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CENTER OF APPLIED SPACE TECHNOLOGY AND MICROGRAVITY ZARM

Quasi-periodic oscillations of a particle in the background of a deformed compact object

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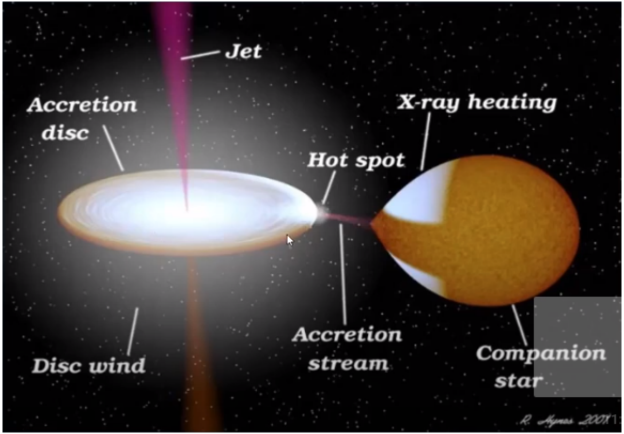
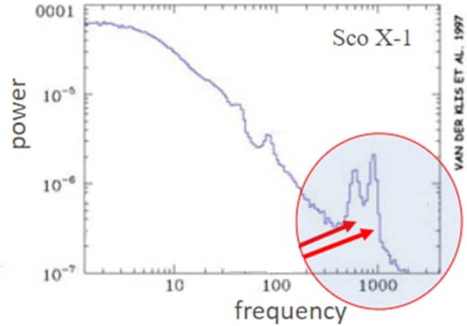
High Frequency QPOs and General Relativity



High Frequency QPOs and General Relativity



QPOs in BH divided in various classes. QPOs in BH XRBs are normally divided into two large groups:

- the low frequency QPOs ~ 50 Hz
- the high-frequency QPOs, above ~ 100Hz up to ~ 500Hz.

Source	GRO 1655 – 40	XTE 1550 – 564	GRS 1915 + 105
ν_U	447 – 453	273 – 279	165 – 171
ν_L	295 – 305	179 – 189	108 – 118
$\frac{M}{M_\odot}$	6.03 – 6.57	8.5 – 9.7	9.6 – 18.4
a	0.65 – 0.75	0.29 – 0.52	0.98 – 1

TABLE II. Observed HF QPO data for the three micro-quasars, independent of the HF QPO measurement, and based on the spectral continuum fitting.

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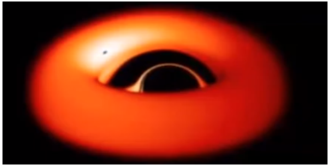
Model to explain HFQPOs

Large variety of ideas to explain the phenomenon of HF QPOs: Main idea is that it is related to the motion of the inner part of accretion disk



- RP Model (Stella)
- TD Model (Cadez)
- WD Model (Kato)
- ER Model (Abramowicz & Kluzniak)

Background metric : q-Metric

A static, axially symmetric metric that is non-spherically symmetric

$$ds^2 = -\left(\frac{x-1}{x+1}\right)^{(1+\alpha)} dt^2 + M^2(x^2-1)\left(\frac{x+1}{x-1}\right)^{(1+\alpha)} \left[\left(\frac{x^2-1}{x^2-y^2}\right)^{\alpha(2+\alpha)}\left(\frac{dx^2}{x^2-1}+\frac{dy^2}{1-y^2}\right)+(1-y^2)d\phi^2\right]. \quad (1)$$

- describes the exterior gravitational field of a static deformed compact object
- α positive for a oblate compact object and negative for an prolate one

Class of orbits that slightly deviate from the circular geodesics

$$\omega_x^2 = \partial_x U^x - \gamma^x_{\eta}\gamma^{\eta}_x = A_x \Omega^2, \quad \omega_y^2 = \partial_y U^y = A_y \Omega^2. \tag{2}$$

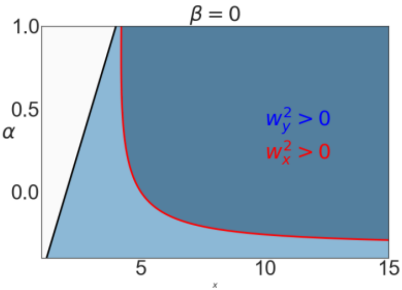
$$A_x = \frac{(1-1/x^2)^{-\alpha(2+\alpha)}}{(x^2-1)}\left[2(2S-x)(S-x) - \frac{(x^2-1)}{S}(1+\alpha)\right]$$

$$A_y = \frac{(1-1/x^2)^{-\alpha(2+\alpha)}}{S}(1+\alpha)$$

Schwarzschild case:

$$\omega_y^2 = \frac{1}{(x+1)^3},$$

$$\omega_x^2 = \frac{1}{(x+1)^3}\left(1 - \frac{6}{x+1}\right).$$

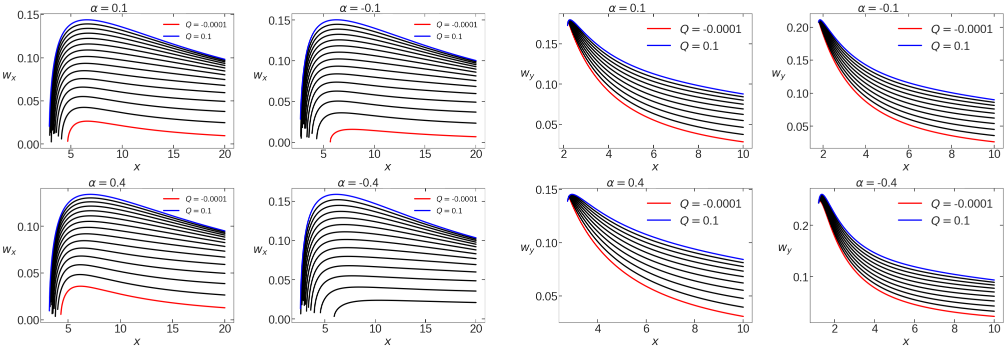


Class of orbits that slightly deviate from the circular geodesics: In a uniform magnetic field

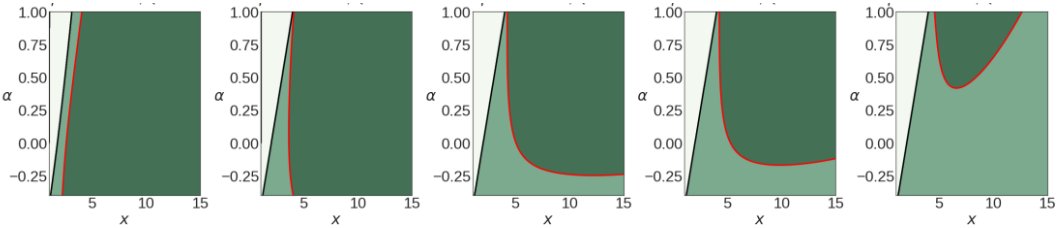
$$\frac{d^2\xi^\mu}{dt^2} + 2\gamma^\mu_\eta \frac{d\xi^\eta}{dt} + \xi^\eta \partial_\eta U^\mu = F^\mu,$$

$$\begin{aligned} w_y^2 &= \left(\frac{x^2-1}{x^2}\right)^{-\alpha(2+\alpha)} \left[\Omega^2 \frac{x f_1(x) + S}{S} + (1 + f_1(x))\Omega\omega_\mathrm{B}\right], \\ w_x^2 &= \frac{\Omega^2(1-1/x^2)^{-\alpha(2+\alpha)}}{x(1-x^2)} \left[g_1(x,\alpha)\frac{x-S}{S} + g_2(x,\alpha)\right] \\ &+ \frac{(1-1/x^2)^{-\alpha(2+\alpha)}}{x(1-x^2)} \left[-\omega_B^2 x(S-x)^2 + \Omega\omega_\mathrm{B} g_2(x,\alpha)\right], \end{aligned} \quad (3)$$

Epicyclic frequencies in the background of a deformed compact object



Condition for circular geodesic



Epicyclic resonances - 3:2 frequency ratio

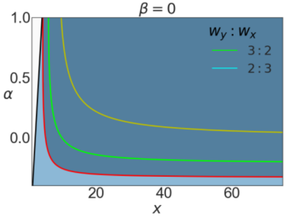
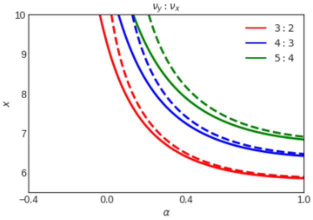
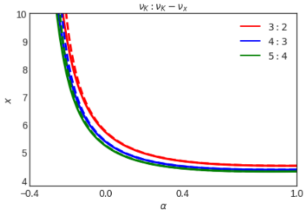


Table 1. Frequency relations corresponding to individual QPO models

Model	ν_U	ν_L
RP	Ω	$\Omega - \omega_x$
Kp	Ω	ω_x
Ep	ω_y	ω_x
TD	$\Omega + \omega_x$	Ω
WD	$2\Omega - \omega_x$	$2\Omega - 2\omega_x$
RP1	ω_y	$\Omega - \omega_x$
RP2	$2\Omega - \omega_y$	$\Omega - \omega_x$





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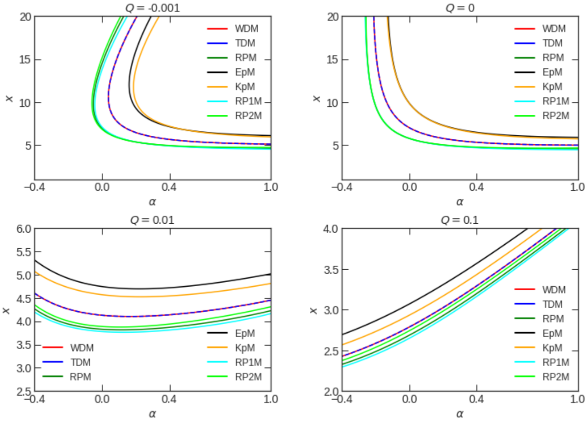
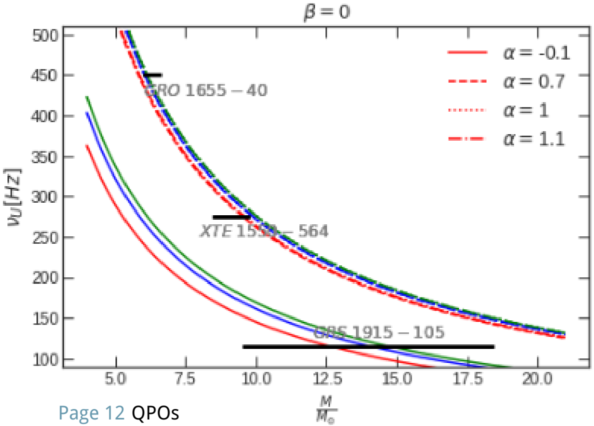
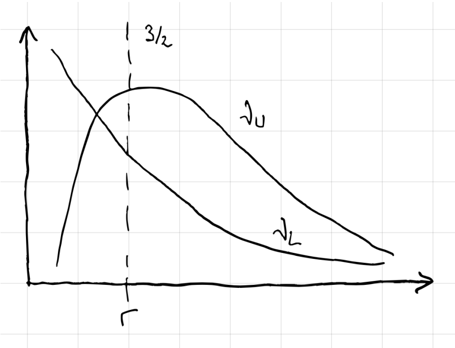
Epicyclic resonances - 3:2 frequency ratio



Data Fitting with Galactic Microquasars: $1/M$ relation



$$\nu_{Up} = \frac{1}{2\pi} \frac{c^3}{GM} \Omega_{Up}$$



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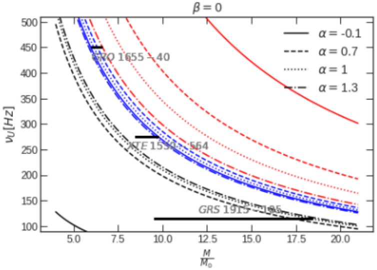

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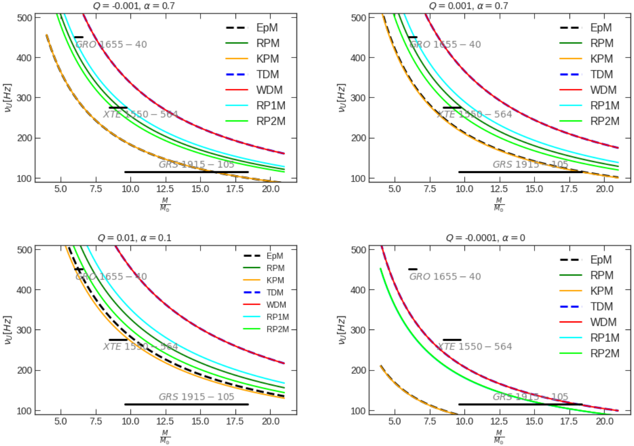

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Conclusion

- The quadrupole moment and magnetic field alter the motion and epicyclic frequencies of charged particles moving in this background.
- cause strong deviation from the corresponding quantities in the Schwarzschild case
- the resonant phenomena of the radial and vertical oscillations at their frequency ratio 3:2 for different parameters can be adequately related to the frequencies of the twin 3:2 HF QPOs observed in the microquasars

Future Work and Questions:

- extend this work from a single particle to a complex system, such as accretion discs
- Spin effect

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Thank you for your attention!

Class of orbits that slightly deviate from the circular geodesics

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{ds}\frac{dx^\rho}{ds} = 0. \quad (5)$$

$x'^\mu = x^\mu + \xi^\mu$ and consider terms up to linear order in ξ^μ

$$\frac{d^2\xi^\mu}{dt^2} + 2\gamma^\mu_{\ \eta}\frac{d\xi^\eta}{dt} + \xi^\eta\partial_\eta U^\mu = 0,$$

$$\gamma^\mu_{\ \eta} = \left[2\Gamma^\mu_{\ \eta\delta}u^\delta(u^0)^{-1}\right]_{y=0},$$

$$U^\mu = \left[\gamma^\mu_\eta u^\eta(u^0)^{-1}\right]_{y=0}.$$

$$\frac{d\xi^\eta}{dt} + \gamma^\eta_{\ \nu}\xi^\nu = 0, \quad \frac{d^2\xi^x}{dt^2} + \omega_x^2\xi^x = 0, \quad (6)$$

$$\frac{d^2\xi^y}{dt^2} + \omega_x^2\xi^y = 0, \quad (7)$$

where in the first equation η can be taken t, or ϕ;

$$\omega_x^2 = \partial_x U^x - \gamma^x_{\ \eta}\gamma^\eta_{\ x}, \quad (8)$$

$$\omega_y^2 = \partial_y U^y. \quad (9)$$