Distinguishing Frames through Entanglement

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Outline

- Perturbative quantum gravity and how to probe it?
- The existence of fifth force through entanglement.
- Implications for de Sitter spacetime.

Reference

- **SC**, Mazumdar and Pradhan, PRD 108, L121505 (2023).
- **SC**, Mazumdar and Pradhan, work in progress.

Why “Quantum” Gravity?

- Massive quantum particle in spatial superposition requires gravity to be quantum as well.

- Where can we ‘see’ such quantum effects? — (a) near the black hole horizon, (b) near the black hole singularity, (c) in the early universe physics, and (d) in the perturbative domain.

- Near the black hole horizon — probed by gravitational waves.

- Early universe physics — probed by scalar power spectrum and primordial GW.

- Perturbative domain — probed by ‘table-top’ experiments involving entanglement between massive objects.

Entanglement between Masses

- Two quantum systems with mass entangle through gravitational interaction, if and only if, gravity is inherently quantum.

[Bose +, Phys. Rev. D 105, 106028 (2022)]

- The two systems are taken to be harmonic oscillators.

$$\hat{H} = \hat{H}_A + \hat{H}_B \; ; \qquad \hat{H}_{A,B} = \frac{1}{2m}\hat{p}^2_{A,B} + \frac{m\omega^2}{2}\delta\hat{x}^2_{A,B}$$

$$\hat{x}_A = -\frac{d}{2} + \delta\hat{x}_A \; ; \; \hat{x}_B = -\frac{d}{2} + \delta\hat{x}_B$$

- Masses interact gravitationally, approximated as a linear system.

$$h_{\mu\nu} = \frac{1}{\kappa}\left(g_{\mu\nu} - \eta_{\mu\nu}\right)$$

$$\gamma_{\mu\nu} := h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu}$$

[Gupta, Proc. Phys. Soc. A65, 161 (1952)]

- The linearized gravitational field is then quantized subsequently.

$$\left[\gamma_{\mu\nu}(x), \gamma_{\alpha\beta}(x')\right] = i\left(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}\right) D\left(x - x'\right)$$

$$\left[\gamma(x), \gamma(x')\right] = -4iD(x - x')$$

$$\gamma_{\mu\nu} = \frac{1}{(2\pi)^{3/2}}\int\frac{\mathrm{d}^3k}{\sqrt{2\omega_{\boldsymbol{k}}}}\left[a_{\mu\nu}\left(\boldsymbol{k}\right)\mathrm{e}^{ikx} + a^\dagger_{\mu\nu}\left(\boldsymbol{k}\right)\mathrm{e}^{-ikx}\right]$$

$$\gamma = \frac{2}{(2\pi)^{3/2}}\int\frac{\mathrm{d}^3k}{\sqrt{2\omega_{\boldsymbol{k}}}}\left[b\left(\boldsymbol{k}\right)\mathrm{e}^{ikx} + b^\dagger\left(\boldsymbol{k}\right)\mathrm{e}^{-ikx}\right]$$

The Interaction Hamiltonian

- The total Lagrangian has both gravity and matter: $L_{\text{total}} = L_{\text{grav}}[g] + L_{\text{mat}}[g, \phi]$
- The Lagrangian can be expanded around flat background.

$$L_{\text{total}} = L_{\text{grav}}[\eta] + \left[\frac{\delta L_{\text{grav}}}{\delta g^{\mu\nu}}\right]_{\eta_{\mu\nu}} \delta g^{\mu\nu} + \frac{1}{2!}\left[\frac{\delta^2 L_{\text{grav}}}{\delta g^{\mu\nu}\delta g^{\alpha\beta}}\right]_{\eta_{\rho\sigma}} \delta g^{\mu\nu}\delta g^{\alpha\beta} + L_{\text{mat}}[\eta, \phi] + \left[\frac{\delta L_{\text{mat}}}{\delta g^{\mu\nu}}\right]_{\eta_{\alpha\beta}} \delta g^{\mu\nu}$$

- This will provide quadratic action for gravitational perturbation and coupling between gravity and matter.
- We will be interested in this coupling and ask if this coupling is same in Einstein and Jordan frames.

$$L_{\text{total}} = L^{(2)}_{\text{grav}}[h] + L_{\text{mat}}[\eta, \phi] + \frac{1}{2}T_{\mu\nu}h^{\mu\nu}$$

- Non-invariance will lead to possible probe of fifth force using quantum entanglement due to gravity.

From Einstein to Jordan

- Two frames are related by conformal transformation: $g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Omega^2(x) g_{\mu\nu}$
- Perturbative gravitational field gets corrected. $$\begin{aligned} \tilde{g}_{\mu\nu} &= \Omega^2 g_{\mu\nu} = (1+\phi)\left(\eta_{\mu\nu} + h_{\mu\nu}\right) \\ &= \eta_{\mu\nu} + \left(h_{\mu\nu} + \phi\eta_{\mu\nu}\right) \ , \end{aligned}$$
- The interaction Hamiltonian now involves an extra (scalar+matter) interaction term.

$$\begin{aligned} \widetilde{H}_{\rm int} &= -\frac{1}{2}\int d^3\mathbf{x}\, \widetilde{T}_{\mu\nu}(t,\mathbf{x})\widetilde{h}^{\mu\nu}(t,\mathbf{x}) \ , \\ &= H_{\rm int} - \frac{1}{2}\int d^3\mathbf{x}\ \phi(t,\mathbf{x})T(t,\mathbf{x}) \ , \end{aligned}$$

$$T_{\mu\nu}(\mathbf{r}) = \frac{p_\mu p_\nu}{E/c^2}\left[\delta(\mathbf{r}-\mathbf{r}_A) + \delta(\mathbf{r}-\mathbf{r}_B)\right]$$

- Thus interaction between matter degrees of freedom is mediated by both gravity and scalar.

[SC +, Phys. Rev. D 108, L121505 (2023)]

An Example: f(R) Theories of Gravity

- The f(R) theories of gravity have a scalar-tensor analog.
- The perturbative theory yields three propagating degrees of freedom — (a) massless spin-2 modes and (b) massive spin 0 mode.

$$\gamma_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu} - F''(0)\Phi\eta_{\mu\nu}$$

- Since background is flat, we must have f(R)=R + F(R). The interaction Hamiltonian becomes:

$$\begin{aligned} \widetilde{H}_{\rm int} &= -\frac{1}{2}\int d^3\mathbf{r}\; h^{\mu\nu}T_{\mu\nu} \\ &= -\frac{1}{2}\int d^3\mathbf{r}\;\left[\gamma^{\mu\nu}T_{\mu\nu} - \frac{1}{2}\gamma T - 2\alpha\Phi T\right] \end{aligned}$$

[SC +, Phys. Rev. D 108, L121505 (2023)]

‘Quantum’ Interaction in Einstein Frame

- The interaction changes the gravitational energy between the masses.

$$\Delta \hat{H}_g \equiv \sum \int d\boldsymbol{k} \, \frac{{}_{\rm g}\langle 0|\hat{H}_{\rm int}|\boldsymbol{k}\rangle_{\rm g}\, {}_{\rm g}\langle \boldsymbol{k}|\hat{H}_{\rm int}|0\rangle_{\rm g}}{E_0^{\rm g} - E_{\boldsymbol{k}}^{\rm g}}$$

$$= -\frac{Gm^2}{|\hat{\mathbf{r}}_A - \hat{\mathbf{r}}_B|} + \frac{4G\hat{p}_A\hat{p}_B}{c^2|\hat{\mathbf{r}}_A - \hat{\mathbf{r}}_B|} - \frac{9G\hat{p}_A^2\hat{p}_B^2}{4c^4m^2|\hat{\mathbf{r}}_A - \hat{\mathbf{r}}_B|}$$

- As a consequence the masses, residing at the ground state of the harmonic oscillator will make transition to excited states.

[Bose +, Phys. Rev. D 105, 106028 (2022)]

- This leads to an entangled state and hence mixed density matrix for either of these harmonic oscillators.
- Existence of an entangled state is a direct evidence of ‘quantum’ gravity.

‘Quantum’ Interaction in Jordan Frame

- In Jordan frame there is an additional scalar degree of freedom. Thus the interaction between the harmonic oscillators will have an extra term.

$$\Delta\hat{H}_\phi = \int d^3\mathbf{k}' \; \frac{{}_\phi\langle 0|\hat{H}^{\phi}_{\rm int}|\mathbf{k}'\rangle_\phi \; {}_\phi\langle \mathbf{k}'|\hat{H}^{\phi}_{\rm int}|0\rangle_\phi}{E^{\phi}_0 - E^{\phi}_{\mathbf{k}'}}$$

- The total interaction between the masses become,

$$\Delta\hat{H}_{\rm AB} = \Delta\hat{H}_g + \Delta\hat{H}_\phi = -\frac{(G + \mathcal{G}e^{-(\hat{\mathbf{r}}_\mathbf{A} - \hat{\mathbf{r}}_\mathbf{B})/L})m^2}{|\hat{\mathbf{r}}_A - \hat{\mathbf{r}}_B|} + \frac{4G\hat{p}_A\hat{p}_B}{c^2|\hat{\mathbf{r}}_A - \hat{\mathbf{r}}_B|} - \frac{9(G + \frac{1}{9}\mathcal{G}e^{-(\hat{\mathbf{r}}_\mathbf{A} - \hat{\mathbf{r}}_\mathbf{B})/L})\hat{p}_A^2\hat{p}_B^2}{4m^2c^4\;|\hat{\mathbf{r}}_A - \hat{\mathbf{r}}_B|}$$

[SC +, Phys. Rev. D 108, L121505 (2023)]

Key Differences

1. The scalar field modifies the interaction.

2. The effect at leading order is a mere change in the gravitational constant by a Yukawa coupling.

3. However, the effects on next-to-leading orders are different — (a) the scalar does not affect terms of order $(1/c^2)$; (b) it affects terms of $\mathcal{O}(1/c^4)$ albeit in a different manner.

4. Thus effect of the scalar is not a simple modification to the gravitational coupling. It is more intricate and depends on the pN order.

Modified State

- The combined two-oscillator system, originally in the ground state, finally becomes an entangled state.

$$|\psi_{\rm f}\rangle_{AB} = \mathcal{N}\Big[|0\rangle_A|0\rangle_B - \frac{g_1+g_2}{2\omega_m}|1\rangle_A|1\rangle_B + \frac{g_3}{2\omega_m}|2\rangle_A|2\rangle_B\Big] ,$$

$$g_1 = \frac{(G+\mathcal{G}e^{-d/L})m}{d^3\omega_m} , \quad g_2 = \frac{2Gm\omega_m}{c^2 d} , \quad g_3 = \frac{(9G+\mathcal{G}e^{-d/L})\hbar\omega_m^2}{16c^4 d}$$

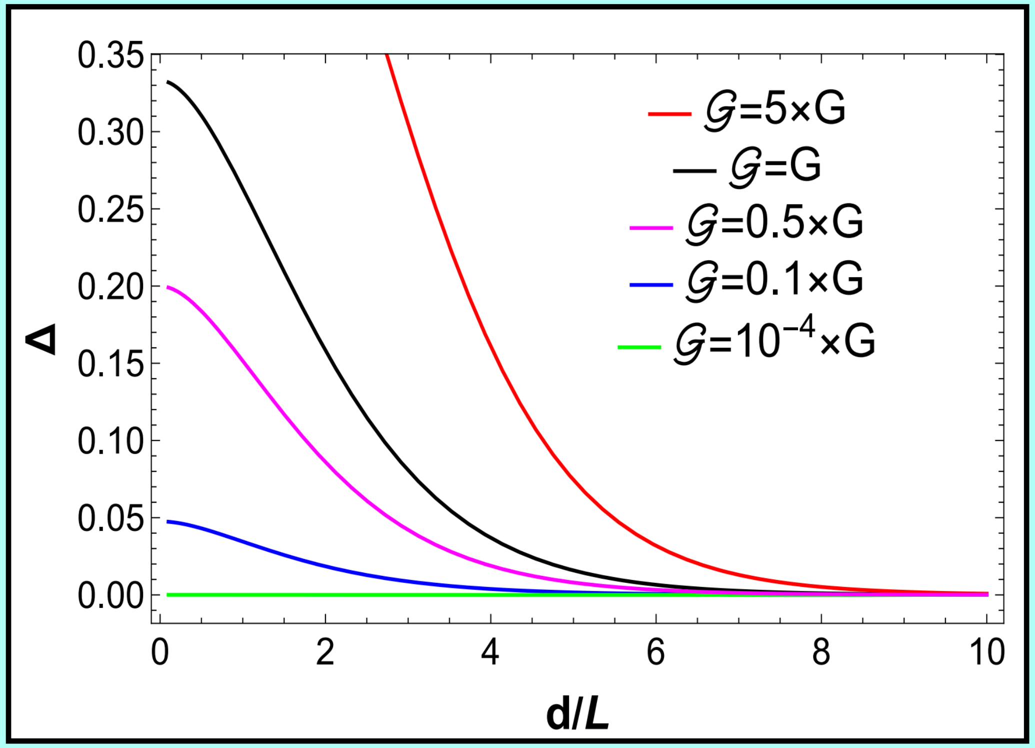
- This leads to a mixed density matrix, from which a non-zero concurrence, for traced over density matrix, can be obtained.
- The presence of an extra scalar degree of freedom enhances the concurrence, leading to stronger entanglement.

$$C = \sqrt{2\left(1 - \mathrm{tr}\left(\hat{\rho}_A^2\right)\right)}$$

Fifth force?

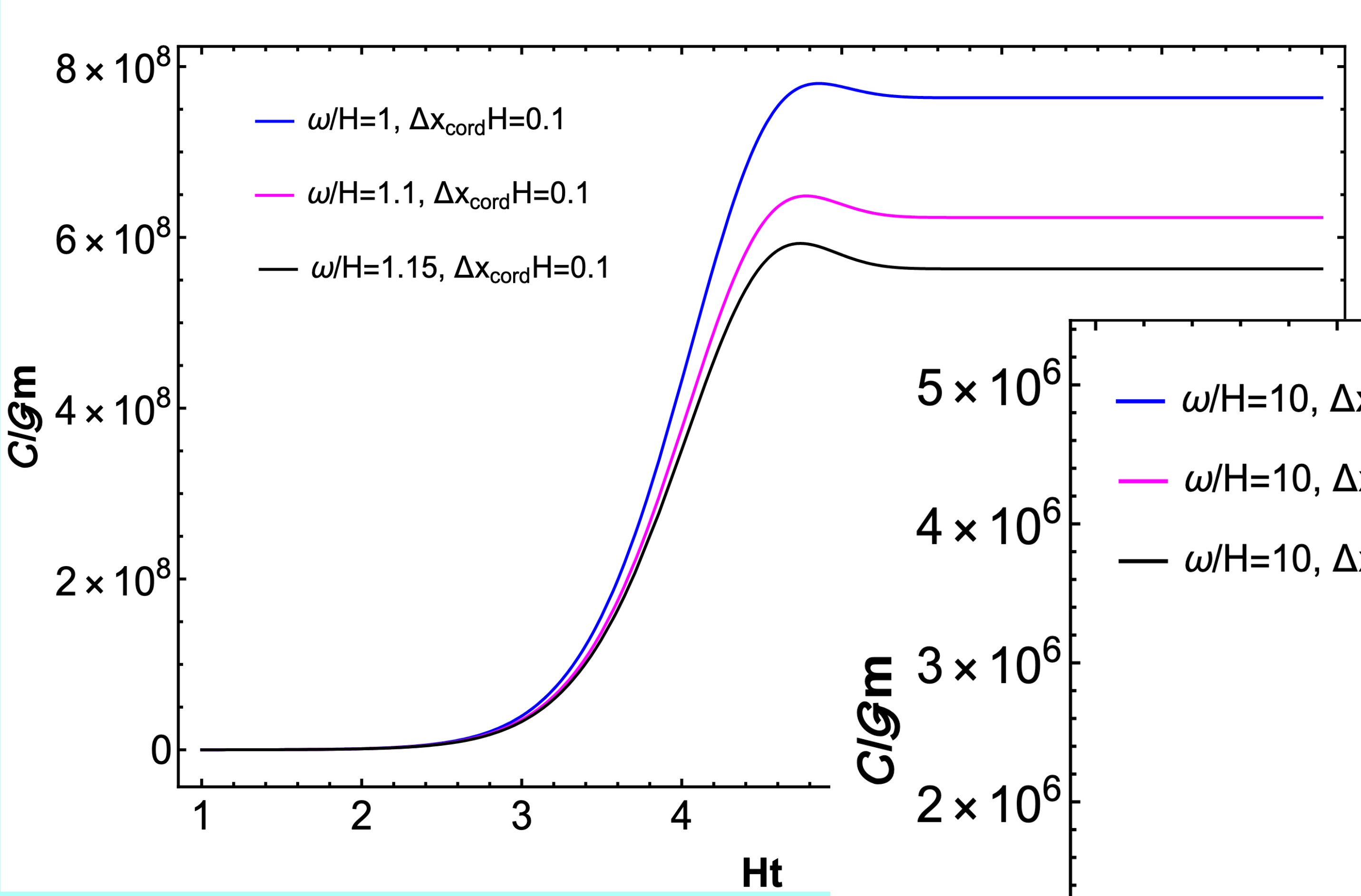
- The concurrence in the absence of scalar field (in Einstein frame) is fixed by Newton's constant and properties of the harmonic oscillators (mass, frequency) and separation.
- In presence of scalar field (in Jordan frame), the concurrence has extra contribution and depends on the strength of the scalar perturbation.

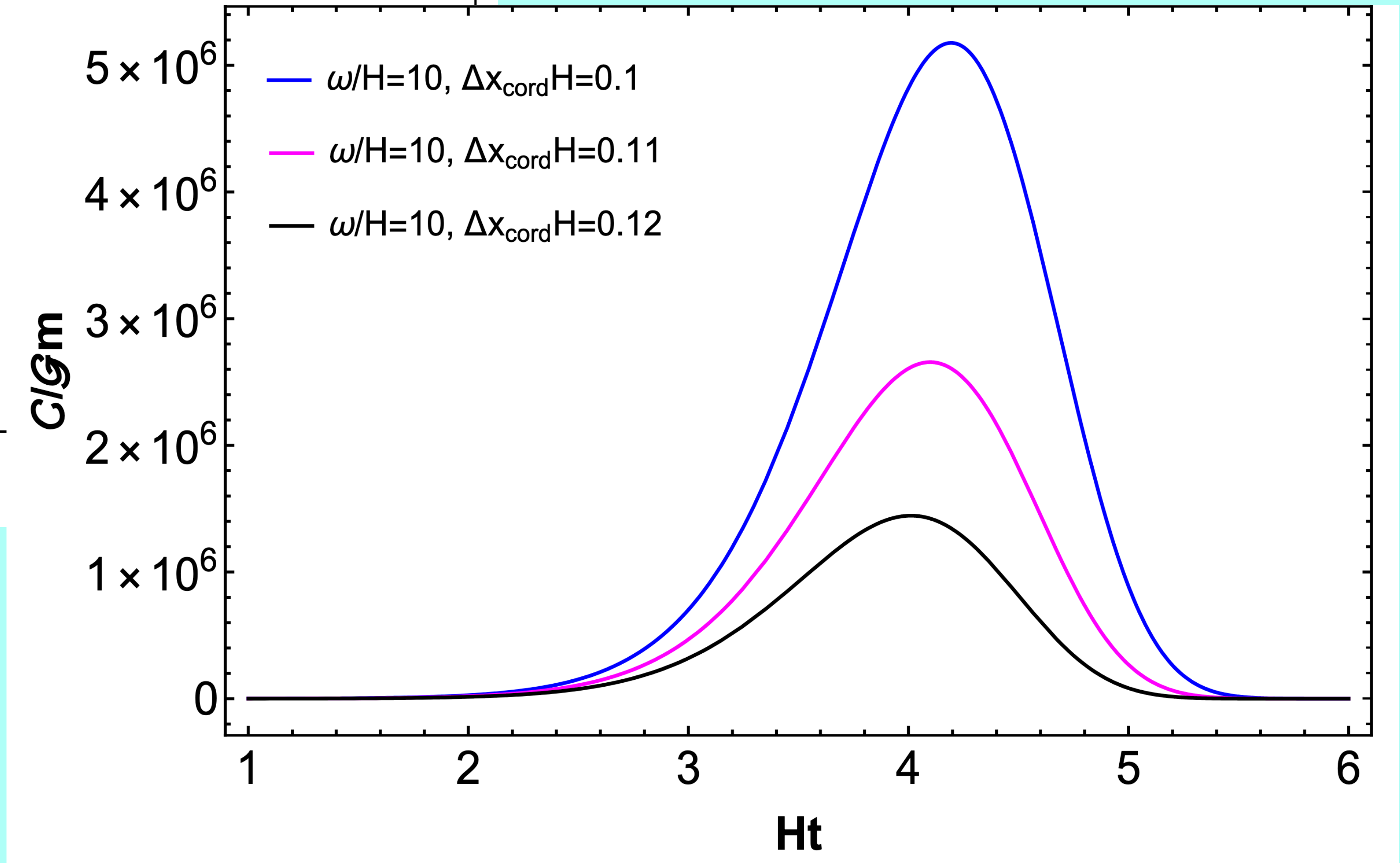
$$\Delta \equiv \frac{C^J(\mathcal{G}) - C^E(\mathcal{G}=0)}{C^J(\mathcal{G}) + C^E(\mathcal{G}=0)}$$



[SC +, Phys. Rev. D 108, L121505 (2023)]

Entanglement in de Sitter





Conclusion

- At linear level, the perturbative ‘quantum’ gravity can distinguish between Einstein and Jordan frame.
- Higher values of concurrence, compared to gravity, signals existence of a fifth force, possibly scalar mediated.
- In de Sitter universe, the concurrence for reasonable oscillators decay with time — implications for quantum to classical transition in the early universe.

Thank You

Restoring Units: GWs as Magnifying Glass

- Let $\Delta A = \ell_{\rm p}^2$. For non-rotating black holes, $A = 16\pi(GM/c^2)^2$.
- Therefore, the frequency associated with a GW making the above change of area becomes,

$$\nu(\text{in Hz}) = \frac{\ell_p^2}{32\pi(G^2/c^6)Mh} = \frac{c^3}{64\pi^2 GM}$$

- For $M = 10M_{\odot}$, the corresponding frequency becomes $32\ \text{Hz}$!
- Precisely in the LIGO frequency band ! **[Agullo +, Phys. Rev. Lett. 126, 041302 (2021)]**
- Thus GWs act as a magnifying glass for quantum effects near the horizon. Information about Hawking Radiation is also encoded !