Plausible detection of rotating magnetized neutron stars by their continuous gravitational waves

arXiv: 2302.03706

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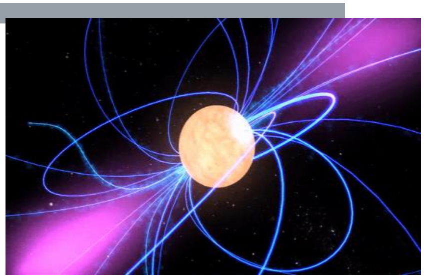
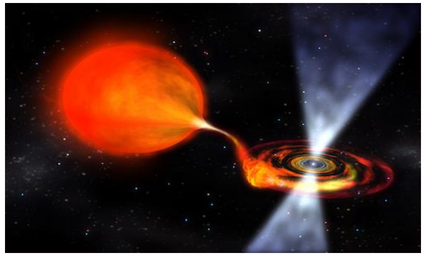
Detection of Neutron stars (NSs):

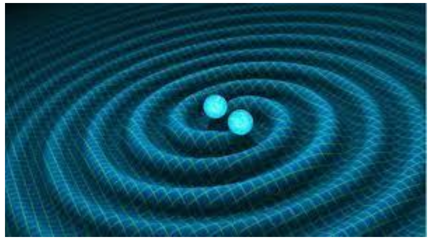
- **Electromagnetic radiation**: **Pulsar** (magnetized rotating NSs) emits beams of electromagnetic radiation out of its magnetic poles. The magnetic poles do not coincide with the rotational axis, so the emission beams are detected as pulse (Lighthouse effect).

- **Accretion-powered NSs**: locate in binary systems and manifest themselves as X-ray sources.

- **Gravitational wave (GW) from Binary merger**: NSs in binary system are strong sources of gravitational waves (due to nonzero time varying quadrupole moment)







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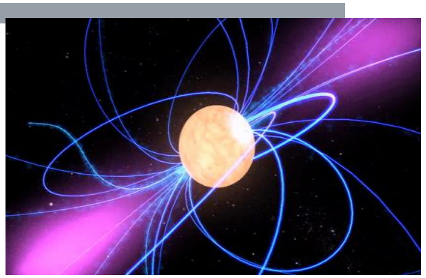
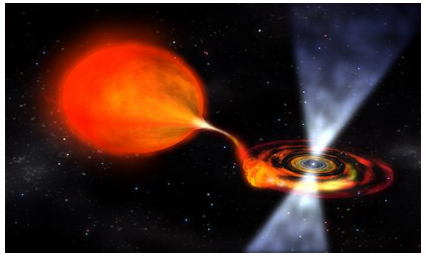
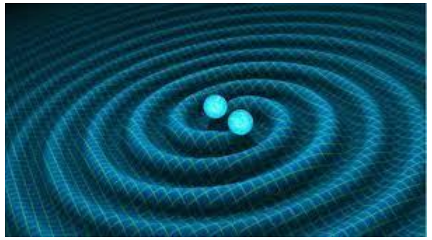






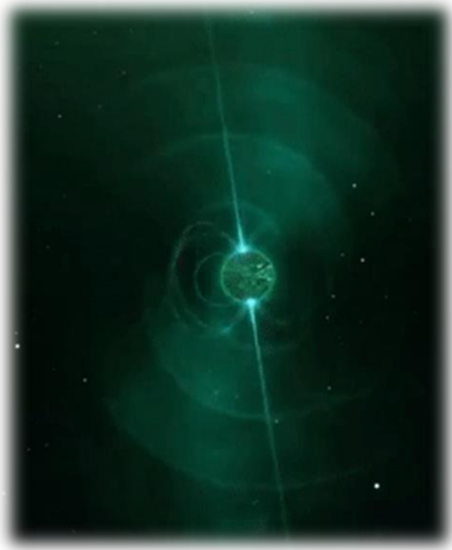
How to Detect 'Isolated' or 'Invisible' NSs and WDs?

Isolated NSs (not in binary system or with negligible electromagnetic radiation) ➡ tiny size and no source of thermal energy ➡ lowly luminous (or pulse not directing towards earth) thus Invisible.

But isolated NSs and WDs can produce **continuous GW!**

GW is emitted continuously, as long as star is magnetized and spinning (like a singer holding a single note for a long time).

Direct detection of invisible stars! ➡ provide idea about their mass, magnetic field, spin and equation of state.



Introduction

NSs are generally born with mass $1.4 - 1.6 \mathrm{M}_{\odot}$. GW from merger GW190814 ($M = 2.50 - 2.67 M_{\odot}$;Abbott et al. 2020) suggests the possible existence of NSs with $M > 2 M_{\odot}$, similar to pulsar timing study of PSR J0740 + 6620 (Cromartie et al. 2020).

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- One of the most exciting possibilities to explain such high mass is by **high magnetic field** along with rotation (Pili et. al. 2014, Das et. al. 2013).
- This in turn would lead them to emit continuous GW (CGW).
- **Origin of strong magnetic field:** Collapse of highly magnetised star → Flux freezing →Dynamo (α-Ω)→ amplify up to ~10^{17-18}G for NS (Thompson & Duncan 1993)

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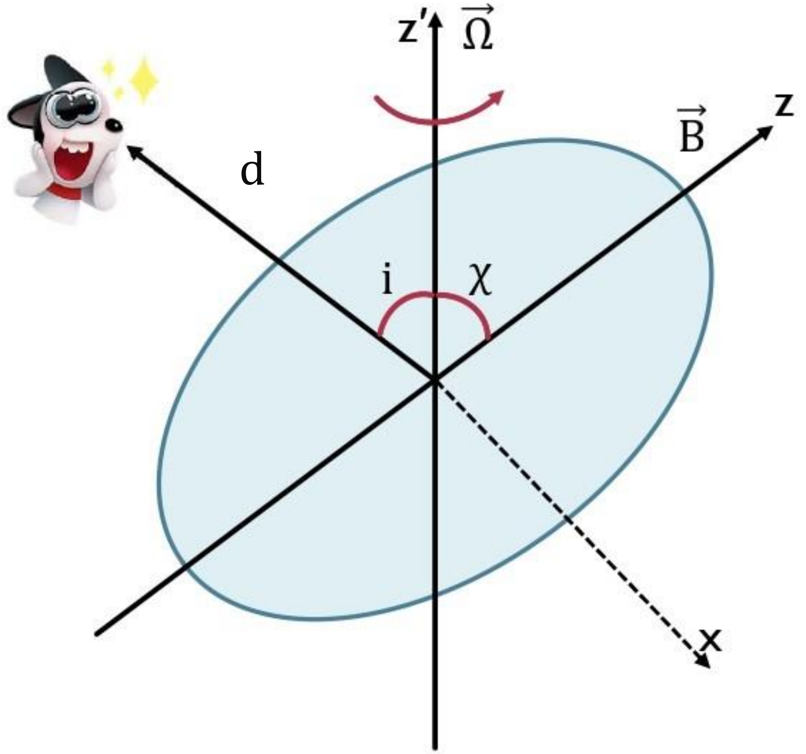
- We simulate CGW from isolated magnetized rotating NSs and WDs and try to understand observation plausibility.

GRAVITATIONAL WAVES FROM PULSATING COMPACT STARS



2D cross-section of an asymmetric NS

Magnetic field
Rotation
→ **Deformation**

Deformation & Misalignment → Triaxial symmetry → Time varying, non zero quadrupole moment → **Continuous** GW

$$h_+ = h_0 \sin\chi \left[\frac{1}{2}\cos i \sin i \cos\chi \cos\Omega t - \frac{1+\cos^2 i}{2}\sin\chi \cos 2\Omega t\right]$$

$$h_\times = h_0 \sin\chi \left[\frac{1}{2}\sin i \cos\chi \sin\Omega t - \cos i \sin\chi \sin 2\Omega t\right]$$

$$\chi \to 0 \qquad h_0 = \frac{4G}{c^4}\Omega^2 \epsilon \frac{I_{xx}}{d}, \qquad \epsilon = \frac{I_{zz}-I_{xx}}{I_{xx}}$$

Bonazzola & Gourgoulhon 1996

MODELLING NS USING XNS Pili et al. 2014

Einstein's equation solver in GRMHD

XNS (A code to study magnetized NSs)

Einstein's equation (describes space-time metric)

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$ds^2 = -\alpha^2\, dt^2 + \psi^4 \left[dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta \left(d\phi + \beta^{\phi}\, dt\right)^2\right]$$

+

Magneto-Hydrostatic Equilibrium

(provides distribution of matter/energy)

$$T^{\mu\nu} = (e + p + b^2)u^{\mu}u^{\nu} - b^{\mu}b^{\nu} + \left(p + \frac{b^2}{2}\right) g^{\mu\nu}$$

→ TOV eq

Momentum-energy conservation: $\nabla_{\mu} T^{\mu\nu} = 0$

→

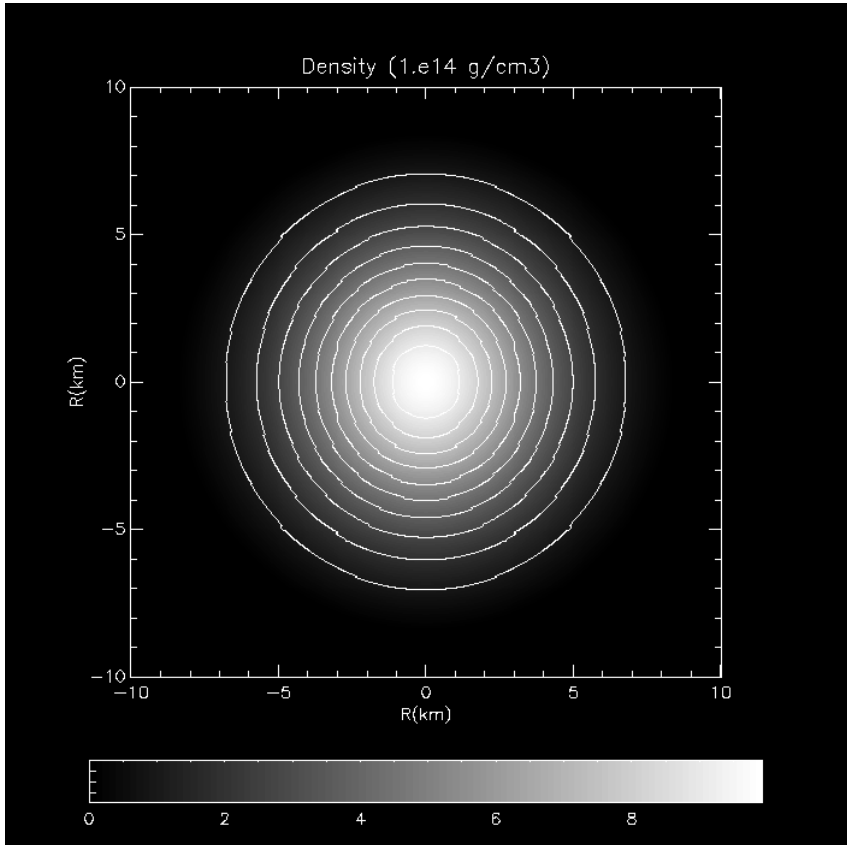
Axisymmetric equilibrium configuration (Uniformly/differentially rotating & Poloidal/Toroidal/mixed field) of NSs in GRMHD

Input ➡

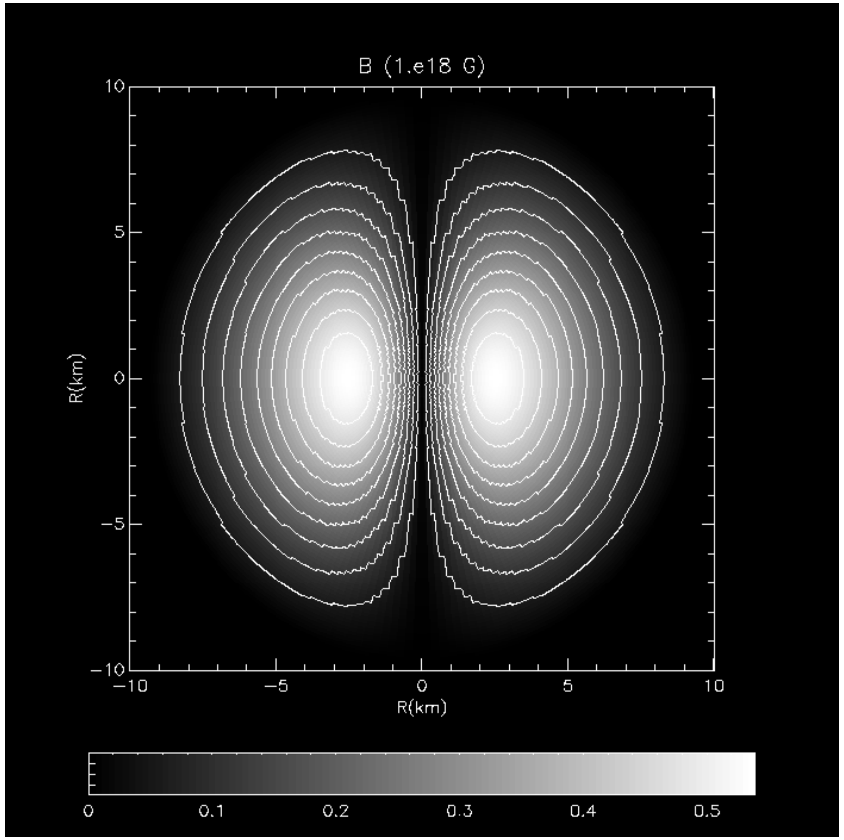
- EOS: polytropic law $P = k\rho^{1+\frac{1}{n}}$, adiabatic index γ = 1 + 1/n (We use K=100, γ=2)
- The differential rotation: $j(\Omega) = A^2(\Omega_C - \Omega)$, Ω = Ω_C (A → ∞)
- Magnetic field: magnetic polytropic law $B \sim k_m \rho^m$, m =polytropic index, k_m= magnetization index.

Output ➡ M, R, I_{xx}, I_{zz}

Toroidal Magnetic field ($\vec{B} = B_\phi \hat{\phi}$) with rotation



Density isocontour



Magnetic field isocontour

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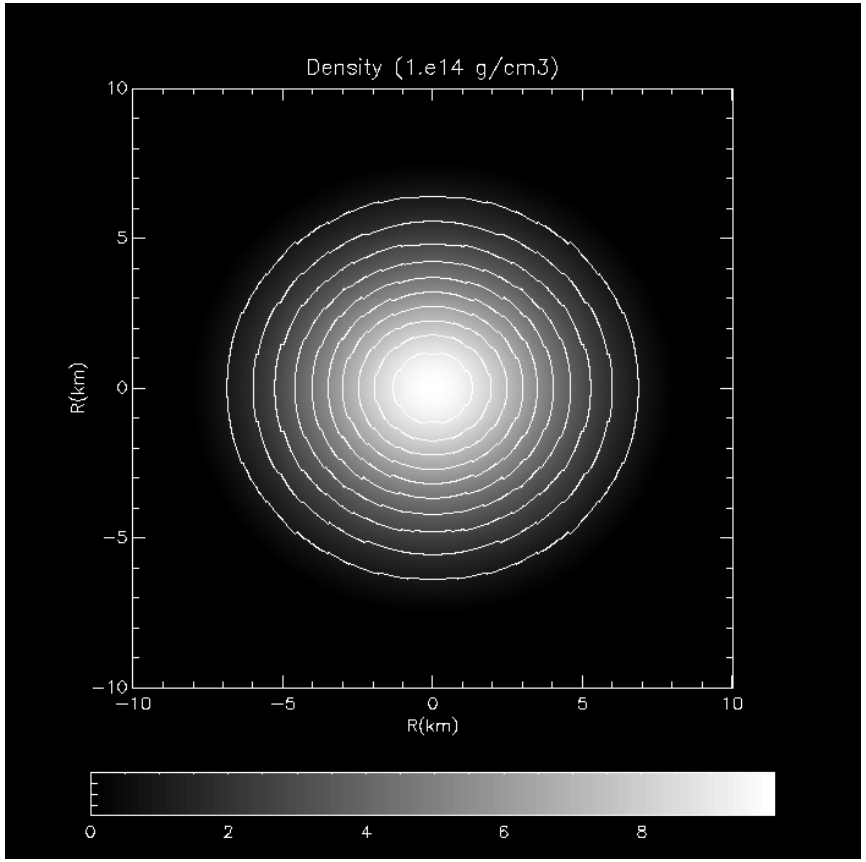
arXiv:2302.03706

$\rho_c = 10^{15} g/cc, B_{max} = 5 \times 10^{17} G, \nu = 500 Hz$ ➡ $M = 2M_\odot, R_E = 14km, R_P/R_E = 0.92, ME/GE = 0.056$

Poloidal Magnetic field ($\vec{B} = B_r\hat{r} + B_\theta\hat{\theta}$) with rotation



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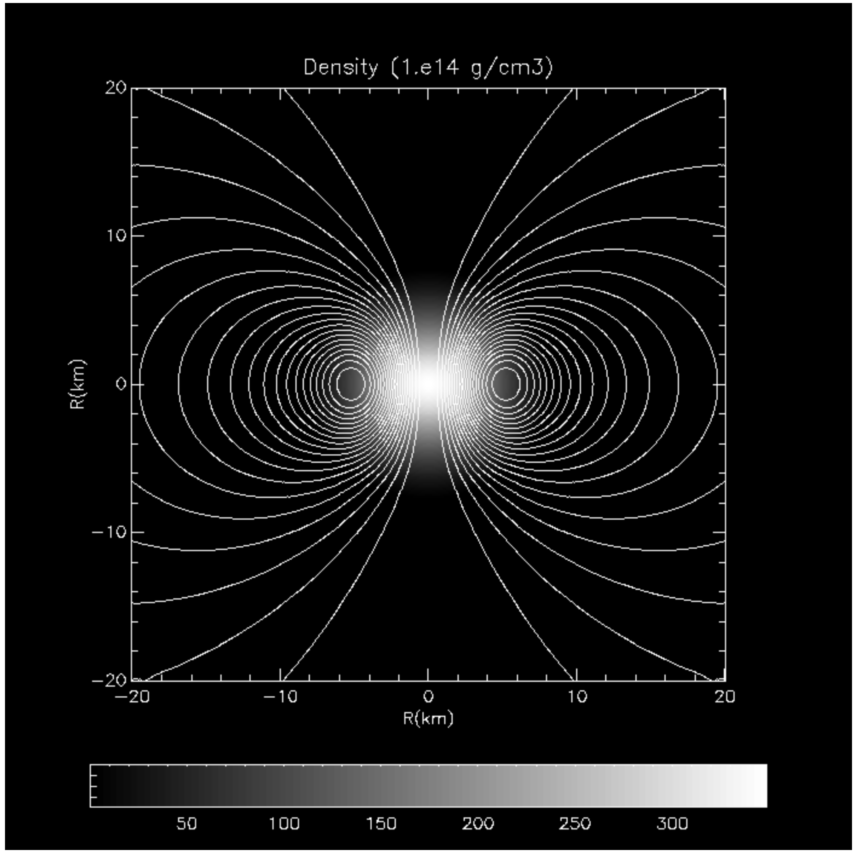


Magnetic field isocontour

MD & Mukhopadhyay 2023)
arXiv:2302.03706

$\rho_c = 10^{15} g/cc, B_{max} = 5 \times 10^{17} G, \nu = 50 Hz,$ ➡ $M = 2M_\odot, R_E = 12km, R_P/R_E = 0.81, ME/GE = 0.02$

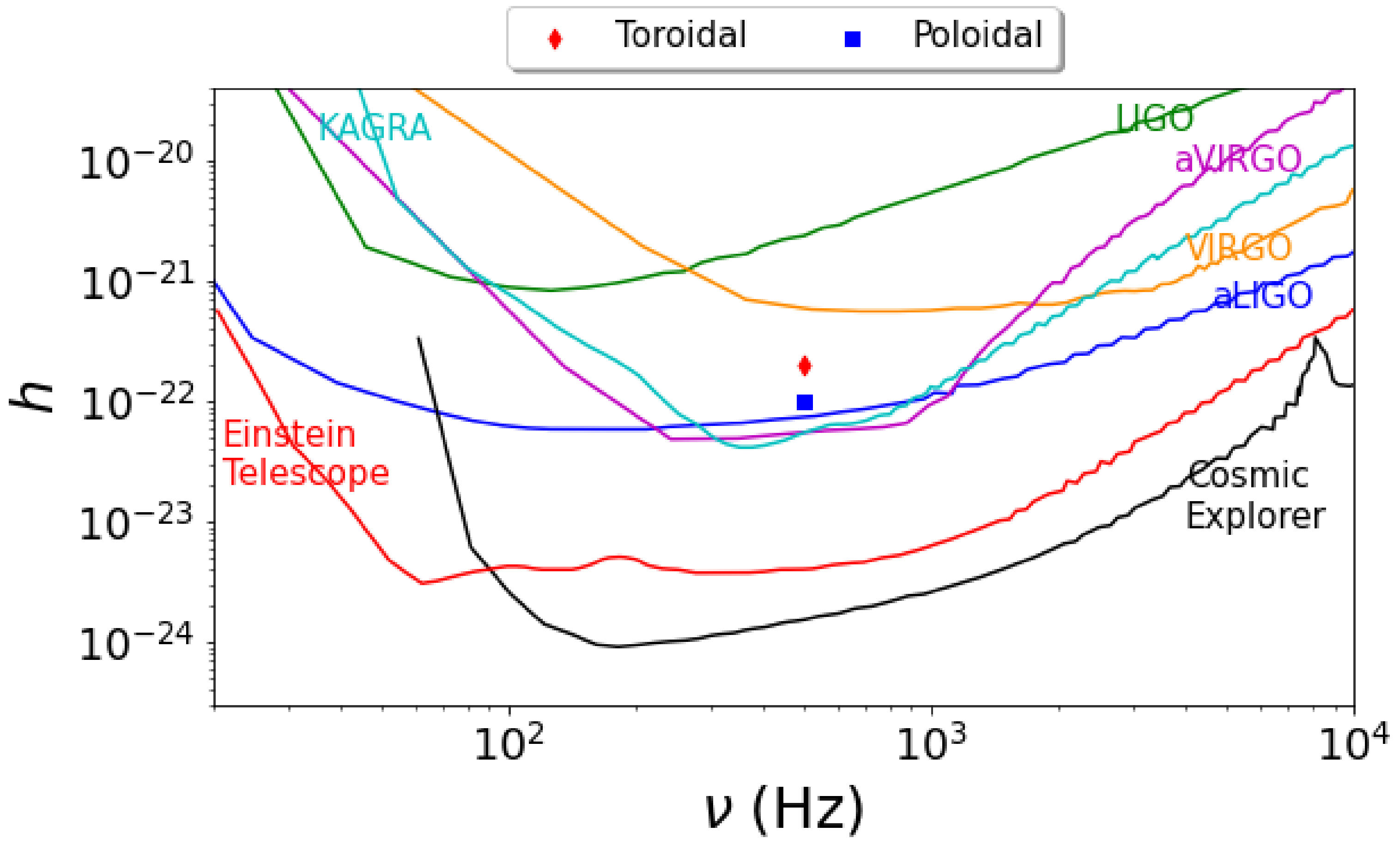
GRAVITATIONAL WAVE DETECTION

GW strain for various magnetic configuration (modelled from XNS) as a function of frequency along with the sensitivity curves of various detectors.

h = 0.0110297h_0 for χ = 3◦,

d=10 kpc

$$B_{max} = 5 \times 10^{17} G,$$
$$\nu = 500 Hz$$



Actually not detected yet, REASON?

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GW strain (h) ∝ Rotation frequency (Ω)
Obliquity angle (χ)
Magnetic field strength (B) } Decays with time!

Then how long we can detect them?

GW strain (h) ∝ Rotation frequency (Ω), Obliquity angle (χ), Magnetic field strength (B) } Decays with time!

Then how long we can detect them?

Ω and χ evolution:

Electromagnetic (dipole) radiation & GW (quadrupole) radiation : Ω, χ ⬇ (Timescale~years)

Viscous & thermal effect: Negligible, in few hours χ increases 3% and saturates

$$\frac{d(\Omega I_{z'z'})}{dt} = -\frac{2G}{5c^5}(I_{zz}-I_{xx})^2\Omega^5\sin^2\chi(1+15\sin^2\chi) - \frac{B_p^2R_p^6\Omega^3}{2c^3}\sin^2\chi F(x_0)$$

$$I_{z'z'}\frac{d\chi}{dt} = -\frac{12G}{5c^5}(I_{zz}-I_{xx})^2\Omega^4\sin^3\chi\cos\chi - \frac{B_p^2R_p^6\Omega^2}{2c^3}\times\sin\chi\cos\chi F(x_0) + \zeta{\epsilon_\Omega}^2\epsilon R^3\frac{g(\chi)}{I_{zz}\sin\chi\cos\chi}$$

$$\zeta=\zeta(T), x_0=x_0(\Omega), T(t)=\left(\frac{6N^s}{C}t+\frac{1}{T_0^6}\right)^{-1/6},$$

$$F(x_0)=\frac{x_0^4}{5(x_0^6-3x_0^4+36)}+\frac{1}{3(x_0^2+1)}.$$

Chau & Henriksen 1970

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Magnetic field decay:

Ohmic dissipation

$$t_{ohmic} \sim 2\times10^{11}\frac{L_5^2}{T_8^2}\left(\frac{\rho}{\rho_{nuc}}\right)^3 \text{ yr},$$

Ambipolar diffusior

$$t_{ambipolar} \sim \frac{5\times10^{15}}{T_8^6 B_{12}^2}\text{ yr} + t^s_{ambipolar},$$

where

$$t^s_{ambipolar} \sim 3\times10^9\frac{L_5^2T_8^2}{B_{12}^2}\text{ yr},$$

and

Hall drift

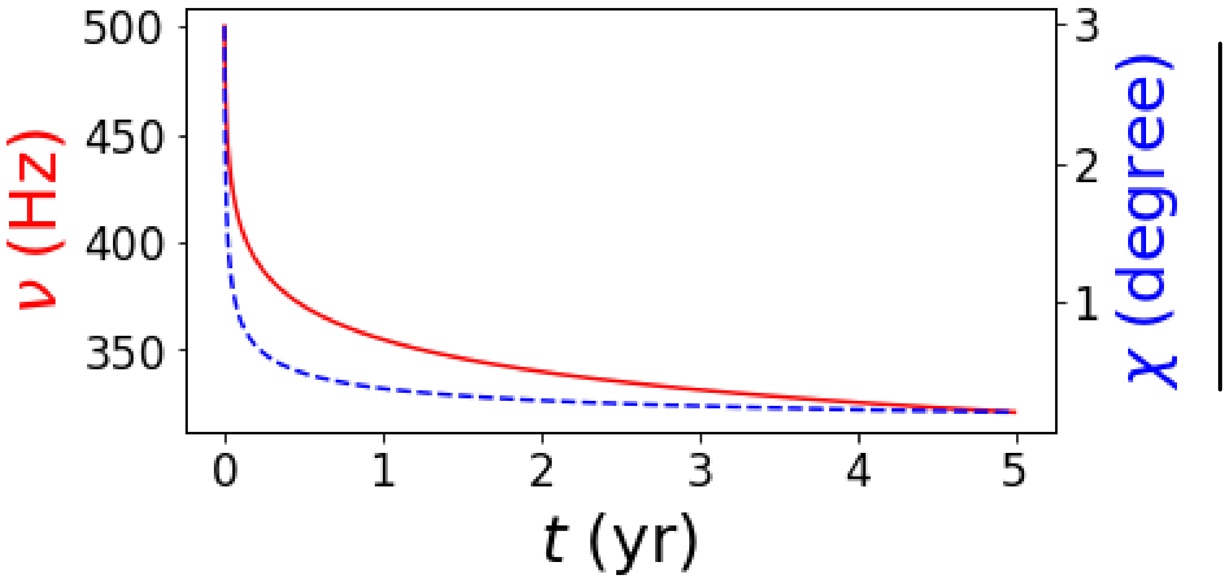
$$t_{Hall} \sim 5\times10^8\frac{L_5^2}{B_{12}}\left(\frac{\rho}{\rho_{nuc}}\right)\text{ yr},$$

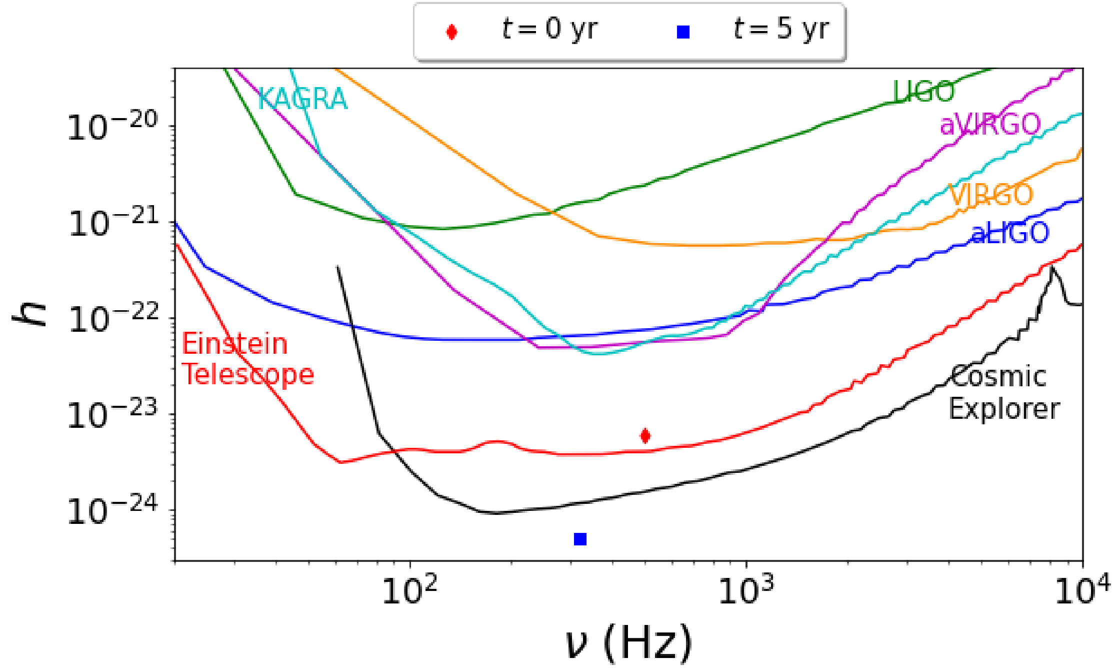
$$\frac{dB}{dt} = -B\left(\frac{1}{t_{ohmic}} + \frac{1}{t_{ambipolar}} + \frac{1}{t_{Hall}}\right)$$

Heyl & Kulkarni 1998

Goldreich & Reisenegger 1992

Evolution of ν and χ (B constant): Toroidal magnetic field





$$M = 2.02 M_\odot, B_{max}^{Toroidal}(initial) = 1.4 \times 10^{17} G, \nu(initial) = 500 Hz$$

However, for $\mathbf{B_{max}^{Poloidal}(initial) = 10^{15} G}$, Ω, χ decays in few days of time,
but stable NSs are actually toroidally dominated. (Wickramasinghe et al. 2014)

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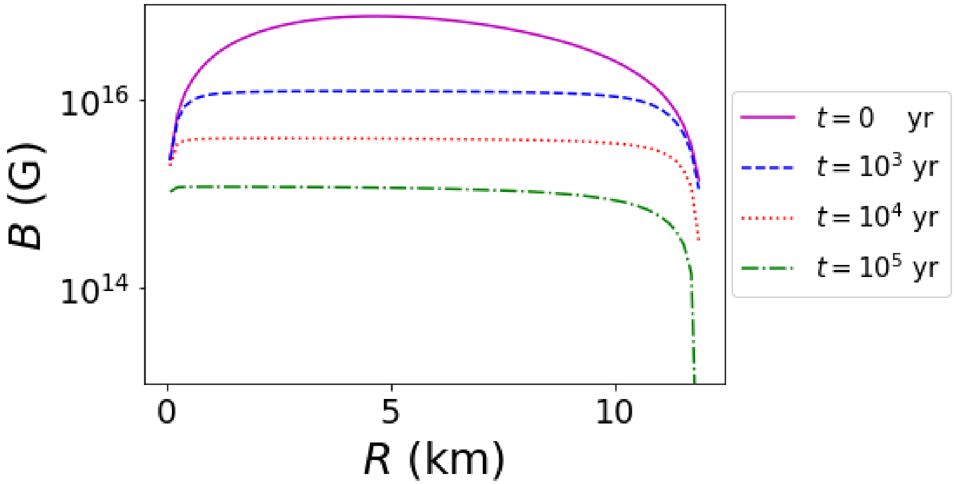
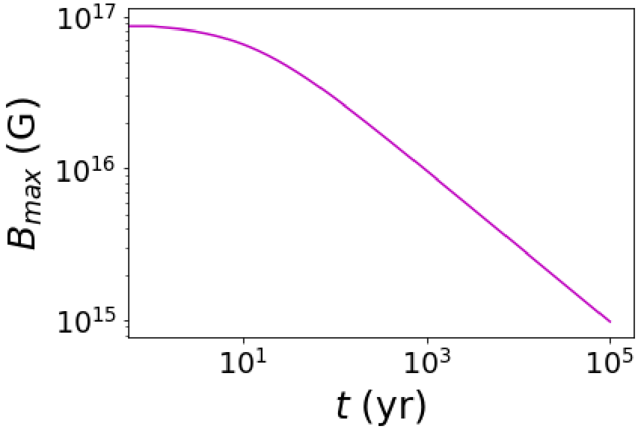
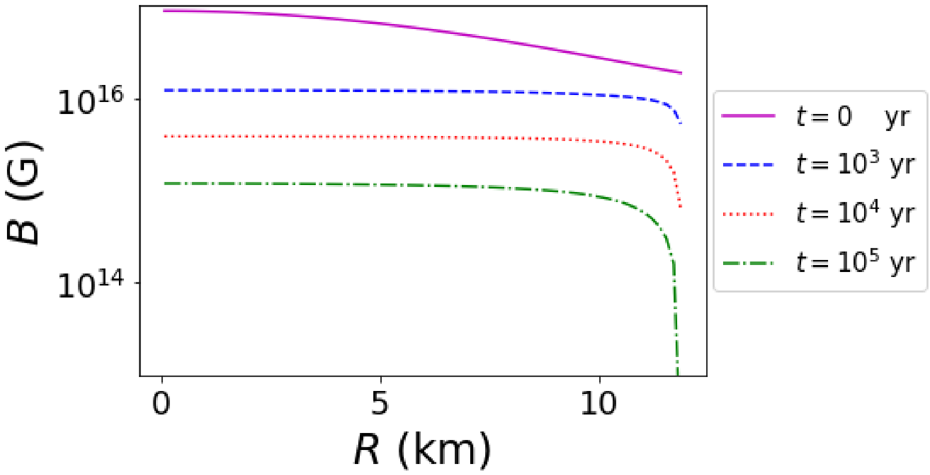
Evolution of B (ν, χ constant)

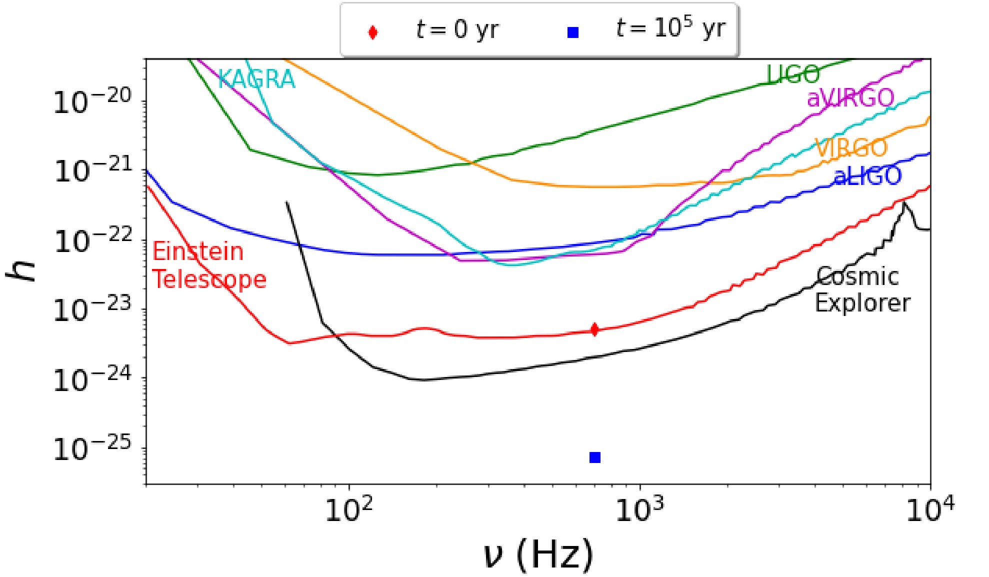
Toroidal

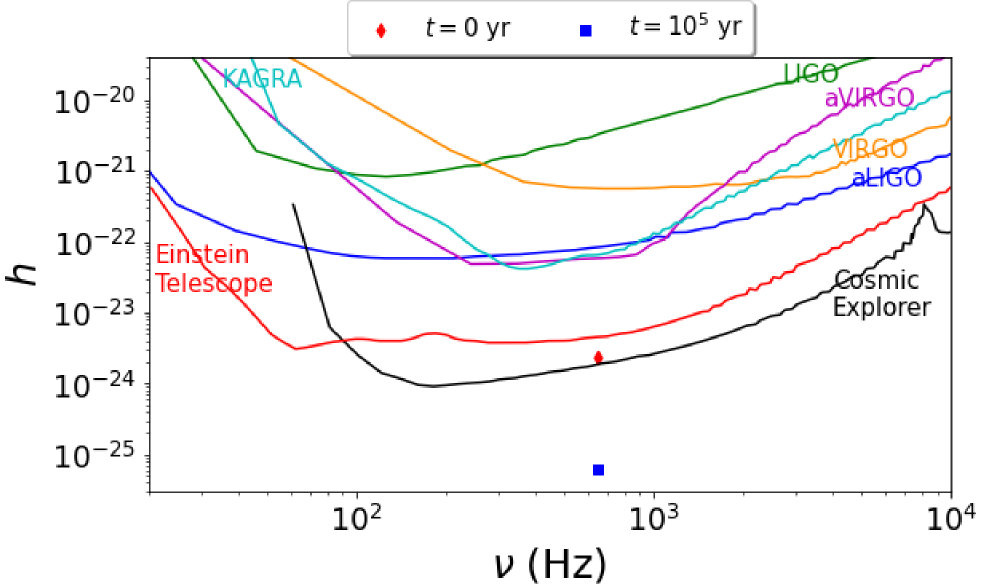
Poloidal











$$M = 2.02 M_\odot \,, B_{max}^{Toroidal/poloidal}(initial) = 9 \times 10^{16} G$$

$\nu = 700 Hz,$ $\nu = 650 Hz$

MD & Mukhopadhyay 2023
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Timescale for ν, χ decay << Timescale for B decay

Decay of ν, χ is more important to study GW amplitude decay

TO INCREASE GW DETECTION POSSIBILITY:

Signal to noise ratio (SNR) for 1 Year integration time

$$\langle S/N \rangle = \sqrt{\langle S/N_{\Omega}^2 \rangle + \langle S/N_{2\Omega}^2 \rangle},$$

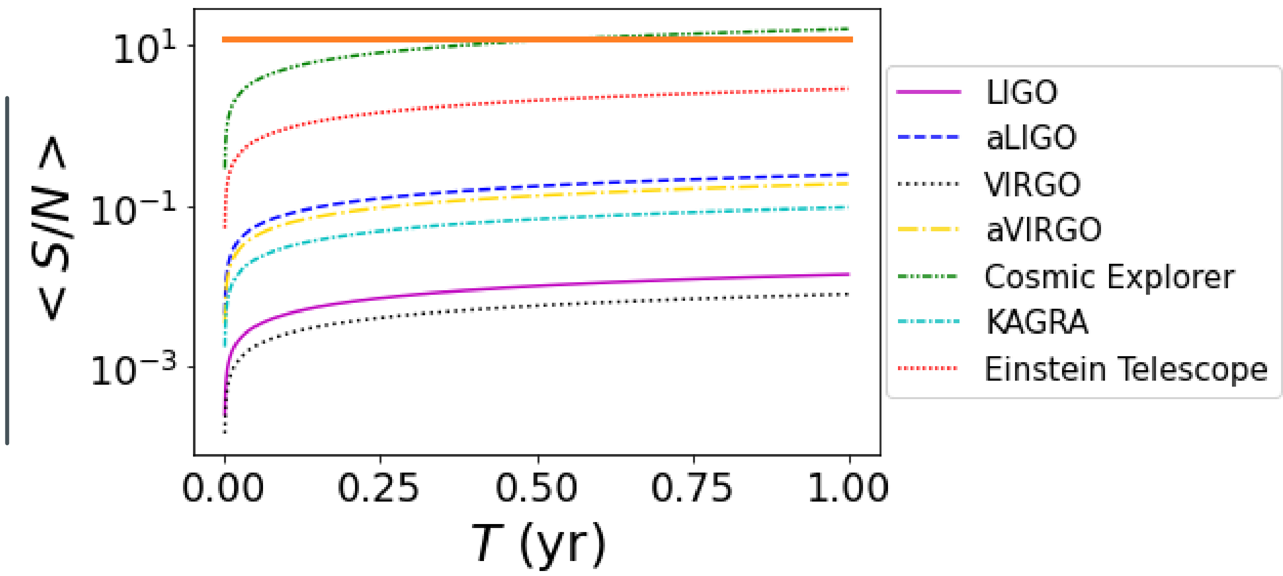
where

$$\langle S/N_{\Omega}^2 \rangle = \frac{\sin^2 \zeta}{100} \frac{h_0^2 \sqrt{N} T_{stack} \sin^2 2\chi}{S_n(\nu)} = \frac{\sin^2 \zeta}{100} \frac{h_0^2 T \sin^2 2\chi}{\sqrt{N} S_n(\nu)}$$

and

$$\langle S/N_{2\Omega}^2 \rangle = \frac{4 \sin^2 \zeta}{25} \frac{h_0^2 \sqrt{N} T_{stack} \sin^4 \chi}{S_n(2\nu)} = \frac{4 \sin^2 \zeta}{25} \frac{h_0^2 T \sin^4 \chi}{\sqrt{N} S_n(2\nu)}$$

Maggiore et al. 2020



$M = 2M_{\odot}, B_{max}^{Toroidal} = 9 \times 10^{16} G, \nu(initial) = 200Hz$

$SNR(threshold) = 12$

MD & Mukhopadhyay; 2023
arXiv:2302.03706

The signal is not detectable instantaneously.
After one month of integration time, it will be detectable!!

CONCLUSION

Highly magnetized rotating NSs can explain existence of **massive** NS >$2M_{\odot}$ (and also possibly the NS candidate of GW190814 belonging to mass gap $2.5M_{\odot} < M < 5M_{\odot}$).

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Uniformly rotating massive NSs, which are detectable after birth, may not be detectable forever. Because its GW amplitude decays due to decay of B, Ω and χ (**that we have first time studied simultaneously**). Perhaps this is why we have **not yet detected CGW** from NSs by aLIGO, aVIRGO, KAGRA.

We can try to **increase the detection** probability by calculating SNR over 1 year leading to direct detection of NSs (which cumulatively adds up the SNR, thus can have better probability to detect after some time).

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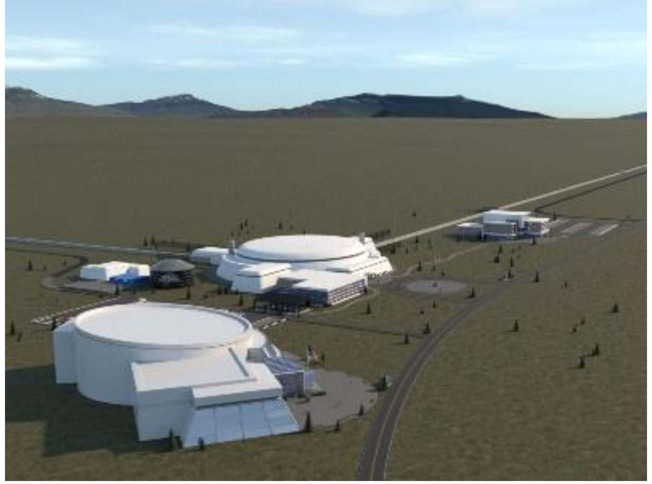
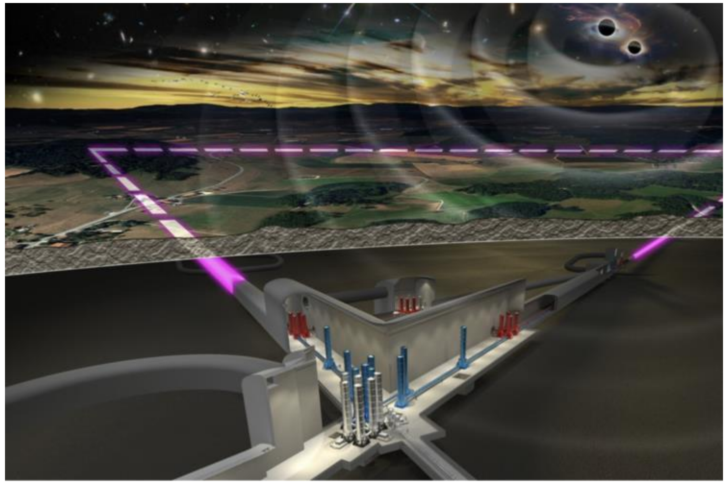
Further, we have shown in the paper, that differentially rotating massive NSs does not remain massive for long and settles into a uniformly rotating, **less massive** NS due to magnetic braking in (0.01-100 seconds).

Future gravitational wave missions with Einstein Telescope and Cosmic Explorer should be planned accordingly to detect such massive NSs, which, if successful can provide us an idea about its spin, magnetic field, as well as about the equation of state.

Future Objectives



Ligo-Virgo collaboration have not yet detected any CGW



Einstein telescope and cosmic explorer may detect them instantaneously or after 1 year of integration time

THANK YOU

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Appendix D

$$\frac{dv_i}{dt} + \frac{1}{\rho}\frac{\partial}{\partial x_j}\left(P_{ij} + M_{ij}\right) = F_i$$

$$M^{ij} = \frac{B^2}{8\pi}\delta^{ij} - \frac{B^i B^j}{4\pi}\,, \qquad T_f^{\mu\nu} = \frac{B^2}{4\pi}u^\mu u^\nu - \frac{B^i B^j}{4\pi} + \frac{B^2}{8\pi}g^{\mu\nu}$$

$$\vec{B} = B^r\hat{r} \Rightarrow \quad M^{ij} = \begin{pmatrix} \frac{-B^2}{8\pi} & 0 & 0 \\ 0 & \frac{B^2}{8\pi} & 0 \\ 0 & 0 & \frac{B^2}{8\pi} \\ 0 & 0 & \end{pmatrix},$$

$$\vec{B} = B^\theta\hat{\theta} \Rightarrow \quad M^{ij} = \begin{pmatrix} \frac{B^2}{8\pi} & 0 & 0 \\ 0 & -\frac{B^2}{8\pi} & 0 \\ 0 & 0 & \frac{B^2}{8\pi} \\ 0 & 0 & \end{pmatrix}$$

$$\vec{B} = B^\emptyset\hat{\Phi} \Rightarrow \quad M^{ij} = \begin{pmatrix} \frac{B^2}{8\pi} & 0 & 0 \\ 0 & \frac{B^2}{8\pi} & 0 \\ 0 & 0 & \frac{-B^2}{8\pi} \\ 0 & 0 & \end{pmatrix}$$

h_+ and $h_\times$ will be suppressed by some factors.

At $\chi = 3$, $t = 0$ and $i = i_{max} = 46.5$

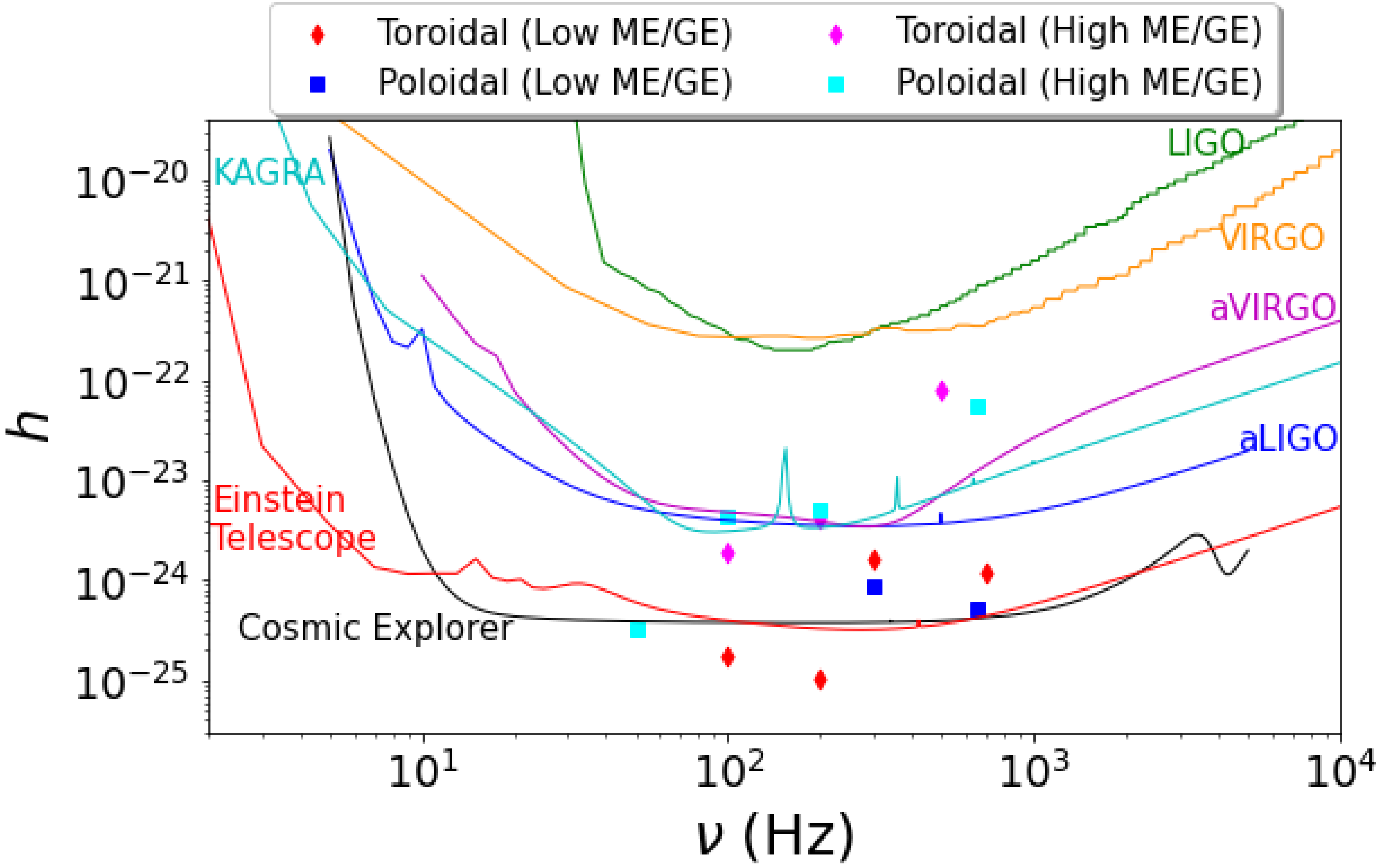
maximum of $\sin\chi\left[\frac{1}{2}\cos i \sin i \cos\chi \cos\Omega t - \frac{1+\cos^2 i}{2}\sin\chi\cos 2\Omega t\right] = 0.011029$

$h_{max} = 0.0110297 h_0$.

We assume the distance between the NS and the detector d=10 kpc

Kalita & Mukhopadhyay 2019

d=10 kpc



Toroidal (Low ME/GE)
Toroidal (High ME/GE)
Poloidal (Low ME/GE)
Poloidal (High ME/GE)
LIGO
KAGRA
VIRGO
aVIRGO
aLIGO
Einstein Telescope
Cosmic Explorer
10^{-20}
10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^{-25}
h
10^{1}
10^{2}
10^{3}
10^{4}
ν (Hz)

Appendix E

The system is closed by Ohm's law for a perfectly conducting plasma, which becomes a constraint for a vanishing electric field in the frame comoving with the fluid i.e., the freely moving charges in a plasma are always supposed to be able to screen any electric field that may arise locally.

$$\vec{E} = \frac{\vec{J}}{\sigma} - \frac{\vec{v} \times \vec{B}}{c}$$

$$\vec{E}' = \left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right), \gamma = 1$$

$$E' \rightarrow 0$$

Grad-Shfranov equation: (in 2D)

Lets consider a pressure balanced magnetic field (in cartesian coordinate) in which variables are independent of z, $\frac{\partial}{\partial z} = 0.$

$\vec{\nabla} \cdot \vec{B} = 0, \quad \vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$ ➡

$$\vec{A} = A\hat{z}$$

$$\vec{B} = B_z(x,y)\hat{z} + \vec{\nabla} \times [\vec{A}(x,y)]$$

Then the magnetic field can be written as,

$$B = \left(\frac{\partial A}{\partial y}, -\frac{\partial A}{\partial x}, B_z(x,y)\right)$$

Pressure Balance condition:

$$\vec{\nabla}P = -\vec{\nabla}\left(\frac{B^2}{8\pi}\right) + \frac{(\vec{B} \cdot \vec{\nabla})\vec{B}}{4\pi}$$

We have to substitude B in pressure balance eq

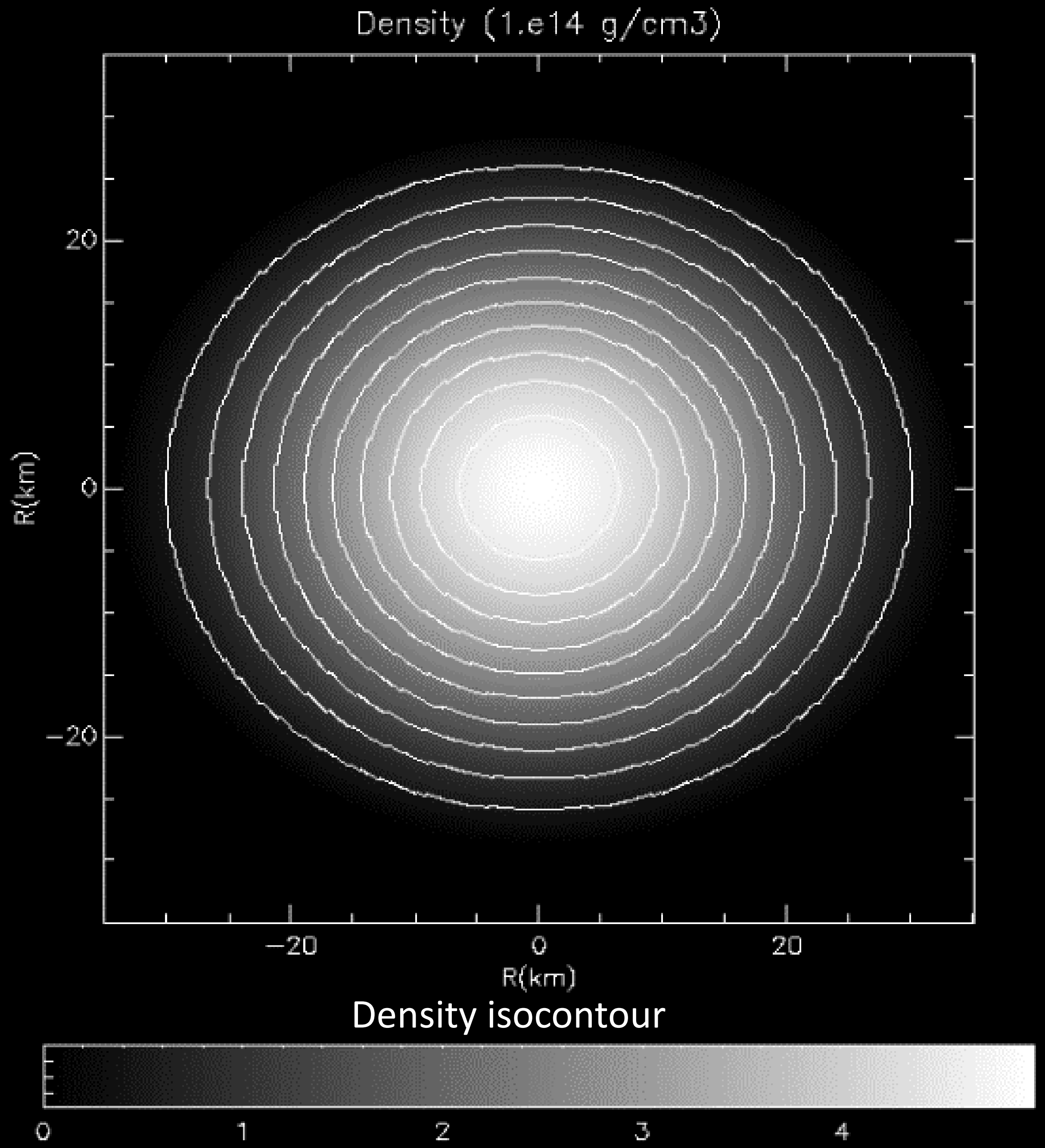
$$\nabla^2 A = -4\pi \frac{\mathrm{d}}{\mathrm{d}A}\left(P + \frac{B_z^2}{8\pi}\right)$$

Detailed sol: PTO

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \vec{F}_g - \frac{1}{\rho}\vec{\nabla}\left(P + \frac{B^2}{8\pi}\right) + \frac{(\vec{B}\cdot\vec{\nabla})\vec{B}}{4\pi\rho}$$

$$\rho\vec{F}_g = \vec{\nabla}\left(p + \frac{B^2}{8\pi}\right) + \frac{(\vec{B}\cdot\vec{\nabla})\vec{B}}{4\pi}$$

Non-Magnetized Uniformly Rotating



Density isocontour

- Centrifugal force makes the star oblate

$$\rho_c = 5.1 \times 10^{14}\, g/c\,c, \nu = 477\ Hz, M = 1.24\ M_\odot$$

⬇

$$R_E = 15\ km,\ \ R_P/R_E = 0.83$$

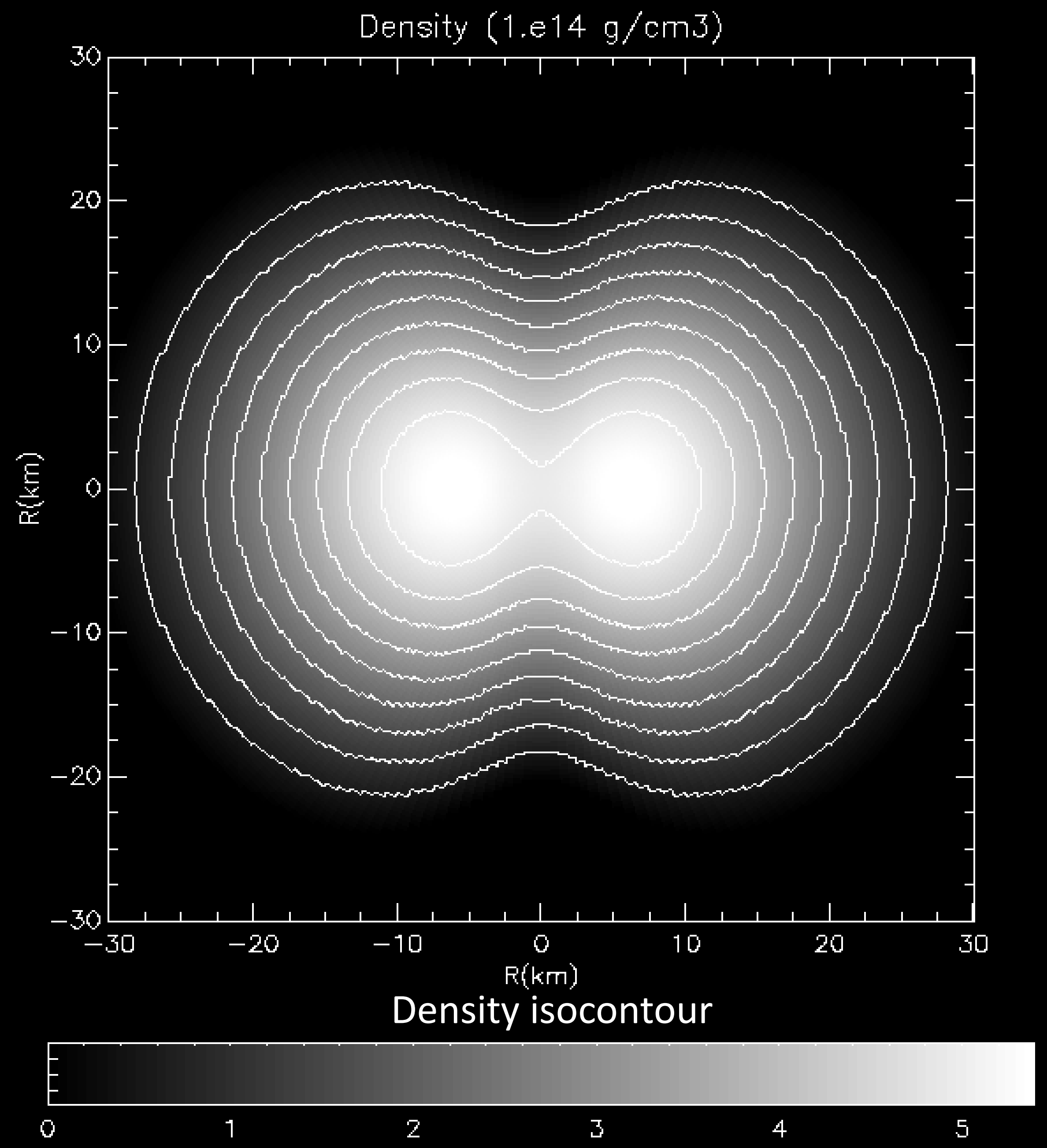
Non-Magnetized Differentially Rotating

- Polar hollow structure.

$$\rho_c = 5.4 \times 10^{14} \frac{g}{cc}, \nu(cen) = 3342\ Hz, \nu(eq) = 282\ Hz$$

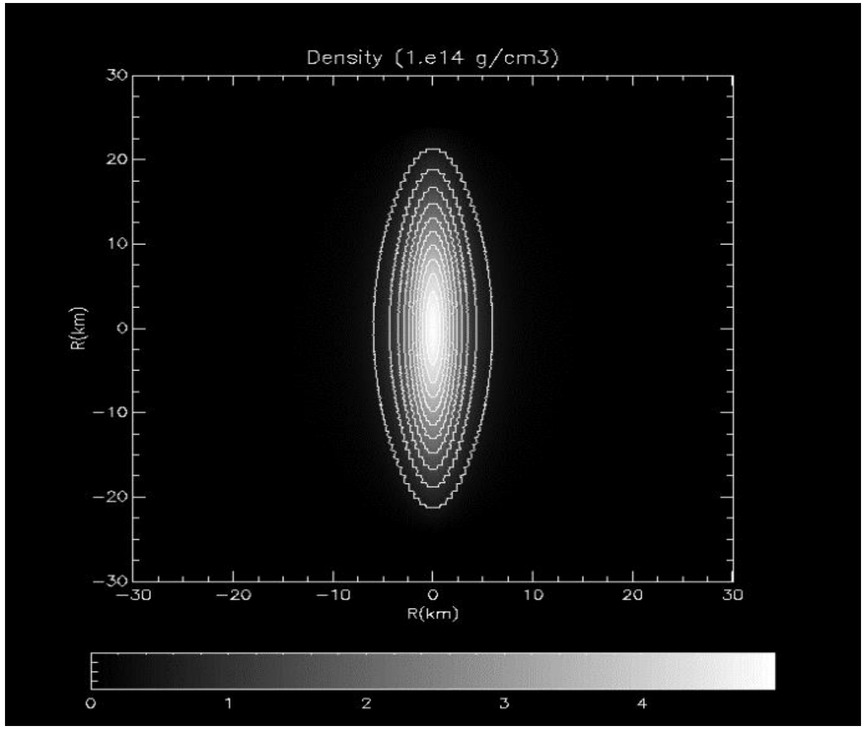
⬇

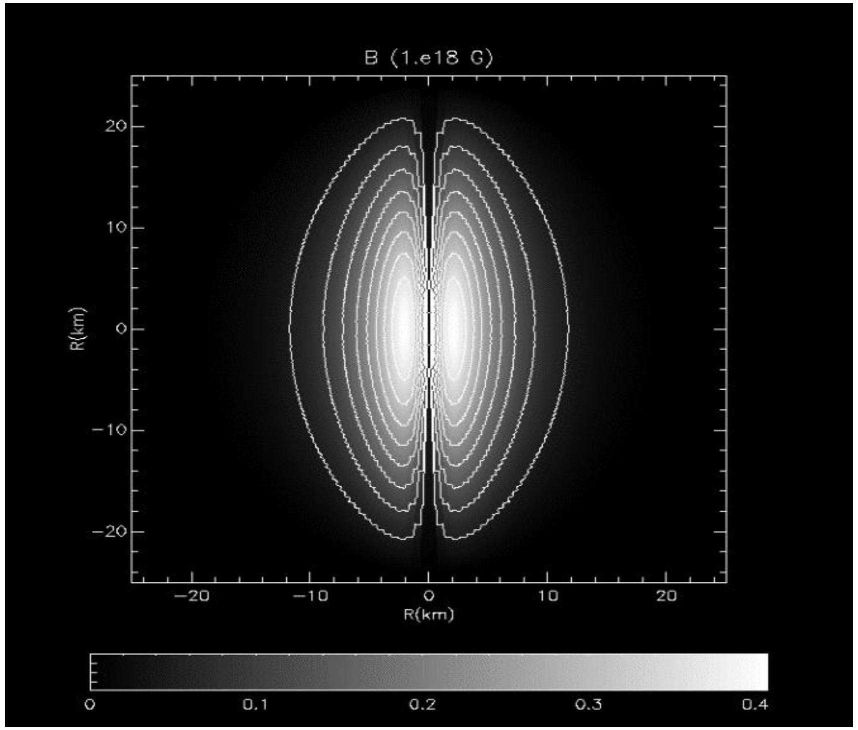
$$M = 1.52 M_{\odot}, R_E = 13.55\ km, R_P/R_E = 0.64$$



Density isocontour

Toroidal Magnetic field ($\vec{B} = B_\phi \hat{\phi}$) without rotation



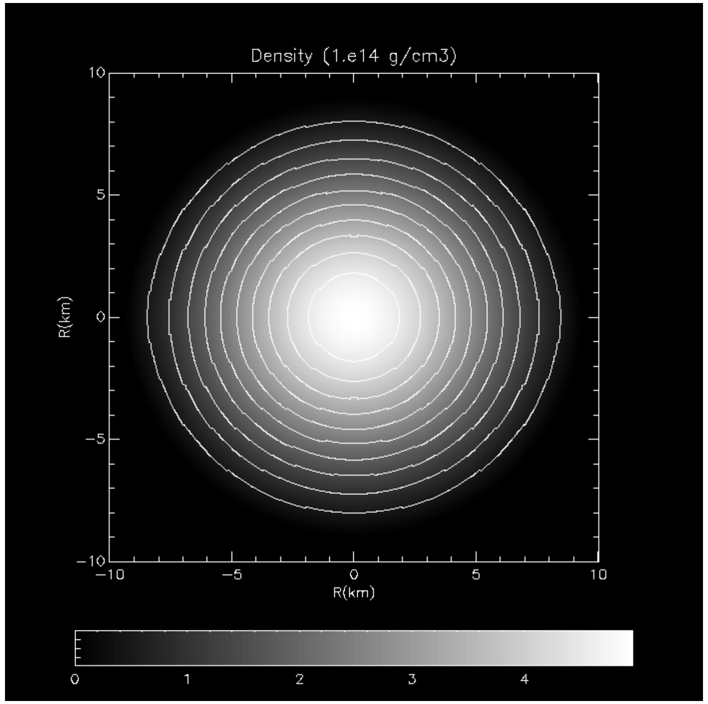


$\boldsymbol{\rho_c} = 5 \times 10^{14} \boldsymbol{g/cc}, \boldsymbol{B_{max}} = \mathbf{4.1 \times 10^{17}} \boldsymbol{G},$ ➡ $\boldsymbol{M} = \mathbf{1.32} \boldsymbol{M_\odot}, \boldsymbol{R_E} = 16km, \boldsymbol{R_P/R_E} = \mathbf{1.4}, ME/GE = 0.45$

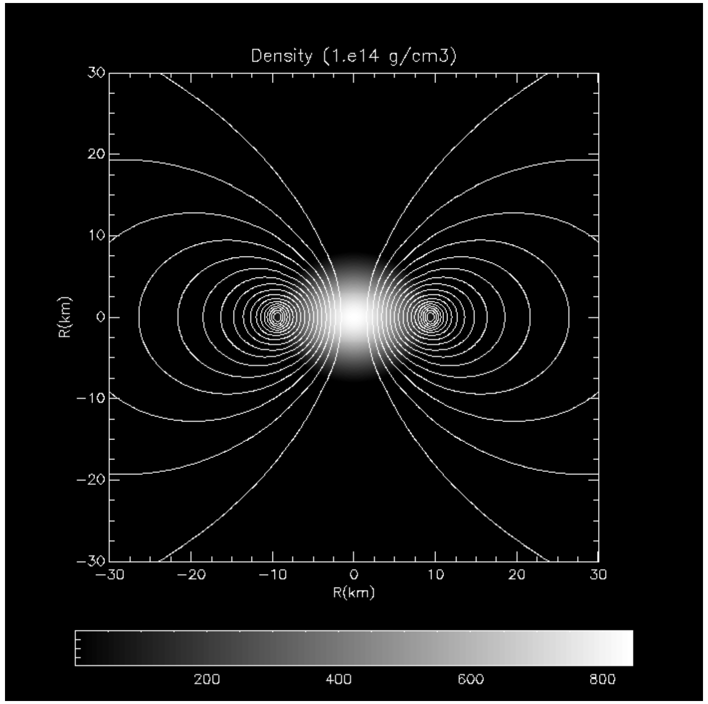
Twisted torus configuration

The poloidal magnetic field extends throughout the star and in the exterior, whereas the toroidal field is confined into a torus shaped region inside the star, where the field lines are closed.

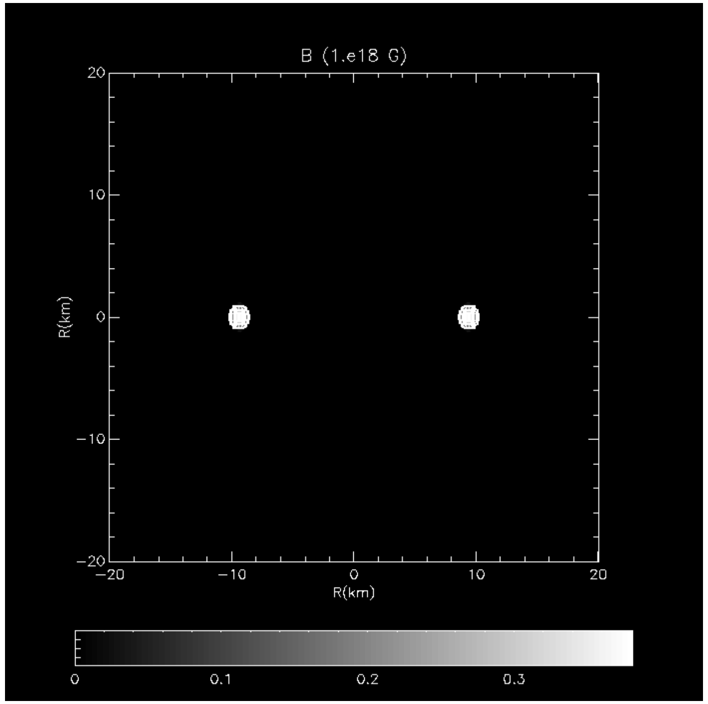
$$\rho_c = 5.1 \times 10^{14}\, g/cc, B_{max} = 1.45 \times 10^{17} G \Rightarrow M = 1.17\, M_{\odot}, R_E = 13.82\, km, R_P/R_E = 0.95, ME/GE = 0.017$$



Density isocontour

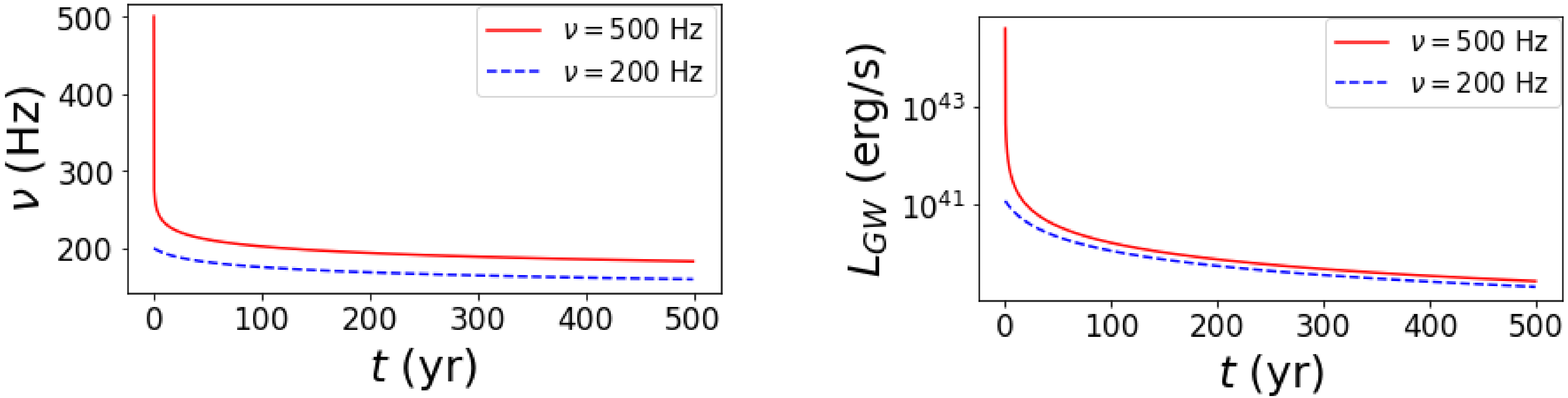


Poloidal Magnetic field isocontour



Toroidal Magnetic field isocontour

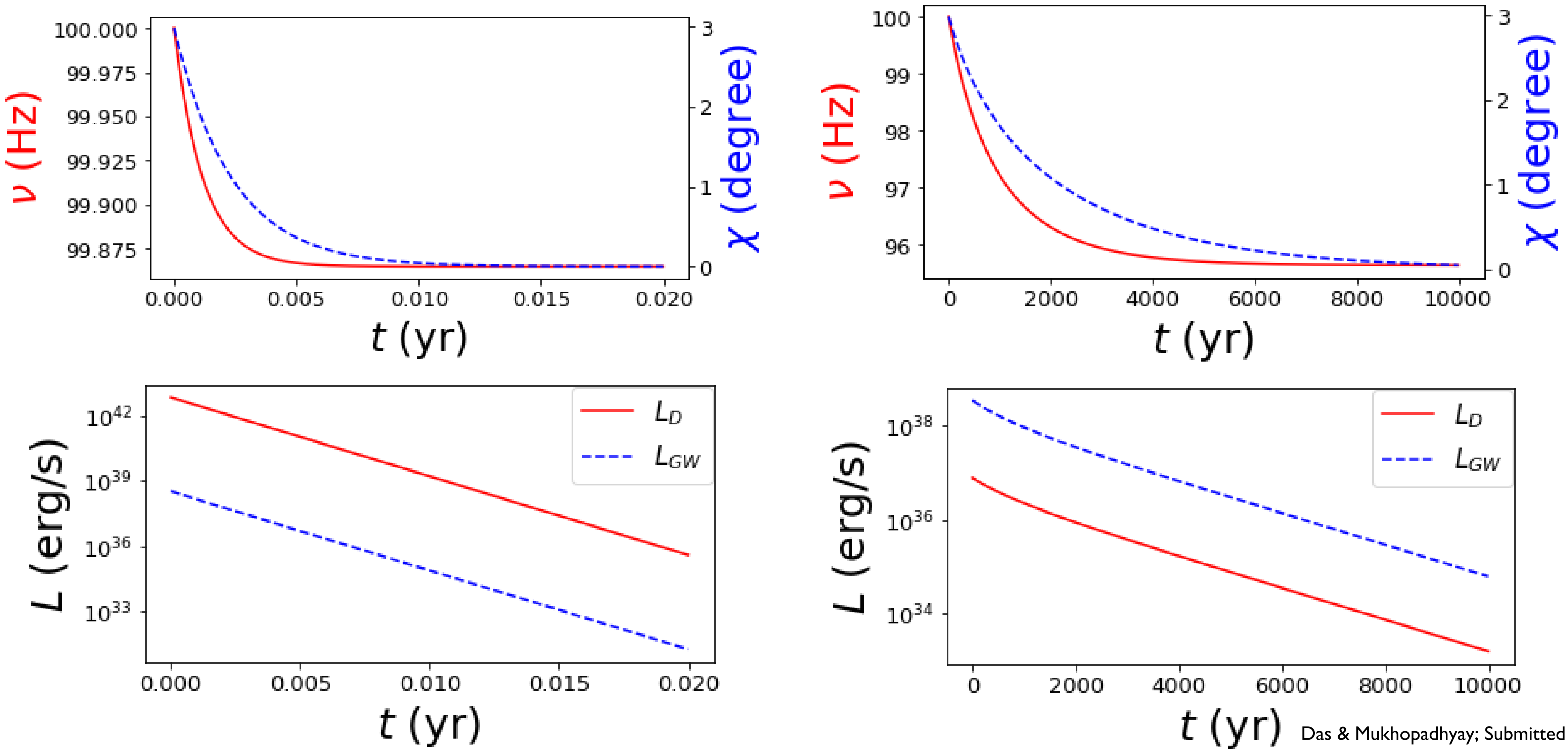
Evolution of ν and χ (B constant): Toroidal magnetic field



$$M = 2.02 M_{\odot}, B_{max}^{Toroidal}(initial) = 1.4 \times 10^{17} G$$

Das & Mukhopadhyay; Submitted (MNRAS)
arXiv:2302.03706

Evolution of ν and χ (B constant): Poloidal magnetic field

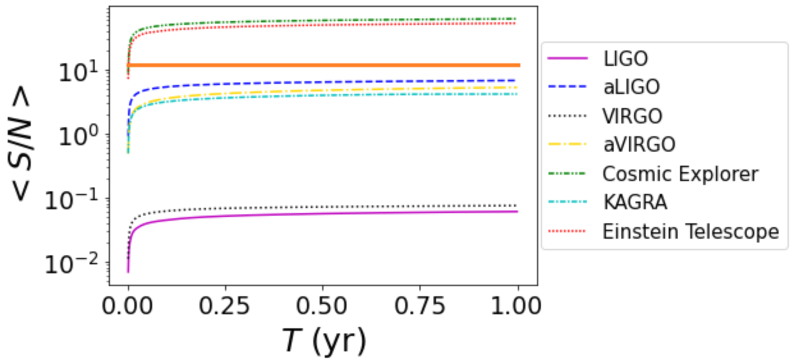


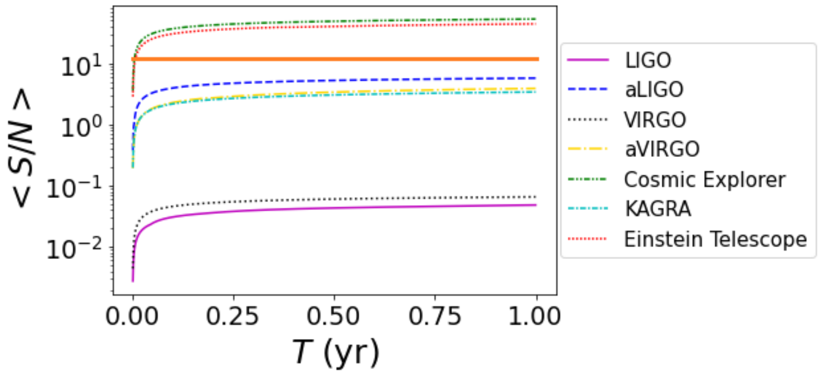
Das & Mukhopadhyay; Submitted (MNRAS)

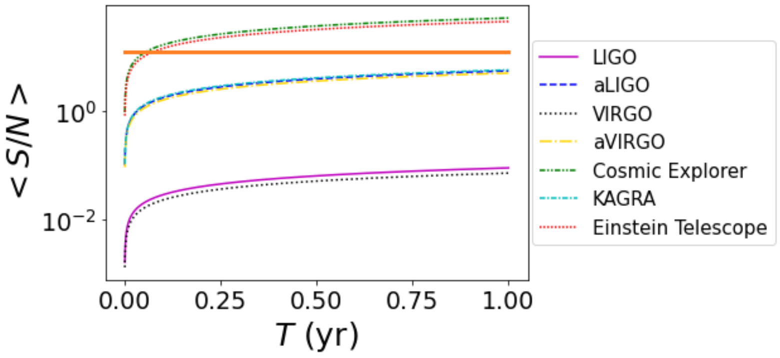
$$M = 2.02M_{\odot}, \nu(initial) = 100Hz$$

$B_{max}^{Poloidal}(initial) = 10^{15}G, L_D \gg L_{GW}$

$B_{max}^{Poloidal}(initial) = 10^{12}G, L_{GW} \gg L_D$

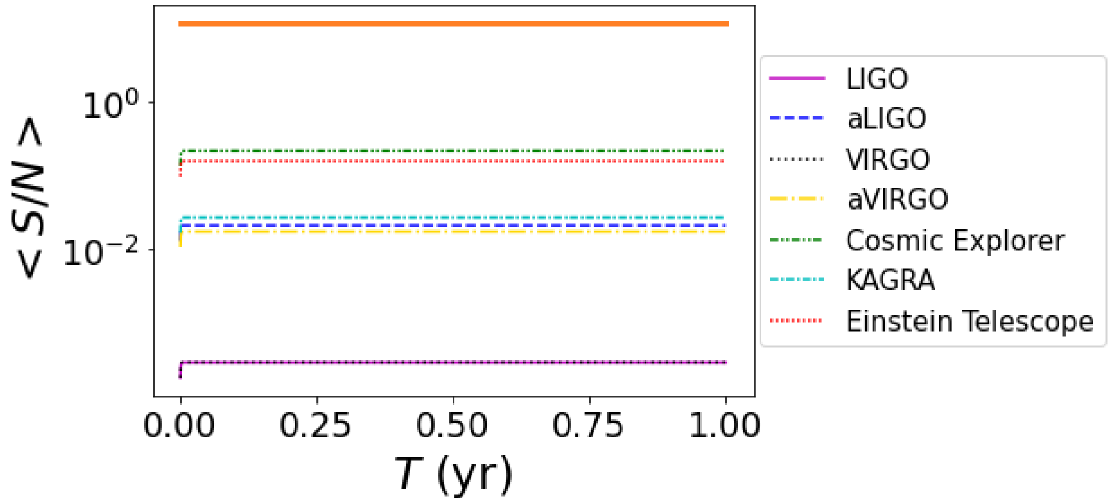


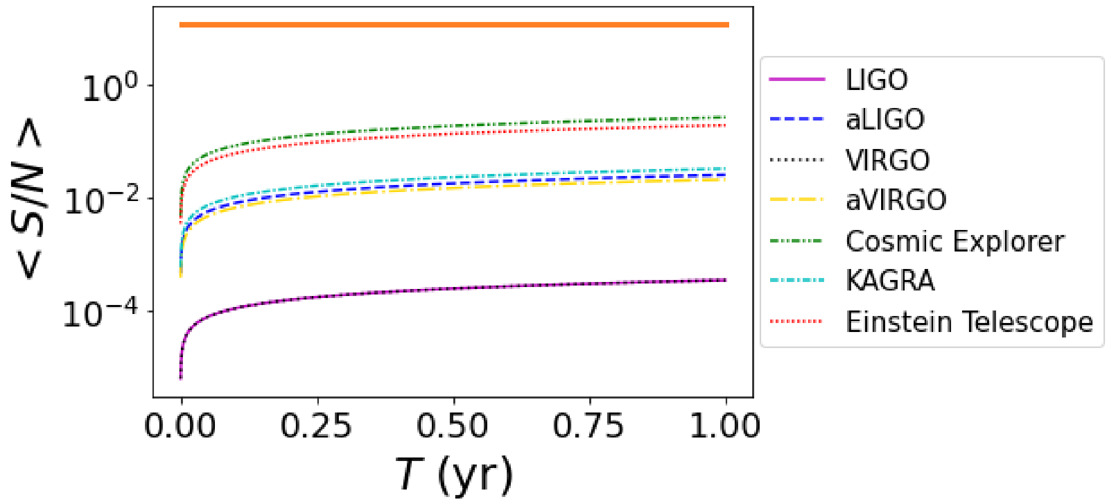




SNR: TOROIDAL MAGNETIC FIELD

Das & Mukhopadhyay; Submitted (MNRAS)
arXiv:2302.03706





SNR: POLOIDAL MAGNETIC FIELD

Das & Mukhopadhyay; Submitted (MNRAS)
arXiv:2302.03706

Magnetic braking

Differentially rotating NS (massive) with seed poloidal field → Magnetic braking and viscous damping → Uniformly rotating (less massive NS)

Magnetic braking: Alfvén timescale

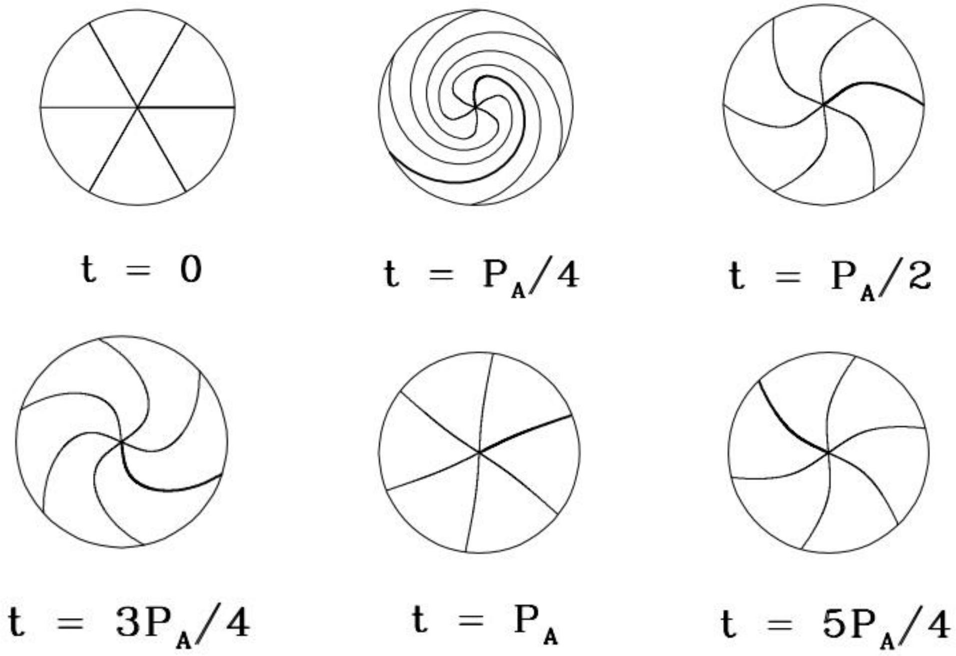
$$T_a(r) = \frac{r}{v_a(r)} = \frac{r\sqrt{4\pi\rho(r)}}{B(r)},$$

Shapiro 2000

Viscous damping: Viscous timescale

$$T_\nu \simeq 63.5 \left(\frac{R}{20km}\right)^{23/4} \left(\frac{T}{10^9 K}\right)^2 \left(\frac{M}{3M_\odot}\right)^{-5/4} yr.$$

Cutler & Lindblom (1987)



Cook et. al 2003

Magnetic field line configurations at selected times. The twisting of the field lines is due to differential rotation. Snapshots are given at time intervals of PA/4, where PA is the standing Alfvén wave period. Each frozen-in field line passes through the same fluid elements at all times. The bold field line is an arbitrarily chosen fiducial line.

MODELLING NS USING XNS

Einstein's equation solver in GRMHD
Pili et al. 2014

Input

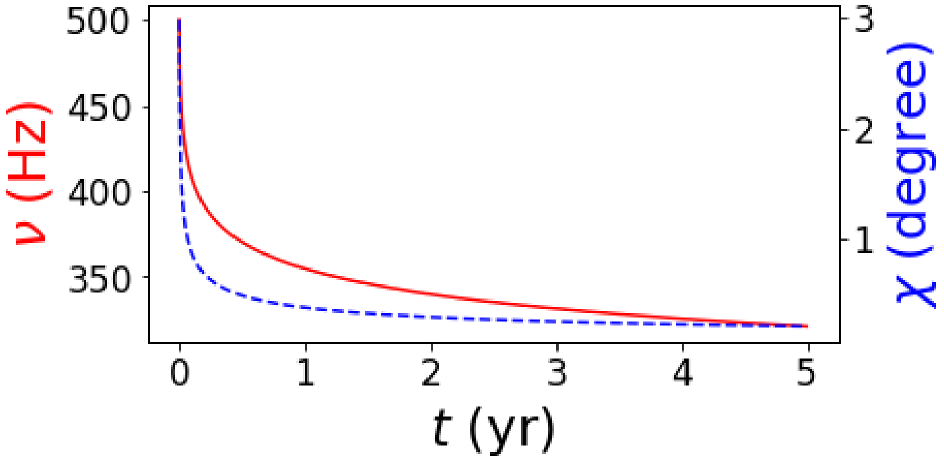
EOS: polytropic law $P = k\rho^{1+\frac{1}{n}}$,

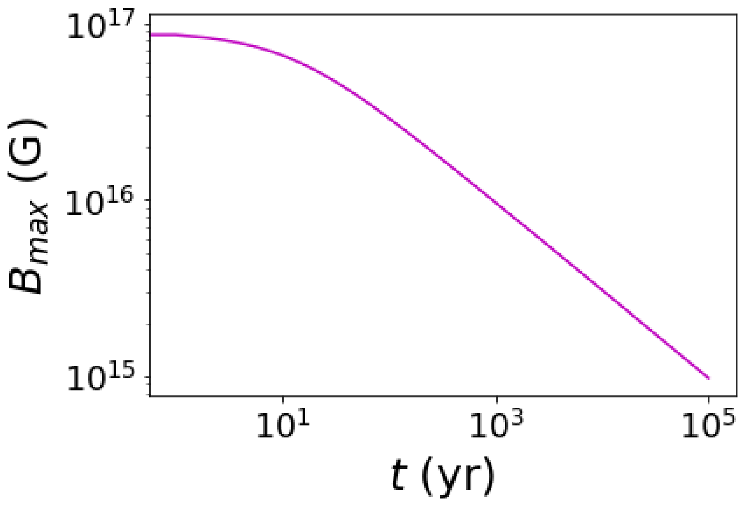
Magnetic field: $B \sim k_m \rho^m$ (Toroidal and/or Poloidal)

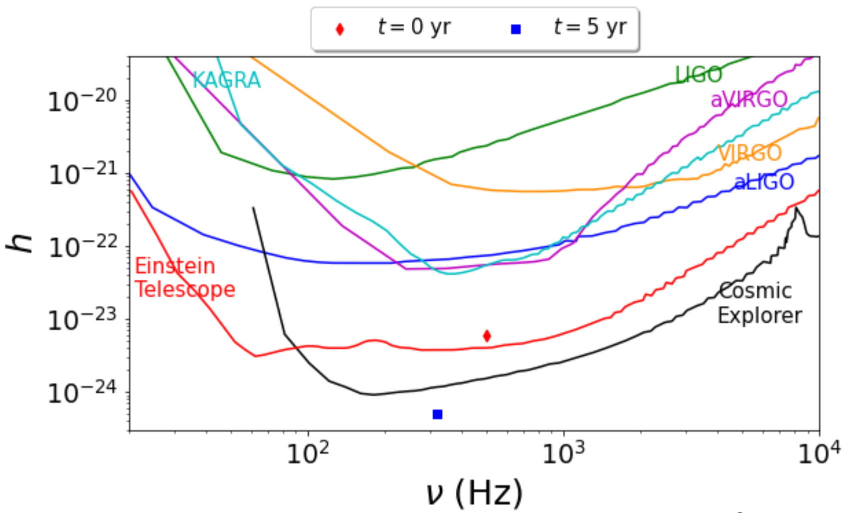
Output M, R, I_{xx}, I_{zz}

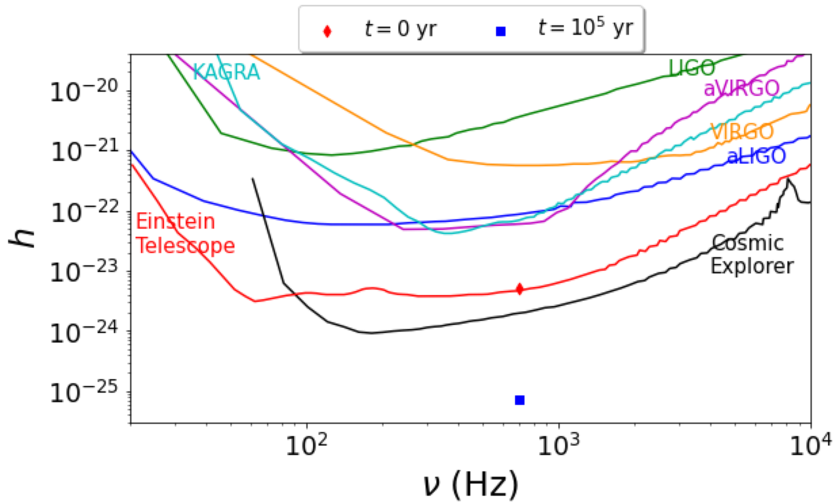
Evolution of ν and χ (B constant)

Evolution of B (ν, χ constant)









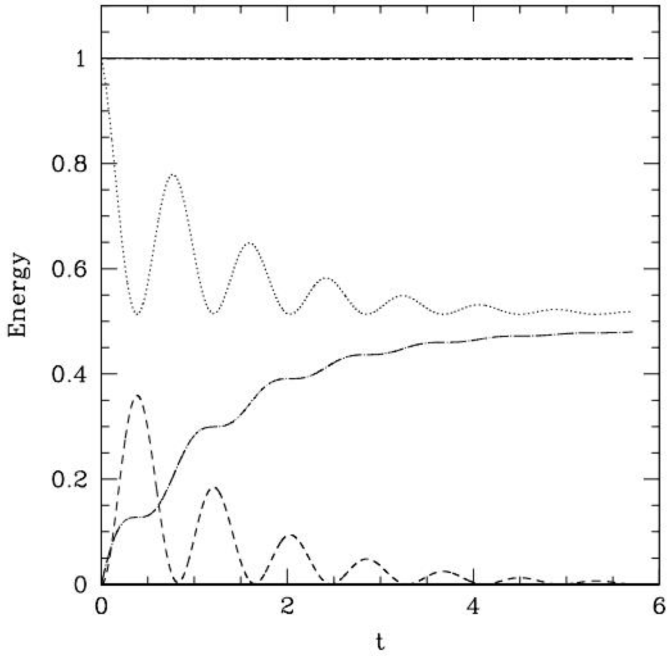
MD & Mukhopadhyay 2023
arXiv:2302.03706

$$M = 2.02 M_{\odot}, B_{max}^{Toroidal}(initial) = 9 \times 10^{16} G, \nu(initial) = 500 Hz$$

Timescale for ν, χ decay << Timescale for B decay

Differentially rotating and uniformly rotating poloidally dominated NSs, where ν_{cen} and ν_{eq} are respectively central and equatorial frequencies.

ρ_c(g/cc)	$M(M_\odot)$	R_E(km)	B_{max}(G)	ν_c(Hz)	ν_{eq}(Hz)	ME/GE	KE/GE	T_a(sec)	T_ν**(year)**
10^{15}	2.08	11.6	0	3800	338	0	5.45×10^{-2}		
10^{15}	1.9	11.99	2×10^{16}	0	0	4.6×10^{-5}	0	0.035	3.5
10^{15}	1.9	11.99	0	0	0	0	0		
10^{15}	1.927	12.14	2×10^{16}	338	338	5×10^{-5}	9.37×10^{-3}		
10^{15}	2.08	11.6	0	3800	338	0	5.45×10^{-2}		
10^{15}	1.9	11.99	1.1×10^{13}	0	0	4.6×10^{-5}	0	70.7	3.5
10^{15}	1.9	11.99	0	0	0	0	0		
10^{15}	1.927	12.14	1.1×10^{13}	338	338	1.2×10^{-11}	9.37×10^{-3}		



Das & Mukhopadhyay; Submitted (MNRAS)
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