Content	Spacetime symmetries	Local symmetries in posets	Regular polytopes	Local symmetries in causets	Conclusion
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## Do causal sets have symmetries?

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<sup>1</sup>Institut für Theoretische Physik, Universität Leipzig

Seventeenth Marcel Grossmann Meeting 8-12 July 2024, Pescara

Content	Spacetime symmetries	Local symmetries in posets	Regular polytopes	Local symmetries in causets	Conclusion
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#### Electronic tools for causal sets and research in their symmetries

- Image: Argon and partially ordered sets in general),
- ② Online tool to help finding the LATEX-macros,
- ③ Preprint "Local symmetries in partially ordered sets".

[CTAN 2020] ctan.org/pkg/causets, [M 2024] c-minz.github.io, [M 2024] arXiv:2406.14533.

Content	Spacetime symmetries	Local symmetries in posets	Regular polytopes	Local symmetries in causets	Conclusion
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- Symmetries of spacetime manifolds vs. sprinkled causal sets
- 2 Local symmetries of (finite) partially ordered sets
- 3 Causal sets of regular geometric polytopes
- 4 Local symmetries in causets

Content 00	Spacetime symmetries	Local symmetries in posets	Regular polytopes OO	Local symmetries in causets 000	Conclusion O
Sprinkling process on sp	pacetime manifolds and (pre-)compact	subsets			

• Probability space  $(Q, \mathcal{B}(Q), \mu)$ 

• 
$$Q := \left\{ S \subset M \; \middle| \; \forall U \subseteq M : |S \cap U| < \infty \right\}$$

• a probability measure  $\mu$  over the Borel  $\sigma\text{-algebra}\;\mathcal{B}(Q)$ 

## A sprinkle on a (pre-)compact subset $U \subset M$

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• 
$$Q_{U,n} := \{ S \subset U \mid |S| = n \}$$

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$$\mu_U(B_n) = \mathrm{e}^{-\rho\nu(U)} \frac{\rho^n}{n!} \nu^n \left( \Sigma_{U,n}^{-1}(B_n) \right)$$

Math. review: [Fewster-Hawkins-M-Rejzner 2021].

 $M = \mathbb{M}^2$ 

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Invariance of the sprin	kling process under spacetime symmetri	es			

#### Invariance under spacetime symmetries

Let  $\Lambda$  be a symmetry transformation of the spacetime. For example,  $\Lambda \in \mathcal{P}_+^{\uparrow}$ , a proper orthochronous Poincaré transformation in Minkowski spacetime.

The volume measure is invariant:

 $\nu \circ \Lambda = \nu \qquad \qquad \mu_{\Lambda U} = \mu_U \,.$ 

A sprinkle in Minkowski spacetime does not pick out a preferred frame of reference [Bombelli–Henson–Sorkin 2006].

Remark: A preferred past structure assigns a unique direction to each element in a causal set, but this is a random distribution on the hyperboloid, for all elements of a sprinkle. IDable-Heath-Fewster-Reizner-Woods 2020. FHMR 2023



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Let P be a poset. Two elements  $a,b \in P$  are singleton-symmetric if

$$L^{\pm}(a) = L^{\pm}(b) \qquad \left( \Leftrightarrow J^{\pm}_{*}(a) = J^{\pm}_{*}(b) \right).$$

- ⇒ "Singleton-symmetric" is an equivalence relation.
- $\Rightarrow$  Taking the quotient of a poset P by this symmetry yields a *retract*  $P \oslash \bullet$

#### Example (Antichains)

Elements of antichains are singleton-symmetric

$$\bullet = (\bullet \bullet) \oslash \bullet = (\bullet \bullet \bullet) \oslash \bullet = (\bullet \bullet \bullet) \oslash \bullet = \dots$$



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Quotient by all (Q, r)-symmetries gives a *retract*  $P \oslash_r Q$  (and we drop the index if r = 2).



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The automor. is a (Q, r)-generator if there exists a sequence of r subsets  $S_i \,\subset \, \Sigma(\sigma)$  with  $S_i \cong Q$ , and they are the *smallest, maximally ordered* subsets of  $\Sigma(\sigma)$  with  $\sigma(S_i) = S_{i+1 \mod r}$  $(0 \le i < r)$  that cover  $\Sigma(\sigma)$ . For  $a, b \in P$ ,  $a \sim_0 b$  if a = b;  $a \sim_1 b$  if  $\exists A, B \subset P$  with (Q, r)-generator  $\sigma$  such that  $a \in A$  and  $b = \sigma^q(a) \in B = \sigma^q(A)$  for some  $1 \le q < r$ ;  $a \sim_1 b$  if  $a \prec_i : b$  for any i < r but  $\exists c \in P$  and

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Classification of posets with local symmetries					

For a finite poset Q and  $r \in \mathbb{N}$ , a poset P is locally (Q, r)-symmetric and (Q, r)-retractable to the poset  $\tilde{P}$  if  $\tilde{P} = P \oslash_r Q \neq P$ . The poset P is locally symmetric if there exists some finite poset Q and  $r \geq 2$  such that P is locally (Q, r)-symmetric, P is retractable to the poset  $\tilde{P}$ (the retract of P) if there exist some sequence of  $(Q_i, r_i)$ -symmetries such that  $\tilde{P} = P \oslash_{r_1} Q_1 \oslash_{r_2} Q_2 \oslash_{r_3} \ldots \neq P$ , and P is

All posets that are (Q, r)-retractable to some

poset R form a class of symmetry extensions

 $[R \odot_r Q] := \{ P \in \mathfrak{P} \mid P \oslash_r Q = R \neq P \} .$ 

Two elements are prime (Q, r)-symmetric if they are not (Q', r')-symmetric by another smaller  $Q' \subset Q$  or smaller r' < r. For example:

Example (Posets of bipartite graphs) $[I \odot \bullet]' = \left\{ \Lambda, \gamma, \Lambda, \chi, \chi, \gamma, \gamma, \ldots \right\}$ 

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 $\hat{P} = P \otimes_{r_1} Q_1 \otimes_{r_2} Q_2 \otimes_{r_3} \ldots \neq P$ , and P is locally unsymmetric if it is not locally symmetric.

All posets that are (Q, r)-retractable to some poset R form a class of symmetry extensions

 $[R \odot_r Q] := \{ P \in \mathfrak{P} \mid P \oslash_r Q = R \neq P \} .$ 

Two elements are prime (Q, r)-symmetric if they are not (Q', r')-symmetric by another smaller  $Q' \subset Q$  or smaller r' < r. For example:

$$\mathbb{M} \otimes \cdots = \mathbb{M} \otimes \mathbb{A} = \mathbb{M} \otimes \mathfrak{l} = \mathfrak{l}$$
$$\mathbb{M} \otimes \cdot \cdot = \mathfrak{l}.$$

Example (Posets of bipartite graphs)

$$[\mathbf{I} \odot \bullet]' = \left\{ \mathbf{A}, \mathbf{V}, \mathbf{A}, \mathbf{M}, \mathbf{V}, \mathbf{V}, \mathbf{M}, \mathbf{M}, \mathbf{V}, \mathbf{M}, \mathbf{M},$$

Content 00	Spacetime symmetries	Local symmetries in posets	Regular polytopes 00	Local symmetries in causets 000	Conclusion O
Classification of posets with local symmetries					

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$$[I \odot \bullet]' = \left\{ \Lambda, \Psi, \Lambda, H, \Psi, \Psi, \\ \Lambda, M, \Psi, \Psi, \Psi, \cdots \right\}.$$

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00	OO	○○●	00		O
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Content	Spacetime s	y
00	00	

Regular polytopes

Local symmetries in causets

Conclusion O

## Example (Causal sets of polygons)

The (regular) polygons also have a geometrical representation as causal sets embedded in (1+2)-dimensional Minkowski spacetime. Imagine a regular polygon embedded in the Cauchy slice and light pulses being emitted from all corners at t = 0. The light pulses propagate and meet pairwise at the central points of the polygon edges, later all pulses meet at the centre of the polygon (2-face).

Posets of regular polygons embedded in (1+2)-dimensional Minkowski spacetime

## Posets of polygons



Content	Spacetime
00	00

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Posets of regular polygons embedded in (1+2)-dimensional Minkowski spacetime

## Posets of polygons



Content	Spacetim
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Regular polytopes

Local symmetries in causets

e symmetries Posets of regular polygons embedded in (1+2)-dimensional Minkowski spacetime

## Posets of polygons



Content	Spacetime	sy
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00	00

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Posets of regular polygons embedded in (1+2)-dimensional Minkowski spacetime

## Posets of polygons



Content	Spacetime symmetries	Local symmetries in posets	Regular polytopes	Local symmetries in causets	Conclusion	
00	OO	000	○●		O	
Posets of simplices that embed $(1+d)$ -dimensional Minkowski spacetime						



#### Theorem (Simplices)

The d-simplex is (d-2)-simplex-retractable to the (d+2)-chain.

#### Theorem (Preservation of layers)

For any (Q, r)-symmetric poset P, the symmetry quotient P/(Q, r) preserves layers.

Christoph Minz (Leipzig)

Do causal sets have symmetries?

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## In the large n behaviour, posets with a small number of layers dominate [Kleitman-Rothschild 1975].

Example (Some Kleitman–Rothschild orders have local symmetries)

Fig. 1 from [Carlip–Carlip–Surya 2023] is singleton-symmetric, retracting to the (0, 1, 2)-faces subset of the 3-simplex, which in turn retracts to the 3-chain,

$$\langle \mathbf{A} = \mathbf{A} \otimes \mathbf{A} \otimes \mathbf{A} \otimes \mathbf{A} = \mathbf{A} \otimes \mathbf{A}$$

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For any  $k \in \mathbb{N}_0$ , a poset P is k-stable locally unsymmetric if, for every subset  $S \subseteq P$  that has cardinality  $0 \le |S| \le k$ , the poset  $P \smallsetminus S$  is locally unsymmetric. A poset P is total locally unsymmetric if  $P \smallsetminus S$  is k-stable locally unsymmetric for every  $k \le |P|$ .

#### Example

Any chain posets (total order) is total locally unsymmetric.

Posets with more layers are more likely to be (total) locally unsymmetric.

## Numbers by cardinality (row) and layer (column).

- $u_n$  Number of all locally unsymmetric posets.
- s<sub>n</sub> Number of all 1-stable locally unsymmetric posets.

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Nun	nbers	by ca	ardinality	′ (rov	v) and	layer	(co	lumn).
	1	2	3	4	5	6	7	$p_n$
1	1							1
2	1	1						2
3	1	3	1					5
4	1	8	6	1				16
5	1	20	31	10	1			63
6	1	55	162	84	15	1		318
7	1	163	940	734	185	21	1	2045

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	1	2	3	4	5	6	7	$ $ $u_n$
1	1							1
2	0	1						1
3	0	1	1					2
4	0	1	3	1				5
5	0	1	11	6	1			19
6	0	3	47	41	10	1		102
7	0	9	266	332	106	15	1	729

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1	1							1
2	0	1						1
3	0	0	1					1
4	0	0	1	1				2
5	0	0	0	3	1			4
6	0	0	2	8	6	1		17
7	0	0	4	37	36	10	1	88

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Local symmetries vs. s	ocal symmetries vs. sprinkled causal sets							

## A sprinkle in *d*-dimensional Minkowski spacetime is total locally unsymmetric with probability 1.

Proof: Let S be a random sprinkle in Minkowski spacetime  $\mathbb{M}^{1+d}$ , take two separated elements. The probability for  $I_t$  to contain n elements is

$$\Pr\left(|\mathsf{S} \cap I_t| = 0\right) = \frac{\rho^n \nu(I_t)^n}{n!} \mathrm{e}^{-\rho\nu(I_t)}.$$



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Content	Spacetime symmetries	Local symmetries in posets	Regular polytopes	Local symmetries in causets	Conclusion		
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Summary and advertiser	ummary and advertisement						

## (Infinite) sprinkles usually do not have local symmetries.

Are local symmetries relevant or even necessary to model the very early universe in causal set theory?

#### Advertisement: LATEX-package 'causets'

Is part of complete distributions so that it is, for example, available on Overleaf. Just load the package with \usepackage{causets}.

Example (Local symmetries of the wedge) To get  $\Lambda \oslash \bullet = 1$ , write  $\gamma_1, 3$  \oslash \pcauset{1} = \pcauset{1,2}

## Online tool to support the use of the package

Content	Spacetime symmetries	Local symmetries in posets	Regular polytopes	Local symmetries in causets	Conclusion			
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Summary and advertise	ummary and advertisement							

(Infinite) sprinkles usually do not have local symmetries.

Are local symmetries relevant or even necessary to model the very early universe in causal set theory?

#### Advertisement: LATEX-package 'causets'

Is part of complete distributions so that it is, for example, available on Overleaf. Just load the package with \usepackage{causets}.

Example (Local symmetries of the wedge) To get  $\land \oslash \cdot = 1$ , write  $\colored \colored \colore$ 

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To help finding the right macro, go to my website c-minz.github.io/assets/html/ proset-editor.html

#### The PrOSET Editor

Welcome to the PrOSET editor to visualise and modify finite partially ordered sets (posets) represented as their Hasse diagrams. Finite posets are interval subsets of causal sets, for example. This version only supports Hasse diagrams of 2-dimensional posets, represented by a permutation of consecutive integers starting from 1 (given as a comma separated list).



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