Spectrum of quasinormal modes of rapidly rotating Ellis-Bronnikov wormholes

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In collaboration with
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Marcel Grossmann Meeting 17

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I) Introduction

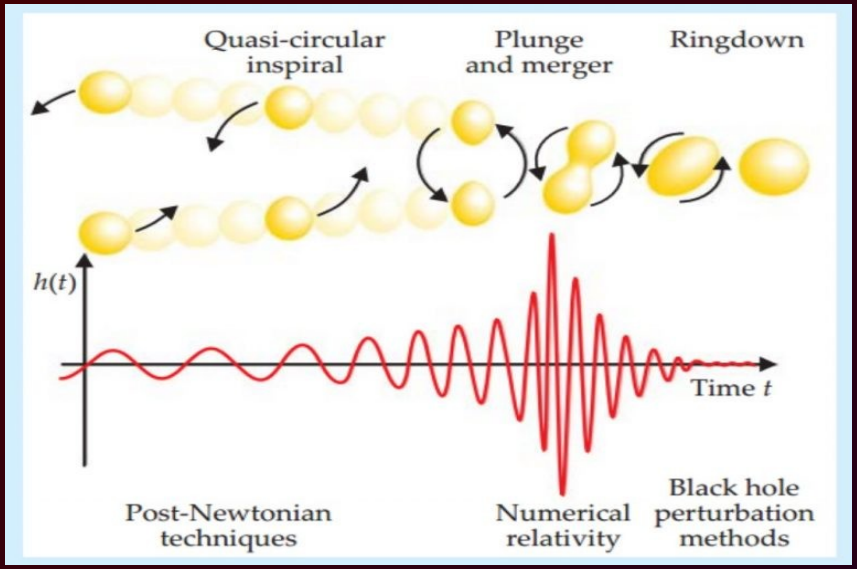
Rotating Wormholes

II) Perturbations and quasinormal modes

Spectral decomposition of the metric perturbations

III) Spectrum of rotating Ellis wormholes.

IV) Conclusions and outlook.



Ringdown phase:
resonant frequencies and
exponential damping

↓

Quasinormal modes

General Relativity:
spectrum depends on M, J, Q and matter composition

Alternative theories:
parameters of the theory // extra charges (scalar fields)

Static background → decoupling of the perturbation equations in ODEs.

Kerr → decoupling in the NP formalism (Teukolsky equation)

Metric perturbations in non-spherical backgrounds result in complicated PDEs

I) Introduction: Ellis Wormhole

GR + Phantom field

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[\mathrm{R} + 2\partial_\mu\Phi\,\partial^\mu\Phi\right]$$

Static and spherically symmetric Ellis Wormhole:

$$ds^2 = -e^f dt^2 + \frac{1}{e^f}\left[dr^2 + (r^2 + r_0^2)(d\theta^2 + \sin^2\theta d\varphi^2)\right]$$

Phantom field:

$$\phi = \frac{Q}{r_0}\left[\tan^{-1}\left(\frac{r}{r_0}\right) - \frac{\pi}{2}\right]$$

$$f = \frac{C}{r_0}\left[\tan^{-1}\left(\frac{r}{r_0}\right) - \frac{\pi}{2}\right]$$

$$4Q^2 = C^2 + 4r_0^2$$

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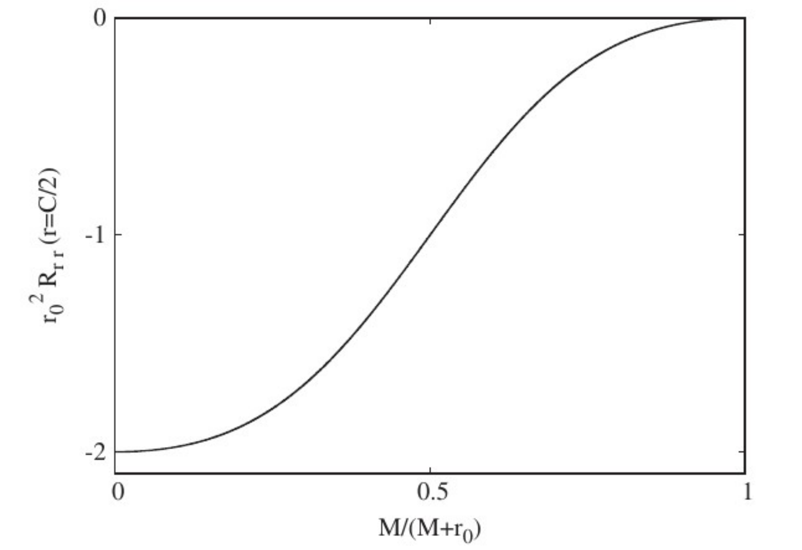
Violation of the WEC and NEC energy conditions:

$$R_{rr} = -\frac{2Q^2}{(r^2 + r_0^2)^2}$$



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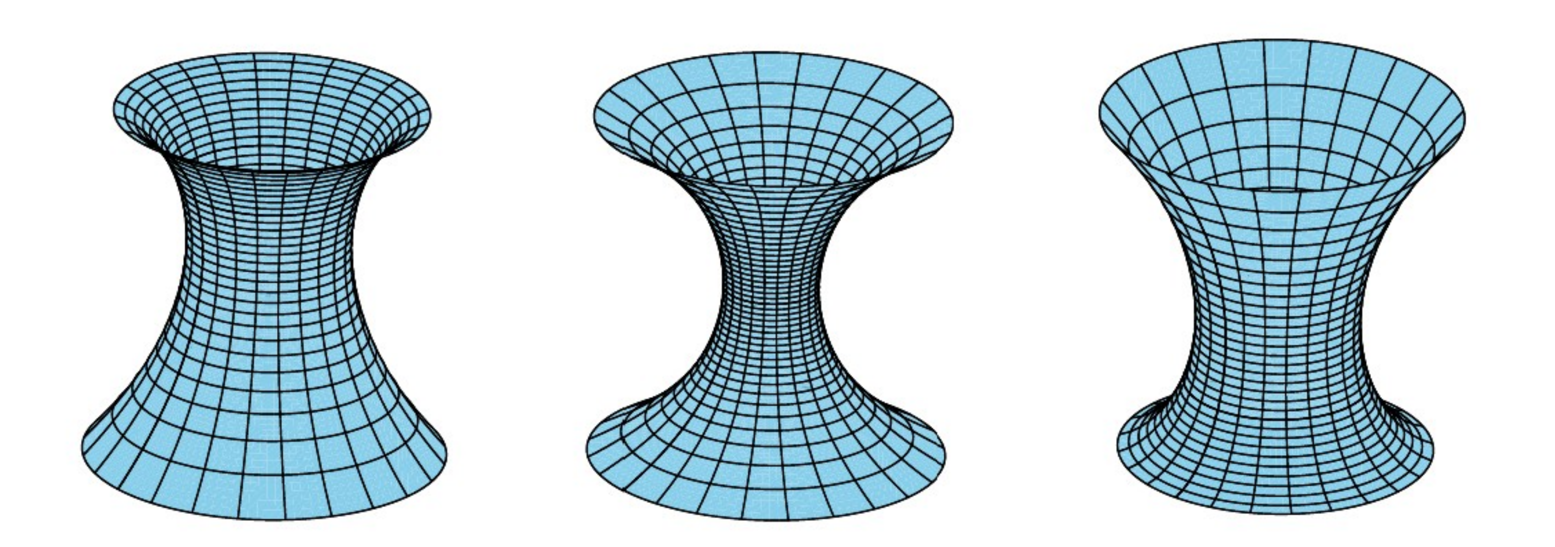
Symmetric and asymmetric wormholes:



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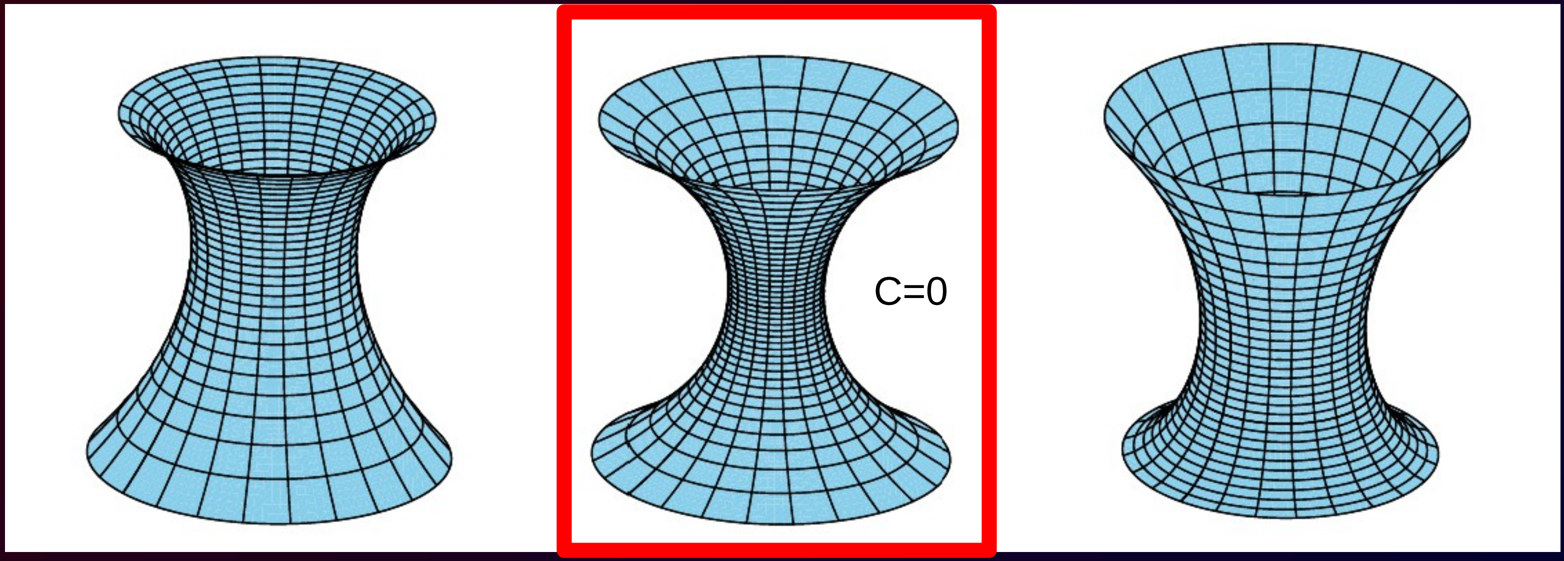


Rapidly rotating Ellis Wormhole:

2014 – B. Kleihaus, J. Kunz - PRD 90 121503 (1409.1503)
2016 – X.Y. Chew, B. Kleihaus, J. Kunz - PRD 94 104031 (1608.05253)

-) Solutions only known numerically

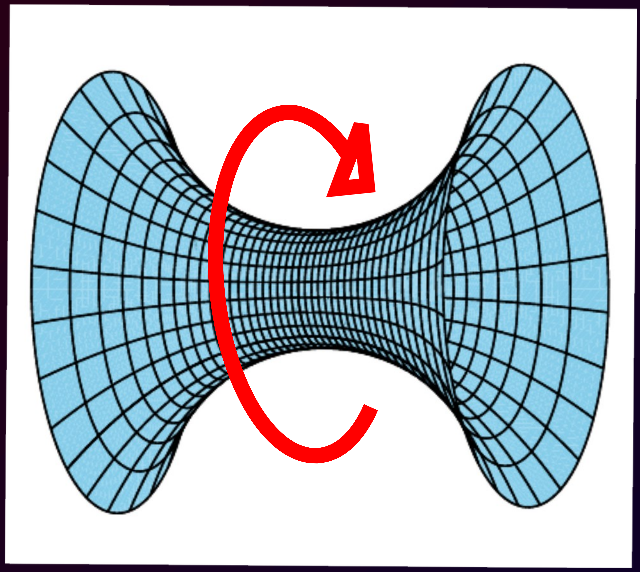
$$
\begin{aligned}
ds^2 &= -\left[e^{f(r,\theta)} - e^{-f(r,\theta)}(r^2+r_0^2)w(r,\theta)^2\sin^2(\theta)\right]dt^2 + e^{\nu(r,\theta)-f(r,\theta)}dr^2 \\
&+ e^{\nu(r,\theta)-f(r,\theta)}(r^2+r_0^2)(d\theta^2+\sin^2(\theta)d\phi^2) \\
&- 2e^{-f(r,\theta)}(r^2+r_0^2)w(r,\theta)\sin^2(\theta)dtd\phi\,.
\end{aligned}
$$

$$\varphi(r,\theta) = \frac{Q}{r_0}\left(\tan^{-1}(r/r_0) - \pi/2\right)$$

-) Focus on symmetric solutions:

$$r \to -\infty$$

$$
\begin{aligned}
f(r,\theta) &= \frac{2M}{r} + \mathcal{O}(r^{-2})\,, \\
\nu(r,\theta) &= \mathcal{O}(r^{-2})\,, \\
w(r,\theta) &= w_{-\infty} + \frac{2J}{r^3} + \mathcal{O}(r^{-4})\,.
\end{aligned}
$$



$$r \to +\infty$$

$$
\begin{aligned}
f(r,\theta) &= -\frac{2M}{r} + \mathcal{O}(r^{-2})\,, \\
\nu(r,\theta) &= \mathcal{O}(r^{-2})\,, \\
w(r,\theta) &= \frac{2J}{r^3} + \mathcal{O}(r^{-4})\,,
\end{aligned}
$$

Some quantities of interest:

Equatorial radius of the throat: $\mathrm{R} = e^{-f(r=0,\theta=\pi/2)/2} r_0$

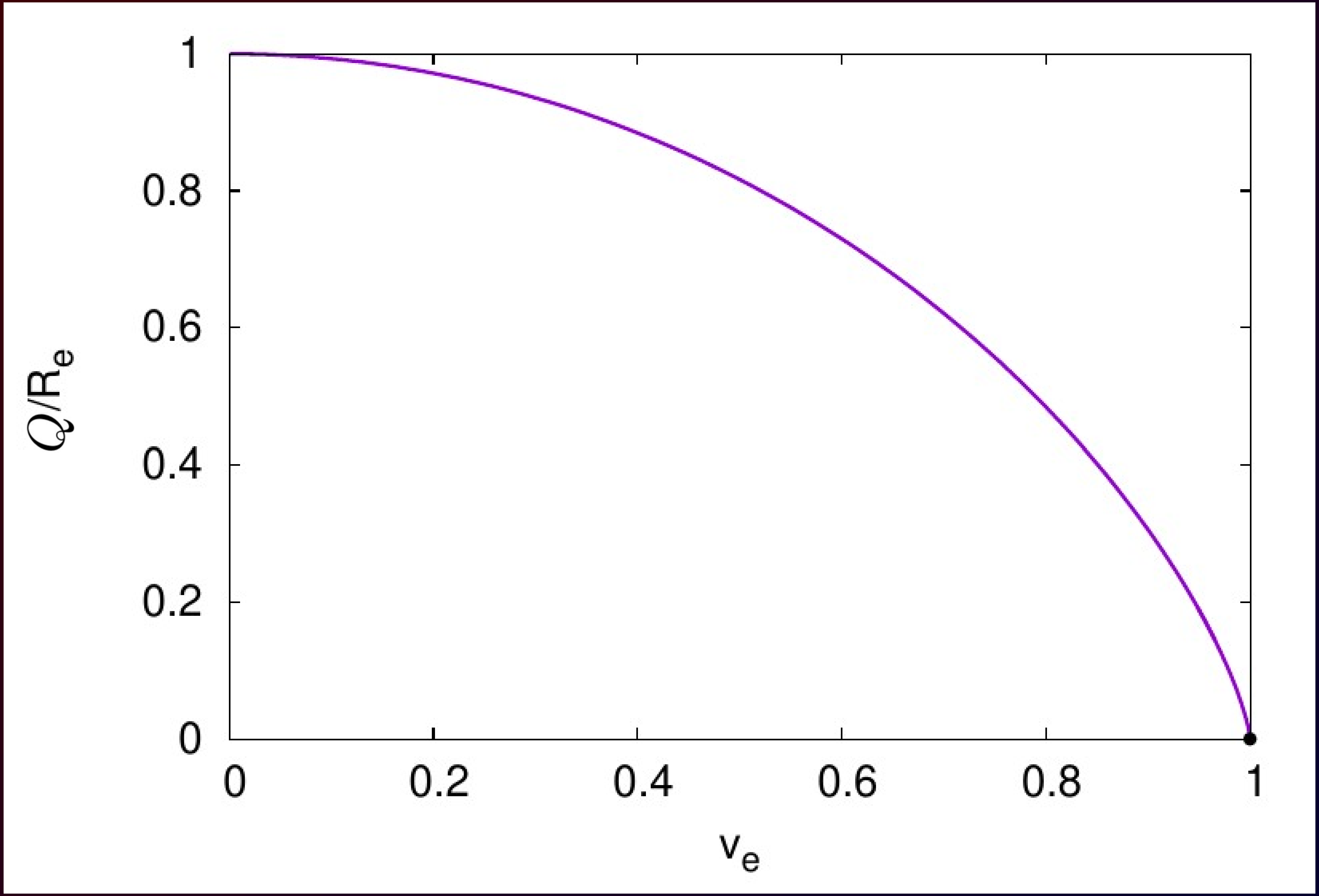
Throat size: $\mathrm{R}_A = \sqrt{A_T/4\pi}$ $\quad A_T = \int_0^{2\pi}\int_0^{\pi} r_0^2 e^{\nu(0,\theta)/2 - f(0,\theta)} \sin(\theta) d\theta d\phi$

Rotational velocity of the throat: $\Omega = w(0,\theta) = w_{-\infty}/2$

Dimensionless rotational velocity: $\mathrm{v}_e = \mathrm{R}\,\Omega$

Scalar charge: Q

For fixed throat size, solutions form a one parameter family:



II) Perturbations and quasinormal modes

Spectral decomposition of the metric perturbations

II) Spectral decomposition of the metric perturbations

Introducing metric perturbations to the metric

$$g_{\mu\nu} = g^{(bg)}_{\mu\nu} + \epsilon\delta h_{\mu\nu}(t, r, \theta, \phi)\,,$$

$$\delta h_{\mu\nu} = \delta h^{(A)}_{\mu\nu} + \delta h^{(P)}_{\mu\nu}\,.$$

In general for axially symmetric background the perturbations have the shape:

$$\delta h^{(A)}_{\mu\nu} = e^{i(M_z\phi-\omega t)}\begin{pmatrix} 0 & 0 & a_1(r,\theta) & a_2(r,\theta) \\ 0 & 0 & a_3(r,\theta) & a_4(r,\theta) \\ a_1(r,\theta) & a_3(r,\theta) & 0 & 0 \\ a_2(r,\theta) & a_4(r,\theta) & 0 & 0 \end{pmatrix}$$

$$\delta h^{(P)}_{\mu\nu} = e^{i(M_z\phi-\omega t)}\begin{pmatrix} N_0(r,\theta) & H_1(r,\theta) & 0 & 0 \\ H_1(r,\theta) & L_0(r,\theta) & 0 & 0 \\ 0 & 0 & T_0(r,\theta) & 0 \\ 0 & 0 & 0 & S_0(r,\theta) \end{pmatrix}$$

Gauge fixing, redefinitions …

$$\begin{aligned}
a_1(r,\theta) &= -iM_z \frac{h_0(r,\theta)}{\sin\theta}\,,\\
a_2(r,\theta) &= \sin\theta\,\partial_\theta h_0(r,\theta)\,,\\
a_3(r,\theta) &= -iM_z \frac{h_1(r,\theta)}{\sin\theta}\,,\\
a_4(r,\theta) &= \sin\theta\,\partial_\theta h_1\,,\\
N_0(r,\theta) &= \left(g_{rr}^{(bg)}(r,\theta)\right)^{-1} N(r,\theta)\,,\\
L_0(r,\theta) &= \left(g_{rr}^{(bg)}(r,\theta)\right) L(r,\theta)\,,\\
T_0(r,\theta) &= \left(g_{\theta\theta}^{(bg)}(r,\theta)\right) T(r,\theta)\,,\\
S_0(r,\theta) &= \left(g_{\phi\phi}^{(bg)}(r,\theta)\right) T(r,\theta)\,.
\end{aligned}$$

Scalar field:

$$\varphi = \varphi^{(bg)} + \epsilon\delta\varphi(t,r,\theta,\phi) = \epsilon e^{i(M_z\phi-\omega t)}\Phi(r,\theta)\,.$$

II) Spectral decomposition of the metric perturbations

We tackle the full 2D problem.
Compactified coordinates e.g. for wormholes:

$$x = \frac{2}{\pi}\tan^{-1}\left(\frac{r}{r_0}\right), \quad y = \cos\theta$$

Convenient parametrization (for a wormhole bkg)

$$\begin{aligned}
H_1 &= \widetilde{H}_1(x,y)\frac{1}{(1-x)(1+x)^2}(1-y^2)^{M_z/2}e^{i\hat{R}}, \\
T &= \widetilde{T}(x,y)(1-y^2)^{M_z/2}e^{i\hat{R}}, \\
N &= \widetilde{N}(x,y)\frac{1}{(1-x)(1+x)^2}(1-y^2)^{M_z/2}e^{i\hat{R}}, \\
L &= \widetilde{L}(x,y)\frac{1}{1-x^2}(1-y^2)^{M_z/2}e^{i\hat{R}}, \\
h_0 &= \widetilde{h}_0(x,y)\frac{1}{(1-x)(1+x)^2}(1-y^2)^{M_z/2}e^{i\hat{R}}, \\
h_1 &= \widetilde{h}_1(x,y)\frac{1}{1-x^2}(1-y^2)^{M_z/2}e^{i\hat{R}}, \\
\Phi &= \widetilde{P}(x,y)(1-x^2)(1-y^2)^{M_z/2}e^{i\hat{R}}.
\end{aligned}$$

System of PDEs for 7 unknown perturbation functions:

$$\vec{X} = [\widetilde{H}_1, \widetilde{T}, \widetilde{N}, \widetilde{L}, \widetilde{h}_0, \widetilde{h}_1, \widetilde{\Phi}_1]$$

Choose 7 equations from the field equations:

$$\{\delta\mathcal{G}_{tr}, \delta\mathcal{G}_{t\theta}, \delta\mathcal{G}_{rr}, \delta\mathcal{G}_{r\theta}, \delta\mathcal{G}_{r\phi}, \delta\mathcal{G}_{\theta\phi}\}$$ and $$\delta\mathcal{S}$$

These are essentially linear equations:

$$\mathcal{D}_{\rm I}(x,y)\vec{X}(x,y) = 0, \qquad {\rm I} = 1, ..., 7\,.$$

$$\mathcal{A}_{\rm I}(x,y)\vec{X}(x,y)|_{x=1} = 0, \qquad {\rm I} = 1, ..., 7\,,$$
$$\mathcal{B}_{\rm I}(x,y)\vec{X}(x,y)|_{x=-1} = 0\,, \quad {\rm I} = 1, ..., 7\,,$$

← Outgoing wave at infinity

← Outgoing at negative infinity

$$\alpha_{\rm I}(x,y)\vec{X}(x,y)|_{y=1} = 0, \qquad {\rm I} = 1, ..., 7\,.$$
$$\beta_{\rm I}(x,y)\vec{X}(x,y)|_{y=-1} = 0, \qquad {\rm I} = 1, ..., 7\,.$$

Regularity at the poles

Complicated PDE system...

Spectral method: Decomposition of the perturbation functions

$$
\begin{aligned}
\widetilde{H}_1(x,y) &= \sum_{k=0}^{N_x-1} \sum_{l=|M_z|}^{N_y+|M_z|-1} C_{1,k,l}\, T_k(x)\, P_l^{M_z}(y)\, (1-y^2)^{-M_z/2}\,, \\
\widetilde{T}(x,y) &= \sum_{k=0}^{N_x-1} \sum_{l=|M_z|}^{N_y+|M_z|-1} C_{2,k,l}\, T_k(x)\, P_l^{M_z}(y)\, (1-y^2)^{-M_z/2}\,, \\
\widetilde{L}(x,y) &= \sum_{k=0}^{N_x-1} \sum_{l=|M_z|}^{N_y+|M_z|-1} C_{3,k,l}\, T_k(x)\, P_l^{M_z}(y)\, (1-y^2)^{-M_z/2}\,, \\
\widetilde{N}(x,y) &= \sum_{k=0}^{N_x-1} \sum_{l=|M_z|}^{N_y+|M_z|-1} C_{4,k,l}\, T_k(x)\, P_l^{M_z}(y)\, (1-y^2)^{-M_z/2}\,, \\
\widetilde{h}_0(x,y) &= \sum_{k=0}^{N_x-1} \sum_{l=|M_z|}^{N_y+|M_z|-1} C_{5,k,l}\, T_k(x)\, P_l^{M_z}(y)\, (1-y^2)^{-M_z/2}\,, \\
\widetilde{h}_1(x,y) &= \sum_{k=0}^{N_x-1} \sum_{l=|M_z|}^{N_y+|M_z|-1} C_{6,k,l}\, T_k(x)\, P_l^{M_z}(y)\, (1-y^2)^{-M_z/2}\,. \\
\widetilde{\Phi}_1(x,y) &= \sum_{k=0}^{N_x-1} \sum_{l=|M_z|}^{N_y+|M_z|-1} C_{7,k,l}\, T_k(x)\, P_l^{M_z}(y)\, (1-y^2)^{-M_z/2}\,.
\end{aligned}
$$

Discretization of the domain:

Gauss-Lobato in x – homogeneus in y

$$x_I = \cos\left(\frac{I-1}{N_x-1}\pi\right), \quad I = 1, ..., N_x\,,$$

$$y_K = 2\frac{K-1}{N_y-1} - 1\,, \quad K = 1, ..., N_y\,.$$

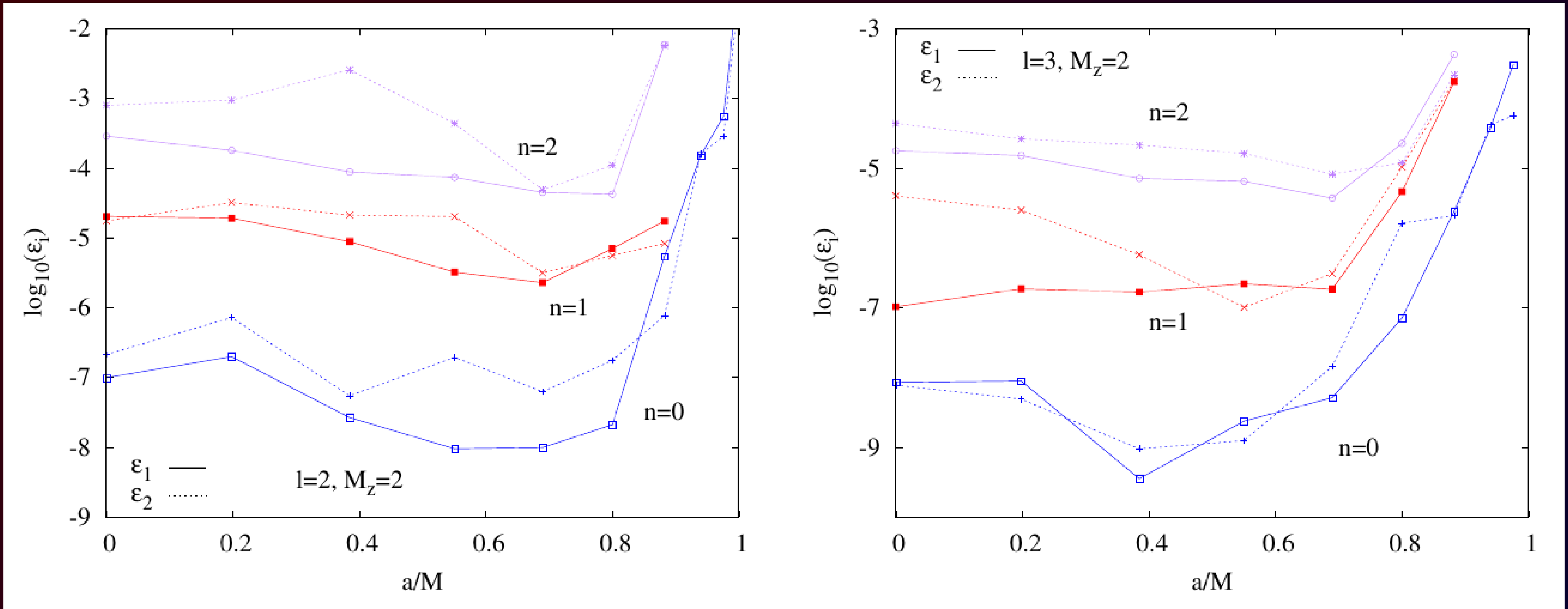
The system of equations and BC can be cast in matrix form

$$\left(\mathcal{M}_0 + \mathcal{M}_1\omega + \mathcal{M}_2\omega^2\right)\vec{C} = 0\,.$$

For scalar + metric perturbations we the matrices have dimension

$$(7 \times N_x \times N_y) \times (7 \times N_x \times N_y)$$

We have applied this method to Kerr black holes with excellent results:



PRD 109 (2024) 6, 064028 • e-Print: 2312.10754 [gr-qc]

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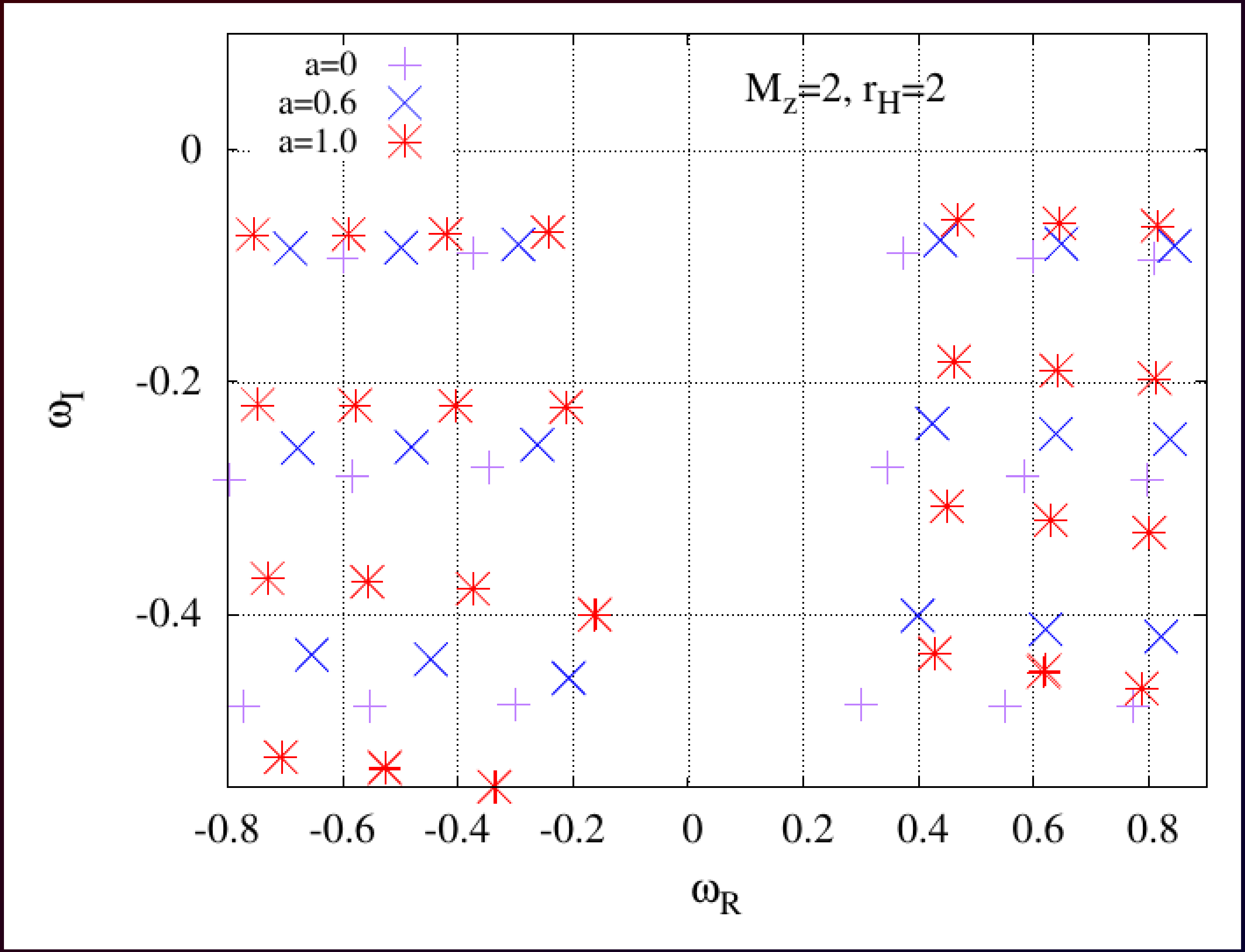

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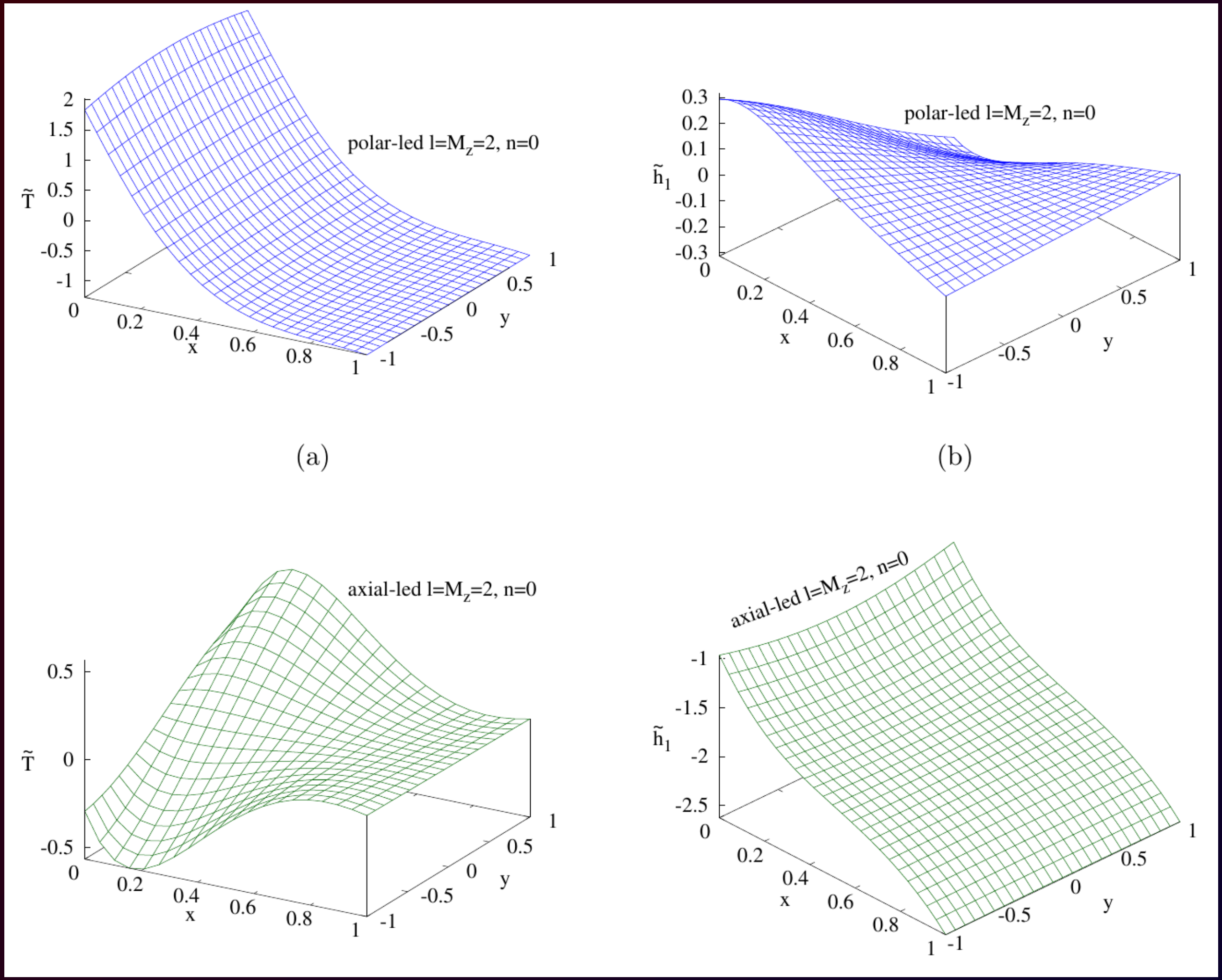

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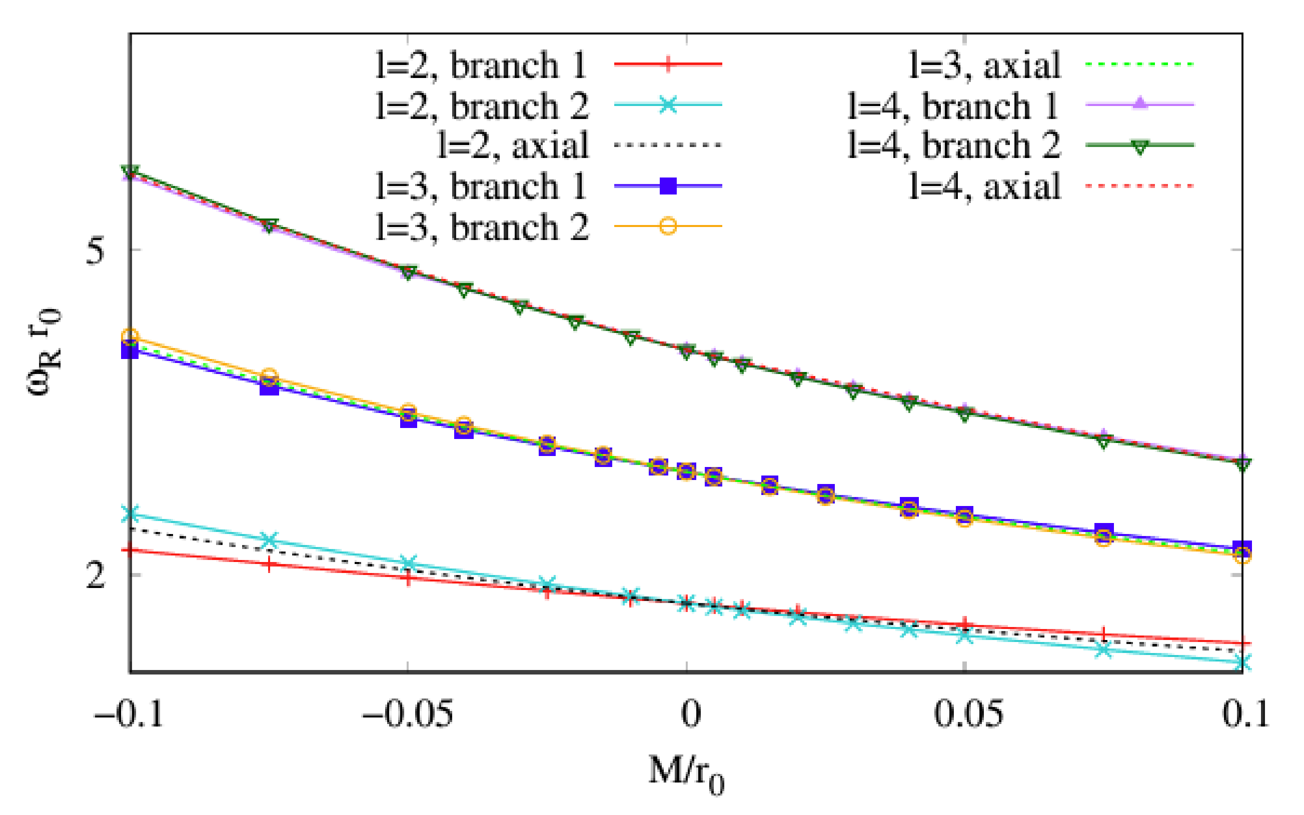
III) Spectrum of rotating Ellis wormholes.

Static and symmetric wormholes: triple isospectrality [B. Azad et al PRD 107 (2023) 084024]



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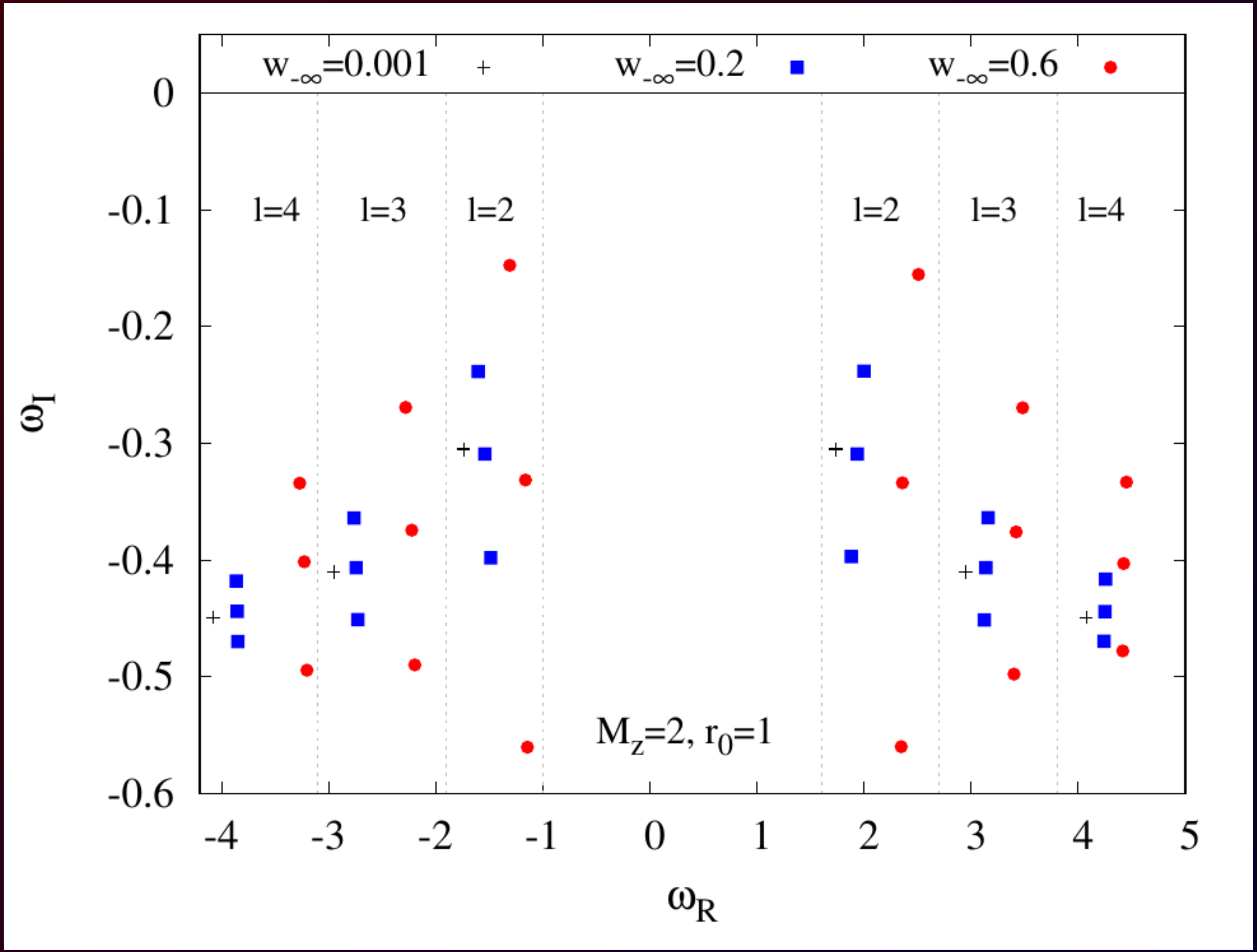

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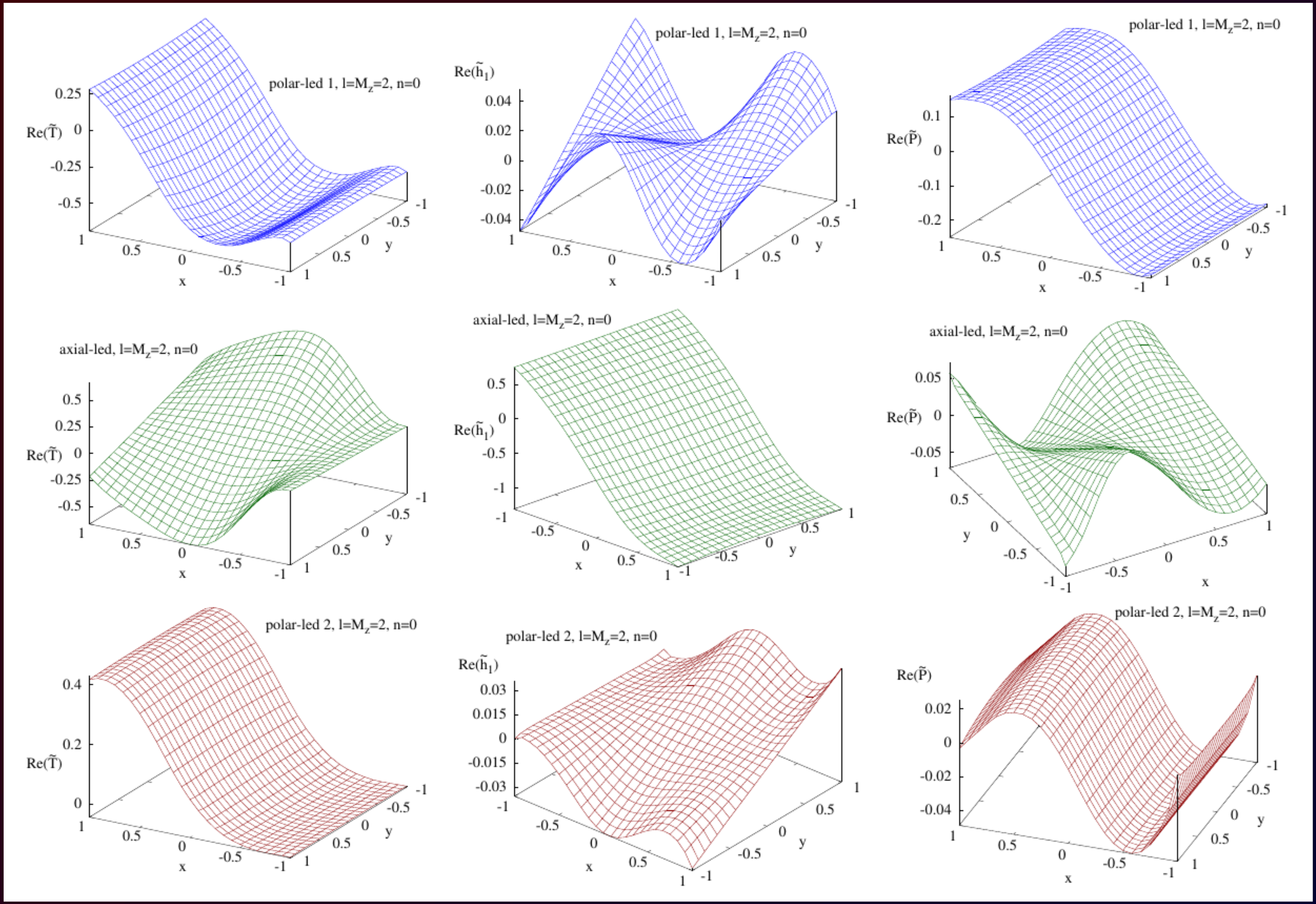

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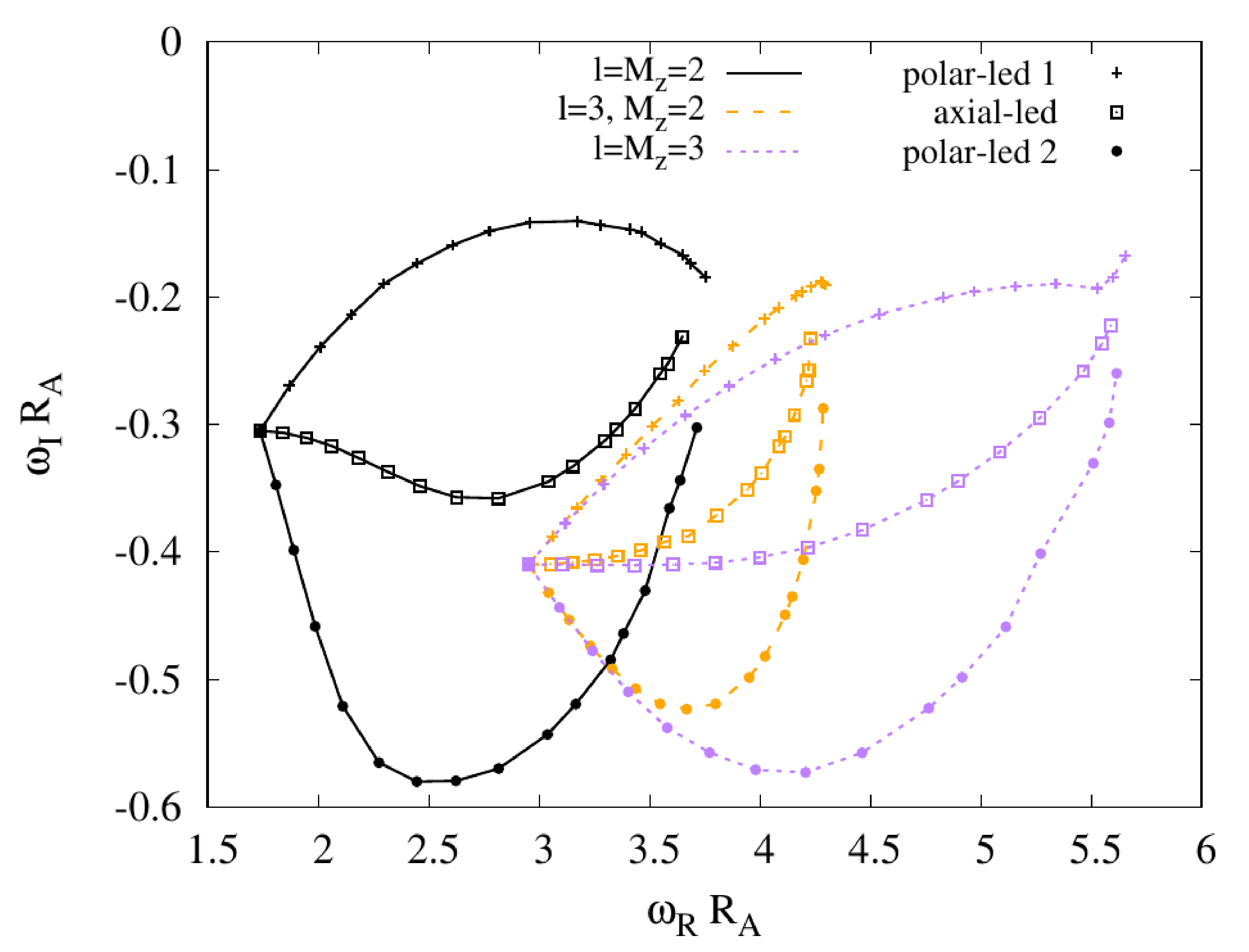

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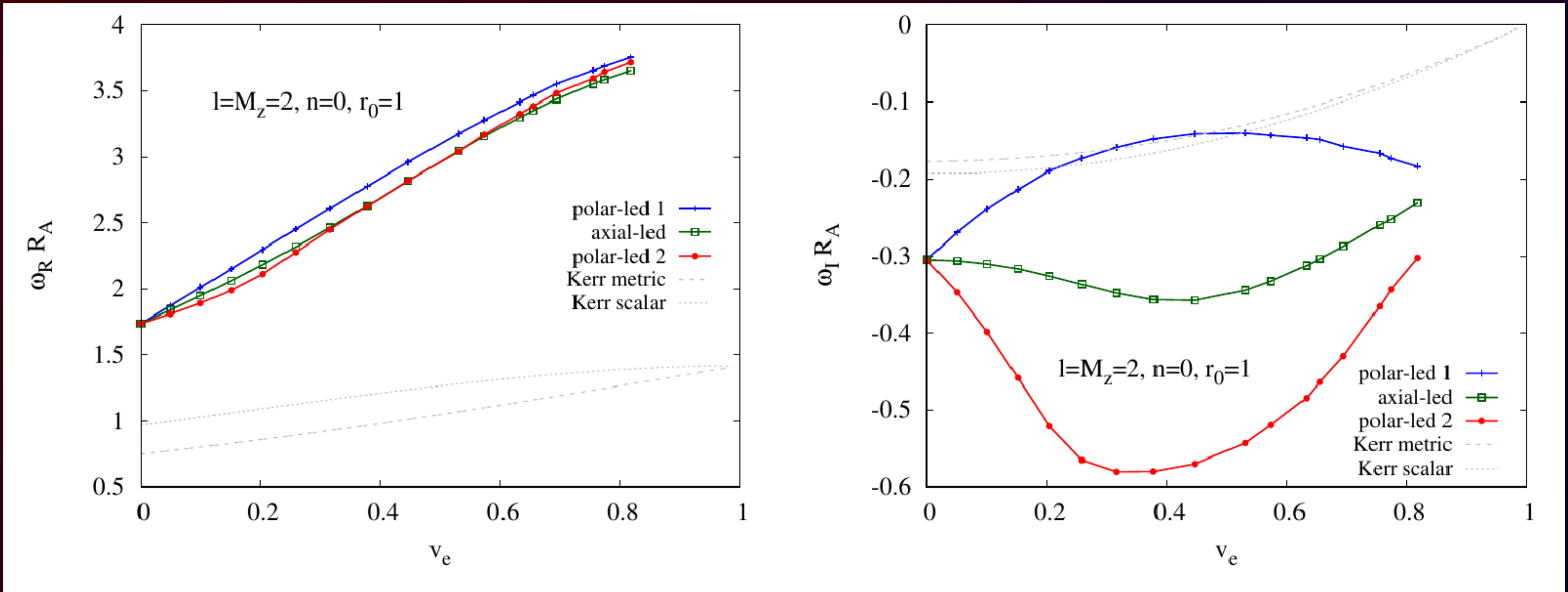

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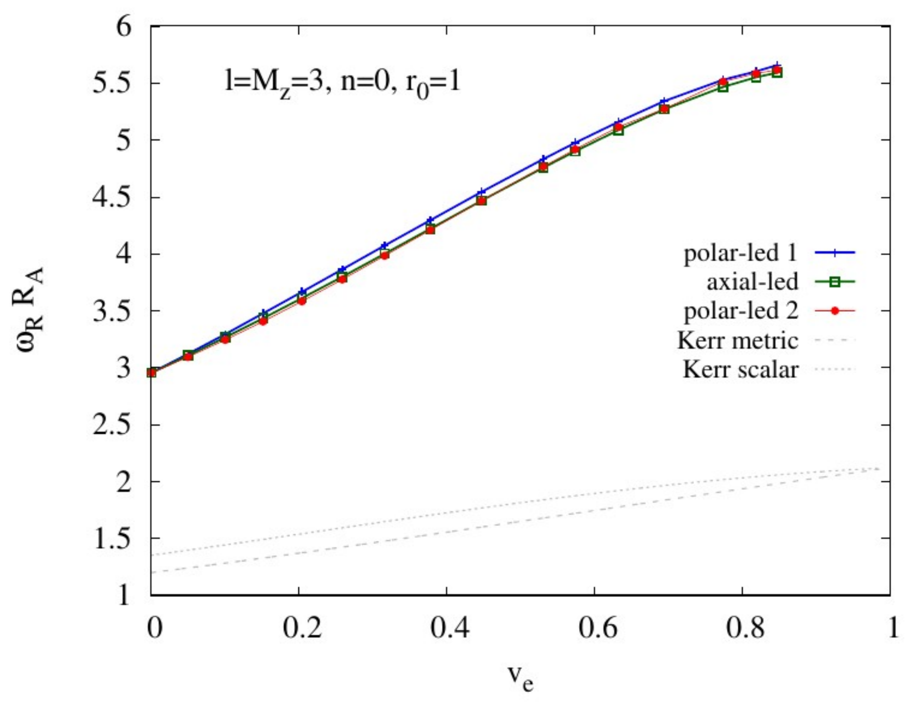

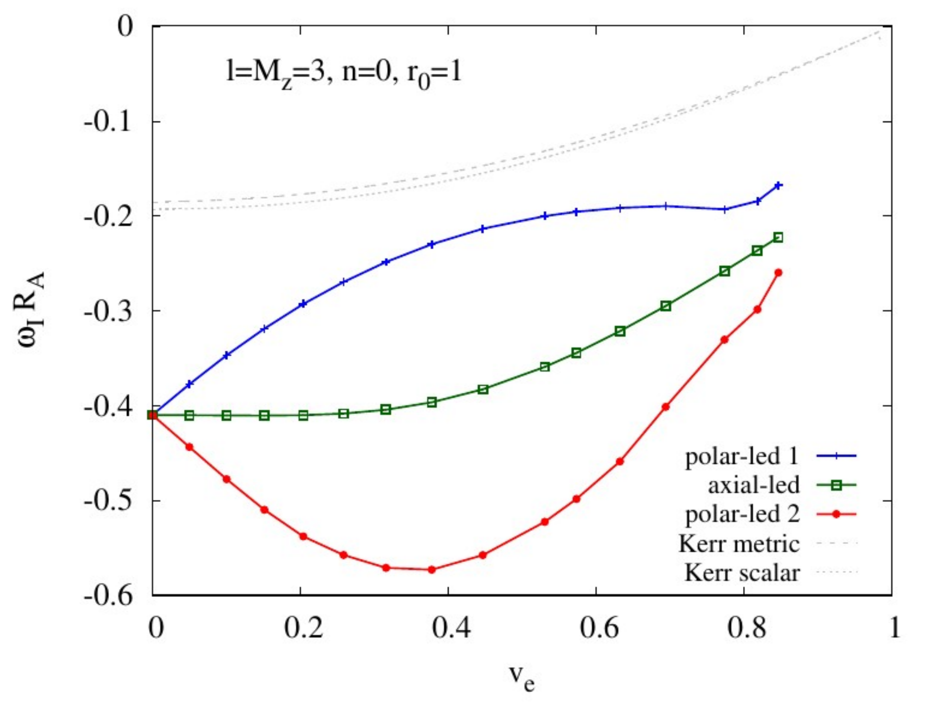

Conclusions and Outlook

- New method to calculate the QNM spectrum of rotating compact objects.

Spectral decomposition of the metric perturbations.

- First calculation of the quasinormal modes of rotating wormholes.

Angular momentum breaks isospectrality.

Outlook:

- Wormhole stability? (see next talk by F.S. Khoo)

- Other compact objects: black holes, neutron stars, boson stars …

- Alternative theories.

Some results on the QNMs of rotating BHs in alternative theories:

MG17 BH2 Session – Friday 12 – F.S. Khoo