How Relic Black Holes , in the early universe in terms of a quantum number n as specified are linked to Torsion , cosmological constant and also adding quantum hair to Quantum Black holes

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Abstract

Our idea is to state that a particular set of values and reformulation of initial conditions for relic black holes, as stated in this manuscript , will enable using the idea of Torsion to formulate a cosmological constant and resultant Dark Energy. Relic Planck sized black holes will allow for a spin density term which presents an opportunity to modify a brilliant argument given as to cancelling Torsion as given by de Sabbata and Sirvaram, Erice 1990 .. Meantime speculation given by Corda replaces traditional firewalls in relic Black holes with a different formulation are included with a quantum number, n. In addition and most important, we have that there may be a solution to showing the incompleteness of the black holes have no hair theorem, for reasons which require quantum number, n, and also BEC representation of Black holes in primordial conditions.

Key words, Inflation, Gravitational waves, Penrose CCC

I. Introduction, the origins of the black holes have no hair theorem and a preview of what we will be trying to modify

Our supposition which will eventually end in terms of challenging the black holes have no hair idea start off with a very simple idea. That is we begin with the simple intuitive model as to how a star mass, M, could attrit a loss of its essence via the following rule. Here, M is a mass, T is temperature, and $\tilde{a}$ is a proportionality term which is assumed to be a constant

$$\frac{dM}{dt} = -\tilde{a} \cdot T^4 \tag{1}$$

In terms of having T as temperature related to star mass we will make the simple rule as follows'

$$T = \frac{\hbar c^3}{8\pi k_B G M} \tag{2}$$

This leads to , if indeed Eq.(1) is observed as far as star mass loss over time to be approximately

$$M^5(loss) = \left(\frac{-5}{(64)^2} \cdot \tilde{a} \right) \cdot \left(\frac{\hbar^4 c^{12}}{\pi^4 k_B^4 G^4} \right) \cdot t \tag{3}$$

In order to parameterize this further in terms of our model as to how we can observe a violation of the black holes have no hair idea we will need to do some parameterization of a mass M, for black holes, in terms of the following inputs for our article

II. Recounting the parameters of black hole physics used in this essay, and where Torion may allow for understanding new bounds as to black hole models, as well as the importance of a quantum number n

Following [1] [2] we do the following, namely for our problem using the substitutions outlined so we can re do the introduction of black hole physics in terms of a quantum number n. To begin this first look at the following for dynamical scaling as far as initial black hole physics in the primordial moments in the regime of Planckian physics to the onset of the big bang. We then get.

$$\begin{aligned} \sqrt{\Lambda} &= \frac{k_B E}{\hbar c S_{entropy}} \\ S_{entropy} &= k_B N_{particles} \end{aligned} \tag{4}$$

And then its reference to the BEC condensate given by[1][3] as to scaling

$$
\begin{aligned}
& m \approx \frac{M_P}{\sqrt{N_{gravitons}}} \\
& M_{BH} \approx \sqrt{N_{gravitons}} \cdot M_P \\
& R_{BH} \approx \sqrt{N_{gravitons}} \cdot l_P \\
& S_{BH} \approx k_B \cdot N_{gravitons} \\
& T_{BH} \approx \frac{T_P}{\sqrt{N_{gravitons}}}
\end{aligned} \tag{5}
$$

This is promising but there is one more step which will utilize the importance of [4] in which we make use of the following Energy expression. First a time step which we make use of , namely

$$
\tau \approx \sqrt{GM\delta r} \tag{6}
$$

Making use of the simplest version of the HUP [5] (NOT the version we finally use) we can use Eq. (3) for an energy of [4] for radiation of a particle pair from a black hole, namely

$$
|E| \approx \left(\sqrt{GM\delta r}\right)^{-1} \hbar \tag{7}
$$

Here we use some approximations, namely we assert that the range of applicability of the spatial variation goes as $\delta r \approx \ell_P$ (8)

This is of a Plank length, whereas we also assume in Eq. (7) that the mass is of roughly a Planck sized black hole mass.

$$
M \approx \alpha M_P \tag{9}
$$

If so, we transform Eq. (4) to roughly be of the form for a 'particle' pair of say an electron positron pair radiating form a black hole as given in Carlip

$$
|E| \approx \left(\sqrt{G \cdot (\alpha M_P) \cdot \ell_P}\right)^{-1} \hbar \qquad (10)'
$$

'

We argue that for very small black holes, of the order of Planck Mass, that we are talking about intense radiation from a Planck sized (or roughly similar sized) black hole, so we approximate this Eq. (10) for a tiny black hole as roughly equivalent to the effective mass of a relic black hole, so up to a point we use the Carlip energy expression as roughly equivalent to the mass of a micro sized black hole

Now using the following normalization of Planck units, i.e. [6], as

$$G = M_P = \hbar = k_B = \ell_P = c = 1 \quad (11)$$

And, also, the initial treatment of energy, E as given for a black hole, to scale as [7]

$$E_{Bh} = -\frac{n_{quantum}}{2} \quad (12)$$

We then can provisionally use for a Black hole the following scaling, namely

$$\begin{aligned} &|E| \approx \left(\sqrt{G \cdot (\alpha M_P) \cdot \ell_P} \right)^{-1} \hbar \\ &\xrightarrow[G=M_P=\hbar=k_B=\ell_P=c=1]{} (1/M_{BH})^{1/2} \approx \frac{n_{quantum}}{2} \end{aligned} \quad (13)$$

We can then go and reference Eq. (5) to observe the following, namely.

$$\begin{aligned} &M_{BH} \approx \sqrt{N_{gravitons}} M_P \\ &\Rightarrow (1/M_{BH})^{1/2} \approx \frac{n_{quantum}}{2} \approx \frac{1}{(N_{gravitons})^{1/4}} \\ &\Rightarrow n_{quantum} \approx \frac{2}{(N_{gravitons})^{1/4}} \end{aligned} \quad (14)$$

This is a stunning result. i.e. Eq. (5) is BEC theory, but it says that if we have a small number of gravitons per black hole, i.e. say due to micro sized black holes, that we are assuming that the number of the quantum number, n associated goes way UP.

Question to be raised. Black hole temperature increases dramatically if we have smaller and smaller black holes. Is this implying that corresponding increases in quantum number, per black hole, n, are commensurate with increasing temperature?

Obviously, this is a preliminary result, but it ties in with what we can say about the following table.

We start off with a given area as to re do the problem of primordial black holes in the universe, and we start off with the following table.

Table 1 from [2] assuming Penrose recycling of the Universe as stated in that document.

End of Prior Universe time frame	Mass (black hole) : super massive end of time BH 1.98910^+41 to about 10^44 grams	Number (black holes) 10^6 to 10^9 of them usually from center of galaxies
Planck era Black hole formation Assuming start of merging of micro black hole pairs	Mass (black hole) 10^-5 to 10^-4 grams (an order of magnitude of the Planck mass value)	Number (black holes) 10^40 to about 10^45, assuming that there was not too much destruction of matter-energy from the Pre Planck conditions to Planck conditions
Post Planck era black holes with the possibility of using Eq. (1) and Eq. (2) to have say 10^10 gravitons/second released per black hole	Mass (black hole) 10 grams to say 10^6 grams per black hole	Number (black holes) Due to repeated Black hole pair forming a single black hole multiple time. 10^20 to at most 10^25

The reason for using this table is because of the following modification of Dark Energy and the cosmological constant [1] [2] [3] [4] To begin this look at [2] which purports to show a global cancellation of a vacuum energy term, which is akin, as we discuss later to the following completely [2][8]

$$\rho_{\Lambda}c^2 = \int_0^{E_{Plank}/c} \frac{4\pi p^2 dp}{(2\pi\hbar)^3}\cdot\left(\frac{1}{2}\cdot\sqrt{p^2c^2+m^2c^4}\right) \approx \frac{\left(3\times10^{19}GeV\right)^4}{(2\pi\hbar)^3} \xrightarrow[E_{Plank}/c\to 10^{\wedge}-30]{} \frac{\left(2.5\times10^{-11}GeV\right)^4}{(2\pi\hbar)^3} \qquad (15)$$

In [2], the first line is the vacuum energy which is completely cancelled in their formulation of application of Torsion. In our article we are arguing for the second line . In fact, in our formulation our reduction to the second line of Eq. (15) will be to confirm the following change in the Planck energy term given by [2]

$$\frac{\Delta E}{c} = 10^{18}GeV - \frac{n_{quantum}}{2c} \simeq 10^{-12}GeV \qquad (16)$$

The term n (quantum) comes from a Corda derived expression as to energy level of relic black holes [7]

We argue that our application of [1] [2] will be commensurate with Eq. (15) which uses the value given in [2] as to the following .i.e. relic black holes will contribute to the generation of a cut off of the energy of the integral given in Eq. (15) whereas what is done in Eq.(15) by [1] [2] is restricted to a different venue which is reproduced below, namely cancellation of the following by Torsion

$$\rho_{\Lambda} c^2 = \int_0^{E_{Plank}/c} \frac{4\pi p^2 dp}{(2\pi\hbar)^3} \cdot \left(\frac{1}{2} \cdot \sqrt{p^2 c^2 + m^2 c^4} \right) \approx \frac{\left(3\times 10^{19} GeV\right)^4}{(2\pi\hbar)^3} \tag{17}$$

Furthermore, the claim in [2] is that there is no cosmological constant, i.e. that Torsion always cancelling Eq. (17) which we view is incommensurate with Table 1 as of [2] . We claim that the influence of Torsion will aid in the decomposition of what is given in Table 1 and will furthermore lead to the influx of primordial black holes which we claim is responsible for the behavior of Eq. (17) above

III. Stating what black hole physics will be useful for in our modeling of Dark Energy. I.e. inputs into the Torsion Spin Density term

In [9] we have the following, i.e., we have a spin density term of [1][9]. And this will be what we input black hole physics into as to forming a spin density term from primordial black holes.

$$\sigma_{Pl} = n_{Pl}\hbar \approx 10^{71} \tag{18}$$

IV. Now for the statement of the Torsion problem as given in [1] [2] [9]

The author is very much aware as to quack science as to purported torsion physics presentations and wishes to state that the torsion problem is not linked to anything other than disruption as to the initial configuration of the expansion of the universe and cosmology, more in the spirit of [9]and is nothing else. Hence, in saying this we wish to delve into what was given in [9] with a subsequent follow up and modification:

To do this, note that in [9] the vacuum energy density is stated to be

$$\rho_{vac} = \Lambda_{eff} c^4 / 8\pi G \tag{19}$$

Whereas the application is given in terms of an antisymmetric field strength $S_{\alpha\beta\gamma}$ [9]

In [2] due to the Einstein Cartan action , in terms of a SL(2,C) gauge theory, we write from [9]

$$L = -R/(16\pi G) + S_{\alpha\beta\gamma} S^{\alpha\beta\gamma} / 2\pi G \tag{20}$$

R here is with regards to Ricci scalar and Tensor notation and $S_{\alpha\beta\gamma}$ is related to a conserved current closing in on the SL(2,C) algebra as given by

$$J^{\mu} = J^{\mu} + 1/(16\pi G)\varepsilon^{\mu\alpha\beta\gamma} S_{\alpha\beta\gamma} \tag{21}$$

This is where we define

$$S_{\alpha\beta\gamma} = c_{\alpha} \times f_{\beta\gamma} \tag{22}$$

Where c_{α} is the structure constant for the group SL(2,C), and

$$f_{\beta\gamma} \cdot \bar{g} = F_{\beta\gamma} \quad (23)$$

Where

$$\bar{g} = \left(g_1, g_2, g_3 \right) \quad (24)$$

Is for tangent vectors to the gauge generators of SL(2,C) , and also for Gauge fields A_γ

$$F_{\beta\gamma} = \partial_\beta A_\gamma - \partial_\gamma A_\beta + \left[A_\beta, A_\gamma \right] \quad (25)$$

And that there is furthermore the restriction that

$$\partial_\rho \left(\varepsilon^{\rho\alpha\beta\gamma} S_{\alpha\beta\gamma} \right) = 0 \quad (26)$$

Finally in the case of massless particles with torsion present we have a space time metric

$$ds^2 = d\tau^2 + a^2 \left(\tau \right) d^2\Omega_3 \quad (27)$$

Where $d^2\Omega_3$ is the metric of S^3

Then the Einstein field equations reduce to in this torsion application, (no mass to particles) as

$$(da / d\tau)^2 = \left[1 - \left(r_{\min}^4 / a^4 \right) \right] \quad (28)$$

With , if S is the so called spin scalar and identified as the basic $\hbar$ unit of spin

$$r_{\min}^4 = 3G^2 S^2 / 8c^4 \quad (29)$$

V. How to modify Eq. (28) in the presence of matter via Yang Mills fields $F_{\mu\nu}^{\beta}$

First of all, this involves a change of Eq. (20) to read

$$L = -R / (16\pi G) + S_{\alpha\beta\gamma} S^{\alpha\beta\gamma} / 2\pi G + (1/4g^2) F_{\mu\nu}^{\beta} F_{\beta}^{\mu\nu} \quad (30)$$

And eventually we have a re do of Eq. (28) to read as

$$(da / d\tau)^2 = \left[1 - \left(\beta_1 / a^2 \right) - \left(\beta_2 / a^4 \right) \right] \quad (31)$$

If $g = \hbar c$ we have $\beta_1 = r_{\min}^2, \beta_2 = r_{\min}^4$, and the minimum radius is identified with a Planck Radius so then

$$(da / d\tau)^2 = \left[1 - \left(\left(\beta_1 = \ell_P^2 \right) / a^2 \right) - \left(\left(\beta_2 = \ell_P^4 \right) / a^4 \right) \right] \quad (32)$$

Eventually in the case of an unpolarized spinning fluid in the immediate aftermath of the big bang, we would see a Roberson Walker universe given as, if σ is a torsion spin term added due to [9] as

$$\left(\frac{\dot{\tilde{R}}}{\tilde{R}} \right)^2 = \left(\frac{8\pi G}{3} \right) \cdot \left[\rho - \frac{2\pi G \sigma^2}{3c^4} \right] + \frac{\Lambda c^2}{3} - \frac{\tilde{k} c^2}{\tilde{R}^2} \quad (33)$$

VI. What [9] does as to Eq. (33) versus what we would do and why

In the case of [1] we would see σ be identified as due to torsion so that Eq. (33) reduces to

$$\left(\frac{\dot{\tilde{R}}}{\tilde{R}}\right)^2 = \left(\frac{8\pi G}{3}\right)\cdot[\rho] - \frac{\tilde{k}c^2}{\tilde{R}^2} \quad (34)$$

The claim is made in [2] that this is due to spinning particles which remain invariant so the cosmological vacuum energy, or cosmological constant is always cancelled.

Our approach instead will yield [9]

$$\left(\frac{\dot{\tilde{R}}}{\tilde{R}}\right)^2 = \left(\frac{8\pi G}{3}\right)\cdot[\rho] + \frac{\Lambda_{0bserved}c^2}{3} - \frac{\tilde{k}c^2}{\tilde{R}^2} \quad (35)$$

I.e. the observed cosmological constant $\Lambda_{0bserved}$ is 10^-122 times smaller than the initial vacuum energy

The main reason for the difference in the Eq. (34) and Eq. (35) is in the following observation. We will go to Table 1 and make the following assertion

Mainly that the reason for the existence of σ^2 is due to the dynamics of spinning black holes in the precursor to the big bang, to the Planckian regime, of space time, whereas in the aftermath of the big bang, we would have a vanishing of the torsion spin term. i.e. the Table 1 dynamics in the aftermath of the Planckian regime of space time would largely eliminate the σ^2 term

VII. Filling in the details of the Eq. (34) collapse of the cosmological term, versus the situation given in Eq. (35) via numerical values

First look at numbers provided by [9] as to inputs, i.e. these are very revealing

$$\Lambda_{Pl}c^2 \approx 10^{87} \quad (36)$$

This is the number for the vacuum energy and this enormous value is 10^122 times larger than the observed cosmological constant. Torsion physics, as given by [9] is solely to remove this giant number .

In order to remove it, the reference [1][9] proceeds to make the following identification, namely

$$\left(\frac{8\pi G}{3}\right)\cdot\left[-\frac{2\pi G\sigma^2}{3c^4}\right] + \frac{\Lambda c^2}{3} = 0 \quad (37)$$

What we are arguing is that instead, one is seeing, instead[9]

$$\left(\frac{8\pi G}{3}\right)\cdot\left[-\frac{2\pi G\sigma^2}{3c^4}\right]+\frac{\Lambda_{Pl}c^2}{3}\approx 10^{-122}\times\left(\frac{\Lambda_{Pl}c^2}{3}\right) \quad (38)$$

Our timing as to Eq. (36) is to unleash a Planck time interval t about 10^-43 seconds

As to Eq. (37) versus Eq. (38) the creation of the torsion term is due to a presumed particle density of

$$n_{Pl} \approx 10^{98} cm^{-3} \quad (39)$$

Finally, we have a spin density term of $\sigma_{Pl} = n_{Pl}\hbar \approx 10^{71}$ which is due to innumerable black holes initially

VIII. Future works to be commenced as to derivational tasks

We will assume for the moment that Eq. (36) and Eq. (37) share in common Eq. (39)

It appears to be trivial, a mere round off, but I can assure you the difference is anything but trivial. And this is where Table 1 really plays a role in terms of why there is a torsion term to begin with, i.e. will make the following determination, i.e.

The term of 'spin density' in Eq. (36) by Eq. (39) is defined to be an ad hoc creation, as to [3]. No description as to its origins is really offered

1st

We state that in the future a task will be to derive in a coherent fashion the following, i.e. the term of

$\left(\frac{8\pi G}{3}\right)\cdot\left[-\frac{2\pi G\sigma^2}{3c^4}\right]$ arising as a result of the dynamics of Table 1, as given in the manuscript

2nd,

We state that the term $\left(\frac{8\pi G}{3}\right)\cdot\left[-\frac{2\pi G\sigma^2}{3c^4}\right]$ is due to initial micro black holes, as to the creation of a Cosmological term.

In the case of Pre Planckian space-time the idea is to do the following [9] ,i.e. if we have an inflaton field [9][10] [11][12][13][14][15][16][17]

$$\left|dp_\alpha dx^\alpha\right| \approx \frac{L}{l}\cdot\frac{h}{c}\cdot\left[\frac{dl}{l}\right]^2$$
$$\xrightarrow[\alpha=0]{}\left|dp_0 dx^0\right| \simeq \left|\Delta E \Delta t\right| \approx \left(h / a_{init}{}^2 \phi(t)\right) \qquad (40)$$
$$\Rightarrow \frac{L}{l}\cdot\frac{h}{c}\cdot\left[\frac{dl}{l}\right]^2 \approx \left(h / a_{init}{}^2 \phi(t_{init})\right)$$

Making use of all this leads to[10]to making sense of the quantum number n as given by reference to black holes, [7] $E_{Bh} = -\frac{n_{quantum}}{2}$

3rd

The conclusion of [1] states that Eq.(40) would remain invariant for the life of the evolution of the universe. We make no such assumption. We assume that, as will be followed up later that Eq. (38) is due to relic black holes with the suppression of the initially gigantic cosmological vacuum energy,

The details of what follow after this initial period of inflation remain a task to be completed in full generality but we are still assuming as a given the following inputs [9] [14]

$$a(t) = a_{initial} t^\nu$$
$$\Rightarrow \phi = \ln\left(\sqrt{\frac{8\pi G V_0}{\nu\cdot(3\nu-1)}}\cdot t\right)^{\sqrt{\frac{\nu}{16\pi G}}}$$
$$\Rightarrow \dot{\phi} = \sqrt{\frac{\nu}{4\pi G}}\cdot t^{-1} \qquad (41)$$
$$\Rightarrow \frac{H^2}{\dot{\phi}} \approx \sqrt{\frac{4\pi G}{\nu}}\cdot t \;\; \cdot T^4 \cdot \frac{(1.66)^2\cdot g_*}{m_P^2} \approx 10^{-5}$$

A possible future endeavor can also make sense of [15] as well

IX. 1st CONCLUSION, how meeting conditions for applying Torsion to obtain the cosmological constant and DE modifies black hole physics in the early universe.

First of all, it puts a premium upon our Table 1 as given and is shown in [9]. Secondly it means utilization of Equation (16) which takes into account the black hole energy equation given by Corda in [7] and it also means that the spin density term as given in Equation (18) is freely utilized.

We refer to black hole creation as given by torsion this way as a correction to [1] largely due to the insufficiency of black hole theory as eloquently given in [16] which we will cite their page 366 admonition as to the insufficiency of current theory

Quote

Black holes of masses sufficiency smaller than a solar mass cannot be formed by gravitational collapse of a star; such miniholes can only form in the early stages of the universe, from fluctuations in the very dense primordial matter

End of quote

Our torsion argument is directly due to this acknowledgement and is due to the sterility of much theoretical thinking, as well as the tremendously important Eq. (12) which is due to Corda [7]

Furthermore, in order to obtain more details of Equation (12) being utilized for black holes, we state that a quantum state of the early universe will utilize [17] and its discussion, page 184, as to how Feynman visualized the quantization of the Gravitational field , i.e. Equations 9.121 and 9.122 of [17] for an early wavefunction path integral treatment for quantized gravity and its use for black holes. Corda himself [7] has alluded to a path forward in such treatment of how black holes can be modeled which lead to Equation (40)

In addition we outlined the stunning result as given as of Eq. (14) as far as a more than an inverse relationship between graviton number, per generated black hole (presumably primordial) and a quantum number n, attached to a black hole as due to [7]. What we see is that if we have small black holes, with BEC characteristics with small number of gravitons, per primordial black hole, that the quantum number n climbs dramatically. We need to obtain the complete dynamics of this relationship as it pertains to how very small black holes have high quantum number n, which we presume is commensurate with initially high temperatures

The details of this development as well as its tie into the dynamics of table 1 as given and Torsion have to be fine tuned

More work needs to be done so we can turn early universe gravitational generation and black hole physics into an empirical science

2nd CONCLUSION, looking directly at a modification of the Black holes have no hair theorem, via the inputs of this document.

In [18] we have the essential black holes have no hair theorem which can be seen roughly as

Quote

The idea is that beyond mass, charge and spin, black holes don't have distinguishing features — no hairstyle, cut or color to tell them apart.

End of quote

How do we get about this ? Note that in [19] there is a pseudo extension which we can chalk up to Hawking before he died; but in order to apply an even more direct treatment we go to what is given in [20]

i.e. we go to formula 65 of that reference. This will give a variation of the radius of a black hole, over the radius, according to a quantum number n AGAIN. Before we get there we will do some initial work up to that quantum number, n as used in formula 65 of reference [20]

i.e. using our equation (14) for N and also the Planck scale normalization as given by

$\hbar = k_B = c = G = M_p = \ell_p = 1$, and if we take $\tilde{a}$ approximately scaled to 1 as well we have that if

$$|N| \approx |N_{gravitons}| \approx \left(\frac{5t}{(64)^2 \pi^4} \right)^{2/5} \qquad (42)$$

Due to using [3]

$$M \approx \sqrt{N} M_p \qquad (43)$$

M here being linked to the mass of a BEC black hole, and also using Eq. (3) for the loss of a black hole, over time

Also use

$$|N_{gravitons}|^{5/2} \times \left(M_p \equiv 1 \right)^{5/2} \approx \left(\frac{5t}{(64)^2 \pi^4} \right) \qquad (44)$$

Then use the last equation of Eq. (14) to obtain, a quantum number associated with a graviton just outside a BEC primordial black hole

$$n_{graviton-quantum-number} \equiv n_{graviton} \approx \left[\frac{2 \cdot (64)^{1/10} \pi^{1/5}}{5^{1/20} \cdot t^{1/20}} \right] \approx \frac{2.16245415907}{t^{1/20}} \qquad (45)$$

Assuming Planck scale time, or close to it, and renormalization to have Planck time as set to 1

This means then that the quantum number, n associated with a graviton with respect to a Planck sized black hole would be close to 2, initially

If so then, and this is for primordial black holes, we then associate this graviton number, n for a graviton as linked to the following from [20], i.e. their so called Eq. (65) so we have for the radius of a BEC black hole as deformed by this quantum number n, a small change

$$\frac{\Delta R_n}{R_n} \equiv \frac{\sqrt{n^2 + 2}}{3n} \qquad (46)'$$

‘If we use the value of n = 2.16245415907 for a graviton “quantum number” at about normalized Planck time, scaled to about 1, and we have according to [20] an ADM mass variance of M so then there is, due to gravitons, a rough change in initial Planck sized black holes

$$\Delta R_n = \left(\frac{\sqrt{n^2+2}}{3n} \right) \cdot R_n \approx \left(\frac{\sqrt{n^2+2}}{3n} \right) \Bigg|_{n=2.16245415907} \times R_n \qquad (47)$$

Where $n \geq (1-\varepsilon) \cdot \left(M / M_p \right)^2$ and we can compare our value of R, as given in Eq. (5) with [20] having a different scale for R, as given in their equation 60

Needless to say, graviton number n, as specified, due to the processes within the primordial black hole we assert would lead to a violation of the black holes have no hair theorem , of [19]

We assert that this value of n, so obtained, as to gravitons would be as to the Corda result on Eq(12) the following

$$n(black-holes) = N(graviton-number-per-black-hole) \times n\left(quantum-number-per-graviton\right) \ (48)$$

The left hand side of Eq. (48) would be fully commensurate with Eq. (12) of Corda’s black hole quantum number

The right hand side of Eq. (48) would be commensurate with n being for a quantum number per graviton associated per black hole

If there are a lot of gravitons, associated with a primordial black hole, this would commence with a very high initial quantum number, n (black holes) associated Cordas great result, as of [7]

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