$$

$$H = H_{\rm infl}\simeq\text{const.}$$

Constant anomaly during inflation, no transplanckian modes !

NB:

$$\Theta\equiv\sqrt{\frac{2}{3}}\frac{\alpha'\kappa}{12}H\dot{\bar{b}}\ll 1.$$

$$\dot{\bar{b}}\ll H/\kappa$$

$$H/M_s\ll 3.83\;,\;H\simeq(10^{-5}-10^{-4})M_{\rm Pl}$$

$$\frac{M_{\rm Pl}}{M_s}\ll 3.83\times(10^4-10^5).$$

$$M_s\le 10^{-4}\,M_{\rm Pl}$$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\Big[\sqrt{-g}\Big(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\mathcal{K}^\alpha(t)\Big)\Big] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\mathcal{K}^0 \sim \text{constant}$$

Using **slow-roll assumption** *b*

n* of O(1), otherwise free parameter, can set $M_s = \mu$

$$2.6 \times 10^{-5}\, M_{Pl} < M_s \leq 10^{-4}\, M_{Pl}$$

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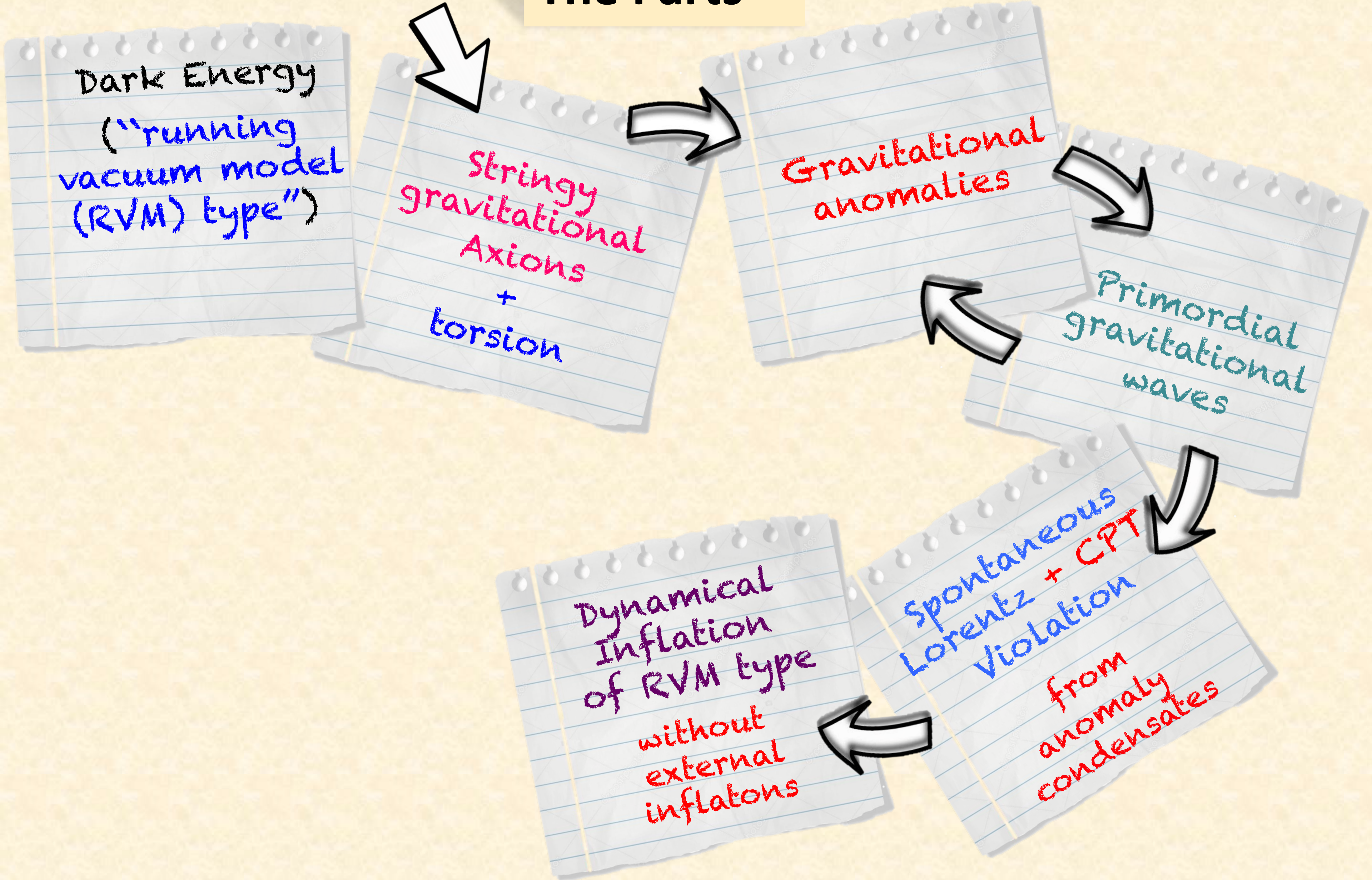
$H/M_s \ll 3.83$, $H \simeq (10^{-5} - 10^{-4}) M_{\rm Pl}$

$$\frac{M_{\rm Pl}}{M_s} \ll 3.83 \times (10^4 - 10^5).$$

$M_s \leq 10^{-4}\, M_{\rm Pl}$

The Parts


Dark Energy
("running vacuum model (RVM) type")
Stringy gravitational Axions
+
torsion
Gravitational anomalies
Primordial gravitational waves
Spontaneous Lorentz + CPT Violation
from anomaly condensates
Dynamical Inflation of RVM type
without external inflatons

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\mathcal{K}^0 \sim \text{constant}$$

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$$H = H_{\rm infl} \simeq \text{const.}$$

@ end of Inflationary era

$$b_{\rm end} \sim b_{\rm initial} + 0.14\,M_{\rm Pl}\,H_{\rm infl}\,t_{\rm end},$$

$$t_{\rm end}H_{\rm infl} \sim \mathcal{N} = \text{e} - \text{foldings}$$

~ 55-70

Fix $b_{initial}$ to arrange approx. constant condensate during appropriate time period (inflation)

Axion Monodromy like potentials

NEM, Universe 7 (2021) 12, 480,
e-Print: 2111.05675 [hep-th]

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle\, b(x)$

Recall: approximately de Sitter provided during the duration of inflation

$$b(t) = \overline{b}(0) + 0.14 M_{\rm Pl}\, H\, t_{end} \simeq \overline{b}(0)$$

order of magnitude

< 0

N=e-folds

beginning of inflation

$$|\overline{b}(0)| \gtrsim \mathcal{O}(10)\, M_{\rm Pl}$$

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↓

$$|\overline{b}(0)| \gtrsim \mathcal{O}(10)\, M_{\rm Pl}$$

Ooguri, Vafa, …Palti

Distance-swampland conjectures?

Axion Monodromy like potentials

NEM, Universe 7 (2021) 12, 480, e-Print: 2111.05675 [hep-th]

BUT....

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$ =

$$-\sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\int d^4x \sqrt{-g}\,\langle \partial_\mu b\, \mathcal{K}^\mu\rangle$$

$$|\dot{b}| \sim \sqrt{2\epsilon}\, H\, M_{\rm Pl} \ll M_{\rm Pl}^2$$

$$\epsilon \ll 1, \quad H \sim 10^{-5}\, M_{\rm Pl}$$

Distance-swampland conjectures avoided ?

Gravitational Anomaly Condensates → Dynamical Inflation

Basilakos, NEM, Solà

$$\Lambda \equiv \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle \simeq 5.86 \times 10^7\, \epsilon \mathcal{N}\, H^4 > 0$$

e-foldings

Positive
Cosmological
Constant-like

Positive total vacuum energy density since Λ-term dominates

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^7 \,(\frac{H}{M_{\rm Pl}})^4 \Big] > 0$$

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Dark Energy

("running vacuum model (RVM) type")

Gravitational Anomaly Condensates → Dynamical Inflation

NEM, Solà

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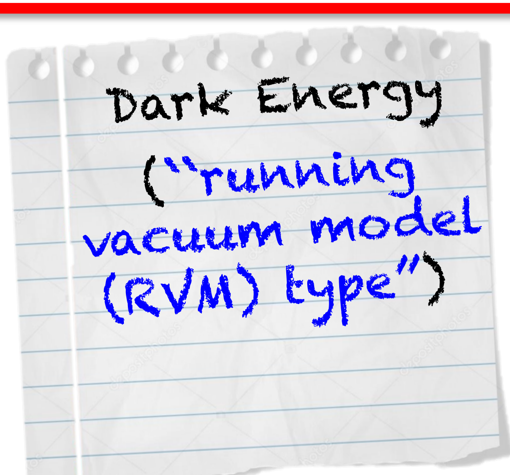


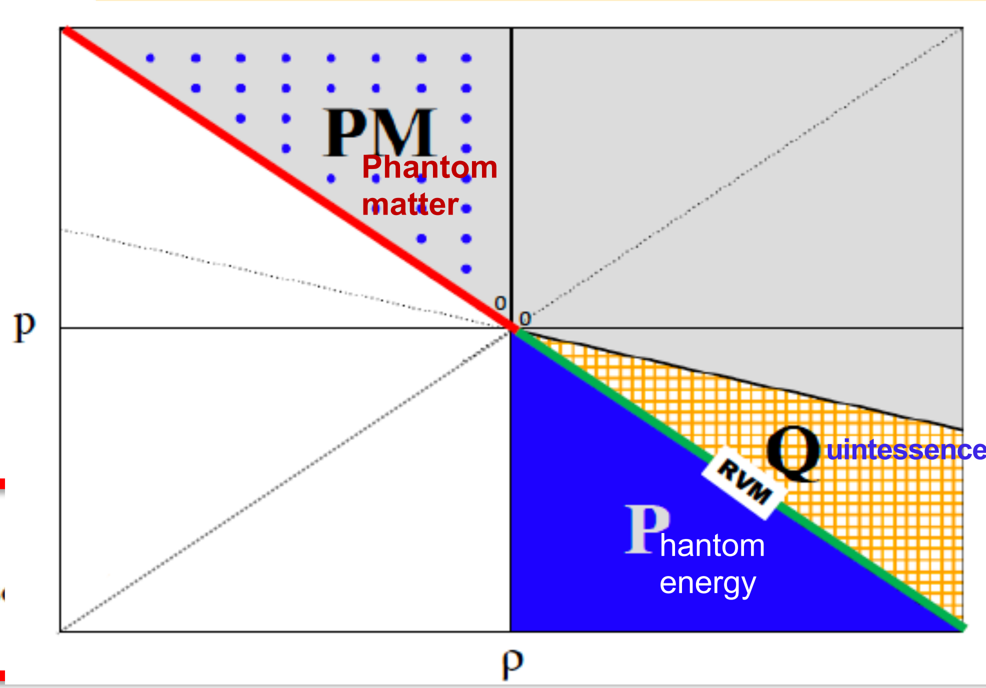
Equation of state :

$0 > \rho_b + \rho_{gCS} = - (p_b + p_{gCS})$ cf. phantom "matter"

$0 < \rho_\Lambda = -p_\Lambda$ → dominates →

$0 < \rho_b + \rho_{gCS} + \rho_\Lambda = - (p_b + p_{gCS} + p_\Lambda)$ true RVM vacuum

Gravitational Anomaly Condensates → Dynamical Inflation

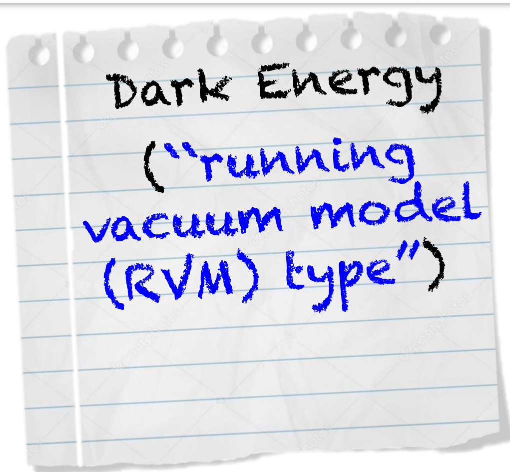


NEM, Solà

$$10^7 \, \epsilon \mathcal{N} \, H^4 > 0$$

Positive Cosmological Constant-like

ρ_{t} … $)^2 + \big(1.17 - 1.37\big) \times 10^7 \, (\frac{H}{M_{\rm Pl}})^4] > 0$



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RVM-like terms drive inflation contain scalar d.o.f. from the anomaly condensate

Dark Energy ("running vacuum model (RVM) type")

But slow roll is due to the KR axion field $\epsilon \sim \frac{1}{2}\frac{1}{(H M_{\rm Pl})^2}\dot{b}^2 \sim 10^{-2}$

Cosmological Evolution of RVM

Basilakos, Lima, Sola + Gomez Valent + ... (2013 - 2018)

$$\omega = \rho_m / p_m$$ *m* = matter, radiation

$$\nabla^\mu T_{\mu\nu} = 0 \quad \Rightarrow \quad \dot{\rho}_m + 3(1+\omega) H \rho_m = -\dot{\rho}^{\Lambda}_{\rm RVM}$$

↔ *m* + RVM

Cosmological Evolution of RVM

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$$\nabla^\mu T_{\mu\nu} = 0$$

m + RVM

$$\dot{\rho}_m + 3(1+\omega) H \rho_m = -\dot{\rho}^{\Lambda}_{\rm RVM} = \dot{\rho}_{\rm total}$$

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(-1.7×10^{-3}: ν; $1.17 - 1.37$: α)

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$$\nabla^\mu T_{\mu\nu} = 0 \Rightarrow \dot{\rho}_m + 3(1+\omega)H\rho_m = -\dot{\rho}^{\Lambda}_{\rm RVM}$$

$c_0 = 0$

$$\dot{H} + \frac{3}{2}(1+\omega)H^2\left(1 - \nu - \frac{c_0}{H^2} - \alpha\frac{H^2}{H_I^2}\right) = 0$$

Solution without fundamental inflatons

$$H(a) = \left(\frac{1-\nu}{\alpha}\right)^{1/2} \frac{H_I}{\sqrt{D\, a^{3(1-\nu)(1+\omega_m)} + 1}}$$

$D > 0$

Early de Sitter (unstable) $Da^{4(1-\nu)} \ll 1 \Rightarrow H^2 = (1-\nu)H_I^2/\alpha$

Radiation $Da^{4(1-\nu)} \gg 1 \Rightarrow H^2 \sim a^{3(1-\nu)(1+\omega_m)} \sim a^{-4}$

$\omega = 1/3$

Late dark-Energy dominated era $H^2(a) = H_0^2\left[\tilde{\Omega}_{m0}\, a^{-3(1-\nu)} + \tilde{\Omega}_{\Lambda 0}\right]$ $\tilde{\Omega}_{\Lambda 0}$ dominant

Cosmological Evolution of RVM

Basilakos, Lima, Sola + Gomez Valent + ... (2013 - 2018)

$\omega = \rho_m / p_m$ m = matter, radiation

$$\nabla^\mu T_{\mu\nu} = 0 \quad \Rightarrow \quad \dot{\rho}_m + 3(1+\omega) H \rho_m = -\dot{\rho}^\Lambda_{\rm RVM}$$

$$\dot{H} + \frac{3}{2}(1+\omega) H^2 \left(1 - \nu - \frac{c_0}{H^2} - \alpha \frac{H^2}{H_I^2}\right) = 0$$

Solution

$$H(a) = \left(\frac{1-\nu}{\alpha}\right)^{1/2} \frac{H_I}{\sqrt{D\, a^{3(1-\nu)(1+\omega_m)} + 1}}$$

$D > 0$

Early de Sitter (unstable)

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Radiation

$$D a^{4(1-\nu)} \gg 1 \quad \Rightarrow \quad H^2 \sim a^{3(1-\nu)(1+\omega_m)} \sim a^{-4}$$

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$$H^2(a) = H_0^2 \left[\tilde{\Omega}_{m0}\, a^{-3(1-\nu)} + \tilde{\Omega}_{\Lambda 0}\right]$$

$\tilde{\Omega}_{\Lambda 0}$ dominant

Gravitational Anomaly Condensates → Dynamical Inflation

NEM, Solà

Cannot obtain such terms
in **ordinary Quantum Field Theories**
You need the **condensate of**
the gravitational anomalies
which have **CP-violating couplings**
with the **gravitational axions**

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \big(1.17 - 1.37\big) \times 10^7 \,(\frac{H}{M_{\rm Pl}})^4\Big] > 0$$

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("running
vacuum model
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RVM-like terms drive inflation contain scalar d.o.f. from the anomaly condensate

But slow roll is due to the KR axion field $\epsilon \sim \frac{1}{2}\frac{1}{(HM_{\rm Pl})^2}\dot{\overline{b}}^2 \sim 10^{-2}$

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Another important role of CP-violation in Early Universe

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Basilakos, NEM, Solà

$$\Lambda \equiv \langle b(x)\, R_{\mu\mu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle \simeq 5.86 \times 10^{7}\, \epsilon \mathcal{N}\, H^4 > 0$$

Positive Cosmological Constant-like

Positive total energy density since Λ-term dominates

$$\rho_{\rm total} = \rho_b + \rho_{gCS} + \rho_\Lambda \simeq 3 M_{\rm Pl}^4 \Big[-1.7 \times 10^{-3} \Big(\frac{H}{M_{\rm Pl}}\Big)^2 + \Big(1.17 - 1.37\Big) \times 10^7 \,(\frac{H}{M_{\rm Pl}})^4 \Big] > 0$$

Negative coefficient ν < 0 due to CS anomaly in early Universe, unlike late-era RVM

RVM-like terms drive inflation contain scalar d.o.f. from the anomaly condensate

But slow roll is due to the KR axion field $\epsilon \sim \frac{1}{2} \frac{1}{(H M_{\rm Pl})^2} \dot{\overline{b}}^2 \sim 10^{-2}$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha\bar{b}-\sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^\alpha(t)\right)\right]=0 \quad\Rightarrow\quad \dot{\bar{b}}=\sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa}\mathcal{K}^0\sim\text{constant}$$

Undiluted KR axion background at the end of Inflation

@ end of Inflationary era

$$\dot{\bar{b}}\sim\sqrt{2\varepsilon}\,M_{\rm Pl}\,H\sim 0.14\,M_{\rm Pl}\,H$$

$$H = H_{\rm infl} \simeq \text{const.}$$

Solutions (backgrounds) to the Eqs of Motion

$$\partial_\alpha\left[\sqrt{-g}\left(\partial^\alpha \bar{b} - \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\mathcal{K}^\alpha(t)\right)\right] = 0 \quad \Rightarrow \quad \dot{\bar{b}} = \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\mathcal{K}^0 \sim \text{constant}$$

Undiluted KR axion background at the end of Inflation

@ end of Inflationary era

$$\dot{\bar{b}} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

$$H = H_{\rm infl} \simeq {\rm const.}$$

Important for Leptogenesis @ radiation era

5. Enhanced cosmic perturbations and densities of primordial black holes and Gravitational Waves

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

NEM, Universe 7 (2021) 12, 480, e-Print: 2111.05675 [hep-th]

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$

$$V(b) \simeq b\,\widetilde{\Lambda}_0^4 \sqrt{\frac{2}{3}}\,\frac{M_{\rm Pl}}{96\,M_s^2} \equiv b\,\frac{\widetilde{\Lambda}_0^4}{f_b} \equiv b\,\Lambda_0^3$$

$$\Lambda_0 = 8.4 \times 10^{-4}\, M_{\rm Pl}$$

$$f_b \equiv \Big(\sqrt{\frac{2}{3}}\,\frac{M_{\rm Pl}}{96\,M_s^2}\Big)^{-1} \overset{Eq.(9)}{\simeq} 5.3 \times 10^{-6}\, M_{\rm Pl}$$

Such a potential can also arise in appropriate brane compactifications (eg type IIB strings)

L. McAllister, E. Silverstein and A. Westphal, Phys. Rev. D 82 (2010), 046003 [arXiv:0808.0706 [hep-th]].

We may extend the model to include other stringy axions arising from compactification

$$V^{\rm lin}_{a_I} = a_I(x)\,\frac{f_b}{f_a}\,\Lambda_0^3$$

f_a = axion coupling

(with canonical kinetic terms for a-axions)

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

NEM, Universe 7 (2021) 12, 480, e-Print: 2111.05675 [hep-th]

NEM, Spanos, Stamou, hep-th-2206.07963

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$

world-sheet (non-perturbative) instantons → periodic potential perturbations

$$V^{b}_{\rm wsinst} \simeq \Lambda_b^4 \cos\Big(\frac{b}{f_b}\Big) \qquad \Lambda_b^4 \sim M_s^4\, e^{-S_{\rm wsinst}} \;\rightarrow\; \Lambda_b \ll \Lambda_0\,.$$

$$V^{a_I}_{\rm wsinst} \simeq \Lambda_I^4 \cos\Big(\frac{a_I}{f_{a_I}}\Big) \qquad \Lambda_0 \gg \Lambda_I \neq \Lambda_b,$$

Restrict to $I=1: \;\; a_1 \equiv a$

NB: For $S_{\rm winst} \geq O(40)$: $m_a \leq O(10^{-17})$ eV, still compatible with ultralight axion DM

$$V_{brane-compact.-effects}(a) \ni \Lambda_2^4\, \frac{1}{f_a}\, a + \Lambda_I^4\Big(1 + \xi_a\, \frac{a}{f_a}\Big)\cos\Big(\frac{a}{f_a}\Big)$$

warp factor → ϵ

$$\frac{\Lambda_2^4}{f_a} \sim \frac{\epsilon}{L}\sqrt{\frac{3}{(2\pi)^3}}\, M_s^3$$

L. McAllister, E. Silverstein and A. Westphal, Phys. Rev. D 82 (2010), 046003 [arXiv:0808.0706 [hep-th]].

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

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NEM, Spanos, Stamou, hep-th-2206.07963

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$$V^{a_I}_{\rm wsinst} \simeq \Lambda_I^4 \cos\Big(\frac{a_I}{f_{a_I}}\Big) \qquad \Lambda_0 \gg \Lambda_I \neq \Lambda_b,$$

Restrict to $I = 1:\; a_1 \equiv a$

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World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} \rangle\, b(x)$

world-sheet (non-perturbative) instantons → periodic potential perturbations

$$V(a,\,b) = {\Lambda_1}^4 \left(1 + f_a^{-1}\, \tilde{\xi}_1\, a(x)\right)\, \cos({f_a}^{-1} a(x)) + \frac{1}{f_a}\left(f_b\, {\Lambda_0}^3 + \Lambda_2^4\right) a(x) + {\Lambda_0}^3\, b(x)$$

Case I

$$\Big(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\,\Lambda_0^3}\Big)^{1/3} \Lambda_0 < \Lambda_1 \ll \Lambda_0$$

Inflation driven by b axion

NEM, Sola + Basilakos
NEM, Spanos, Stamou, hep-th-2206.07963

Case II

$$\Lambda_0 \ll \Big(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\,\Lambda_0^3}\Big)^{1/3} \Lambda_0 < \Lambda_1$$

Zhou, Jiang, Cai, Sasaki, Pi, Phys. Rev. D 102 (2020) no.10, 103527

Inflation driven by compactification axions,
Prolonged by b axion

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$

world-sheet (non-perturbative) instantons → periodic potential perturbations

$$V(a,\, b) = \Lambda_1{}^4 \Big(1 + f_a^{-1}\, \tilde{\xi}_1\, a(x)\Big) \cos(f_a{}^{-1} a(x)) + \frac{1}{f_a}\Big(f_b\, \Lambda_0{}^3 + \Lambda_2^4\Big)\, a(x) + \Lambda_0{}^3\, b(x)$$

Case I $\quad \Big(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\, \Lambda_0^3}\Big)^{1/3} \Lambda_0 < \Lambda_1 \ll \Lambda_0$

Enhancement of cosmic perturbations

Case $\quad \Lambda_0 \ll \Big(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\, \Lambda_0^3}\Big)^{1/3} \Lambda_0 < \Lambda_1$

NEM, Sola + Basilakos
NEM, Spanos, Stamou, hep-th-2206.07963

Zhou, Jiang, Cai, Sasaki, Pi, Phys. Rev. D 102 (2020) no.10, 103527

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$

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$$V(a,\,b) = {\Lambda_1}^4 \Big(1 + f_a^{-1}\,\tilde{\xi}_1\, a(x)\Big)\,\cos({f_a}^{-1} a(x)) + \frac{1}{f_a}\Big(f_b\,{\Lambda_0}^3 + \Lambda_2^4\Big)\, a(x) + {\Lambda_0}^3\, b(x)$$

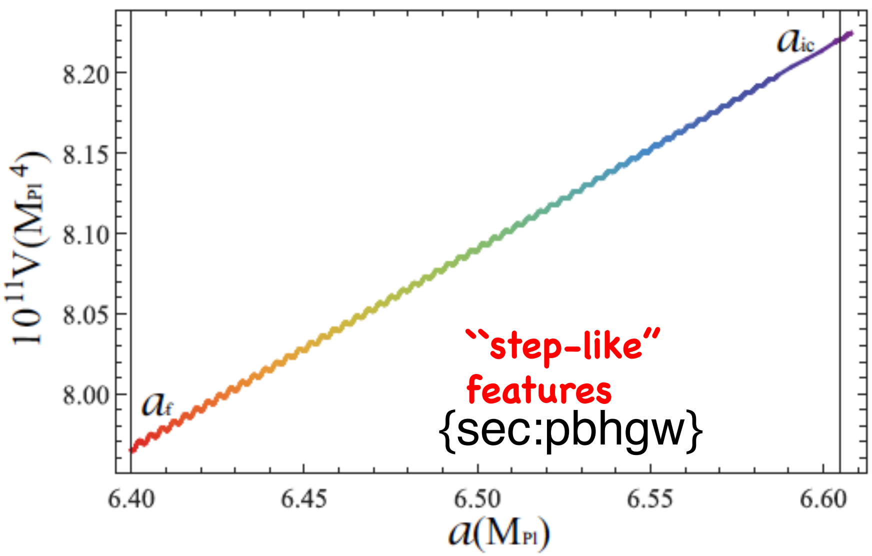
Case I $$\Big(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\,\Lambda_0^3}\Big)^{1/3}\Lambda_0 < \Lambda_1 \ll \Lambda_0$$

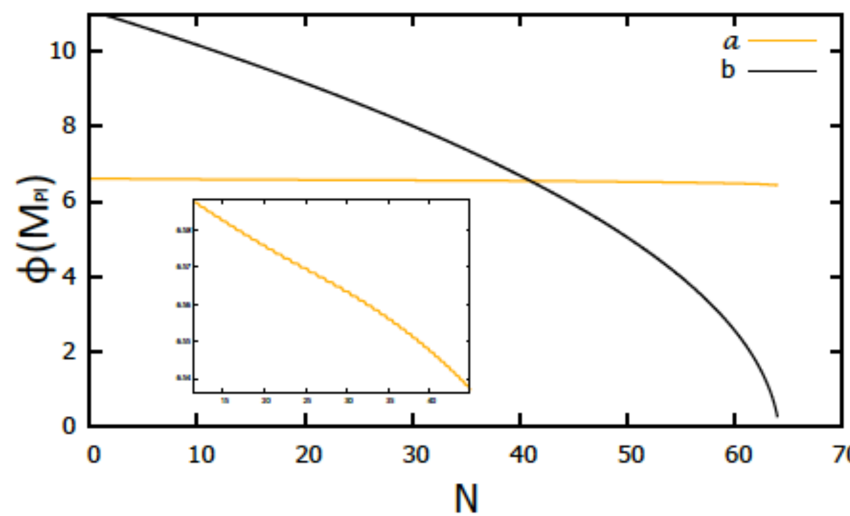
NEM, Spanos, Stamou, hep-th-2206.07963

The enhancement of cosmic perturbations mechanism

NEM, Spanos, Stamou, hep-th-2206.07963

b-field + condensate drive inflation, **a-axion ends inflation**





$$V(a,\, b) = \Lambda_1{}^4 \left(1 + f_a^{-1}\,\tilde{\xi}_1\, a(x)\right)\, \cos(f_a{}^{-1} a(x)) + \frac{1}{f_a}\left(f_b\, \Lambda_0{}^3 + \Lambda_2^4\right) a(x) + \Lambda_0{}^3\, b(x)$$

$$n_s = 1 + \frac{d\ln P_R}{d\ln k} \qquad r = \frac{P_T}{P_R} \qquad P_T = \frac{2}{\pi^2} H^2$$

SET	g_1	g_2	ξ	$f(M_{Pl})$	$\Lambda_0(M_{Pl})$	$\Lambda_1(M_{Pl})$	$\Lambda_3(M_{Pl})$
1	0.021	0.904	-0.15	2.5×10^{-4}	8.4×10^{-4}	8.19×10^{-4}	2.32×10^{-4}
2	0.026	0.774	-0.20	2.5×10^{-4}	8.4×10^{-4}	7.89×10^{-4}	2.49×10^{-4}

SET	a_{ic}	b_{ic}	n_s	r
1	6.605	11.1	0.9638	0.062
2	4.932	11.4	0.9619	0.060

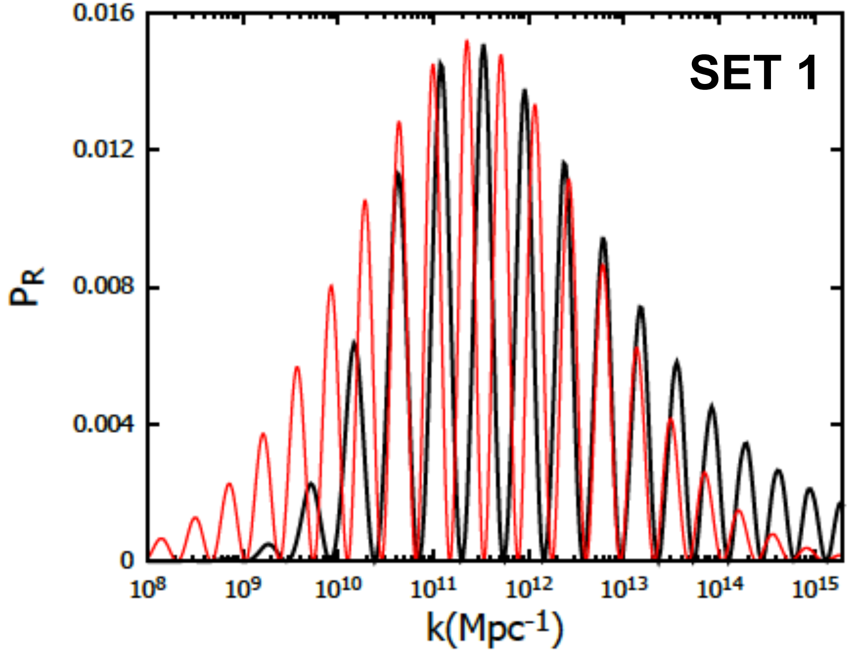
The enhancement of cosmic perturbations mechanism

NEM, Spanos, Stamou,
hep-th-2206.07963

b-field + condensate drive inflation, **a-axion ends inflation**

$$P_{R1}(k) = P_{max} \exp\left(\frac{-1}{2\Delta^2} \log\left(\frac{k}{k_{\rm ref}}\right)^2\right) \left(1 + A_{\log} \cos(\omega_{\log} \ln(k/k_{\rm ref}) + \vartheta_{\log})\right)$$

(Red curve)



Parameters

$\Delta = 3, \quad k_{\rm ref} = 5 \times 10^5$

$\vartheta_{\log} = \pi/4, \ A_{\log} = 1,$

$\omega_c = 4.77 \quad \omega_{\log} = 1.6 \times \omega_c$

$P_{max} = 0.0076$

$\omega_{\log} \geq \omega_c$ resonant peak in Ω_{GW}

Fumagalli, Renaux-Petel, Witkowski,
arXiv: 2105.04861

SET	g_1	g_2	ξ	$f(M_{Pl})$	$\Lambda_0(M_{Pl})$	$\Lambda_1(M_{Pl})$	$\Lambda_3(M_{Pl})$
1	0.021	0.904	-0.15	2.5×10^{-4}	8.4×10^{-4}	8.19×10^{-4}	2.32×10^{-4}
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NEM, Spanos, Stamou, hep-th-2206.07963

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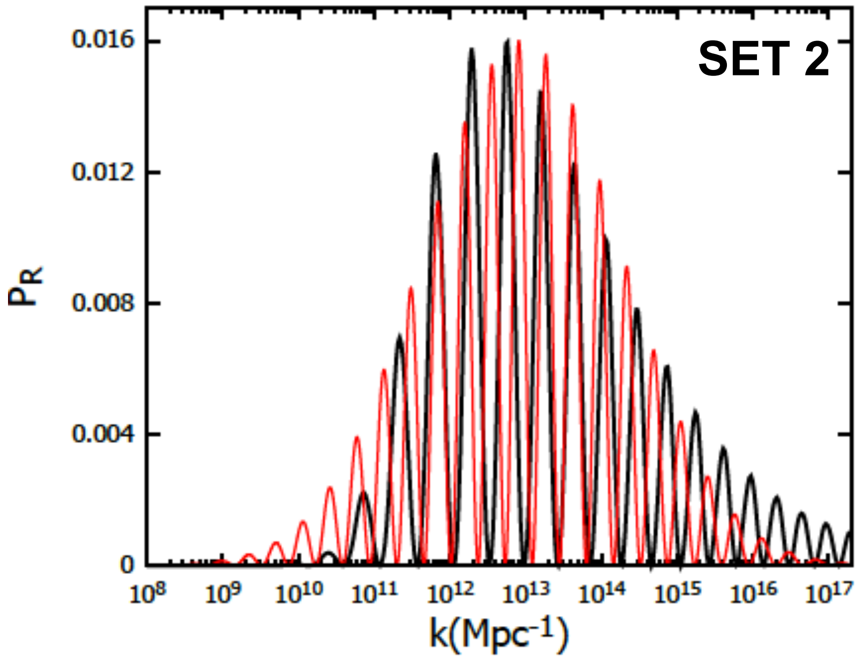
(Red curve)



Parameters

$$\Delta = 3, \;\; k_{\rm ref} = 3 \times 10^6$$

$$\vartheta_{\log} = \pi/4, \; A_{\log} = 1$$

$$\omega_c = 4.77 \quad \omega_{\log} = 1 \times \omega_c$$

$$P_{max} = 0.0080$$

$\omega_{\log} \geq \omega_c$ resonant peak in Ω_{GW}

Fumagalli, Renaux-Petel, Witkowski, arXiv: 2105.04861

SET	g_1	g_2	ξ	$f(M_{Pl})$	$\Lambda_0(M_{Pl})$	$\Lambda_1(M_{Pl})$	$\Lambda_3(M_{Pl})$
1	0.021	0.904	-0.15	2.5×10^{-4}	8.4×10^{-4}	8.19×10^{-4}	2.32×10^{-4}
2	0.026	0.774	-0.20	2.5×10^{-4}	8.4×10^{-4}	7.89×10^{-4}	2.49×10^{-4}

SET	a_{ic}	b_{ic}	n_s	r
1	6.605	11.1	0.9638	0.062
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The enhancement of cosmic perturbations mechanism

NEM, Spanos, Stamou, hep-th-2206.07963

b-field + condensate drive inflation, **a-axion ends inflation**

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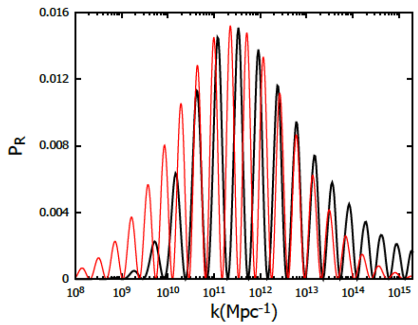
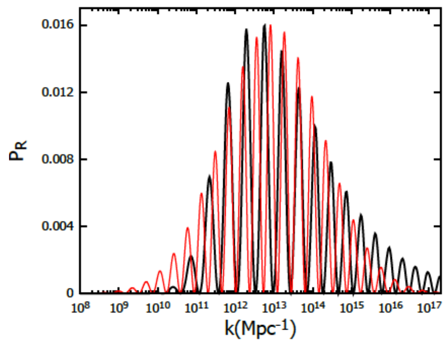


SET 1

SET 2



Parameters

$\Delta = 3, \quad k_{\rm ref} = 5 \times 10^5$

$\theta_{\log} = \pi/4, \; A_{\log} = 1.6,$

$\omega_c = 4.77 \quad \omega_{\log} = 1.6 \times \omega_c$

$P_{max} = 0.0045.$

$\omega_{\log} \geq \omega_c$ resonant peak in Ω_{GW}

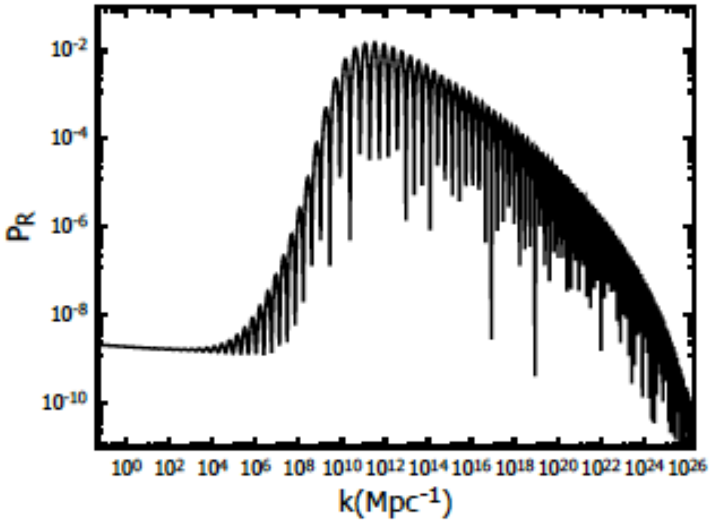
Fumagalli, Renaux-Petel, Witkowski, arXiv: 2105.04861

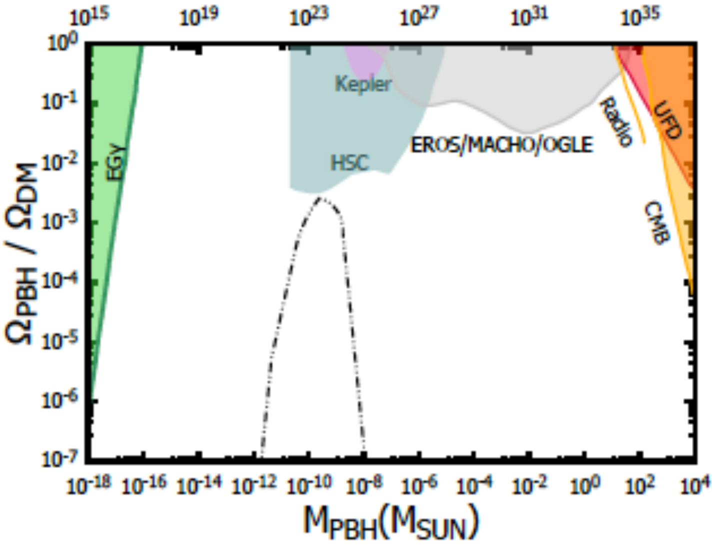
SET	g_1	g_2	ξ	$f(M_{Pl})$	$\Lambda_0(M_{Pl})$	$\Lambda_1(M_{Pl})$	$\Lambda_3(M_{Pl})$
1	0.021	0.904	−0.15	2.5×10^{-4}	8.4×10^{-4}	8.19×10^{-4}	2.32×10^{-4}
2	0.026	0.774	−0.20	2.5×10^{-4}	8.4×10^{-4}	7.89×10^{-4}	2.49×10^{-4}

SET	a_{ic}	b_{ic}	n_s	r
1	6.605	11.1	0.9638	0.062
2	4.932	11.4	0.9619	0.060

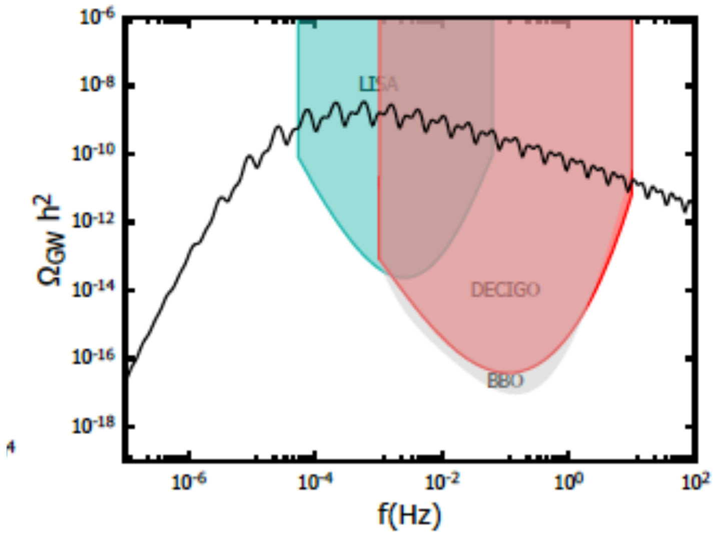
Primordial Black Hole (PBH) and GW enhanced production during inflation

NEM, Spanos, Stamou, hep-th-2206.07963





SET 1



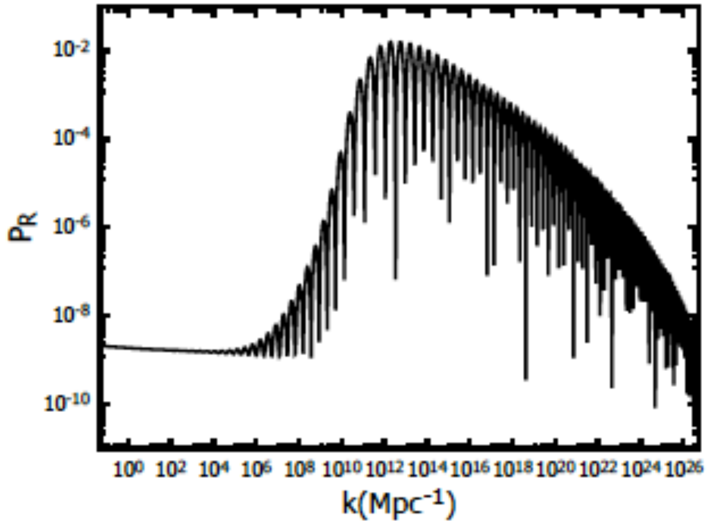
fractional PBH abundance

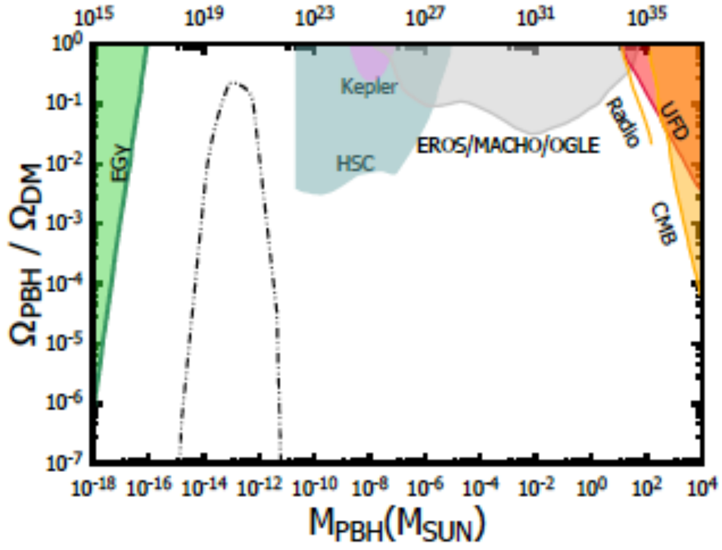
$$f_{PBH} = \int_k dM_{\rm PBH}(k)\, \frac{1}{M_{\rm PBH}(k)}\, \frac{\Omega_{PBH}}{\Omega_{DM}}(M_{\rm PBH}(k))$$

$$f_{PBH} = 0.01$$

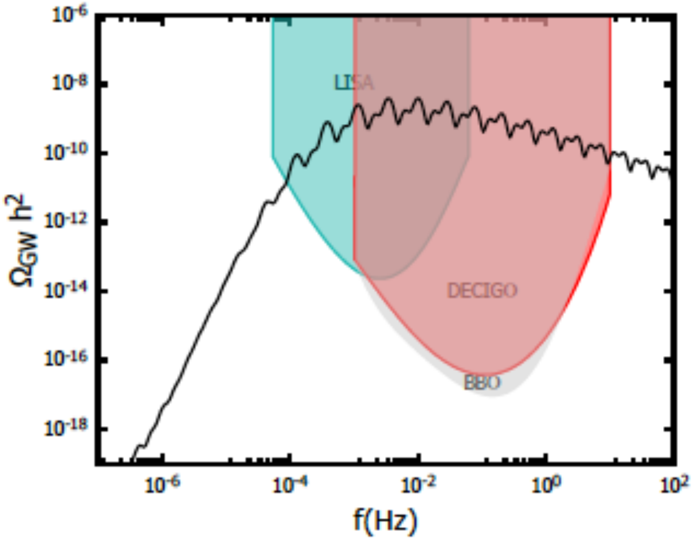
Primordial Black Hole (PBH) and GW enhanced production during inflation

NEM, Spanos, Stamou, hep-th-2206.07963





SET 2



fractional PBH abundance

$$f_{PBH} = \int_k dM_{\rm PBH}(k) \frac{1}{M_{\rm PBH}(k)} \frac{\Omega_{PBH}}{\Omega_{DM}} (M_{\rm PBH}(k))$$

$$f_{PBH} = 0.80.$$

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$

world-sheet (non-perturbative) instantons → periodic potential perturbations

$$V(a,\, b) = {\Lambda_1}^4 \left(1 + f_a^{-1}\,\tilde{\xi}_1\, a(x)\right)\, \cos({f_a}^{-1} a(x)) + \frac{1}{f_a}\left(f_b\, {\Lambda_0}^3 + \Lambda_2^4\right) a(x) + {\Lambda_0}^3\, b(x)$$

Case II $$\Lambda_0 \ll \left(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\,\Lambda_0^3}\right)^{1/3} \Lambda_0 < \Lambda_1$$

Zhou, Jiang, Cai, Sasaki, Pi, Phys. Rev. D 102 (2020) no.10, 103527

NEM, Spanos, Stamou, hep-th-2206.07963

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$

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$$V(a,\, b) = {\Lambda_1}^4 \Big(1 + f_a^{-1}\, \tilde{\xi}_1\, a(x)\Big)\, \cos({f_a}^{-1} a(x)) + \frac{1}{f_a}\Big(f_b\, {\Lambda_0}^3 + \Lambda_2^4\Big)\, a(x) + {\Lambda_0}^3\, b(x)$$

NEM, Spanos, Stamou, hep-th-2206.07963

specific set of parameters enhancement due to **inflection points** in the potential → different enhancement mechanism than in

Zhou, Jiang, Cai, Sasaki, Pi, Phys. Rev. D 102 (2020) no.10, 103527

Case II $\quad \Lambda_0 \ll \left(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\, \Lambda_0^3}\right)^{1/3} \Lambda_0 < \Lambda_1$

World-Sheet Instantons, Axion Monodromy like potentials & deviations from scale invariance

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$

world-sheet (non-perturbative) instantons → periodic potential perturbations

$$V(a,\, b) = {\Lambda_1}^4 \left(1 + f_a^{-1}\, \tilde{\xi}_1\, a(x)\right)\, \cos({f_a}^{-1} a(x)) + \frac{1}{f_a}\left(f_b\, {\Lambda_0}^3 + \Lambda_2^4\right) a(x) + {\Lambda_0}^3\, b(x)$$

$$\Lambda_0 = 8.4\times 10^{-4} M_{\rm Pl}, \quad g_1 = 110, \quad g_2 = 1.779\times 10^4, \quad \xi = -0.09, \quad f = 0.09\, M_{\rm Pl}.$$

SET 3 $(a_{ic}, b_{ic}) = 7.5622, 0.522.$

Case II $\quad \Lambda_0 \ll \left(\frac{f_b}{f_a} + \frac{\Lambda_2^4}{f_a\, \Lambda_0^3}\right)^{1/3} \Lambda_0 < \Lambda_1$

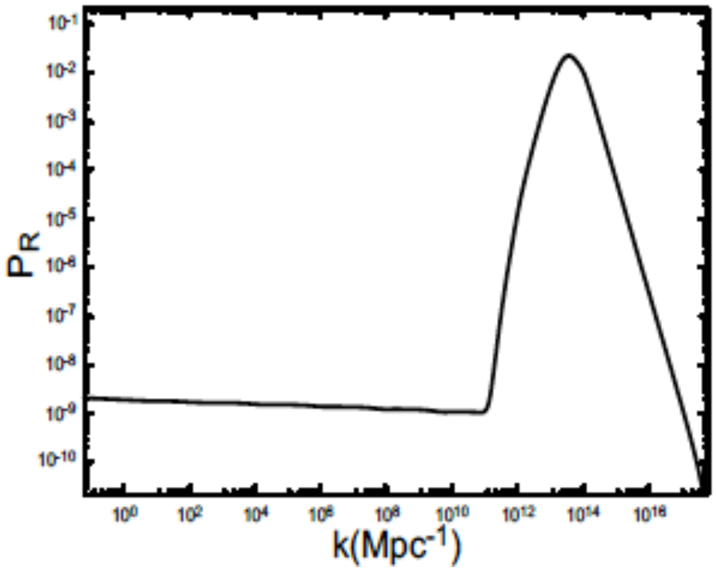
NEM, Spanos, Stamou, hep-th: 2206.07963

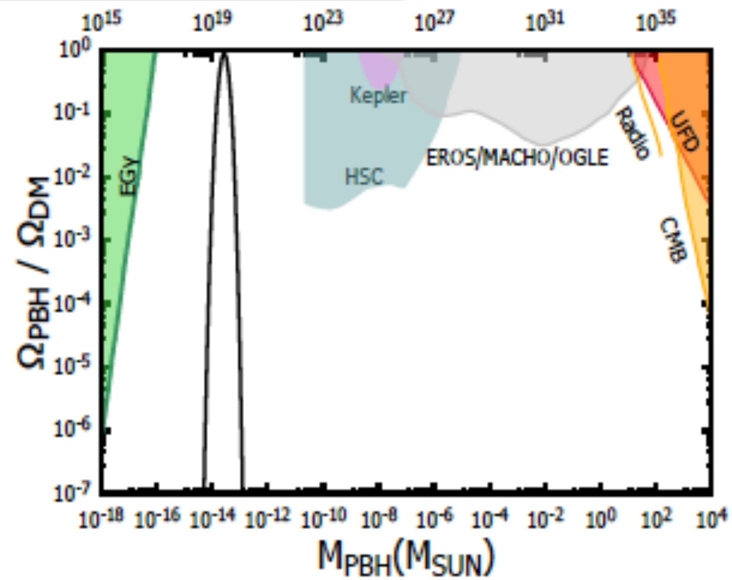
specific set of parameters enhancement due to **inflection points** in the potential → different enhancement mechanism than in

Zhou, Jiang, Cai, Sasaki, Pi, Phys. Rev. D 102 (2020) no.10, 103527

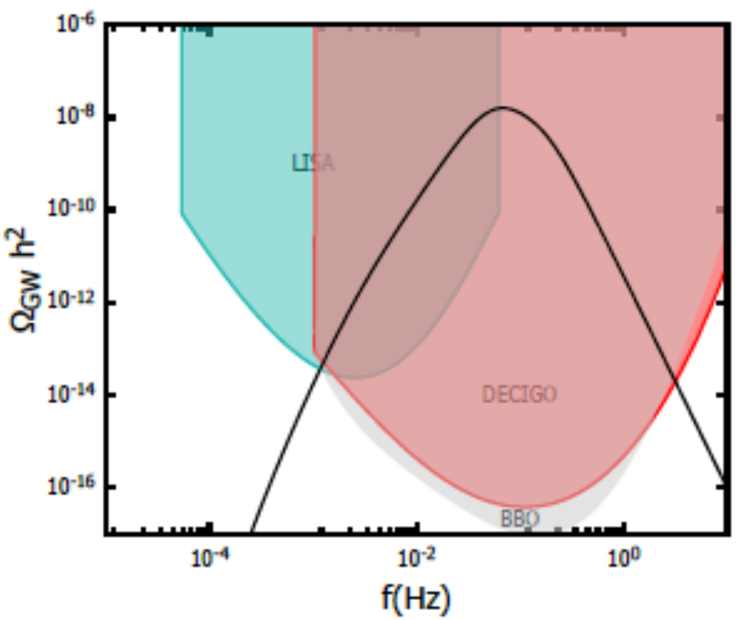
Primordial Black Hole (PBH) and GW enhanced production during inflation in Case 2

NEM, Spanos, Stamou, hep-th-2206.07963.





SET 3



fractional PBH abundance

$$f_{PBH} = \int_k dM_{\rm PBH}(k) \frac{1}{M_{\rm PBH}(k)} \frac{\Omega_{PBH}}{\Omega_{DM}}(M_{\rm PBH}(k))$$

$$f_{PBH} = 0.762$$

SUMMARY: Primordial Black Hole (PBH) and GW enhanced production during inflation in Cases 1 + 2

NEM, Spanos, Stamou, hep-th-2206.07963

SET	P_R^{peak}	$M_{PBH}^{peak}(M_\odot)$	f_{PBH}
1	1.466×10^{-2}	2.394×10^{-10}	0.009
2	1.365×10^{-2}	8.313×10^{-14}	0.799
3	2.24×10^{-2}	1.791×10^{-14}	0.762

Hence in both hierarchies of scales :

Case 1: $\left(\frac{f_b}{f_a}+\frac{\Lambda_2^4}{f_a\Lambda_0^3}\right)^{1/3}\Lambda_0<\Lambda_1\ll\Lambda_0$, **Case 2:** $\Lambda_0\ll\left(\frac{f_b}{f_a}+\frac{\Lambda_2^4}{f_a\Lambda_0^3}\right)^{1/3}\Lambda_0<\Lambda_1$

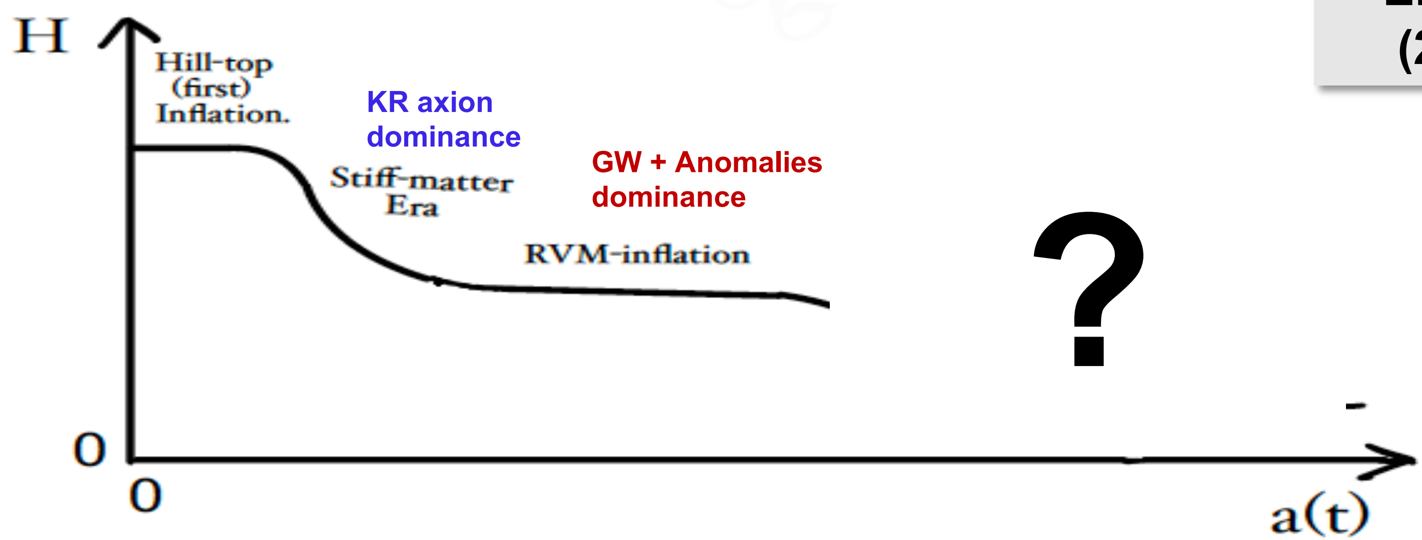
Common: one may get **significant enhancement** of cosmic perturbations, ands PBH production, and thus a **significant portion** of PBH could play **the role of DM**, also, as a result, **profiles of GW** could **change** during radiation.

Difference: In case 1: **intensely oscillating spectra** , case 2: **smooth behaviour**
→ **distrinct behaviour**, in principle **falsifiable predictions** at future **interferometers (e.g. LISA).**

6. Post Inflationary Eras & Cosmic Evolution of the stringy RVM

Post-RVM-Inflation Eras & Evolution

NEM, Solà
EPJ-ST
(2020)



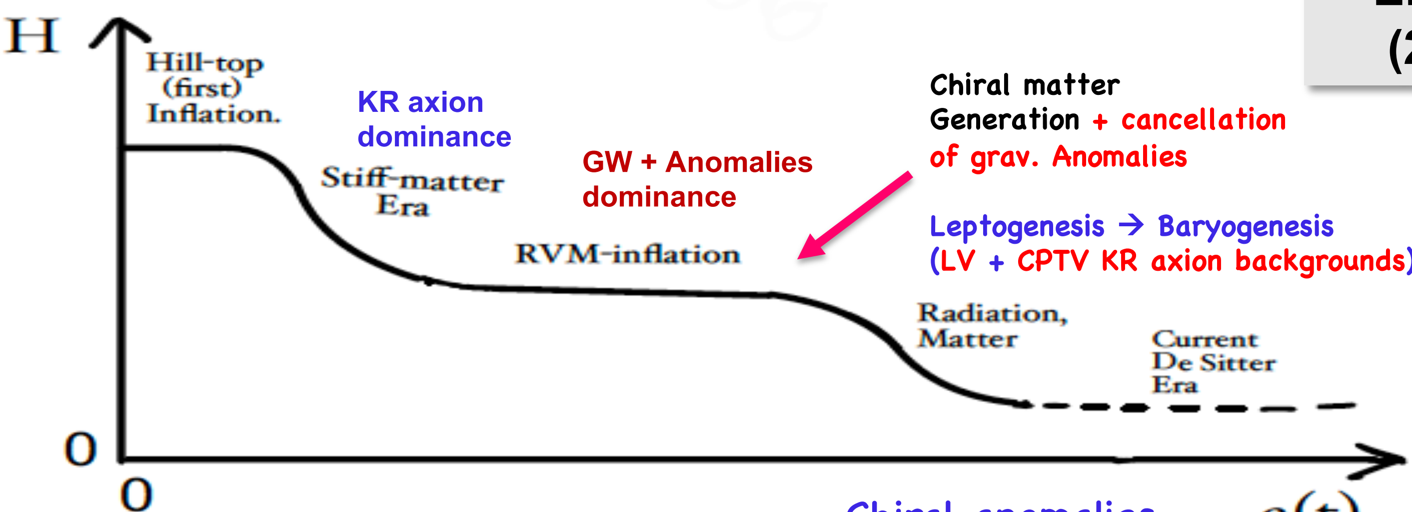
Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation (including sterile ν)

Basilakos, NEM,Solà (2019-20)

Required by consistency of quantum theory of matter and radiation (**diffeomorphism invariance**)

NEM,Solà EPJ-ST (2020)



NEM, Sarkar + De Cesare, Bossingham

Chiral anomalies Remain in matter era

→ KR axion mass generation through QCD instantons (Dark Matter)

Cancellation of Gravitational Anomalies in Radiation Era by:

Chiral Fermionic Matter generation @ end of Inflation (including sterile ν)

Basilakos, NEM,Solà (2019-20)

Required by consistency of quantum theory of matter and radiation (**diffeomorphism invariance**)

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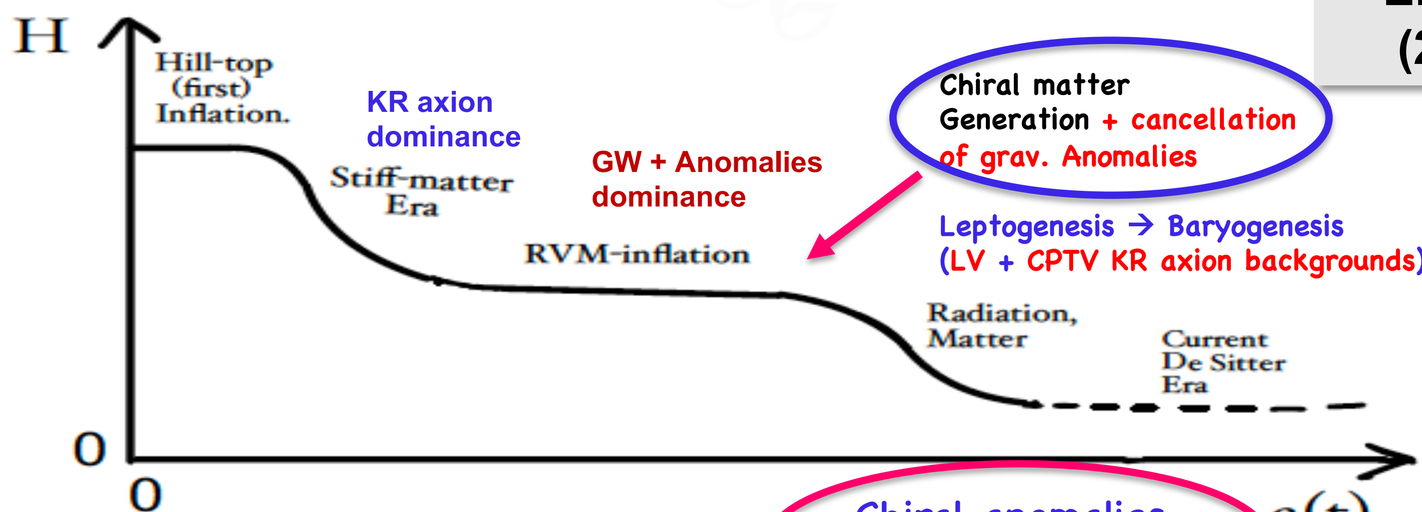


Chiral matter Generation + cancellation of grav. Anomalies

Leptogenesis → Baryogenesis (LV + CPTV KR axion backgrounds)

NEM, Sarkar + De Cesare, Bossingham

Chiral anomalies Remain in matter era

KR axion mass generation through QCD instantons (Dark Matter)

The Whole

Stringy-RVM

Cosmological Evolution

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

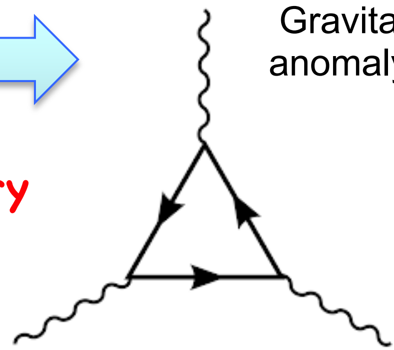
Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves → Gravitational anomaly (GA) →



Undiluted constant KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\,\dot{b}\,\delta_{\mu 0}$$

$$\dot{b} \sim \sqrt{2\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

chiral matter generation @ inflation exit

From a pre-inflationary era after Big-Bang

Cancellation of GA

Radiation Era

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

B-L conserving sphelaron processes → Baryongenesis

Matter Era

Possible potential (mass) generation for b → axion Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$ Phenomenology

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

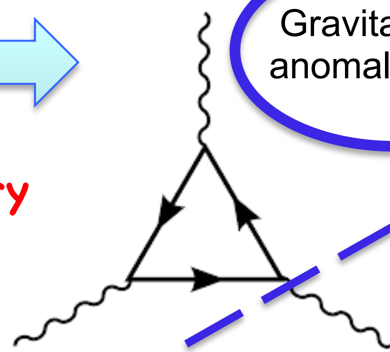
Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



Gravitational anomaly (GA)

Undiluted constant KR axial backround

$B_\mu = M_{\rm Pl}^{-1}\dot{b}\,\delta_{\mu 0}$

$\dot{b} \sim \sqrt{2\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$

From a pre-inflationary era after Big-Bang

chiral matter generation @ inflation exit

Radiation Era

$B_0 \propto T^3$

Cancellation of GA

Leptogenesis induced by RHN (tree-level) decays

$N_I \rightarrow \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$

ΔL in the (approx.) constant LV + CPTV background $B_\mu = M_{\rm Pl}^{-1}\dot{b}\,\delta_{\mu 0}$

NEM, Sarkar + De Cesare, Bossingham

B-L conserving sphelaron processes → Baryongenesis

Matter Era

Possible potential (mass) generation for b → axion Dark matter

NB:

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k \overline{L}_k \tilde{\phi} N + h.c.$$

(approx.) Constant B_0 Background

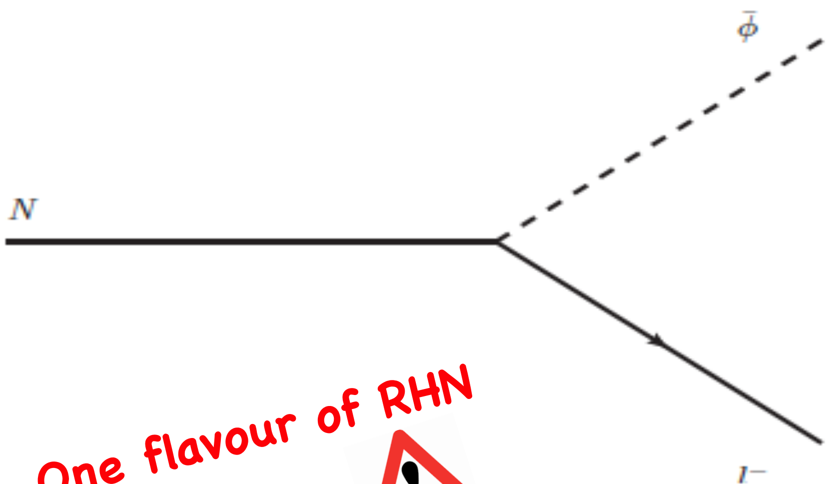
CPT Violation

Early Universe
$T >> T_{EW}$

Lepton number & CP Violations @ tree-level
due to Lorentz/CPTV Background

$$N_I \to \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

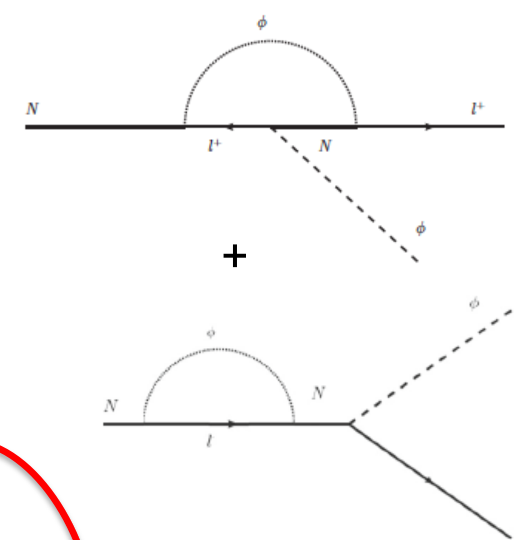
Produce Lepton asymmetry



One flavour of RHN suffices

De Cesare, NEM, Sarkar ,
+ Bossingham

Contrast with one-loop conventional CPV Leptogenesis (in absence of H-torsion)



Fukugita, Yanagida,

CPT Violation

de Cesare, NEM, Sarkar
Eur.Phys.J. C75, 514 (2015)

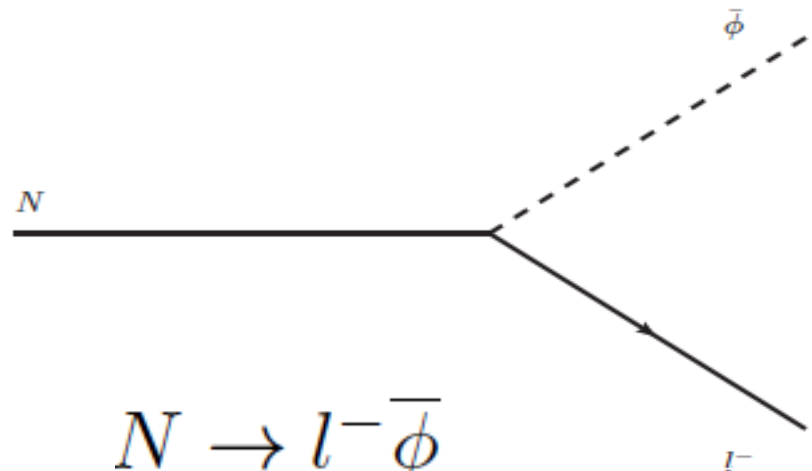
Early Universe
T >> T_{EW}

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k \overline{L}_k \tilde{\phi} N + h.c.$$

Heavy Right-Handed-Neutrinos (N) interact with **axial (approx.) constant background** with only temporal component $B_0 \propto \dot{b} \neq 0$

Produce Lepton asymmetry

Lepton number & CP Violations
@ tree-level due to
Lorentz/CPTV Background



$$N \rightarrow l^+ \phi \qquad\qquad N \rightarrow l^- \overline{\phi}$$

$$\Gamma_1 = \sum_k \frac{|Y_k|^2}{32\pi^2} \frac{m^2}{\Omega} \frac{\Omega + B_0}{\Omega - B_0} \neq \Gamma_2 = \sum_k \frac{|Y_k|^2}{32\pi^2} \frac{m^2}{\Omega} \frac{\Omega - B_0}{\Omega + B_0}$$

$B_0 \neq 0$

CPV & LV

$$\Omega = \sqrt{B_0^2 + m^2}$$

CPTV Thermal

$$\mathcal{L} = i\overline{N}\not{\partial}N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}\not{B}\gamma^5 N - Y_k \overline{L}_k \tilde{\phi} N + h.c.$$

Early Universe
T > 10^5 GeV

CPT Violation

(approx.) Constant $B^0 \neq 0$ background

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \to \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10}, \quad \to \quad \frac{B_0}{m} \simeq 10^{-8}$$

Produce Lepton asymmetry

$$Y_k \sim 10^{-5}$$
$$m \geq 100\text{TeV} \to$$
$$B^0 \sim 1\text{MeV}$$
$$T_D \simeq m \sim 100 \text{ TeV}$$

Solving system of Boltzmann eqs

$$\frac{\Delta L^{TOT}}{s} \simeq \frac{g_N}{7\,e\,(2\pi)^{3/2}}\frac{B_0}{m} \simeq 0.007\,\frac{B_0}{m}$$

Similar order of magnitude estimates if $B^0 \sim T^3$ during Leptogenesis era

Bossingham, NEM, Sarkar

CPTV Thermal

$$\mathcal{L} = i\overline{N}\partial\!\!\!/ N - \frac{m}{2}(\overline{N^c}N + \overline{N}N^c) - \overline{N}B\!\!\!/\gamma^5 N - Y_k\overline{L}_k\tilde{\phi}N + h.c.$$

Early Universe
$T > 10^5$ GeV

CPT Violation

(approx.) Constant $B^0 \neq 0$ background

Lepton number & CP Violations @ tree-level due to Lorentz/CPTV Background

$$N_I \to \overline{\phi}\,\ell\,,\ \phi\,\overline{\ell}$$

$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10},$$

$$\frac{B_0}{m} \simeq 10^{-8}$$

Produce Lepton asymmetry

Equilibrated electroweak B+L violating sphaleron interactions

B-L conserved

Environmental Conditions Dependent

$$L = \frac{2}{M} l_L l_L \phi\phi + \text{H.c.}$$

where

$$l_L = \begin{pmatrix}\nu_e \\ e\end{pmatrix}_L, \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L, \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_L$$

Observed Baryon Asymmetry In the Universe (BAU)

Fukugita, Yanagida,

$$\frac{n_B - \overline{n}_B}{n_B + \overline{n}_B} \sim \frac{n_B - \overline{n}_B}{s} = (8.4 - 8.9) \times 10^{-11}$$

$T > 1$ GeV

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

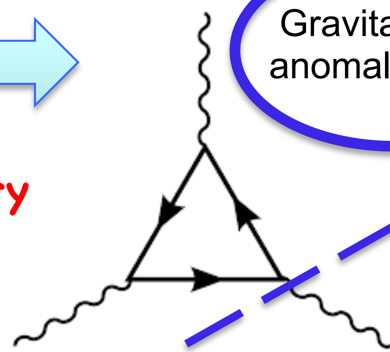
Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



Gravitational anomaly (GA)

Undiluted constant KR axial backround

$$B_\mu = M_{\rm Pl}^{-1}\dot{b}\,\delta_{\mu 0}$$

$$\dot{b} \sim \sqrt{2\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

From a pre-inflationary era after Big-Bang

chiral matter generation @ inflation exit

Radiation Era

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

ΔL in th ... nd $B_\mu = M_{\rm Pl}^{-1}\dot{b}\,\delta_{\mu 0}$

NEM, Sarkar + De Cesare, Bossingham

B-L conserving sphelaron proces...

Important Role of heavy sterile Right-handed neutrinos

Matter Era

Possible pote... ion Dark matter

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

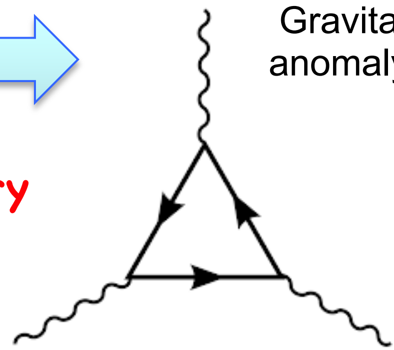
forward direction

Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves → Gravitational anomaly (GA) →



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chiral matter generation @ inflation exit

From a pre-inflationary era after Big-Bang

Cancellation of GA

Radiation Era

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ΔL In the (approx.) constant LV + CPTV background $B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$

B-L conserving sphelaron processes → Baryongenesis

Matter Era

Possible potential (mass) generation for b → axion Dark matter

Chiral anomalies @ QCD era (instantons)

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

forward direction

Basilakos, NEM, Solà

$$2.6 \times 10^{-5}\, M_{Pl} < M_s \leq 10^{-4}\, M_{Pl}$$

$$V_b^{\rm QCD} \simeq \Lambda^4_{\rm QCD}\Big(1 - \cos(\frac{b}{f_b})\Big), \quad f_b \equiv \sqrt{\frac{8}{3}}\,\frac{\kappa}{\alpha'} = \sqrt{\frac{8}{3}}\Big(\frac{M_s}{M_{\rm Pl}}\Big)^2 M_{\rm Pl}$$

$$\Lambda_{\rm QCD} \sim 218 \;\; {\rm MeV}$$

Remaining chiral anomalies

@ QCD Era

$$S_b^{\rm eff} = \int d^4x \sqrt{-g}\Big[\frac{1}{2}\,\partial_\mu b\, \partial^\mu b - \frac{\alpha'}{\kappa}\,\sqrt{\frac{3}{8}}\,\frac{\alpha_s}{8\pi}\, b(x)\, G^a_{\mu\nu}\, \widetilde{G}^{a\mu\nu}\Big]$$

T~ 200 MeV

$$1.17 \times 10^{-5}\,{\rm eV} \gtrsim m_b \gtrsim 1.17 \times 10^{-8}\,{\rm eV}$$

Instanton-effects-induced KR-axion potential and mass due to QCD chiral anomaly

Matter Era

Possible potential (mass) generation for b → axion Dark matter

Cosmic Time

Summary of (stringy-RVM) Cosmological Evolution

Basilakos, NEM, Solà

forward direction

2.6 x 10^{-5} M_{Pl} < M_s ≤ 10^{-4} M_{Pl}

$$V_b^{\rm QCD} \simeq \Lambda_{\rm QCD}^4\Big(1 - \cos(\frac{b}{f_b})\Big), \quad f_b \equiv \sqrt{\frac{8}{3}}\frac{\kappa}{\alpha'} = \sqrt{\frac{8}{3}}\Big(\frac{M_s}{M_{\rm Pl}}\Big)^2 M_{\rm Pl}$$

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Matter Era

Possible potential (mass) generation for b → axion Dark matter

Connection of b axion with torsion ↔ geometric origin of DM

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

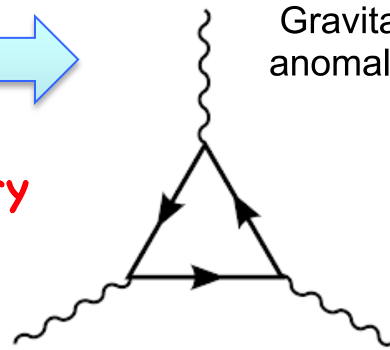
forward direction

Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



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$$\dot{\overline{b}} \sim \sqrt{2\varepsilon}\, M_{\rm Pl}\, H \sim 0.14\, M_{\rm Pl}\, H$$

From a pre-inflationary era after Big-Bang

chiral matter generation @ inflation exit

Radiation Era

Cancellation of GA

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$$N_I \;\rightarrow\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

B-L conserving sphelaron processes → **Baryongenesis**

Matter Era

Possible potential (mass) generation for b → axion Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\, M_{\rm Pl}\, H_0$$

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$ **Phenomenology**

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

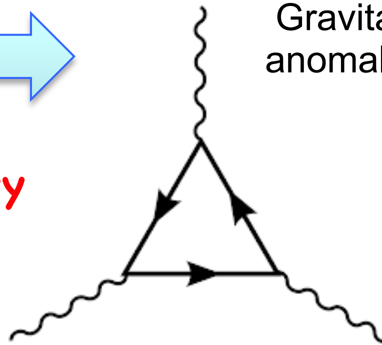
forward direction

Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves →



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chiral matter generation @ inflation exit

From a pre-inflationary era after Big-Bang

Radiation Era

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Leptogenesis induced by RHN (tree-level) decays

$$N_I \;\to\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

B-L conserving sphelaron processes → Baryongenesis

Consistent with current bounds on LV & CPTV
$B_0 < 10^{-2}$ eV,
$B_i < 10^{-22}$ eV

Matter Era Possible potential (mass) generation f b – → axion Dark matter

Modern de-Sitter Era **GA resurfacing**

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$H_0 \sim 10^{-42}$ GeV $\approx 10^{-60}\, M_{\rm Pl} \approx 10^{-33}$ eV

$$\varepsilon' \sim \varepsilon = \mathscr{O}(10^{-2})$$ **Phenomenology**

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

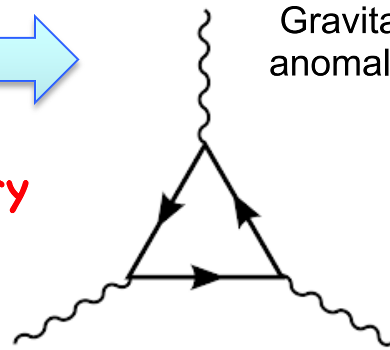
Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves



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$$\dot{\overline{b}} \sim \sqrt{2\,\varepsilon}\,M_{\rm Pl}\,H \sim 0.14\,M_{\rm Pl}\,H$$

From a pre-inflationary era after Big-Bang

chiral matter generation @ inflation exit

Radiation Era

$$B_0 \propto \dot{\overline{b}} \propto T^3 + \text{subleading } (\sim T^2) \text{ chiral U(1) anomaly terms}$$

$$B_0\Big|_{\rm today} \sim 2.435 \times 10^{-34}\,{\rm eV}$$

Consistent with current bounds on LV & CPTV
$B_0 < 10^{-2}$ eV,
$B_i < 10^{-22}$ eV

Matter Era

Possible potential (mass) generation for b – → axion Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$H_0 \sim 10^{-42}$ GeV $\approx 10^{-60}\,M_{\rm Pl} \approx 10^{-33}$ eV

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$

Phenomenology

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

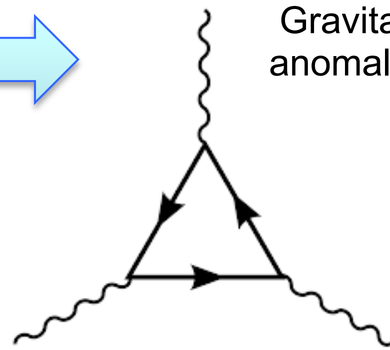
Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gravitational Waves → Gravitational anomaly (GA) →



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chiral matter generation @ inflation exit

Cancellation of GA

Radiation Era

$$B_0 \propto T^3$$

Leptogenesis induced by RHN (tree-level) decays

$$N_I \;\to\; \overline{\phi}\,\ell\,,\;\; \phi\,\overline{\ell}$$

B-L conserving sphel...

Consistent with current bounds on LV & CPTV
$B_0 < 10^{-2}$ eV,
$B_i < 10^{-22}$ eV

Need to understand Modern Era better

Matter Era

Dark matter

Modern de-Sitter Era

GA resurfacing

$$\dot{b}_{\rm today} \sim \sqrt{2\,\varepsilon'}\,M_{\rm Pl}\,H_0$$

$$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$$

$H_0 \sim 10^{-42}$ GeV $\approx 10^{-60}\, M_{\rm Pl} \approx 10^{-33}$ eV

Phenomenology

7. Modern-era phenomenology: deviations from ΛCDM and alleviation of cosmological data tensions?

Summary of (stringy-RVM) Cosmological Evolution

Cosmic Time

Big-Bang, pre-inflationary phase (broken Sugra)

Basilakos, NEM, Solà

forward direction

RVM Inflationary (de Sitter) Phase

Primordial Gr… W…

Gravitational

Undiluted constant KR axial backround

$B_\mu = M_{\rm Pl}^{-1}\,\dot{\overline{b}}\,\delta_{\mu 0}$

$\dot{\overline{b}} \sim \sqrt{2\varepsilon}\, M_{\rm Pl} H \sim 0.14\, M_{\rm Pl} H$

chiral matter generation …lation exit

Ra…

B_0…

Le…

RH…

N_I…

B-L…

Ma…

Modern de-Sitter Era

GA resurfacing

…ion … matter

$\dots \varepsilon' M_{\rm Pl} H_0$

$\varepsilon' \sim \varepsilon = \mathcal{O}(10^{-2})$

Distinguishing feature from ΛCDM

Alleviate data tensions

$$\rho^{\rm today}_{\rm RVM}(H) = 3M_{\rm Pl}^4\left(c_0 + \nu_0\left(\frac{H_0}{M_{\rm Pl}}\right)^2\right)$$

$$0 < \nu_0 = \mathcal{O}(10^{-3})$$

$$\frac{3}{\kappa^2}\,c_0 \simeq 10^{-122}\,M_{\rm Pl}^4$$

Gómez-Valent Solà

session talks

RVM-type Running Dark Energy

Gómez-Valent, Solà

Could **Alleviate** Tensions in Data, e.g. H_0, σ_8 tensions

$$0 < \nu_0 = \mathcal{O}(10^{-3})$$

$$\mathcal{O}(10^{-4}) \lesssim \beta \lesssim \mathcal{O}(1)$$

$$\frac{3}{\kappa_0^2} c_0 \simeq 10^{-122} M_{Pl}^4$$

$$\rho_{RVM}(H) = 3M_{Pl}^4 \left(c_0 + \nu_0 \left(\frac{H_0}{M_{Pl}} \right)^2 + \beta \frac{H_0^4}{M_{Pl}^4} \right), \quad \beta > 0.$$

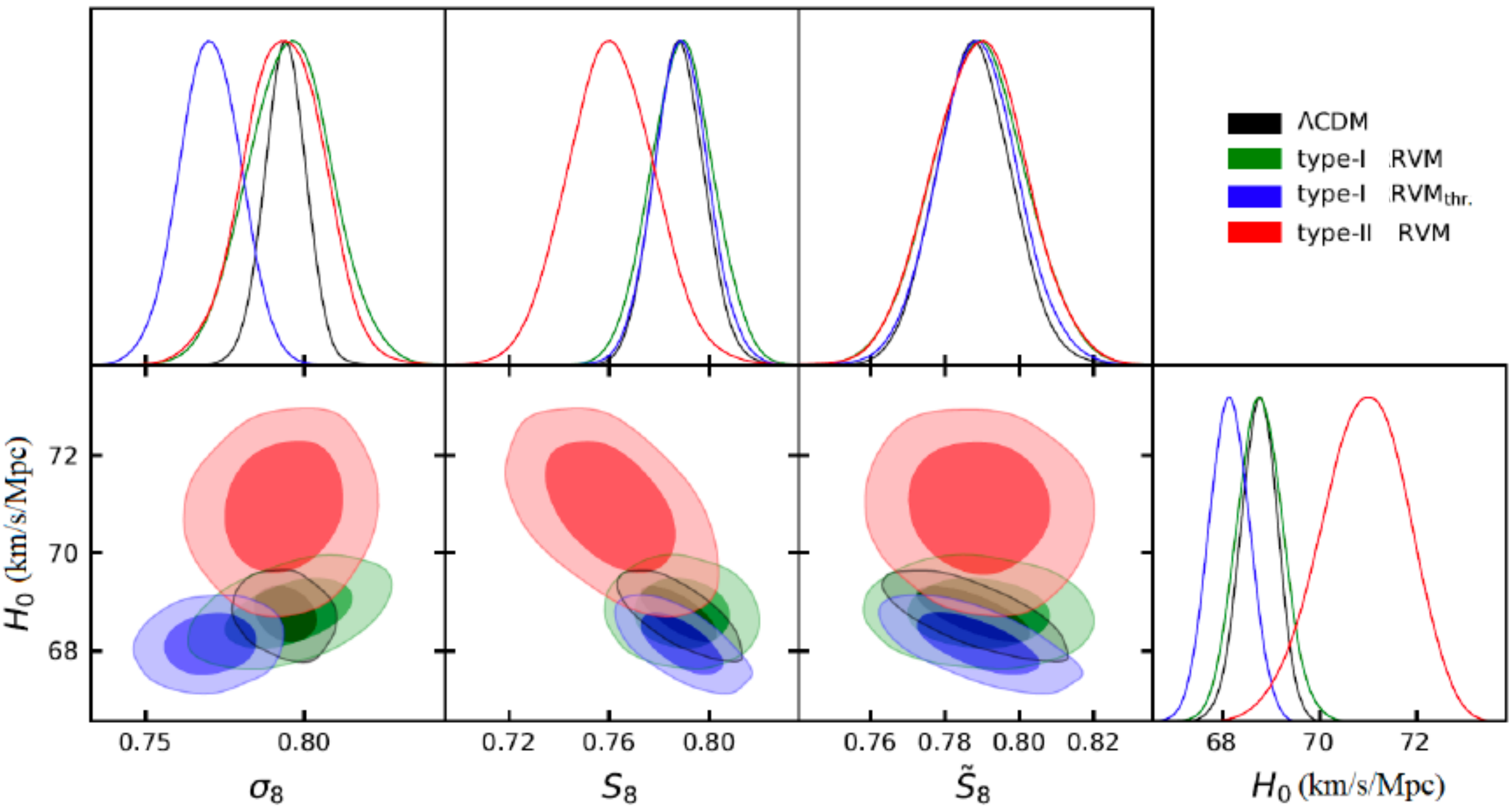
Not dominant today

Running RVM Dark Energy

If tensions are not due to statistics

Solà, Gómez-Valent, De Cruz Perez, Moreno-Pulido, (Planck 2018 data)

Alleviation of the H_0 , σ_8 tension by RVM model

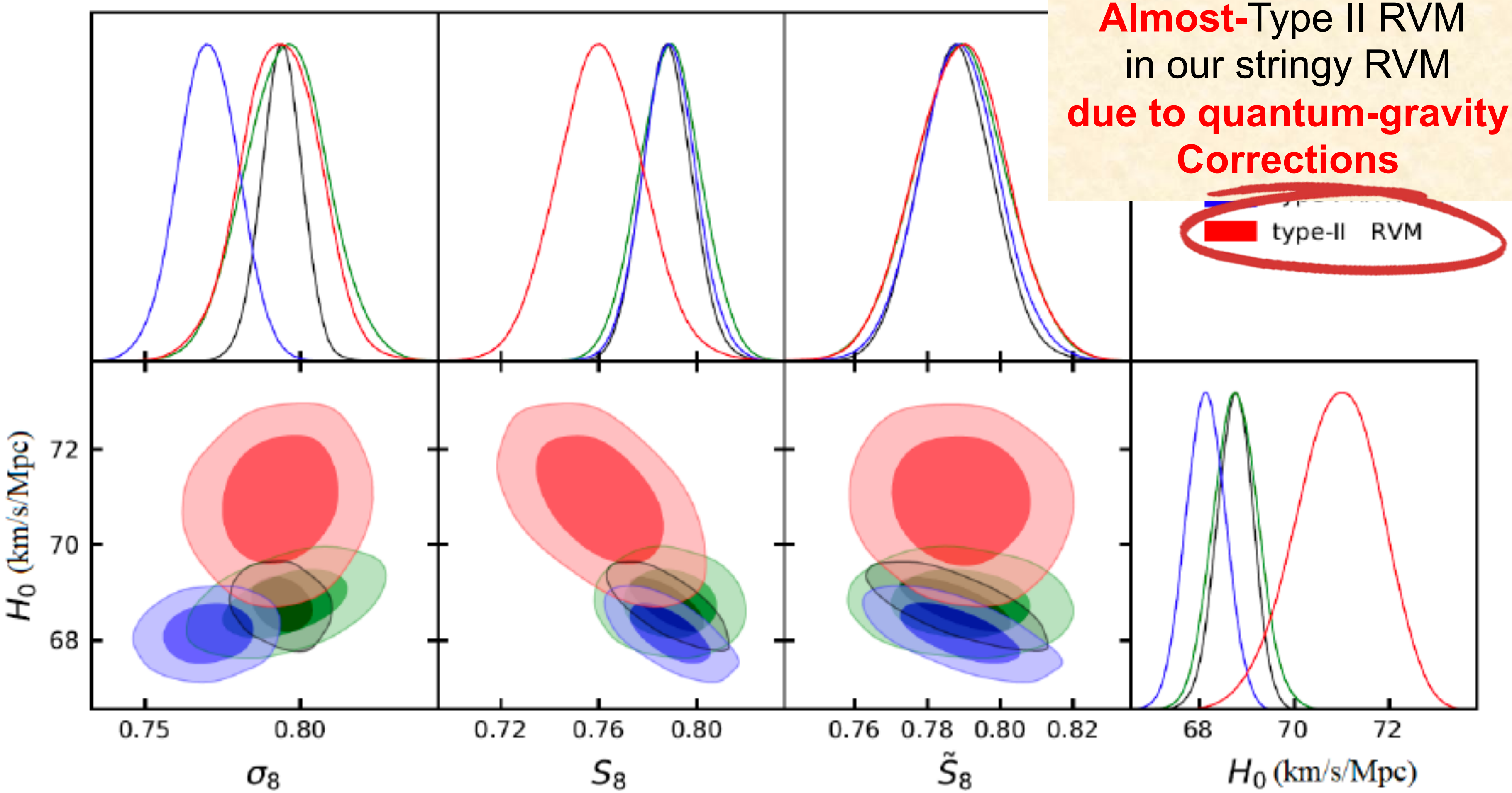


NEM, Solà (2021)

Integrating out graviton flcts

$$\rho \propto (c_1 + c_2 \ln H)\, H^2 + (c_3 + c_4 \ln H)\, H^4 + \Lambda$$

Almost-Type II RVM
in our stringy RVM
due to quantum-gravity Corrections



With Arguelles, Ruffini, Rueda
+ Yunis, Carinci, Krut, Lopez Nacir
Moline, Scoccola

8. Warm Dark Matter in Galaxies:

Potential role of light sterile Neutrinos in galactic structure & their interactions with axions

Dark Matter may consist of
more than one
dominant **species**
depending on the cosmo era!

Self-Interacting Dark Matter (SIDM) & small-scale Cosmology

Early pioneering works in implementing SIDM in N-body simulations

D. N. Spergel and P. J. Steinhardt, PRL 84 , 3760 (2000)

Figure of merit: (total) cross section per unit DM particle mass

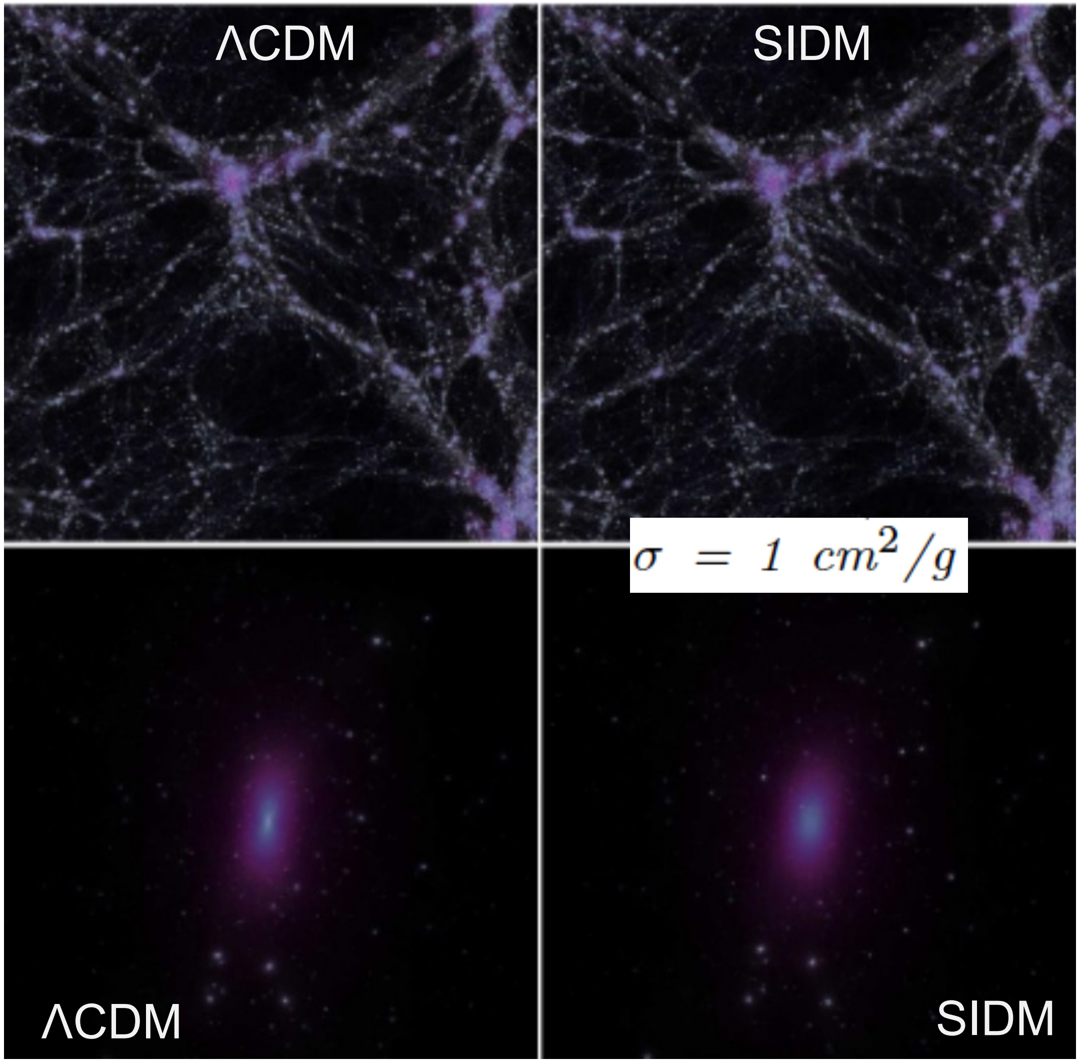
σ/m

Early days: 10 GeV $c^{-2} \geq$ m $\geq$ 1 MeV c^{-2}
in DM haloes with densitie $10^{-2} M_\odot/\mathrm{pc}^3$

$$\sigma/m \sim 0.1 - 100\ \mathrm{cm}^2/\mathrm{g}$$

would imply observational effects in the inner haloes

Self-Interacting Dark Matter (SIDM) & small-scale Cosmology



Large Scale Structure: roughly the same

Individual galaxies: more cored & spherical in SIDM models

M Rocha *et al.* MNRAS 430, 81 (2013)

Self-Interacting Dark Matter (SIDM) & small-scale (galactic) Cosmology

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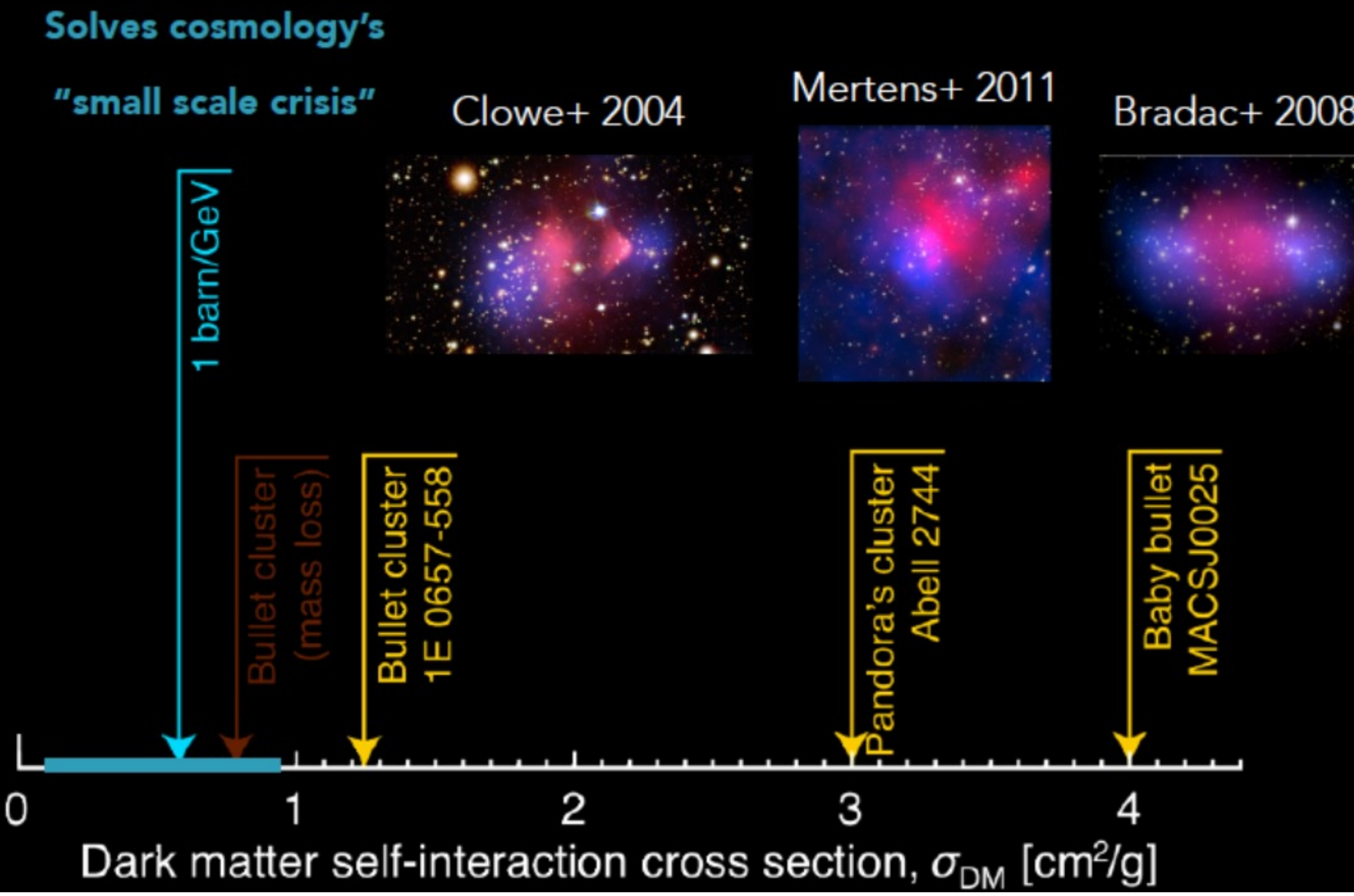
Early days: 10 GeV c^{-2} ≥ m ≥ 1 MeV c^{-2}
in DM haloes with densitie$10^{-2} M_\odot/\mathrm{pc}^3$

$$\sigma/m \sim 0.1 - 100\ \mathrm{cm}^2/\mathrm{g}$$

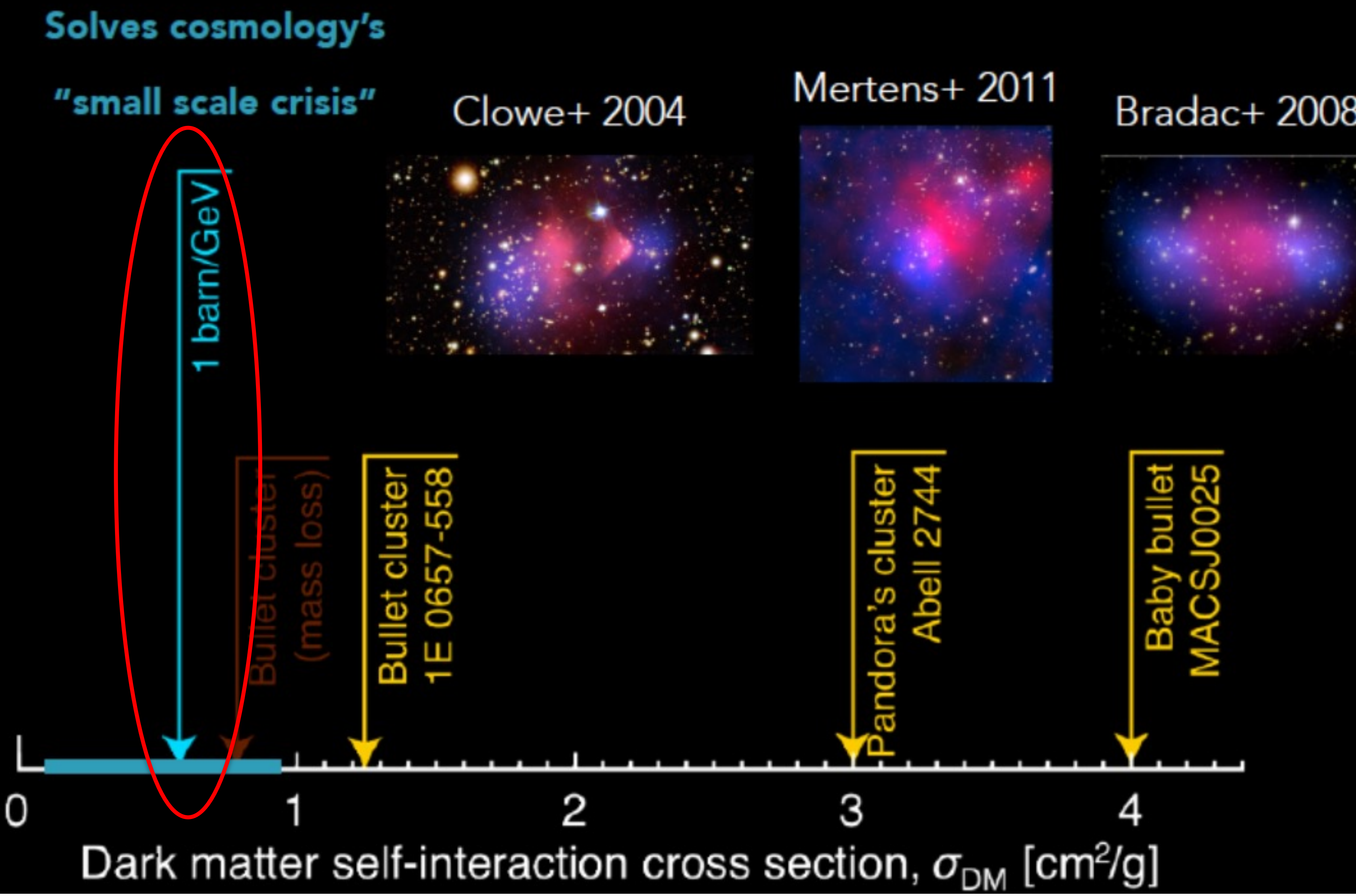
=1 barn/GeV
consistent with all current constraints of GSC

would imply observational effects in the inner haloes

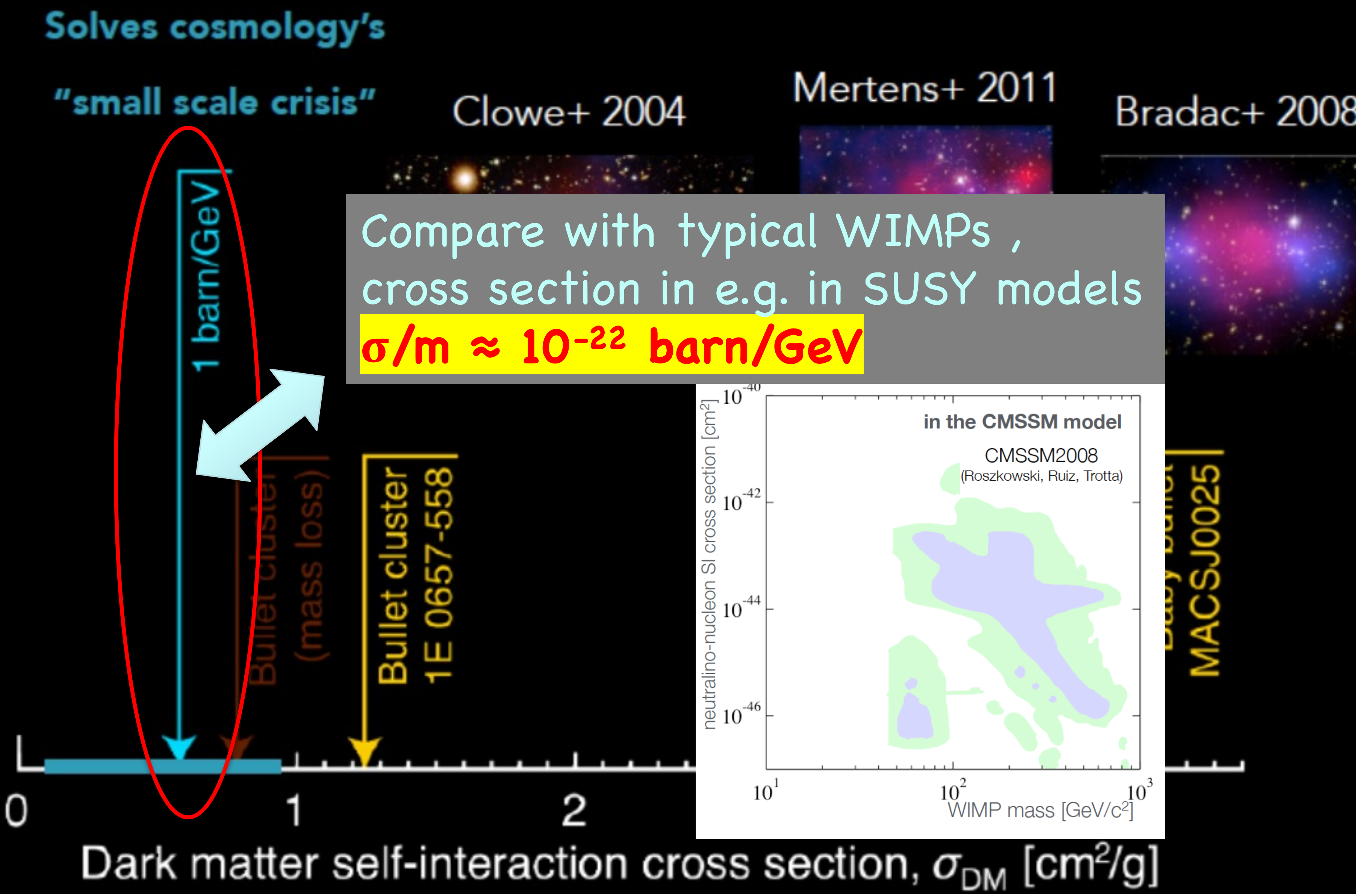
CONSTRAINTS ARE LIMITED



CONSTRAINTS ARE LIMITED

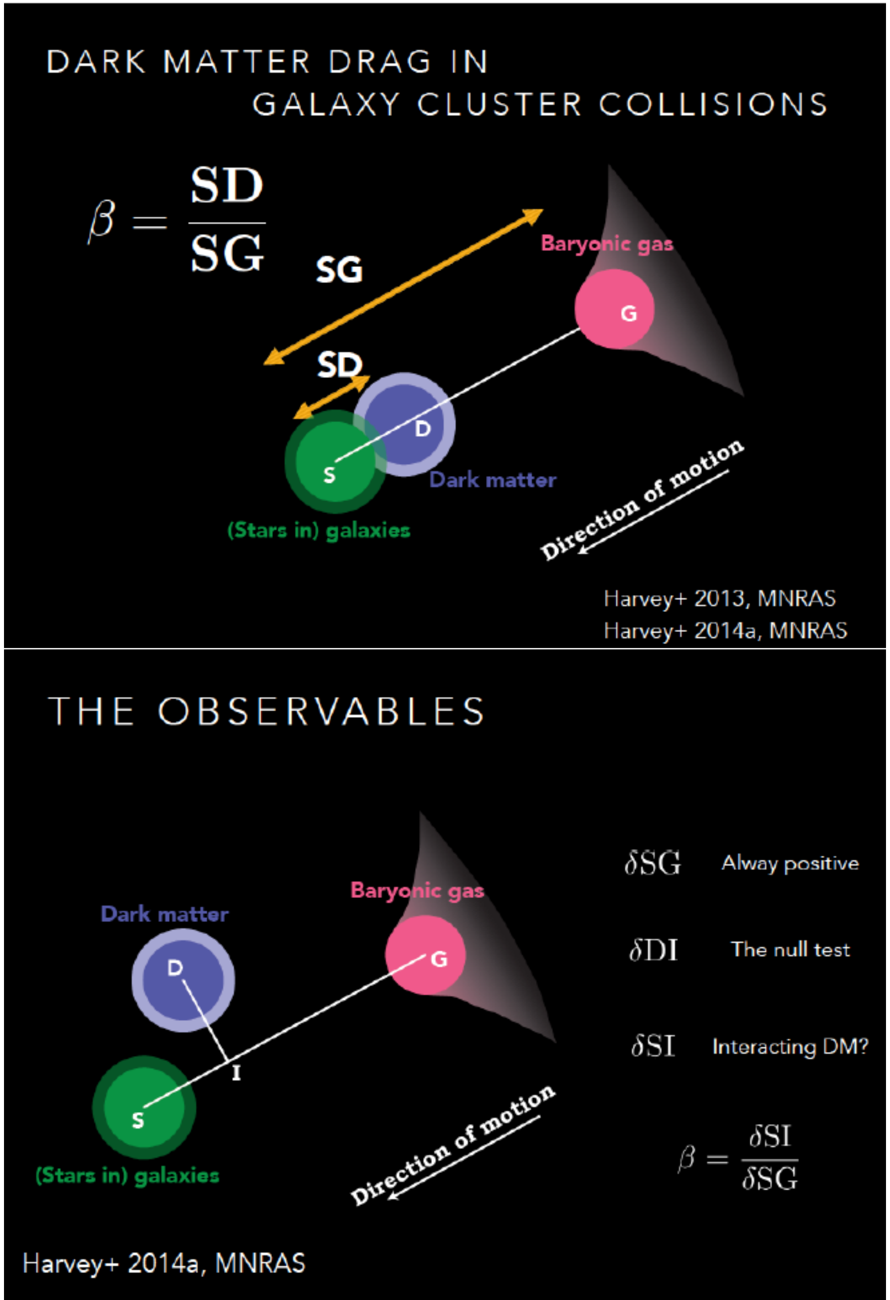


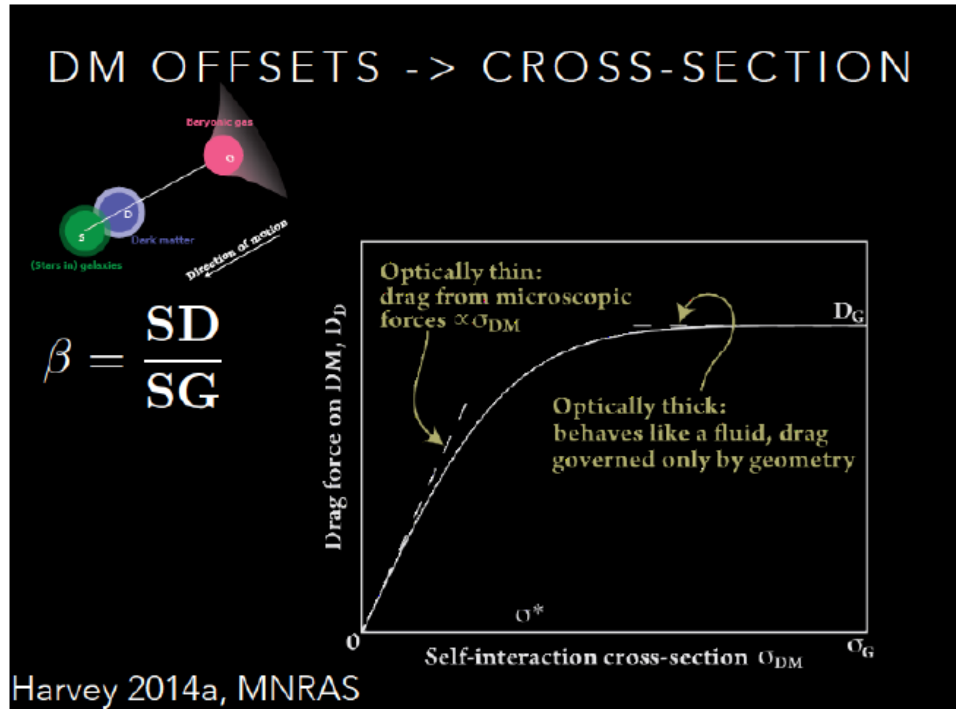
CONSTRAINTS ARE LIMITED



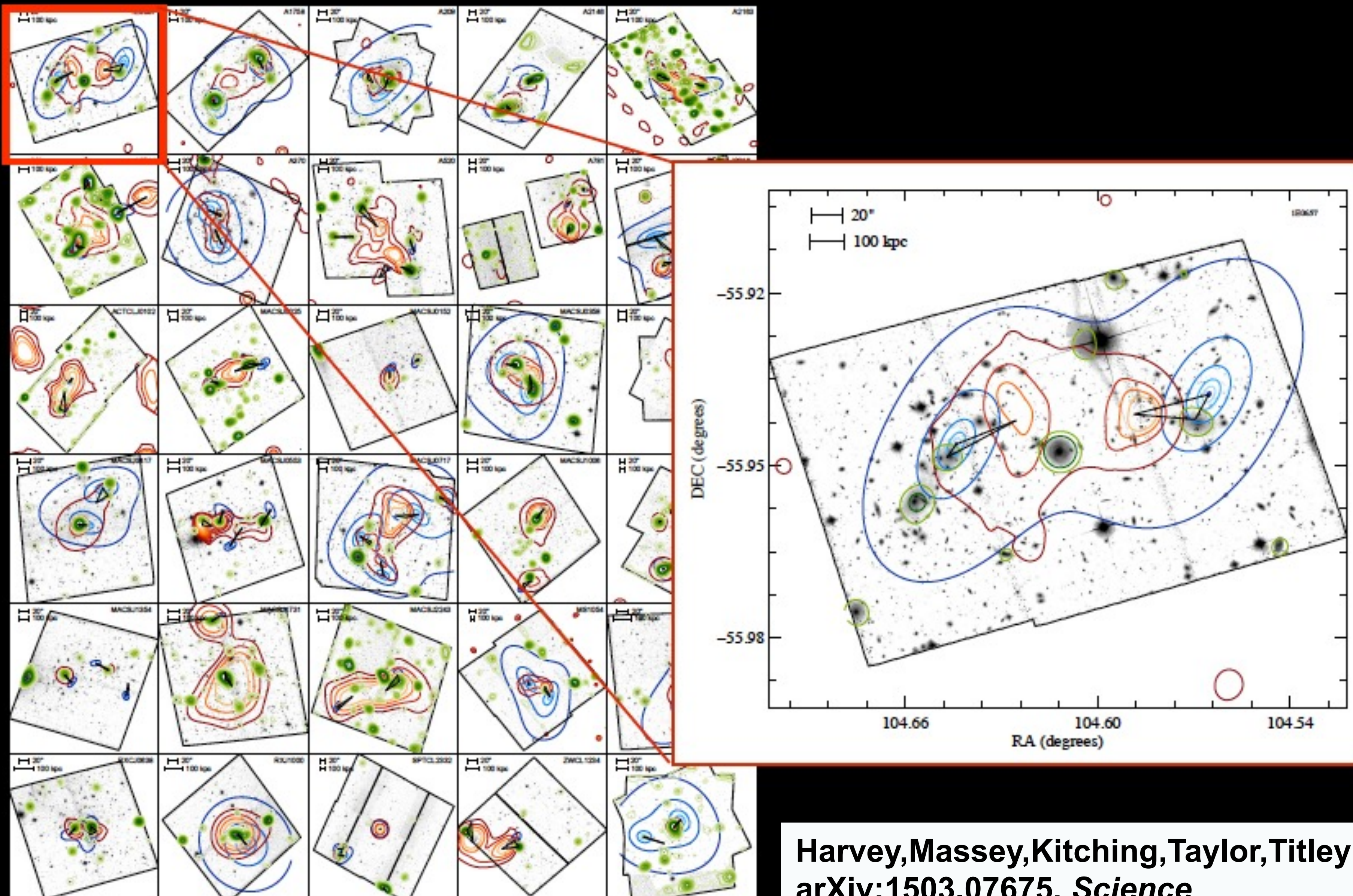
New Observables due to DM drag in collding galaxy clusters

Harvey,Massey,Kitching,Taylor,Titley
arXiv:1503.07675, *Science*



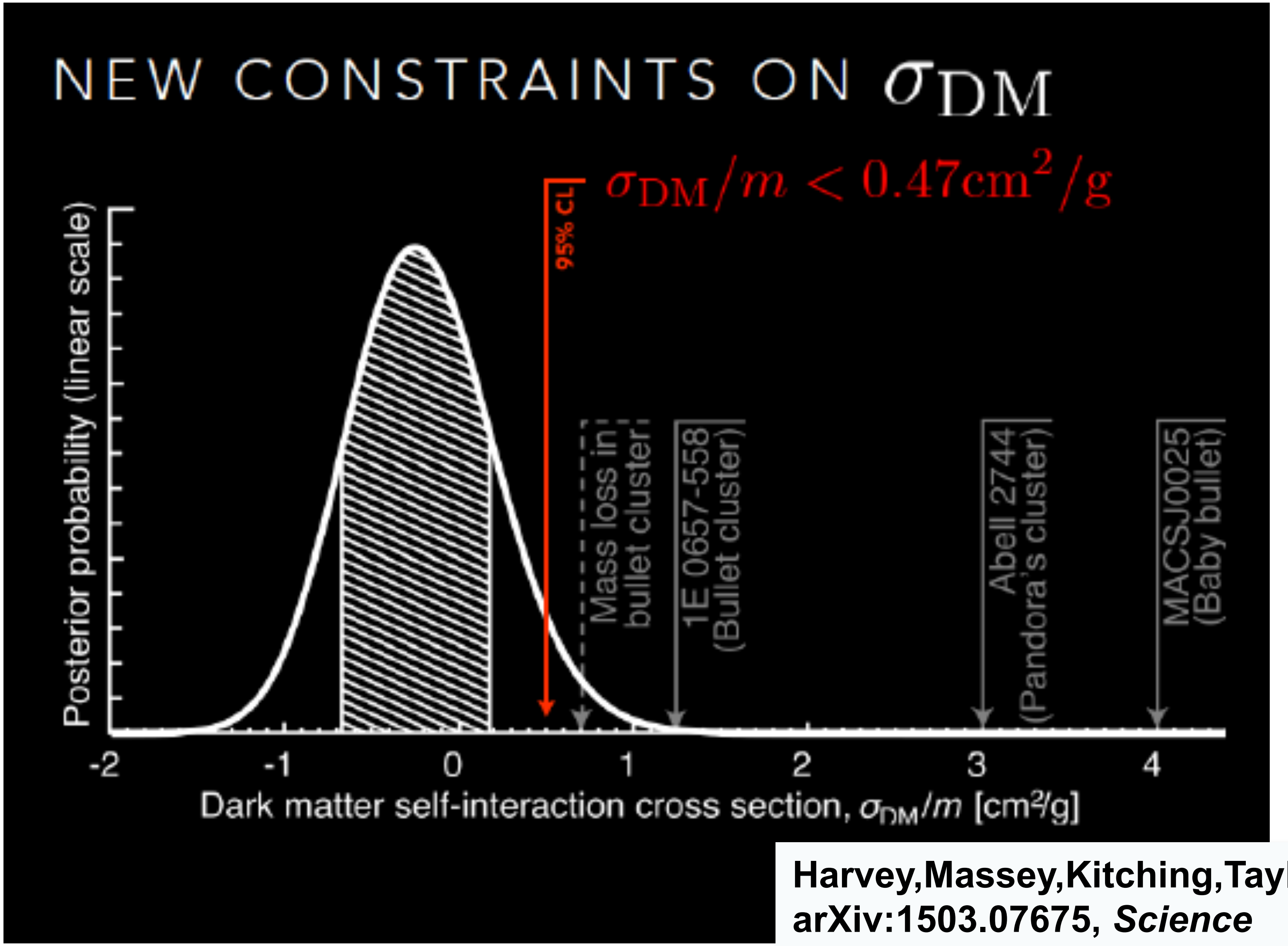


30 MERGING GALAXY CLUSTERS

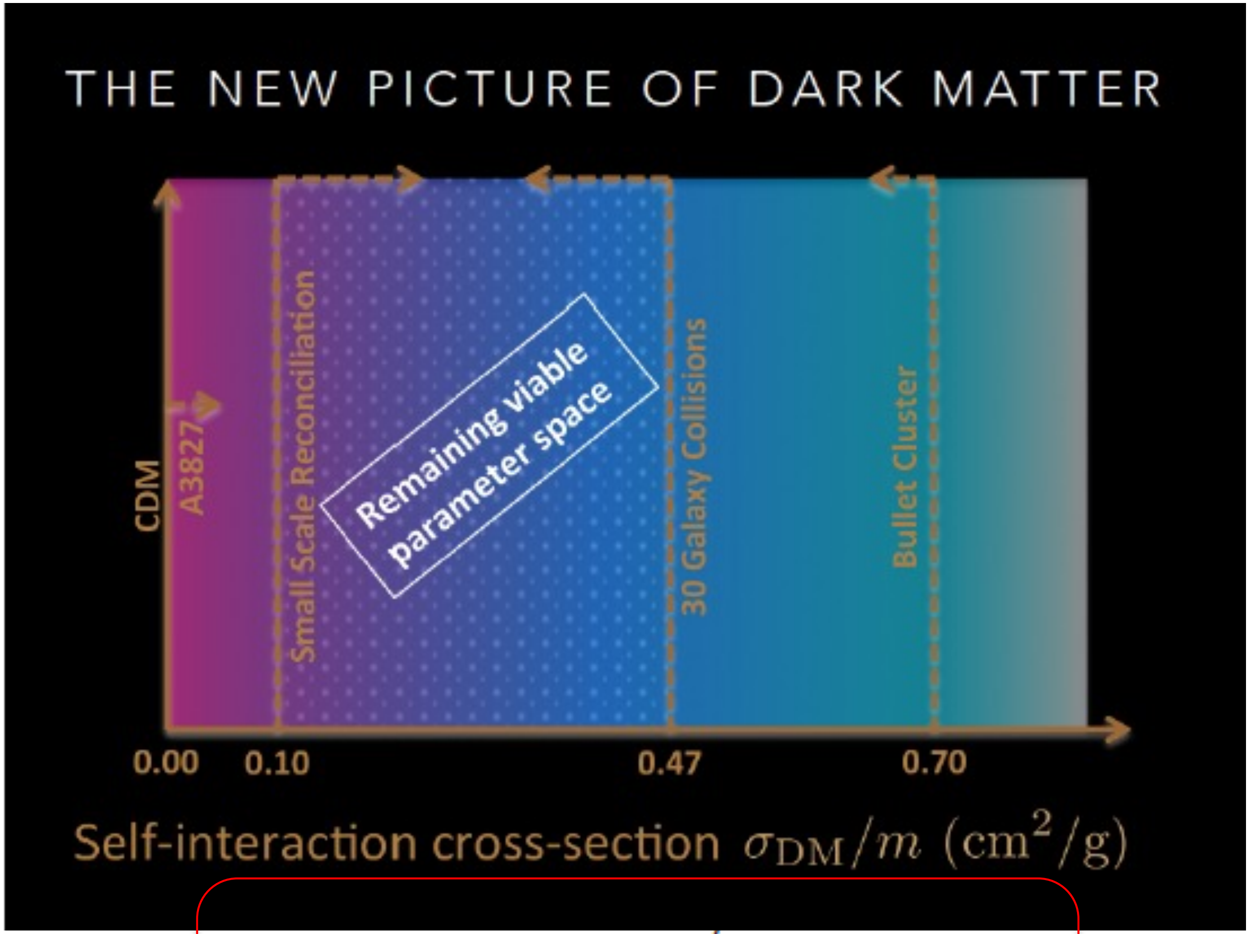


Harvey,Massey,Kitching,Taylor,Titley
arXiv:1503.07675, *Science*

Self-Interacting Dark Matter (SIDM) & small-scale (galactic) Cosmology

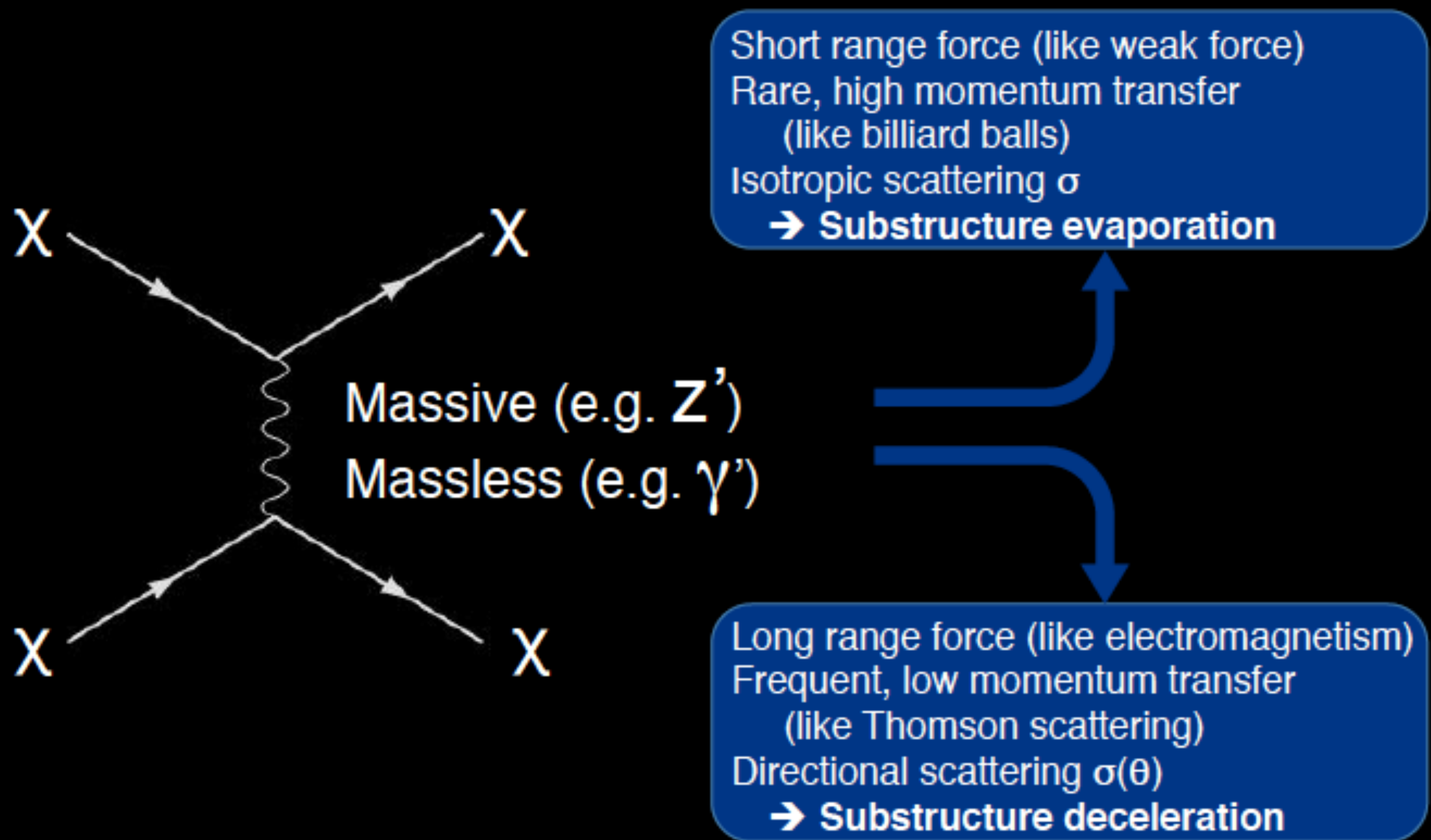


Harvey, Massey, Kitching, Taylor, Titley
arXiv:1503.07675, *Science*



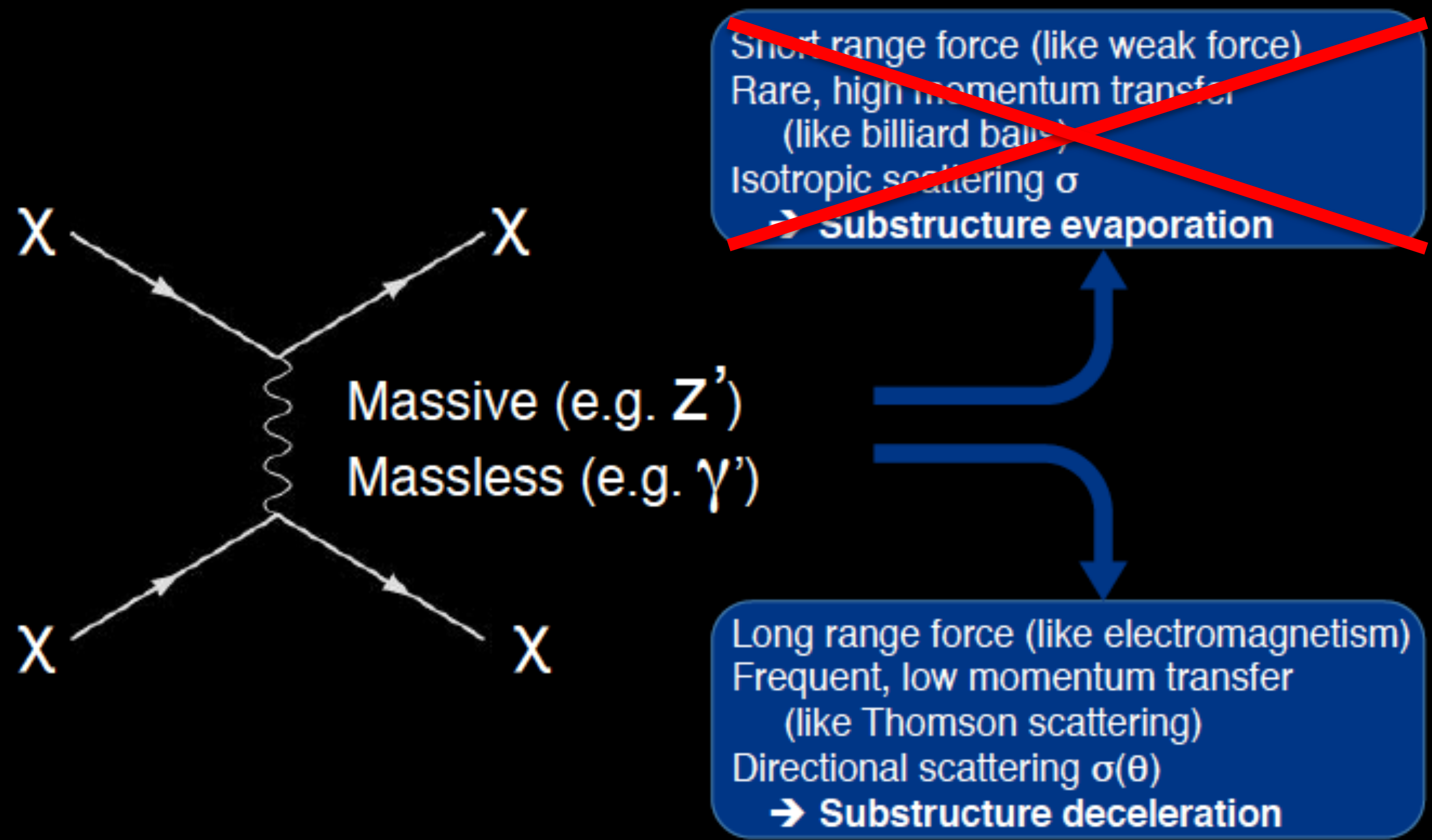
$$0.1 \leq \frac{\sigma_{\rm SIDM}/m}{{\rm cm}^2\, g^{-1}} \leq 0.47$$

OBSERVABLE MANIFESTATION OF SELF-INTERACTIONS IN COLLIDING CLUSTERS



Kahlhoefer et al. 2014, MNRAS 437, 5865
Boehm et al. 2010, PRL 105, 1301

OBSERVABLE MANIFESTATION OF SELF-INTERACTIONS IN COLLIDING CLUSTERS

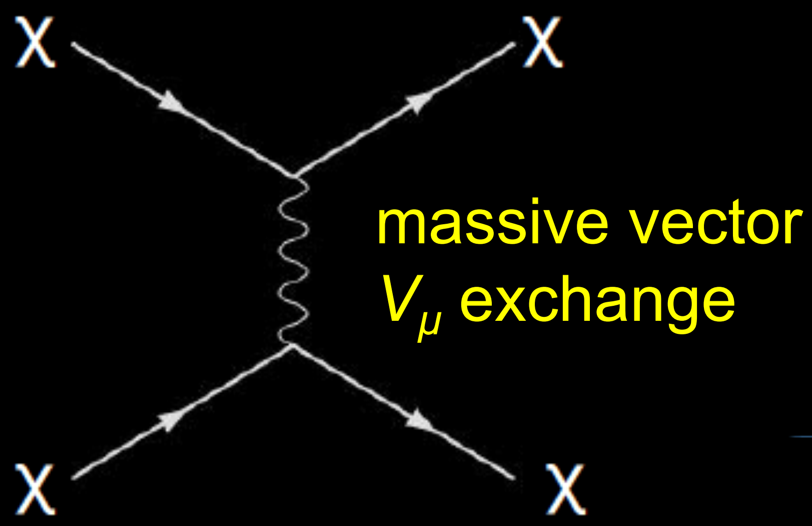


Kahlhoefer et al. 2014, MNRAS 437, 5865

Boehm et al. 2010, PRL 105, 1301

OBSERVABLE MANIFESTATION OF SELF-INTERACTIONS IN COLLIDING CLUSTERS

χ = Right-handed neutrino



In **Right-handed neutrino** WDM:

(i) mass of up to O(50) keV,

(ii) interactions stronger than the weak force, 10^8 G_F

(iii) massive ~ 10^4 keV exchange vector is **OK** for **core-galaxy structure**

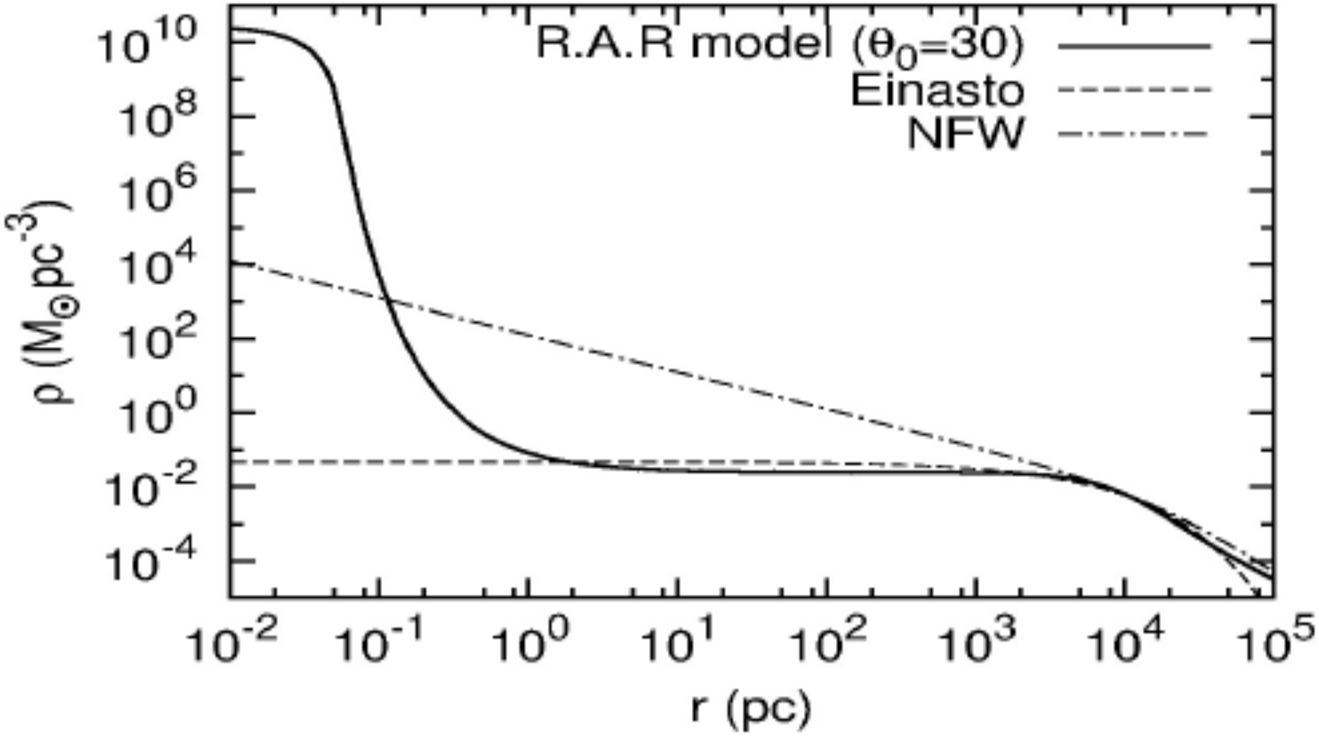
Arguelles, NEM, Ruffini, Rueda, JCAP (2016)

Self-Interacting Right-Handed Neutrino Warm Dark matter & galactic core-halo structures

Earlier Studies:
massive (non-interacting) fermions in galaxies @ a quantum level

m =O(10) keV

Ruffini, Arguelles, Rueda, MNRAS (2015)



In halo region RAR model behaves similar to Einasto or NFW profiles
The core region needs revisiting → **self interacting fermionic dark matter**

A concrete model for SIDM –
Right-handed keV Neutrinos with vector interactions

Arguelles, NEM, Rueda, Ruffini, JCAP 1604, 038 (2016)

- Assume **minimal extension of the Standard Model (non-supersymmetric) with right-handed neutrinos (RHN)** **self interacting** **via massive vector exchange interactions** in the dark sector

- Use models of particle physics, e.g. **νMSM** (**Shaposhnikov *et al*.** or **our stringy RVM** model) with three RHN, but **augment** them with **these self-interactions** among the lightest of the RHN (**quasi stable → DM**)

- Consistency of the **halo-core profile** of dwarf galaxies in Milky Way or large Elliptical → **mass** of lightest RHN in **O(50) keV (WDM)** ← **Cosmological constraints** of νMSM

Sterile neutrinos as warm DM in galaxies_

A concrete model for SIDM –
Right-handed keV Neutrinos with vector interactions

Arguelles, NEM, Rueda, Ruffini, JCAP 1604, 038 (2016)

- Assume **minimal extension of the Standa**
(non-supersymmetric) with righ **RHN)**
self interacting **via mas**
interactions in th

- (
bu
am

- Con **of dwarf galaxies**
in Mi **mass** of lightest RHN
in **O(5** **)** ← **Cosmological constraints** of νMSM

In our stringy RVM for CPTV Leptogenesis

At least one very heavy ($m \geq 10^5$ GEV)
Required

But we may have a hierarchy such that
The lightest is of O(50 keV)

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Sterile neutrinos as warm DM in galaxies_

A concrete model for SIDM –
Right-handed keV Neutrinos with vector interactions

Arguelles, NEM, Rueda, Ruffini, JCAP 1604, 038 (2016)

- Assume **minimal extension of the Standard M**... **(non-supersymmetric) with right-han**... **self interacting via massive v**... **interactions** in the d...
- Use...

...warf galaxies
... **mass** of lightest RHN
... **Cosmological constraints** of νMSM

S... neutrinos as warm DM in galaxies_

Recent (nustar) constraints on warm sterile DM mass

Yunis *et al*. Phys. Dark Univ. 30 (2020) 100699
- e-Print: 2008.08464

Yunis *et al*. MG16 Proceedings, e-Print: 2111.07642

+ ADD INTERACTIONS AMONG STERILE NEUTRINOS

Sterile (right-handed) neutrinos, I = 1,2,3

$$L = L_{SM} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I}\, \bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2}\, \bar{N}_I^c N_I + \text{h.c.}$$

+ self-interactions, e.g. vector type $g_V^{4F}\, \overline{N}_I \gamma^\mu N_I\, \overline{N}_J \gamma_\mu N_J$

Or interacting
with a massive
Dark vector A_μ

$$g_V \overline{N}_I \gamma^\mu N_I\, A_\mu^D$$

$$+ \text{kinetic terms of } \mathrm{A}_\mu^{\mathrm{D}}$$

Right-handed keV Neutrinos with vector self-interactions & galactic structure

Arguelles, NEM, Rueda, Ruffini, JCAP 1604, 038 (2016)

Place the νMSM in **curved space tim** $g_{\mu\nu} = \mathrm{diag}(e^{\nu}, -e^{\lambda}, -r^2, -r^2 \sin^2\varphi)$

ν=ν(r) λ = λ(r)

$$\mathcal{L} = \mathcal{L}_{GR} + \mathcal{L}_{N_{R\,1}} + \mathcal{L}_{V} + \mathcal{L}_{I}$$

$$\mathcal{L}_{GR} = -\frac{R}{16\pi G}, \quad \mathcal{L}_{N_{R\,1}} = i\,\overline{N}_{R\,1}\gamma^{\mu}\nabla_{\mu} N_{R\,1} - \frac{1}{2} m\,\overline{N^c}_{R\,1} N_{R\,1},$$

$$\mathcal{L}_{V} = -\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} m_V^2 V_{\mu} V^{\mu}, \quad \mathcal{L}_{I} = -g_V V_{\mu} J^{\mu}_{V} = -g_V V_{\mu} \overline{N}_{R\,1}\gamma^{\mu} N_{R\,1}$$

$$\nabla_{\mu} = \partial_{\mu} - \frac{i}{8}\,\omega_{\mu}^{ab}[\gamma_a, \gamma_b]$$

Classical fields (eqs of motion) satisfy detailed **thermodynamic equilibrium conditions** in a galaxy at a temperature T < O(keV)

NB: Alternatively one may have four-fermion (attractive) current-current interactions

$$\mathcal{L}_I \ni g_{\rm v} J_V^{\mu} J_{V\,\mu}$$

$$J_V^{\mu} = \overline{N}_{R\,I} \gamma^{\mu} N_{R\,I}$$

Corresponds to a limiting case where vector boson mass *m_V >> momentum scale*

Similar effects on galactic structure for sufficiently strong interaction couplings g_V

Right-handed keV Neutrinos with vector self-interactions & galactic structure

Measure of Strength of self Interactions

$$C_V \equiv g_V^2 / m_V^2$$

$$C_V(r) = \begin{cases} C_0 & \text{at} \quad r < r_m \quad \text{when} \quad \lambda_B / l > 1 \\ 0 & \text{at} \quad r \geqslant r_m \quad \text{when} \quad \lambda_B / l < 1 \end{cases}$$

inter-particle mean distance l
at temperature T

de-Broglie wavelength $\lambda_B = \frac{h}{\sqrt{2\pi m k_B T}}$

Right-handed keV Neutrinos with vector self-interactions & galactic structure

sterile ν mass

m (keV)	$\overline{C}_0$	θ_0	β_0	r_c (pc)	δr (pc)	$\theta(r_m)$
Milky Way ($M_c = 4.4 \times 10^6 M_\odot$)						
47	2	3.70×10^3	1.065×10^{-7}	6.2×10^{-4}	2.1×10^{-4}	-29.3
	10^{14}	3.63×10^3	1.065×10^{-7}	6.2×10^{-4}	2.2×10^{-4}	-29.3
	10^{16}	2.8×10^3	1.065×10^{-7}	6.3×10^{-4}	2.4×10^{-4}	-29.3
350	1	2.40×10^6 (†)	1.431×10^{-7}	1.3×10^{-6}	6.7×10^{-7}	-37.3
	10^{14}	1.27×10^5	1.104×10^{-7}	5.9×10^{-6}	9.4×10^{-7}	-37.3
	4.5×10^{18}	1.7×10^1	1.065×10^{-7}	5.9×10^{-4}	2.0×10^{-4}	-37.3
Elliptical ($M_c^{cr} = 2.3 \times 10^8 M_\odot$)						
47	2	1.76×10^5 (†)	1.7×10^{-6}	7.9×10^{-5}	3.9×10^{-5}	-31.8
	10^{14}	5.8×10^4	1.4×10^{-6}	1.4×10^{-4}	4.8×10^{-5}	-31.8
	10^{16}	1.5×10^4	1.3×10^{-6}	3.0×10^{-4}	7.0×10^{-5}	-31.8
Large Elliptical ($M_c = 1.8 \times 10^9 M_\odot$)						
47	10^{16}	1.02×10^4	3.0×10^{-6}	3.8×10^{-4}	1.8×10^{-5}	-32.8

$$\beta \equiv k_B T/m = \beta_0 \, e^{(\nu_0 - \nu(r))/2}$$

$$\theta \equiv \mu/(k_B T)$$

at the core (β_0, θ_0)

No solution for

$m < 47$ keV$/c^2$

$m > 350$ keV$/c^2$

gravitational collapse

Right-handed keV Neutrinos with vector self-interactions & galactic structure

sterile ν mass

m (keV)	$\overline{C}_0$	θ_0	β_0	r_c (pc)	δr (pc)	$\theta(r_m)$
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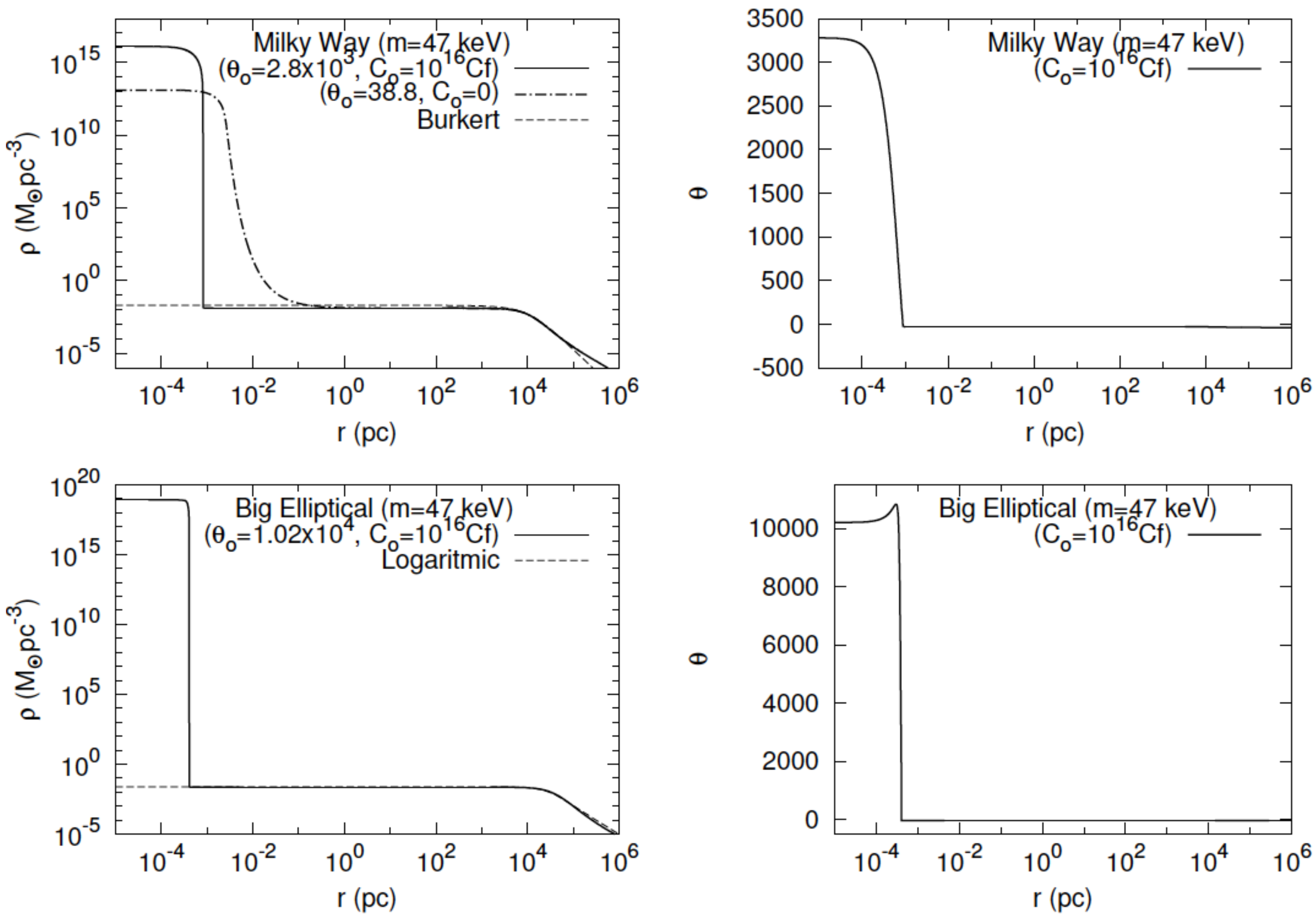
$$\beta \equiv k_B T/m = \beta_0 \, e^{(\nu_0 - \nu(r))/2}$$

$$\theta \equiv \mu/(k_B T)$$

at the core (β_0, θ_0)

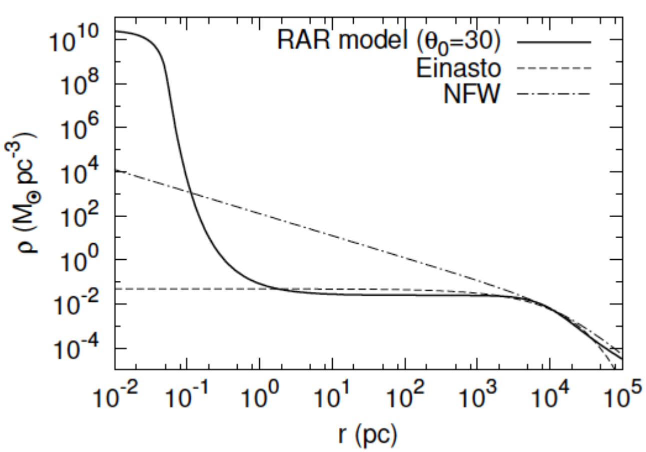
Allowed WDM mass range

47 keV c^{-2} ≤ m ≤ 350 keV c^{-2}



Arguelles, NEM, Rueda, Ruffini, JCAP 1604, 038 (2016)

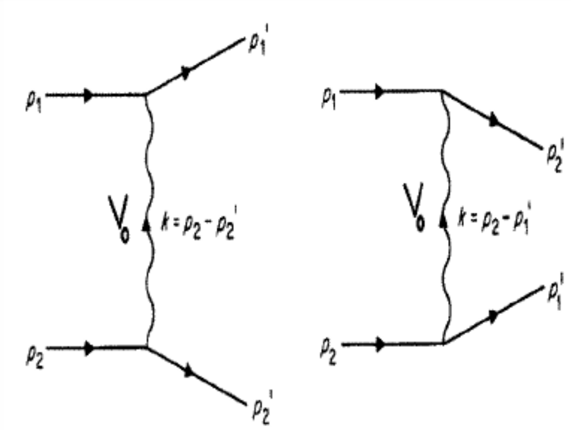
RHN self Interactions make inner Core more compact and increase central degeneracy compared to non-interacting case



Non interacting right-handed neutrino case with m = O(10) keV

Ruffini, Arguelles, Rueda, MNRAS (2015)

N-N Cross sections under massive vector exchange (perturbation theory $g_V < 1$ OK)



$$m \in (47, 350) \text{ keV}$$

$$\sigma^{tot}_{core} \approx \frac{(g_V/m_V)^4}{4^3 \pi} 29 m^2 \qquad (p^2/m^2 \ll 1)$$

Hidden sector vector interactions -> Much stronger than weak interactions in visible sector

$$\overline{C}_V = \left(\frac{g_V}{m_V}\right)^2 G_F^{-1} \Rightarrow \overline{C}_V \in (2.6 \times 10^8, 7 \times 10^8)$$

to resolve issues of small-scale cosmology crisis

MASS OF A_μ^D $\quad m_V \lesssim 3 \times 10^4 \text{ keV}$

Arguelles, NEM, Rueda, Ruffini, JCAP 1604, 038 (2016)

$N \rightarrow H\,\nu$

$$L = L_{SM} + \bar{N}_I i\partial_\mu \gamma^\mu N_I - F_{\alpha I}\,\bar{L}_\alpha N_I \tilde{\phi} - \frac{M_I}{2}\,\bar{N}_I^c N_I + \text{h.c.}$$

Small Mixing angle parametrization sin2θ ≈ 2θ

$$\theta^2 = \sum_{\alpha=e,\mu,\tau} (v^2 F_{\alpha,1})/m_s^2 \,,$$

m_s = Lightest sterile

Light Neutrino Masses through see saw

$$m_\nu = -M^D \frac{1}{M_I}[M^D]^T \,.$$

$$M_D = F_{\alpha I} v$$

$$v = \langle \phi \rangle \sim 175 \text{ GeV}$$

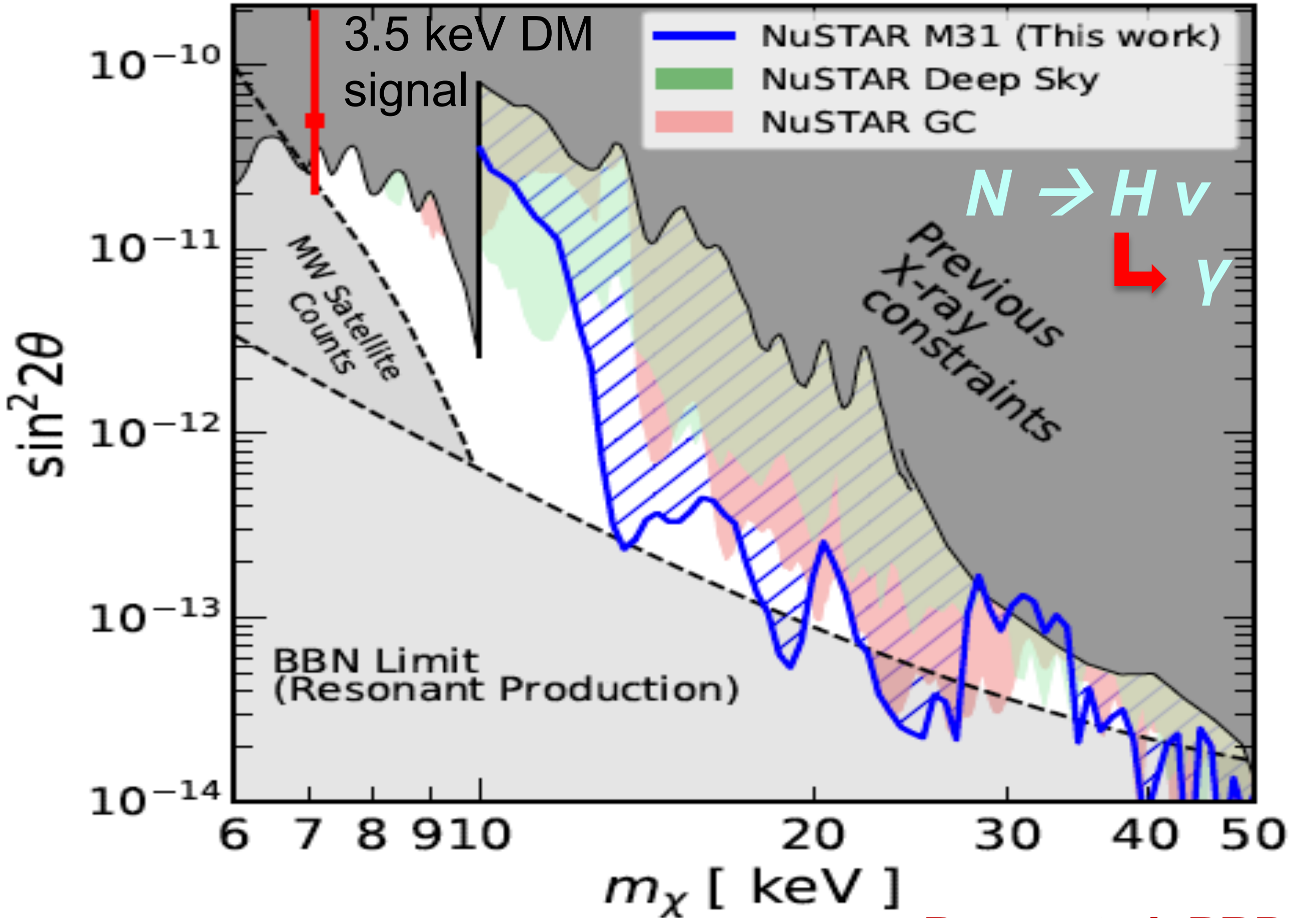
$$M_D \ll M_I$$

$$F_{\alpha 1} \approx 10^{-10} \rightarrow m_\nu^2 \approx 10^{-3}\ \text{eV}^2$$

Boyarski, Ruchayskiy, Shaposhnikov…

νMSM non self interacting

MODEL CONSISTENT WITH BBN, STRUCTURE FORMATION DATA IN THE UNIVERSE & ALL OTHER ASTROPHYSICAL CONSTRAINTS but constrained severely by x-rays due to the Higgs portal

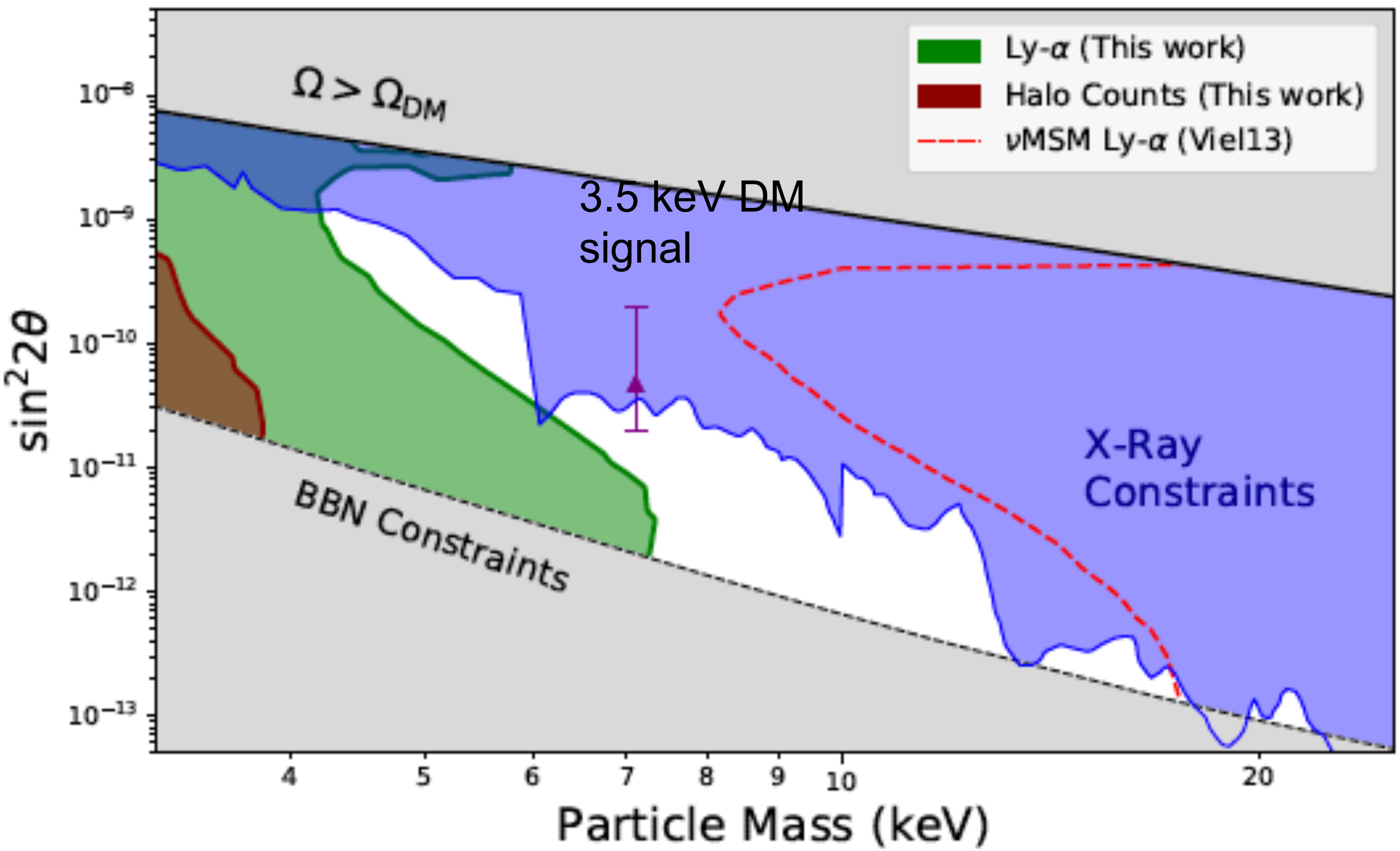


Perez et al. PRD99 (2019) 083005

νMSM self interacting (vector) , RAR profile

Yunis et al., MG16, arXiv:2111.07642

$$\sigma/m \sim 0.144 C_v^2/m^3 = 0.1\mathrm{cm}^2/g$$



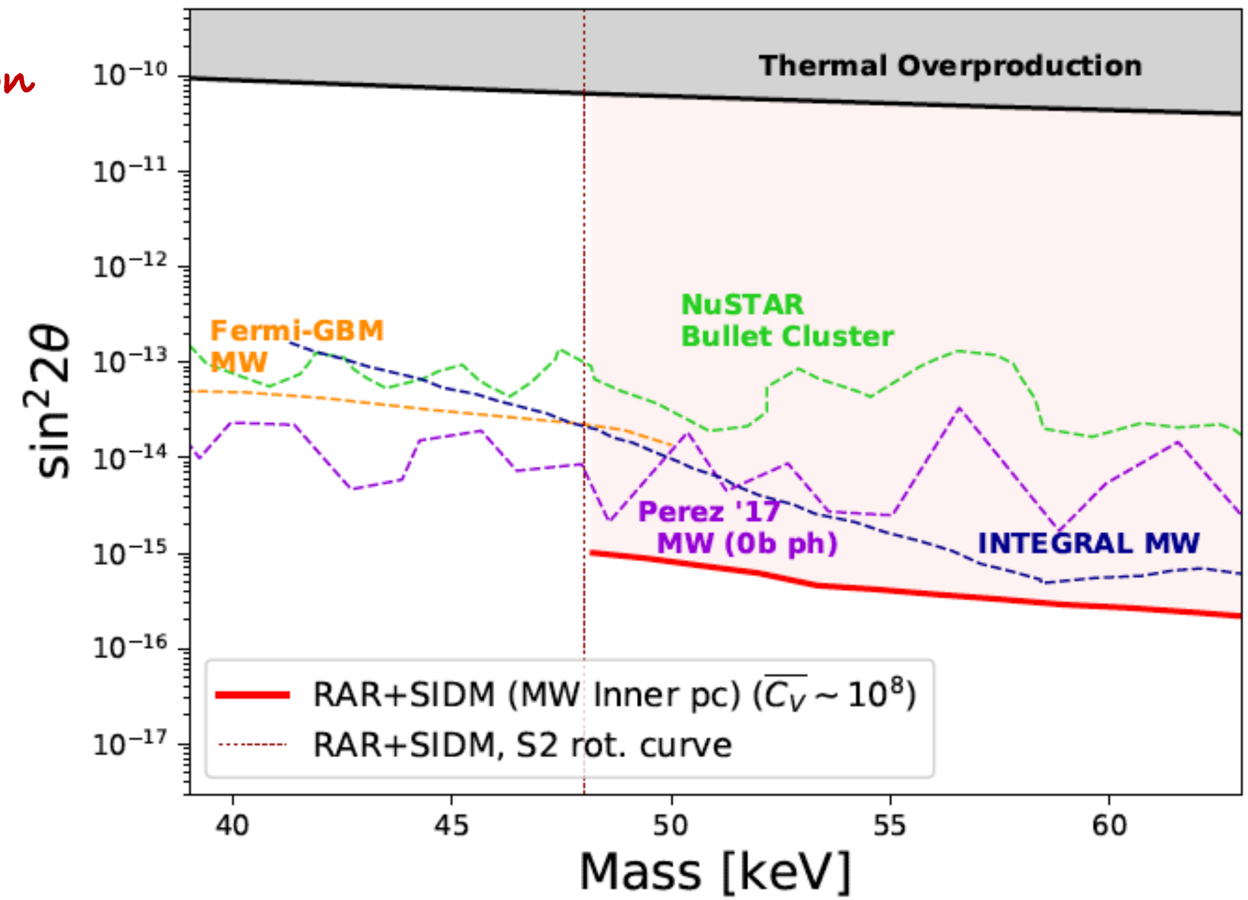
Yunis, Arguelles, Scoccola, Nacir, Giordano

νMSM self interacting

Yunis, Arguelles, NEM, Moline, Krut, Carinci, Rueda, Ruffini,
PDU 30 (2020) 100699 • e-Print: 2008.08464

But ....self interactions (or in general interactions with other DM species, e.g. axions) and modified galaxy profiles (RAR+SIDM) allow for heavier steriles ... But smaller portal mixing

DM production Through Dark vector decays



Axion-Sterile-neutrinos interactions

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\, R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}} b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

$$+\, S_{Dirac}^{Free} + \int d^4x\sqrt{-g}\,\Big(\frac{\kappa}{2}\sqrt{\frac{3}{2}}\,\partial_\mu b\Big)\, J^{5\mu} - \frac{3\kappa^2}{16}\int d^4x\sqrt{-g}\, J^5_\mu J^{5\mu} + \dots\Big] + \dots$$

or Majorana

$$J^{5\,\mu} = \bar{\psi}_j\,\gamma^\mu\,\gamma^5\psi_j$$

j = All fermion species, including sterile neutrinos

Axion-Sterile-neutrinos interactions

$$S^{\rm eff}_B = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \frac{\sqrt{2}\,\alpha'}{96\,\kappa\sqrt{3}}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

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j = All fermion species, including sterile neutrinos

Derivative coupling of axion with fermions – shift symmetry

Suppressed

NEM & Pilaftsis 2012
PRD 86, 124038 arXiv:1209.6387

However, non-perturbative
(eg stringy instanton) effects can
Generate a non-derivative coupling
of axion b with sterile neutrinos
(steriles are singlet under
standard model group, hence there
Is preservation of SM gauge groups)

Mavromatos, Pilaftsis arXiv: 1209.6387

ANOMALOUS MAJORANA NEUTRINO MASS TERMS from QUANTUM TORSION

OUR SCENARIO ***Break*** such ***shift symmetry*** by coupling first b(x) to another pseudoscalar field such as QCD axion a(x) (or e.g. other string axions)

Shift b → b+c symmetric

$$\mathcal{S} = \int d^4x\sqrt{-g}\,\Big[\frac{1}{2}(\partial_\mu b)^2 + \frac{b(x)}{192\pi^2 f_b}R^{\mu\nu\rho\sigma}\widetilde{R}_{\mu\nu\rho\sigma} + \frac{1}{2f_b^2}J^5_\mu J^{5\mu} + \gamma(\partial_\mu b)\,(\partial^\mu a) + \frac{1}{2}(\partial_\mu a)^2 - y_a i a\left(\overline{\psi}_R^{\,C}\psi_R - \overline{\psi}_R\psi_R^{\,C}\right)\Big]\,,$$

Mavromatos, Pilaftsis arXiv: 1209.6387

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Yukawa

neutrino fields

Shift a → a + c non-symmetric

Field redefinition

$$b(x) \to b'(x) \equiv b(x) + \gamma a(x)$$

so, effective action becomes

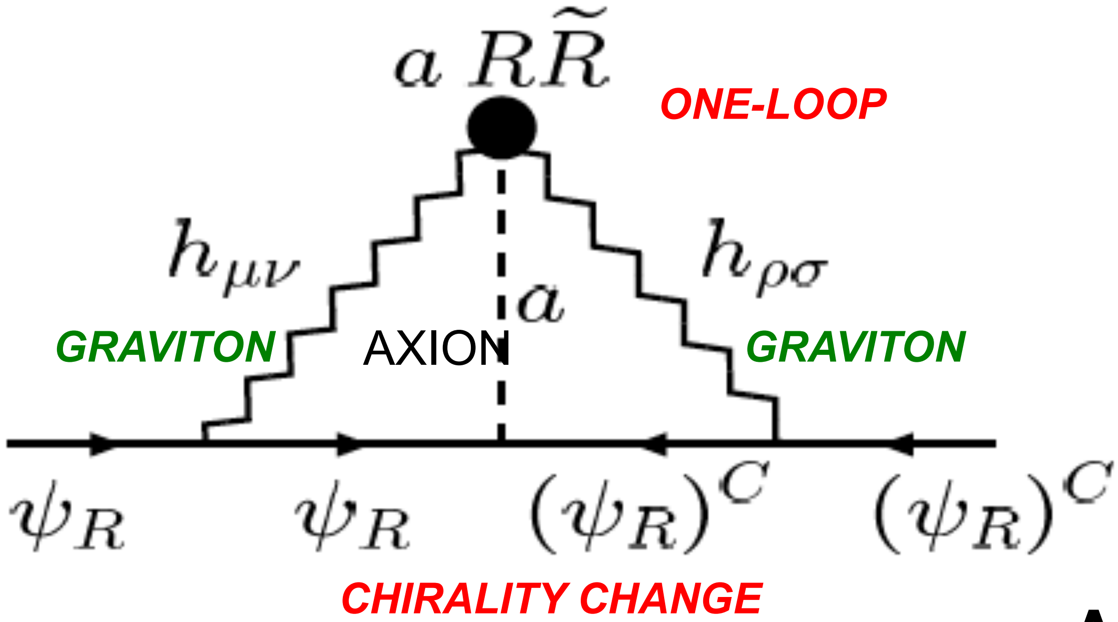
$$\mathcal{S} = \int d^4x \sqrt{-g} \Big[\frac{1}{2}(\partial_\mu b')^2 + \frac{1}{2}\left(1-\gamma^2\right)(\partial_\mu a)^2 + \frac{1}{2f_b^2} J^5_\mu J^{5\mu} + \frac{b'(x) - \gamma a(x)}{192\pi^2 f_b} R^{\mu\nu\rho\sigma} \widetilde{R}_{\mu\nu\rho\sigma} - y_a i a \left(\overline{\psi}^C_R \psi_R - \overline{\psi}_R \psi_R^C \right) \Big] .$$

must have $|\gamma| < 1$ otherwise axion field a(x) appears as a ghost → canonically normalized kinetic terms

$$\mathcal{S}_a = \int d^4x \sqrt{-g} \Big[\frac{1}{2}(\partial_\mu a)^2 - \frac{\gamma a(x)}{192\pi^2 f_b \sqrt{1-\gamma^2}} R^{\mu\nu\rho\sigma} \widetilde{R}_{\mu\nu\rho\sigma} - \frac{i y_a}{\sqrt{1-\gamma^2}} a \left(\overline{\psi}^C_R \psi_R - \overline{\psi}_R \psi_R^C \right) + \frac{1}{2f_b^2} J^5_\mu J^{5\mu} \Big] .$$

CHIRALITY CHANGE

THREE-LOOP ANOMALOUS STERILE NEUTRINO MASS



Λ = UV cutoff

$$M_R \sim \frac{1}{(16\pi^2)^2}\,\frac{y_a\,\gamma\,\kappa^4\Lambda^6}{192\pi^2 f_b(1-\gamma^2)} = \frac{\sqrt{3}\,y_a\,\gamma\,\kappa^5\Lambda^6}{49152\sqrt{8}\,\pi^4(1-\gamma^2)}$$

SOME NUMBERS

$\Lambda = 10^{17}$ GeV

$\gamma = 0.1$

M_R is at the TeV for $y_a = 10^{-3}$

$\Lambda = 10^{16}$ GeV

$M_R \sim 16$ keV, $y_a = \gamma = 10^{-3}$

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INTERESTING WARM DARK MATTER REGIME

Appropriate Hierarchy for the other two massive Right-handed neutrinos for Leptogenesis-Baryogenesis & Dark matter constraints can be arranged by choosing Yukawa couplings

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$M_R \sim 16$ keV, $y_a = \gamma = 10^{-3}$

May be (discrete) **symmetry** reasons force **two** of the heavier **RH neutrinos** to be **degenerate** → dictate patterns for the axion-RH-neutrino Yukawa couplings y_a

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Appropriate Hierarchy for the other two massive Right-handed neutrinos for Leptogenesis-Baryogenesis & Dark matter constraints can be arranged by choosing Yukawa couplings

FINITENESS OF THE MASS

Arvanitaki, Dimopoulos *et al.*

MULTI-AXION SCENARIOS (e.g. string axiverse)

$$\mathcal{S}_a^{\rm kin} = \int d^4x \sqrt{-g} \left[\frac{1}{2} \sum_{i=1}^{n} \left((\partial_\mu a_i)^2 - M_i^2 \right) + \gamma (\partial_\mu b)(\partial^\mu a_1) - \frac{1}{2} \sum_{i=1}^{n-1} \delta M_{i,i+1}^2 \, a_i a_{i+1} \right];$$

$$\delta M_{i,i+1}^2 \; < \; M_i M_{i+1}$$

positive mass spectrum
for all axions

simplifying all mixing equals

$$M_R \sim \frac{\sqrt{3}\, y_a \,\gamma\, \kappa^5 \Lambda^{6-2n} (\delta M_a^2)^n}{49152 \sqrt{8}\, \pi^4 (1-\gamma^2)} \qquad n \le 3$$

$$M_R \sim \frac{\sqrt{3}\, y_a \,\gamma\, \kappa^5 (\delta M_a^2)^3}{49152 \sqrt{8}\, \pi^4 (1-\gamma^2)} \, \frac{(\delta M_a^2)^{n-3}}{(M_a^2)^{n-3}} \qquad n > 3$$

FINITENESS OF THE MASS

Mavromatos, Pilaftsis arXiv: 1209.6387

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$$\mathcal{S}_a^{\rm kin} = \int d^4x \sqrt{-g}\, \Big[\frac{1}{2}\sum_{i=1}^{n} \Big((\partial_\mu a_i)^2 - M_i^2 \Big) + \gamma (\partial_\mu b)(\partial^\mu a_1) - \frac{1}{2}\sum_{i=1}^{n-1} \delta M^2_{i,i+1}\, a_i a_{i+1} \Big] ;$$

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M_R : UV finite for n=3 @ 2-loop, independent of axion mass

Open Issues

Include **non-derivative axion-sterile neutrino** interactions and examine the above constraints **on allowed masses** of Sterile neutrinos

$$y\, a\, \overline{N}^c\, N^c,$$

$$g_V\, \overline{N}\, \gamma^\mu\, N\, A^D_\mu$$

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Recall b axion mass can be that of QCD axion in our scenario

$$1.17 \times 10^{-5}\,\text{eV} \gtrsim m_b \gtrsim 1.17 \times 10^{-8}\,\text{eV}$$

but large masses can also be allowed, depending on the model, also for compactification stringy axions

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May be both axion and sterile neutrino warm DM pay a role in **galactic structure…**

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We also argue that DM may also consist of PBH, whose production can be enhanced during RVM Inflation due to axion potential modulation by world-sheet Instanton effects in our string-inspired model…

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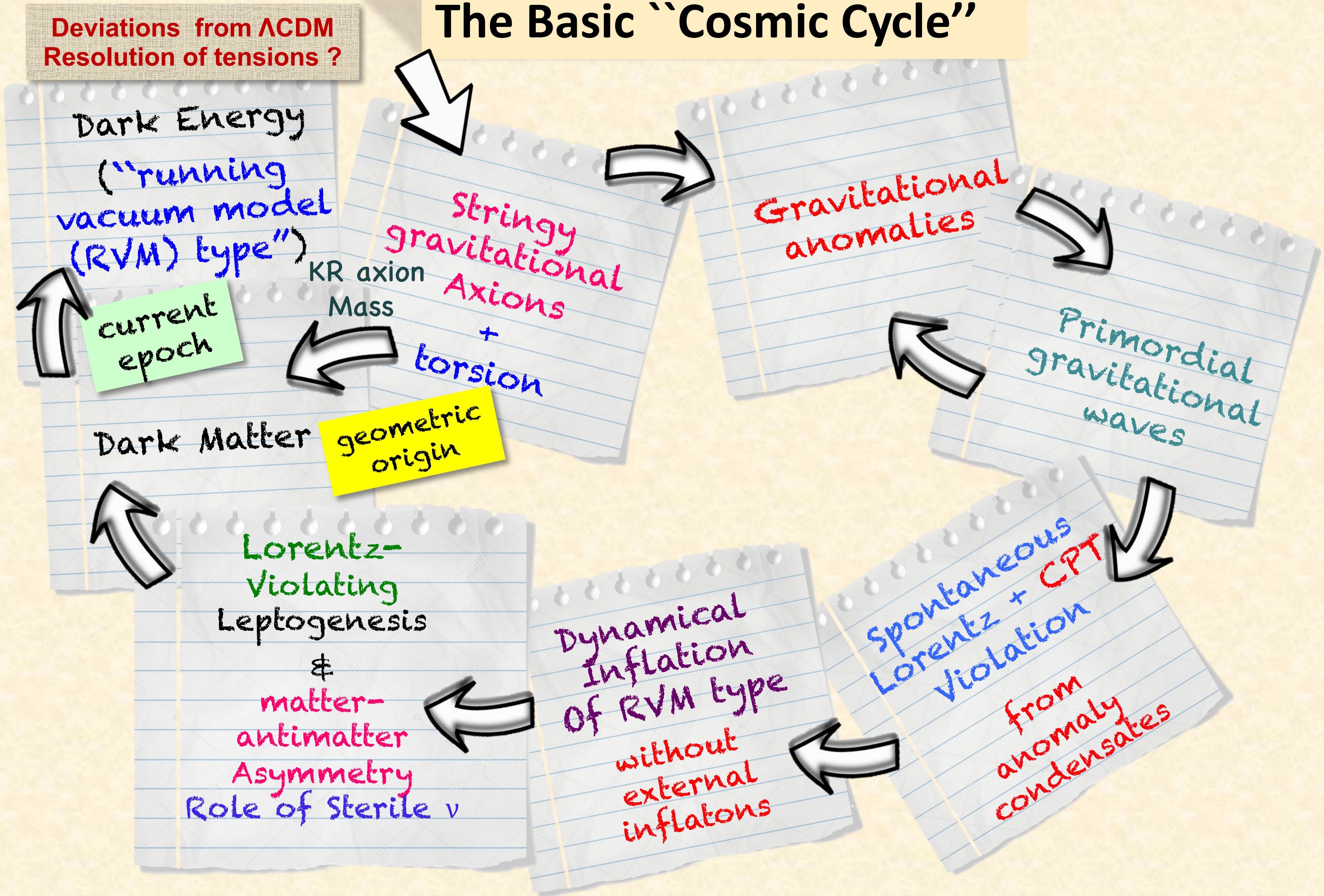
but large masses ca… allowed, depending on the model, also for compacti… stringy axions

To be explored ...

We also argue that DM may also consist of PBH, whose production can be enhanced during RVM Inflation due to axion potential modulation by world-sheet Instanton effects in our string-inspired model…

9. Summary & Outlook

The Basic ``Cosmic Cycle''



The Basic ``Cosmic Cycle''

Deviations from ΛCDM
Resolution of tensions ?

Deviations from ΛCDM
Resolution of tensions ?

The Basic ``Cosmic Cycle''

Dark Energy ("running vacuum model (RVM) type")

current epoch

ROLE OF INTERACTING STRINGY AXIONS & STERILE ν AS DM COMPONENTS IN GALACTIC STRUCTURE

+

PRIMORDIAL BLACK HOLES AS DM COMPONENT (enhanced production during inflation)

without external inflatons

Lorentz Violation

from anomaly condensates

The Basic ``Cosmic Cycle''

Deviations from ΛCDM
Resolution of tensions ?

Dark Energy

Primordial gravitational waves

Outlook:

(i) Search for axion-background -induced Lorentz and CPT Violation in early Universe (CMB, etc)

(ii) Potential search for negative coefficient of H^2 in RVM inflationary energy density due to grav. anomalies

(ii) Axion Cosmology – exclude String-Compactification models phenomenologically

Mehta et al.
e-Print: 2011.08693

Marsh, e-Print: 1510.07633

References:

a microscopic (string-inspired) model for RVM Universe....

Basilakos, NEM, Solà
(i)JCAP 12 (2019) 025
(ii) IJMD28 (2019) 1944002
(iii) Phys.Rev.D 101 (2020) 045001
(iv) Phys.Lett.B 803 (2020) 135342
(v) Universe 2020, 6(11), 218
NEM, Solà
(vi) EPJST 230 (2020), 2077
(vii) EPJPlus 136 (2021), 1152
NEM
(viii) arXiv:2205.07044
(ix) Universe 7 (2021), 480
(x) Phil. Trans. A380 (2022) 2222
NEM, Spanos, Stamou,
(xi) hep-th:2206.07963

Links with : spontaneous Lorentz violation (via (gravitational axion) backgrounds) and Matter-Antimatter Asymmetry in theories with Right-Handed Neutrinos

(i) NEM & Sarben Sarkar, EPJC 73 (2013), 2359
(ii) John Ellis, NEM & Sarkar, PLB 725 (2013), 407
(iii) De Cesare, NEM & Sarkar, EPJC 75 (2015), 514
(iv) Bossingham, NEM & Sarkar, EPJC 78 (2018), 113; 79 (2019), 50
(v) NEM & Sarben Sarkar, EPJC 80 (2020), 558

References:

(Light) Sterile neutinos
&
Galactic Structure

(i) Arguellies, NEM, Rueda, Ruffini, JCAP 1604, 038 (2016)

(ii) Yunis et al., PDU 30 (2020) 100699

(iii) Yunis et al., MG16 talks, e-print: arXiv: 2111.07642

Thank you!

SPARES

Primordial Gravitational Waves (GW)
Potential origins ?

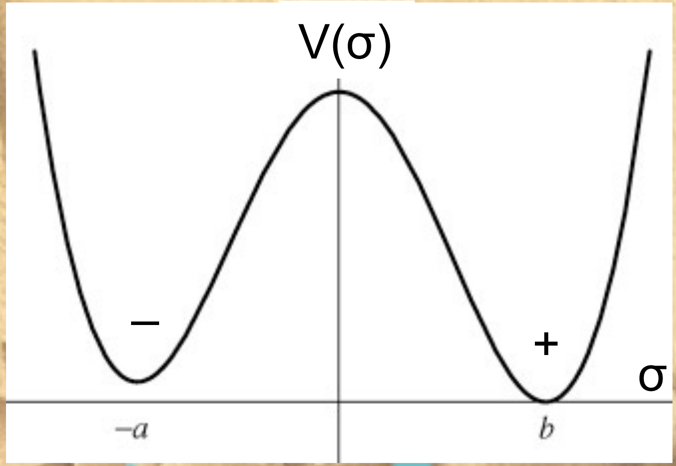
NEM, Sola
EPJ-ST
(2020)

$$p^+ = \int_{\vartheta+\Sigma_\ell}^{+\infty} d\sigma(x)\, \frac{1}{\sqrt{2\pi\,\Delta(\ell)}}\, \exp\Big(- \frac{(\sigma(x) - \vartheta - \Sigma_\ell)^2}{2\,\Delta(\ell)} \Big)$$

$$\Delta(\ell) \simeq \frac{H_i}{4\,\pi^2}\, \ln\Big(\frac{\ell}{\ell_c}\Big) \qquad \Sigma_\ell = \sqrt{\xi(\ell)} \qquad \xi(\ell) \simeq \frac{H_i}{4\,\pi^2}\, \ln\Big(\frac{L}{\ell}\Big)$$

$$H(t)^{-1} \equiv \ell_c(t) \le \lambda \le L$$

L=radius of Universe
ℓ = radius of causal bubble



$\sigma \equiv < \overline{\psi}_\mu\, \psi^\mu > \neq 0$

Not equal probabilities
for occupying + or − vacua

$p^+ \neq p^- = 1 - p^+$

→ percolating unstable domain walls → **GW**

Lalak, Ovrut, Lola, G. Ross, Thomas

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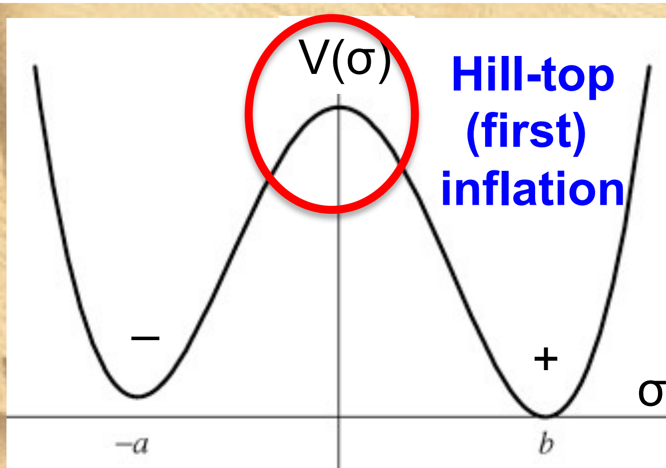

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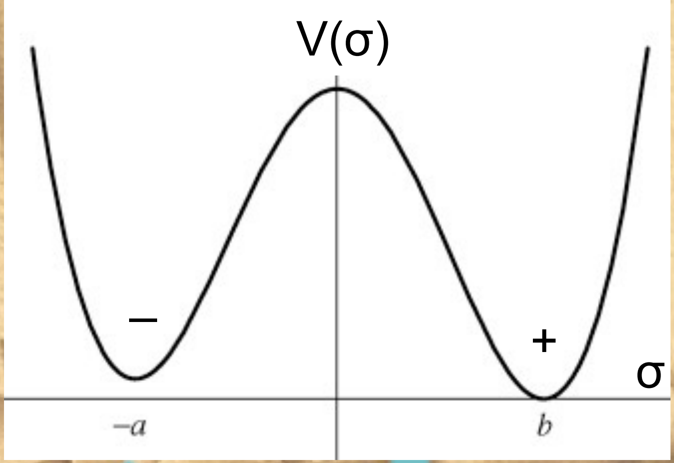
$$\sigma(x) = \sigma_{\rm cl} + \sigma_q(x) = \vartheta + \sigma_q(x)$$

$$\sigma_{\rm cl} \simeq \vartheta = \text{constant}$$

$$\ddot\sigma + 3H\dot\sigma = \frac{\partial V}{\partial \sigma} \simeq 0$$



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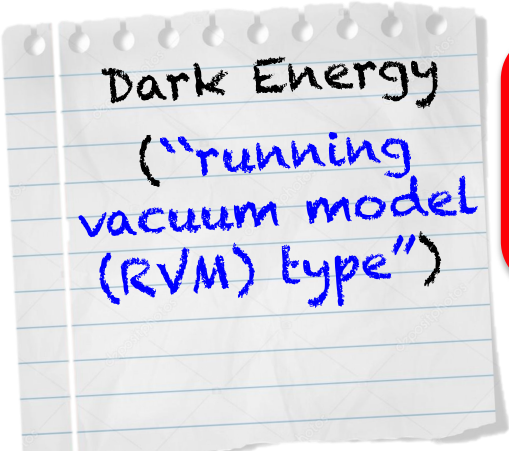
Lalak, Ovrut, Lola, G. Ross, Thomas

RVM

Shapiro + Solà
Solà, ...



$$\rho_{\Lambda}^{\rm RVM} = \kappa^{-2}\,\Lambda + c_1 H^2 + c_2 H^4 + \dots \equiv \kappa^{-2}\,\Lambda(t)$$

$$\Lambda \equiv 3\, c_0 \qquad c_1 = 3\nu\kappa^{-2},\ c_2 = 3\alpha\kappa^{-2}\, H_I^{-2},$$
$$H_I \sim 10^{-5}\kappa^{-1}\ \text{(current pheno)}$$

Vacuum energy density assumed de Sitter like but with time-dependent Cosmological parameter Λ(t) :

$$\rho^{\Lambda}_{\rm RVM}(t) = \Lambda(t)/\kappa^2 \qquad \kappa = \sqrt{8\pi {\rm G}} = M_{\rm Pl}^{-1}$$

$$p(t)_{\rm RVM} = -\rho^{\Lambda(t)}_{\rm RVM}(t)$$

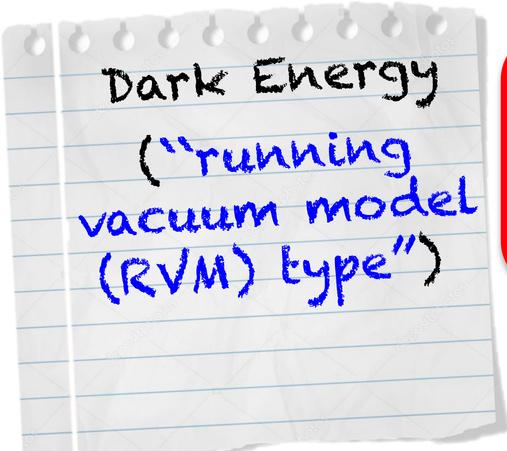
Renormalization-Group-like equation for the evolution of **vacuum energy density**
Hubble parameter H(t) ←→ RG scale *μ*

$$\frac{d\,\rho_{\Lambda}^{\rm RVM}(t)}{d\ln H^2} = \frac{1}{(4\pi)^2}\sum_{i=F,B}\left[a_i M_i^2 H^2 + b_i H^4 + \mathcal{O}\left(\frac{H^6}{M_i^2}\right)\right]$$

general covariance →
even powers of *H*

RVM

Shapiro + Solà
Solà, ...



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$c_1 = 3\nu\kappa^{-2}$, $c_2 = 3\alpha\kappa^{-2}\,H_I^{-2}$,

$H_I \sim 10^{-5}\kappa^{-1}$ (current pheno)

Vacuum energy den... ...me-dependent Cosmological parameter $\Lambda(t)$ ... $\sqrt{8\pi G} = M_{\rm Pl}^{-1}$

Any **dH/dt** $\approx -(\mathbf{1}+\mathbf{q})\,\mathbf{H^2}$,
decel. parameter q ≈ const
in each cosmic epoch

$$p(t)_{\rm RVM} = -\rho^{\Lambda(t)}_{\rm RVM}(t)$$

Renormal... ...up-like equation for the evolution of **vacuum energy density**
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RVM

Shapiro + Solà
Solà, ... (> 2000)

Dark Energy
("running vacuum model (RVM) type")

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Also: **Ellis, NEM, Nanopoulos (1998) – in non critical strings**

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Hubble parameter H(t) ←→ RG scale μ

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general covariance →
even powers of *H*

Axion Monodromy like potentials

NEM, Universe 7 (2021) 12, 480,
e-Print: 2111.05675 [hep-th]

Anomaly condensate → **linear axion potential** $V_{\rm eff} \ni \langle R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}\rangle\, b(x)$

Recall: approximately de Sitter provided during the duration of inflation

$$b(t) = \overline{b}(0) + 0.14 M_{\rm Pl}\, H\, t_{end} \simeq \overline{b}(0)$$

order of magnitude

< 0

N=e-folds

beginning
of inflation

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \frac{1}{2}\sum_i(\partial_\mu\chi_i\partial^\mu\chi_i - g^2(\phi-\phi_i)^2\chi_i^2)$$

$$V(\phi) = \mu^3\phi$$

Ooguri, Vafa, ...Palti

Distance-swampland conjectures?

Trapped inflation scenarios (moduli production @ enhanced symmetry points)

Potential ways out:
Jin, Brandenberger, Heisenberg,
Eur. Phys. J. C (2021) 81:162

Earlier Studies:
massive (non-interacting) fermions in galaxies @ a quantum level

Collisionless Relaxation mechanics in galaxies (**King Model**)

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} - \nabla\Phi(\mathbf{r},t)\frac{\partial f}{\partial \mathbf{v}} = 0 \qquad \triangle\Phi = 4\pi G \int f d^3\mathbf{v}$$

$f \rightarrow \bar{f}$

average

Violent relaxation (**Lynden Bell (1967)**) $\frac{dE}{dt} = \frac{\partial \Phi}{\partial t}|_{r(t)}$

total **energy not conserved**

$$S = \int \rho(\mathbf{r},\mathbf{v},\eta)\ln\rho(\mathbf{r},\mathbf{v},\eta)d\eta d^3\mathbf{r}d^3\mathbf{v} \qquad \bar{f}(\mathbf{r},\mathbf{v}) = \int \rho(\mathbf{r},\mathbf{v},\eta)\eta d\eta$$

entropy maximization at fixed total mass &energy

$$\delta S = 0 \Rightarrow \bar{f} = \frac{1}{e^{\beta[\epsilon(p)-\alpha]}+1}$$

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Ruffini & Stella, A & A (1983)

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$f \rightarrow \bar{f}$ **average**

$$f(v) = \frac{1 - \exp\left[-j^2(v_e^2 - v^2)\right]}{\exp\left[j^2(v^2 - \bar{\mu})\right] + 1}, \quad v \leq v_e$$
$$= 0, \qquad v > v_e,$$

rotational velocities

$$j^2 = m/(2kT),\ \bar{\mu} = 2\mu/m \text{ and } \theta = j^2\bar{\mu}.$$

$\theta \rightarrow -\infty \Rightarrow$ dilute limit (King distribution at classical level)

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massive (non-interacting) fermions in galaxies @ a quantum level

Gao, Merafina, Ruffini, A & A (1990)

Collisionless Relaxation mechanics in galaxies

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} - \nabla\Phi(\mathbf{r},t)\frac{\partial f}{\partial \mathbf{v}} = 0 \qquad \triangle\Phi = 4\pi G\int f d^3\mathbf{v}$$

$$f(p) = \frac{1}{e^{\frac{\epsilon(p)-\mu}{kT}}+1}, \qquad \epsilon(p) = \sqrt{c^2p^2+m^2c^4} - mc^2$$

Fermi distribution
Pauli exclusion principle

Equation of State

$$\rho = m\frac{2}{h^3}\int f(p)\left[1+\frac{\epsilon(p)}{mc^2}\right] d^3p,$$

$$P = \frac{1}{3}\frac{2}{h^3}\int f(p)\left[1+\frac{\epsilon(p)}{mc^2}\right]^{-1}\left[1+\frac{\epsilon(p)}{2mc^2}\right]\epsilon\, d^3p,$$

in curved metric

$$ds^2 = e^{\nu}c^2dt^2 - e^{\lambda}dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\phi^2$$

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Gao, Merafina, Ruffini, A & A (1990)

Einstein equations

$$e^{-\lambda} = 1 - \frac{2GM}{c^2 r}.$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho,$$

$$\frac{dP}{dr} = -\frac{1}{2}\frac{d\nu}{dr}(c^2\rho + P), \quad \frac{d\nu}{dr} = \frac{2G}{c^2}\frac{M + 4\pi r^3 P/c^2}{r^2[1 - 2GM/(c^2 r)]}$$

First law of **thermodynamics** (**Klein conditions**)

$$e^{\nu/2} T = \text{constant},$$

$$e^{\nu/2}(\mu + mc^2) = \text{constant}.$$

Earlier Studies:
massive (non-interacting) fermions in galaxies @ a quantum level

Gao, Merafina, Ruffini, A & A (1990)
Ruffini, Arguelles, Rueda, MNRAS (2015)

Dimensionless form of equations $(\hat{r} = r/\chi,\ \chi \propto m^{-2})$ m=fermion mass (``**ino**'')

$$\frac{d\hat{M}}{d\hat{r}} = 4\pi\hat{r}^2\hat{\rho},$$

$$\frac{d\theta}{d\hat{r}} = -\frac{1-\beta_0(\theta-\theta_0)}{\beta_0}\frac{\hat{M}+4\pi\hat{P}\hat{r}^3}{\hat{r}^2(1-2\hat{M}/\hat{r})},$$

$$\frac{d\nu}{d\hat{r}} = \frac{\hat{M}+4\pi\hat{P}\hat{r}^3}{\hat{r}^2(1-2\hat{M}/\hat{r})},$$

$$\beta(r) = \beta_0 e^{-\frac{\nu(r)+\nu_0}{2}}$$

Free parameters: $\beta_0 = kT_0/mc^2$, $\theta_0 = \mu_0/kT_0$ and m

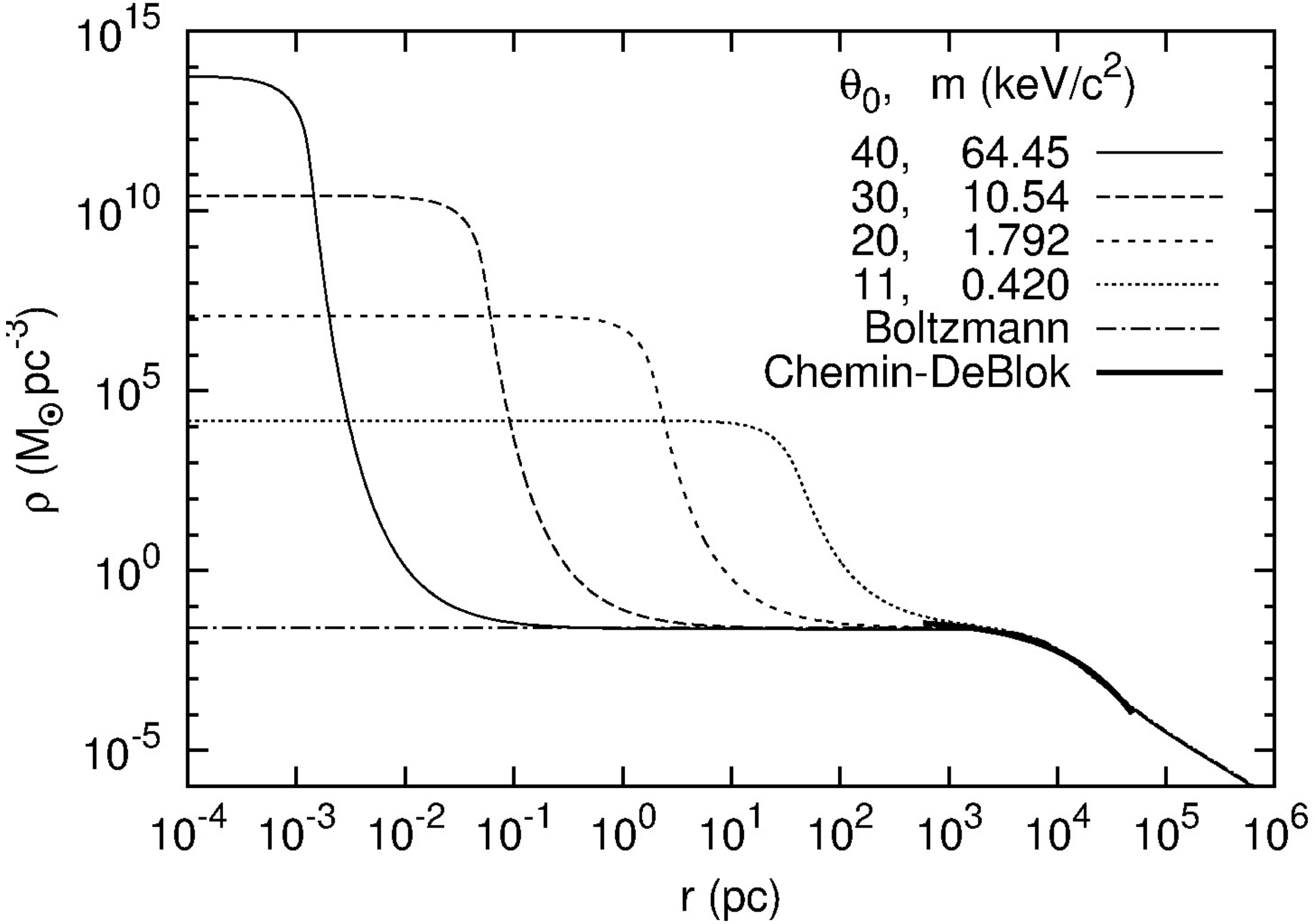
Initial conditions $M(0) = 0;\quad \nu_0 = 0;\quad \theta(0) = \theta_0 > 0;\quad \beta(0) = \beta_0;$

Dark matter halo observables of spiral galaxies (**boundary condition**

$$r_h = 25\,\mathrm{Kpc};\ \mathrm{v_h} = 168\,\mathrm{km/s};$$
$$\mathrm{M_h} = 1.6\times10^{11}\mathrm{M_\odot}$$

Earlier Studies:
massive (non-interacting) fermions in galaxies @ a quantum level

Ruffini, Arguelles, Rueda (RAR), MNRAS (2015)

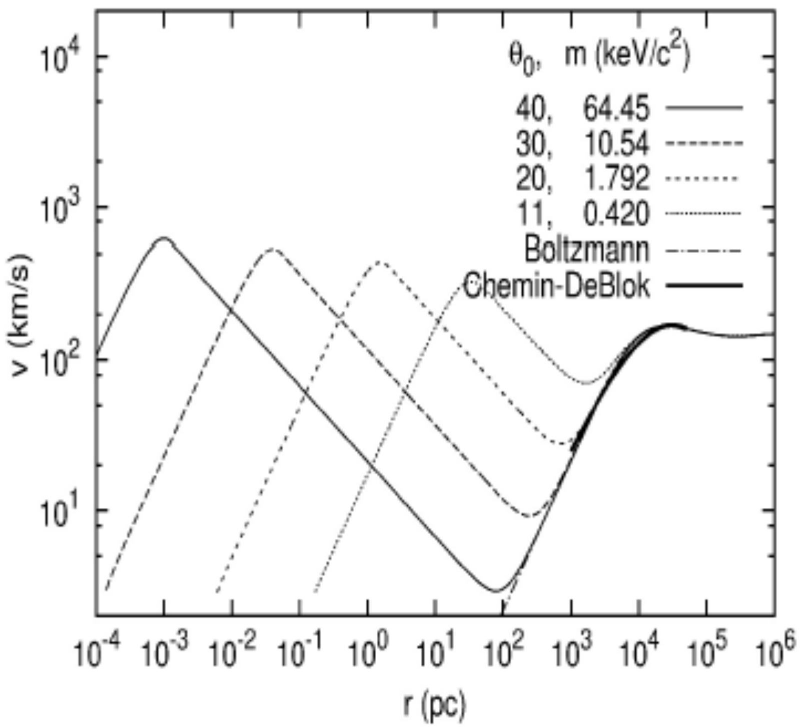


Density profiles of galaxies

Earlier Studies:
massive (non-interacting) fermions in galaxies @ a quantum level

Ruffini, Arguelles, Rueda (RAR), MNRAS (2015)

- ROTATION CURVES AND THE CORE CHARACTERISTICS
- m is strongly dependent ONLY on the core characteristics!
- For $m \sim 10\text{keV}/c^2 \rightarrow M_c \sim 10^6 M_\odot$ (SgrA* candidate)



θ_0	$m(keV/c^2)$	$r_c(pc)$	$M_c(M_\odot)$
11	0.420	3.3×10^1	8.5×10^8
25	4.323	2.5×10^{-1}	1.4×10^7
30	10.540	4.0×10^{-2}	2.7×10^6
40	64.450	1.0×10^{-3}	8.9×10^4
58.4	2.0×10^3	9.3×10^{-7}	1.2×10^2
98.5	3.2×10^6	3.2×10^{-13}	7.2×10^{-5}