ON THE UNIQUENESS OF ΛCDM-LIKE EVOLUTION FOR HOMOGENEOUS AND ISOTROPIC COSMOLOGY IN GENERAL RELATIVITY

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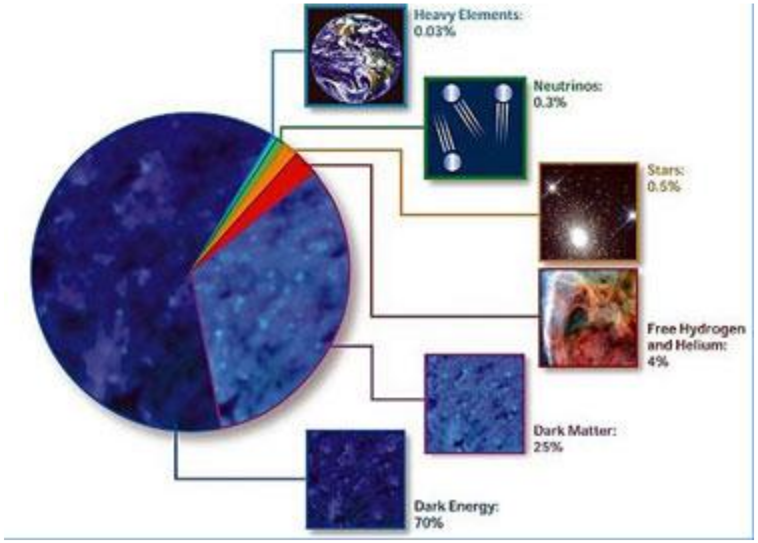
THE SUCCESS OF THE ΛCDM MODEL

- The Universe is spatially flat, homogeneous and isotropic on large scales

- The Universe is dominated by dark energy which may be accounted for by a cosmological constant term and dark matter

- Only six parameters needed for this concordance model: density of dark matter, density of baryons, expansion rate of the Universe, amplitude of the primordial fluctuations, their scale dependence, optical depth of the Universe



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→ The Universe is well described by a Friedmann-Robertson-Walker metric, whose gravity source is a mixture of non interacting fluids including a cosmological constant

P. J. E. Peebles, *Principles of Physical Cosmology* (Princeton University Press, Princeton, NJ, 1993).

THE COINCIDENCE PROBLEM AND THE HUBBLE TENSION

- Cosmological coincidence problem: why do we live in a cosmic epoch in which the abundance of dark energy and dark matter are of the same order of magnitude? From data we infer $\Omega_{\Lambda 0} \sim 0.7$ and $\Omega_{m0} \sim 0.3$
- There is a discrepancy between the early vs. late time (e.g. considering datasets at different redshift) measurements of the Hubble constant: $H_0 = (73.2 \pm 1.3)$ km s^{-1} Mpc^{-1} vs. $H_0 = (67.27 \pm 0.60)$ km s^{-1} Mpc^{-1}
- Relationship between Hubble function and matter parameters (non-interacting scenario): $H^2(z) = H_0 \left[\Omega_{m0}(1+z)^3 + \Omega_{\mathrm{DE}0}\exp\left(3\int_0^z \frac{1+w(z')}{1+z'}dz'\right)\right]$
- A recent review on the Hubble tension problem arXiv:2103.01183 [astro-ph.CO] contains more than 1000 references: many studies in the literature

BASIC IDEA OF OUR PAPER

- Recall also the $f\sigma_8$ tension: it deals with the evolution of matter perturbations

- If we assume that these measurements of the Hubble constant are not affected by systematics (gravitational lensing effects on the cosmic microwave background angular spectrum and calibration and reddening issues for supernovae), we would be required to add some new ingredients into the standard model of cosmology

- Our idea: try to construct a theoretical model whose cosmic history is as ΛCDM at the background level. The model though should not be based on a cosmological constant and dust dark matter, but for example, on some fluids $p = w(\rho)\rho$ or scalar fields, possibly interacting. These extra degrees of freedom (free model parameters) should then be constrained at the perturbative level

- In our paper, we assumed the Copernican principle (homogeneity and isotropy) and General Relativity

OUR METHOD: COSMOGRAPHY

Recall the cosmographic expansions for the luminosity distance and for the Hubble function in terms of the redshift z:

$$d_L(z) \simeq \frac{z}{H_0}\Big[1 + \frac{(1-q_0)z}{2} + \frac{(-1+q_0+3q_0^2+j_0)z^2}{6} + \frac{(2-2q_0-15q_0^2-15q_0^3+5j_0+10q_0j_0+s_0)z^3}{24}\Big]$$

$$H(z) \simeq H_0\left[1 + (1+q_0)z + \frac{(j_0-q_0^2)z^2}{2} + \frac{(3q_0^2+3q_0^3-j_0(3+4q_0)-s_0)z^3}{6}\right]$$

in which we have introduced the deceleration, jerk, and snap parameters:

$$q := -1 - \frac{\dot{H}}{H^2}$$

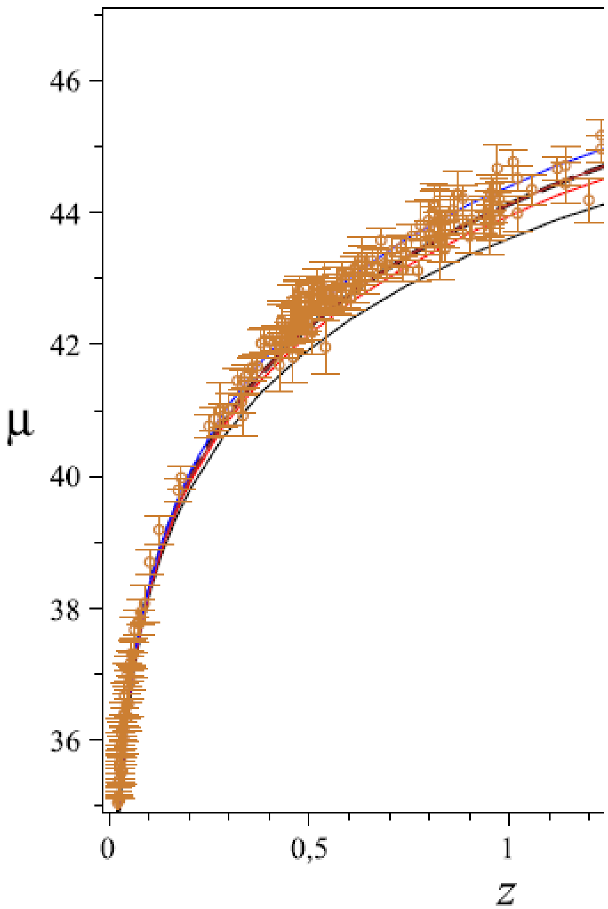
$$j := \frac{\ddot{H}}{H^3} - 3q - 2$$

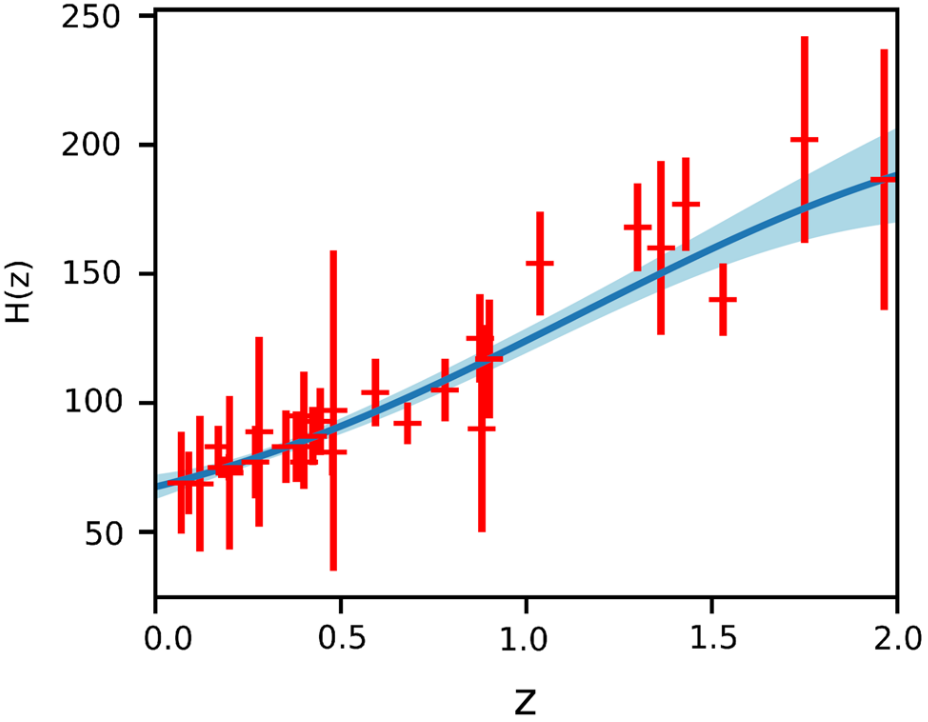
$$s := \frac{\dddot{H}}{H^4} + 4j + 3q(q+4) + 6$$

and a subscript 0 indicates that the quantity is evaluated at the present time.

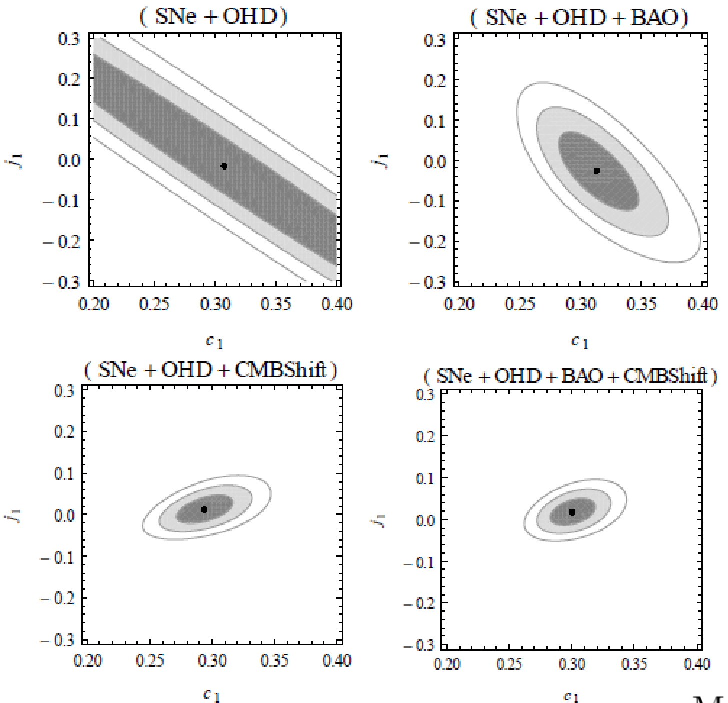
COSMOGRAPHIC ANALYSIS

The luminosity distance can be assessed through observations of type Ia supernovae, while the evolution of the Hubble function can be reconstructed using the distance to passively evolving galaxies (cosmic chronometers).

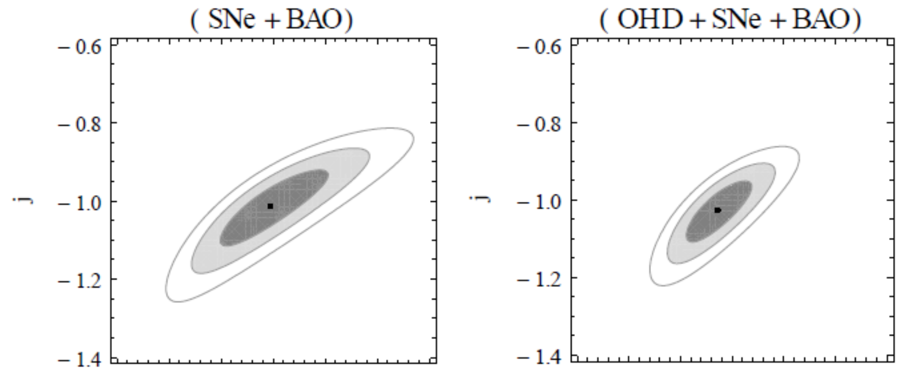


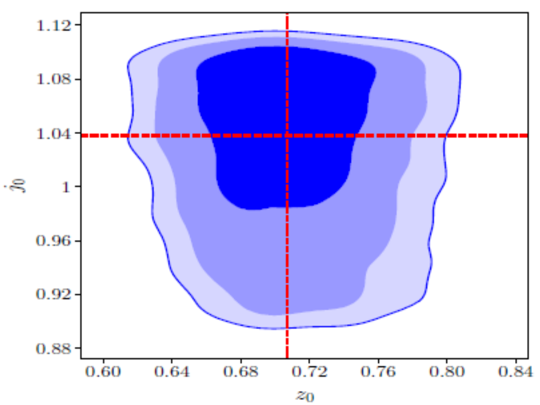


ON THE JERK PARAMETER



$$\frac{H^2(z)}{H_0^2} = c_1(1+z)^3 + c_2 + j_1(1+z)^2,$$





- Mukherjee, Banerjee, "Parametric reconstruction of the cosmological jerk from diverse observational data sets," PRD (2016), arXiv:1601.05172 [gr-qc].
- Zhai, Zhang, Zhang, Liu, Zhang, "Reconstruction and constraining of the jerk parameter from OHD and SNe Ia observations," PLB (2013), arXiv:1303.1620 [astro-ph.CO].
- Amirhashchi, Amirhashchi, "Recovering ΛCDM model from a cosmographic study," GRG (2020), arXiv:1811.05400 [astro-ph.CO].
- Mukherjee, Banerjee, "In search of the dark matter dark energy interaction: a kinematic approach," CQG (2017), arXiv:1610.04419 [astro-ph.CO].

THEORETICAL RECONSTRUCTION FROM JERK j=1

$$j(z) = \frac{H(z)(1+z)^2 \frac{d^2H(z)}{dz^2} + \left[(1+z)\frac{dH(z)}{dz} - H(z)\right]^2}{H^2(z)}$$

For ΛCDM a direct computation delivers $j = 1$.

$\neq$

Arbitrary mathematical constants

Solving $j(z) = 1$ we obtain:

$$h^2(z) \equiv \frac{H^2(z)}{H_0^2} = \mathcal{C}_1(1+z)^3 + \mathcal{C}_2,$$

where $\mathcal{C}_1$ and $\mathcal{C}_2$ are two arbitrary constants satisfying $\mathcal{C}_1 + \mathcal{C}_2 = 1$. In general, for nonzero $\mathcal{C}_1$ and $\mathcal{C}_2$, this family of solutions specifies a ΛCDM-like evolution history in the sense that it has two clearly defined limits: an *effective* CDM limit at asymptotic past $z \to \infty$ where the cosmic evolution goes like $h^2 \sim (1+z)^3$, and an *effective* Λ-limit at asymptotic future $z \to -1$ where the evolution goes like $h^2 \sim constant$. The **particular** solution of the above family, specified by $\mathcal{C}_1 = \Omega_{m0}$ and $\mathcal{C}_2 = \Omega_{\Lambda 0}$, corresponds to the particular ΛCDM cosmic history that our universe is going through.

SOME WORDS OF WARNING

- The spatial curvature parameter k enters the Friedmann equation, but not the continuity equations. Its evolution is thus not affected if we consider interactions and energy flows, and it remains as $\sim (1+z)^2$. Thus, $j = 1$ can correspond to a spatially flat universe only.

- The cosmographic parameter j is equivalent to the statefinder diagnostic parameter r. Our analysis with $r = 1$ can cover all the cases but that with $q = \frac{1}{2}$ and $q = -1$

- The relationship between the integration constants and the matter abundances, $\mathcal{C}_1 = \mathcal{C}_1(\Omega_{\rm DE0}, \Omega_{m0})$ and $\mathcal{C}_2 = \mathcal{C}_2(\Omega_{\rm DE0}, \Omega_{m0})$, is model-dependent.

For fully specifying a physical model we need **both** an evolution equation **and** some initial conditions: *kinematics* vs. *dynamics*

COUPLED FLUID-FLUID MODEL

The governing equations of the system are:

$$\begin{aligned} &\text{Friedmann constraint:} && 3H^2 = \rho_m + \rho_{\rm DE}, \\ &\text{Raychaudhuri equation:} && 2\dot{H} + 3H^2 = -w_{\rm DE}\rho_{\rm DE}, \\ &\text{Continuity equation:} && \dot{\rho}_m + 3H\rho_m = -Q, \\ &\text{Continuity equation:} && \dot{\rho}_{\rm DE} + 3H(1 + w_{\rm DE})\rho_{\rm DE} = Q. \end{aligned}$$

We define the matter abundance variables:

$$\Omega_m \equiv \frac{\rho_m}{3H^2}, \qquad \Omega_{\rm DE} \equiv \frac{\rho_{\rm DE}}{3H^2}$$

which are related by the algebraic constraint

$$\Omega_m + \Omega_{\rm DE} = 1.$$

We can allow $w_{\rm DE} = w_{\rm DE}(\rho)$ and $Q \neq 0$: for a time-evolving equation of state of dark energy and some energy flows (interactions) between dark energy and dark matter.

COUPLED FLUID-FLUID MODEL

$$j = 1 + \frac{9}{2} w_{\rm DE}(1 + w_{\rm DE})(1 - \Omega_m) - \frac{3}{2}(1 - \Omega_m) w'_{\rm DE} - \frac{w_{\rm DE} Q}{2H^3}$$

By imposing $j = 1$ (background degeneracy with ΛCDM) we can reconstruct the interaction term

$$\frac{Q}{3H^3} = \left[3(1 + w_{\rm DE}) - \frac{w'_{\rm DE}}{w_{\rm DE}}\right](1 - \Omega_m)$$

The evolution of dark energy abundance is governed by

$$\Omega'_{\rm DE} = \Omega_{\rm DE}\left[3(1 + w_{\rm DE}\Omega_{\rm DE}) - \frac{w'_{\rm DE}}{w_{\rm DE}}\right]$$

which admits the equilibria:

$$\mathcal{P}_1 : (\Omega_m, \Omega_{\rm DE}) = (1, 0), \qquad \mathcal{P}_2 : (\Omega_m, \Omega_{\rm DE}) = \left(\frac{1 + w_{{\rm DE}*}}{w_{{\rm DE}*}}, -\frac{1}{w_{{\rm DE}*}}\right)$$

COUPLED FLUID-FLUID MODEL

$$\mathcal{P}_2 : (\Omega_m, \Omega_{\rm DE}) = \left(\frac{1 + w_{\rm DE*}}{w_{\rm DE*}}, -\frac{1}{w_{\rm DE*}} \right)$$

Here $w_{\rm DE*}$ is a constant whose explicit value would be known from solving $(w'_{\rm DE})_{\rm eq} = 0$. $\mathcal{P}_2$ might actually represent more than one equilibrium point depending on the number of different roots of the evolution equation for the dark energy equation of state parameter.

Even without knowing those solutions explicitly, we can conclude that the equilibrium point $\mathcal{P}_2$ can exist only if dark energy, at $\mathcal{P}_2$, comes in the form of a phantom fluid, e.g. with $w_{\rm DE*} < -1$, violating the null energy condition.

This claim is independent on how many free parameters the equation of state of dark energy will be based

COUPLED CANONICAL QUINTESSENCE MODEL

The energy density and pressure of dark energy are:

$$\rho_{\rm DE} = \frac{\dot{\phi}^2}{2} + V(\phi)\,, \qquad p_{\rm DE} = \frac{\dot{\phi}^2}{2} - V(\phi)$$

Kinetic energy and potential of the scalar field

The Friedmann equation and the continuity equations are now given by:

$$\text{Friedmann constraint:} \quad 3H^2 = \rho_m + \frac{\dot{\phi}^2}{2} + V(\phi),$$

$$\text{Raychaudhuri equation:} \quad \dot{H} = -\frac{\rho_m + \dot{\phi}^2}{2},$$

$$\text{Continuity equation:} \quad \dot{\rho}_m + 3H\rho_m = -Q,$$

$$\text{Continuity equation:} \quad \dot{\phi}\ddot{\phi} + V_{,\phi}\dot{\phi} + 3H\dot{\phi}^2 = Q.$$

COUPLED CANONICAL QUINTESSENCE MODEL

We introduce the dimensionless variables (arXiv:1712.03107 [gr-qc]):

$$x := \frac{\dot{\phi}}{\sqrt{6}H}\,, \qquad y := \frac{\sqrt{|V(\phi)|}}{\sqrt{3}H}\,, \qquad \Omega_\phi := \frac{\rho_{\rm DE}}{3H^2} = x^2 + y^2\,, \qquad \Omega_m := \frac{\rho_m}{3H^2}\,,$$

and the auxiliary quantities

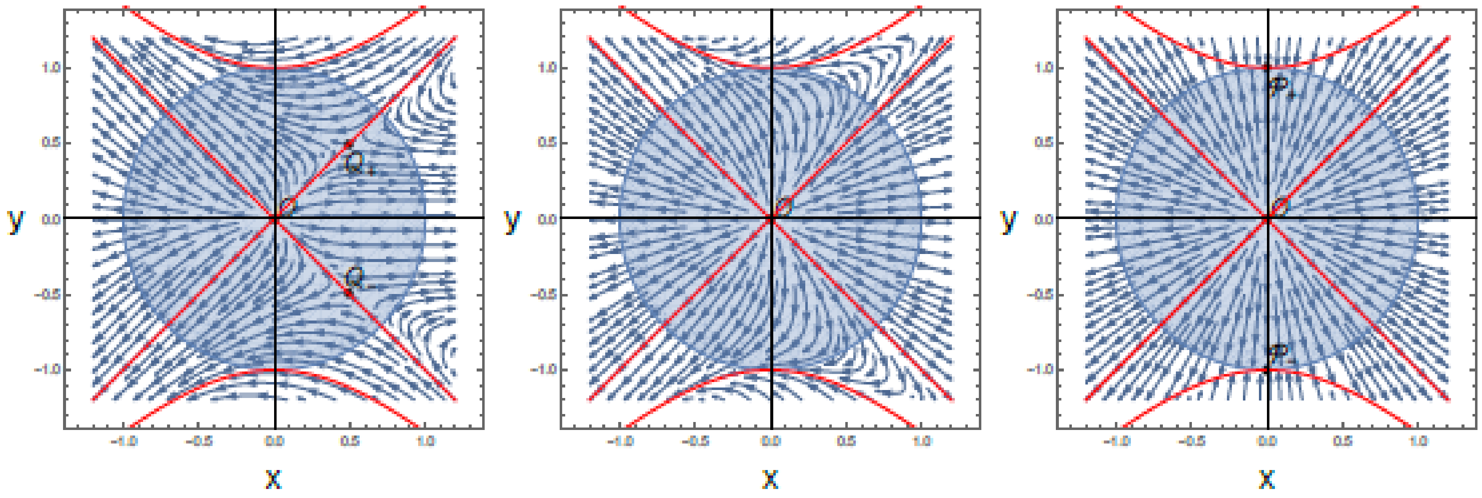
$$\lambda := -\frac{V_{,\phi}}{V}\,, \qquad \Gamma := \frac{V V_{,\phi\phi}}{V_{,\phi}^2}\,.$$

Some interesting possibilities:

- $\Gamma = 1$ can be integrated to give the exponential potential $V(\phi) = V_0 e^{-\lambda\phi}$
- $\Gamma = \mathcal{A} \neq 1$ delivers $V(\phi) = \left(\frac{1}{(1-\mathcal{A})(\mathcal{A}_1\phi+\mathcal{A}_2)}\right)^{\frac{1}{\mathcal{A}-1}}$. For $\phi \to 0$ the potential approaches a constant, while for $\phi \to \infty$ the potential can either diverge (if $\mathcal{A} < 1$) or asymptotically vanish (if $\mathcal{A} > 1$) with the value $\mathcal{A} = 1$ constituting a bifurcation
- The case $\lambda = 0$ delivers a constant potential

COUPLED CANONICAL QUINTESSENCE MODEL

Same strategy: by imposing $j = 1$ we reconstruct the interaction term Q



(a) (b) (c)

The dynamical equations make the system leave the physically allowed region during its evolution but for very specific (a set of measure zero) initial conditions

DISCUSSION AND INTERPRETATION

- There is an active research program in literature trying to construct cosmological models degenerate with ΛCDM at the background level, but with different evolutions of the perturbations

- Should we impose jerk $j = 1$ we can have a ΛCDM-like cosmic history

- We clarify that $j = 1$ for a homogeneous and isotropic universe in General Relativity necessarily corresponds to spatially flatness

- We should distinguish between *kinematical* vs. *dynamical* evolution of a cosmological model

- A coupled fluid-fluid cosmological model with $j = 1$ would violate the null energy condition since it exhibits a phantom epoch

- A coupled canonical quintessence cosmological model kinematically degenerate with ΛCDM requires very fine-tuned initial conditions

- Under the assumptions of General Relativity, homogeneity and isotropy, the ΛCDM although not "unique", it seems quite "special"