TESTING SCALE-INVARIANT INFLATION AGAINST COSMOLOGICAL DATA

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MG17, Pescara

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Outline

1. Scale invariance as a fundamental symmetry
2. Scale-invariant quadratic gravity
3. Inflationary predictions

Inflation

How to realise naturally flat inflationary potentials without fine-tuning?

Naturalness problem

How to make divergent quantum corrections naturally small?

Renormalizability

Can we find a highly predictive criterion beyond renormalizability?

A fundamental QFT does not involve any intrinsic parameter with dimension mass or length

- We introduce an explicit mass scale k

CANONICAL FIELD $\leftarrow$ $\phi = k\,\tilde{\phi}$ $\rightarrow$ SCALE-INVARIANT FIELD

- The corresponding effective action obeys

$$k\partial_k\Gamma_k[\phi] = \zeta_k[\phi]$$

General solution

$$\boxed{k\partial_k\Gamma_k[\tilde{\phi}] = 0}$$

Particular, scaling solution holding when the canonical fields are expressed in terms of the scale-invariant ones

Naturally flat potentials for inflation

- Scale-invariant theory non-minimally coupled to gravity

$$\mathcal{L}_J = \sqrt{-g}\left[\xi\phi^2 R - \lambda\phi^4 - \frac{1}{2}(\partial\phi)^2\right]$$

- Weyl rescaling from the Jordan frame to the Einstein frame

$$\mathcal{L}_E = \sqrt{-\tilde{g}}\left[\frac{M_{Pl}^2}{2}\tilde{R} - M_{Pl}^4\frac{\lambda}{\xi^2} - \frac{1}{2}(\partial\tilde{\phi})^2\right]$$

$$\downarrow$$

The potential is FLAT at tree-level: no fine-tuning

- Scale symmetry breaking can occur from quantum corrections

Fundamental SI

SOLUTION TO THE NATURALNESS PROBLEM

- Coefficients of super-renormalizable terms

  $\downarrow$

  Power-law divergences

MASS DIMENSION < 4
IN 3+1 DIMENSIONS

Fundamental scale invariance requires only mass dimension 4 Lagrangian terms

- Indication that Nature may prefer dimension-4 operators

Fundamental SI

A criterion beyond renormalizability

- For general renormalisable theories the effective action remains well defined in the continuum limit if one employs renormalised fields

RENORMALIZED FIELDS $\leftarrow$ $\phi_{R,i}(x) = k^{d_i} f_i(k)\, \tilde{\phi}_i(x)$ $\rightarrow$ SCALE-INVARIANT FIELDS

Theories with fundamental scale invariance:

1. Renormalizable
2. For some choice of the fields the effective action becomes k-independent
3. Exact scaling solutions: no free parameters.

   $\rightarrow$ High predictive power

The model

- $\mathcal{L}_{EH} \rightarrow f(R, \phi)$: scalar-tensor theory

$$\mathcal{L}_J = \sqrt{-g}\left[\frac{\alpha}{36}R^2 + \frac{\xi}{6}\phi^2 R - \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4\right], \qquad \alpha, \lambda, \xi > 0$$

$\downarrow$ Higher order term in R

$\downarrow$ Scalar field

- Two additional scalar degrees of freedom

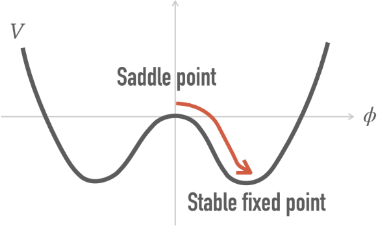
SCALE TRANSFORMATION:

- $\bar{g}_{\mu\nu}(x) = g_{\mu\nu}(lx)$
- $\bar{\phi}(x) = l\phi(lx) \qquad \rightarrow \qquad \bar{\mathcal{L}} = \mathcal{L}$

Scale-invariant quadratic gravity

JORDAN FRAME

- The field ϕ is subjected to $V_{\text{eff}}(\phi) = -\frac{\xi}{6}\phi^2 R + \frac{\lambda}{4}\phi^4$



CLASSICAL SCALE-SYMMETRY BREAKING

The scalar field takes a non-zero VEV at the minimum

$$\langle\phi_0^2\rangle = \frac{\xi R}{3\lambda}$$

DYNAMICAL GENERATION OF MASS SCALE

Natural identification with the Planck mass

$$\frac{\xi}{6}\phi_0^2 R \equiv \frac{1}{2} M_{Pl}^2 R$$

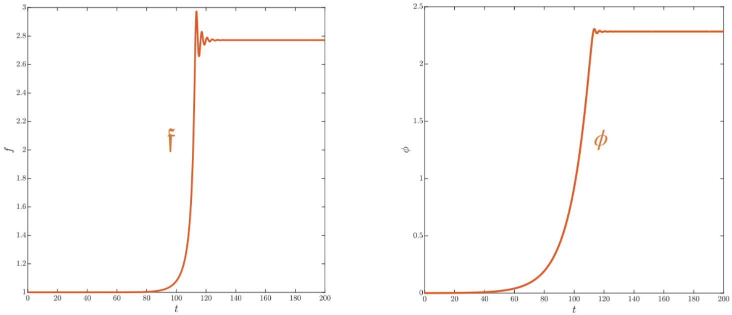
EINSTEIN FRAME

$$g^{*}_{\mu\nu} = \Omega^2 g_{\mu\nu}$$

- Two dynamical degrees of freedom: are we in multi-field inflation?

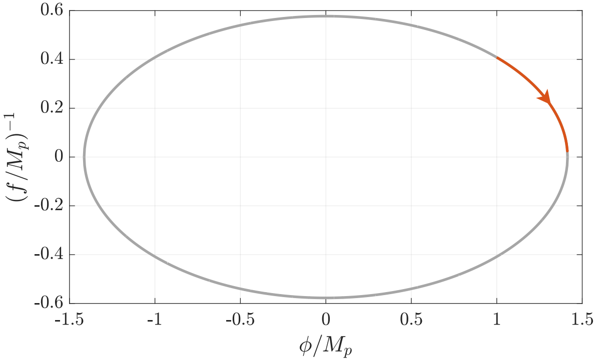
$$\mathcal{L}_E = \sqrt{-g}\left[\frac{M^2}{2}R - \frac{3M^2}{f^2}(\partial f)^2 - \frac{f^2}{2M^2}(\partial\phi)^2 - V(f,\phi)\right]$$

Note that M is a redundant parameter



Einstein frame: Noether's current conserved

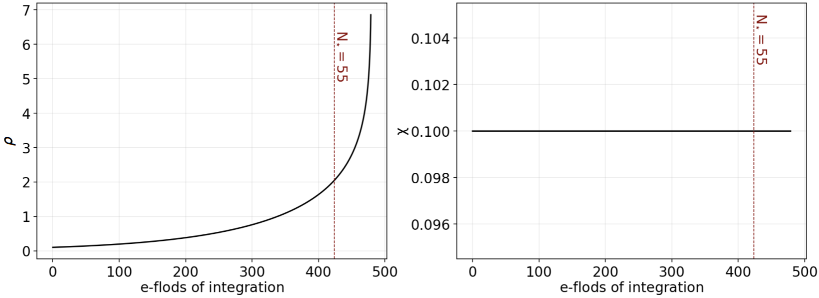
- Constraint on the two-fields dynamics $f = \frac{\sqrt{6}M_{Pl}^2}{\sqrt{2M_{Pl}^2 - \phi^2}}$



NOETHER'S CURRENT CONSERVED

- Its conservation can be employed to shift all the dynamics on one field $\rho = \rho(f, \phi), \quad \chi = \chi(f, \phi)$

$$\mathcal{L}_E = \sqrt{-g}\left(\frac{M^2}{2}R - \frac{1}{2}\partial_\mu\rho\partial^\mu\rho - 3\cosh\left[\frac{\rho}{\sqrt{6}M}\right]^2\partial_\mu\chi\partial^\mu\chi - V(\rho)\right)$$

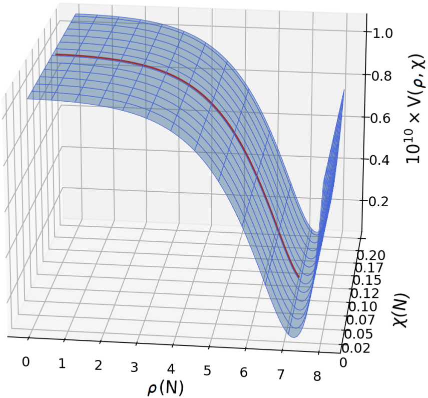


NOETHER'S CURRENT CONSERVED: single field inflation

- Naturally flat plateau: no fine-tuning
- Non-vanishing at the minima

$\chi \rightarrow$ GOLDSTONE BOSON

$\rho \rightarrow$ INFLATON



Scale-invariant quadratic gravity

Noether's current conserved: entropy perturbations

- Employing Noether's current conservation we show that

$$\boxed{\delta s = 0}$$

Scale invariance protects from any form of geometrical destabilization ✓

GEOMETRICAL DESTABILIZATION OF INFLATION

S. Renaux-Petel, K. Turzyński Phys.Rev.Lett.117 (2016)

- Multi-field inflation
- Hyperbolic fields' space geometry $\rightarrow \quad m^2_{s(\text{eff})} < 0$

$\rightarrow$ Instability prematurely ending inflation

Numerical analysis

We dynamically calculate ϵ until end of inflation ($|\epsilon| = 1$)

$\downarrow$

Sufficiently long inflation? $\rightarrow$ DISCARD

$\downarrow$

$$\boxed{\text{Compute } A_s, n_s, \alpha_s, r}$$

$\downarrow$

Are they within some reasonably chosen ranges? $\rightarrow$ DISCARD

$\downarrow$

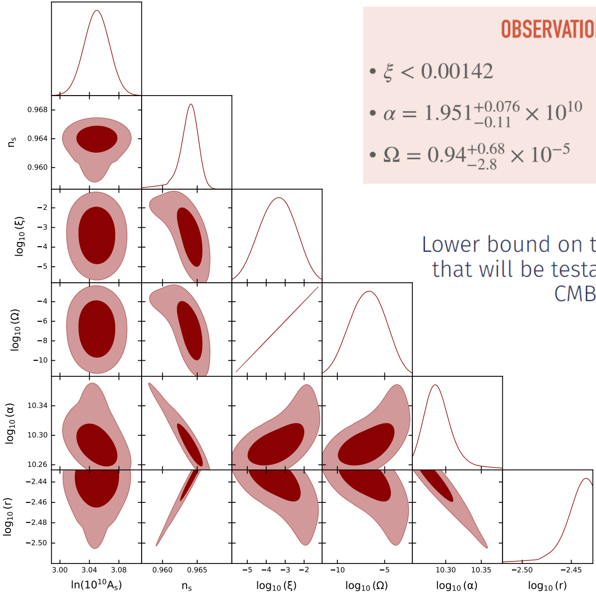
Implement CAMB and assign a likelihood based on how well the model agrees with recent observations:

- Planck2018 (TT-TE-EE+Lensing)
- BICEP/KECK (BB)

Inflationary predictions

Initial conditions	**Constraints**	**Uniform prior ranges**
$\rho_{\rm ini}/M_p$	*(unconstrained)*	$\rho_{\rm ini}/M_p \in [0.1, 2]$
$\chi_{\rm ini}/M_p$	*(unconstrained)*	$\chi_{\rm ini}/M_p \in [0.1, 10]$
Model parameters	**Constraints**	**Uniform prior ranges**
ξ	< 0.00142	$\log_{10}(\xi) \in [-5, -1]$
α	$1.951^{+0.076}_{-0.11} \times 10^{10}$	$10^{-10} \times \alpha \in [1, 3]$
Ω	$0.93^{+0.72}_{-2.8} \times 10^{-5}$	$\Omega \in [\xi^2, 2\xi^2]$
Primordial spectra parameters	**Constraints**	
A_s	$(\,2.112 \pm 0.033\,) \cdot 10^{-9}$	*(derived)*
n_s	$0.9638^{+0.0015}_{-0.0010}$	*(derived)*
α_s	$< 1.2 \times 10^{-4}$	*(derived)*
r	> 0.00332	*(derived)*

Inflationary predictions



OBSERVATIONAL CONSTRAINTS

- $\xi < 0.00142$
- $\alpha = 1.951^{+0.076}_{-0.11} \times 10^{10}$
- $\Omega = 0.94^{+0.68}_{-2.8} \times 10^{-5}$
- $A_S = (2.112 \pm 0.033) \times 10^{-9}$
- $n_S = 0.9638^{+0.0015}_{-0.0010}$
- $r > 0.00332$

$\downarrow$

Lower bound on the tensor-to-scalar ratio r, that will be testable from next generation CMB experiments

$$\Omega \equiv \alpha\lambda + \xi^2$$

Inflationary predictions

Observational constraints

- Strong correlation between Ω and ξ to avoid eternal inflation;
- Overall insensitivity to initial conditions;
- $\xi < 0.00142$ (95% C.L.): conformal invariance is ruled out.

Inflationary predictions

Scale invariance vs Starobinsky

$$\mathcal{L} = \sqrt{-g}\frac{M_{Pl}^2}{2}\left[R + \frac{R^2}{6M^2}\right]$$

$\downarrow$ SCALE-INVARIANT

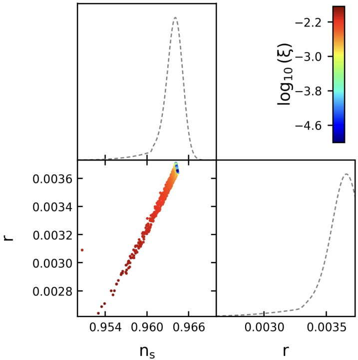
Starobinsky's model is scale-invariant when the R^2 term dominates!

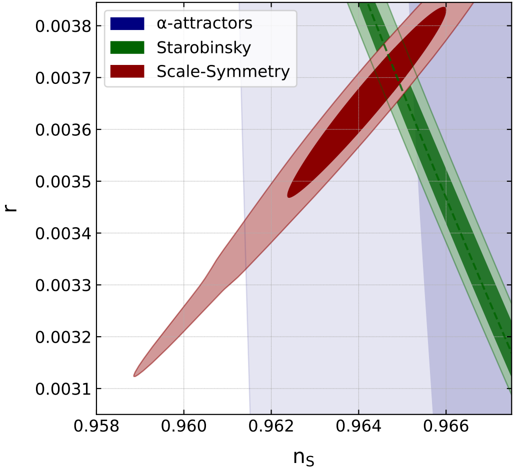
- Can we discriminate between the two models?

Inflationary predictions

SCALE INVARIANCE VS STAROBINSKY

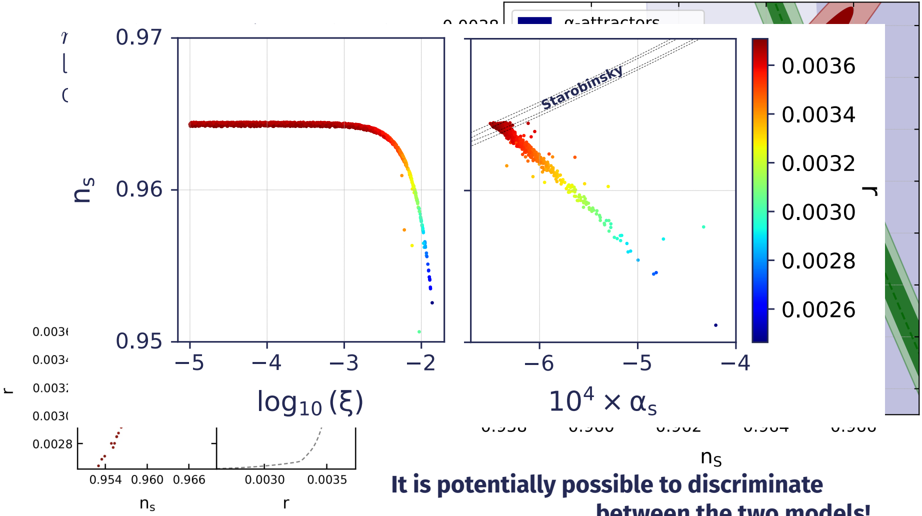
n_s and r are anti-correlated like in Starobinsky's model only at fixed ξ.





It is potentially possible to discriminate between the two models!

Scale invariance vs Starobinsky



It is potentially possible to discriminate between the two models!

Inflationary predictions

- Scale-invariant inflation

$$n_s \simeq 1 - \frac{1}{3}\sqrt{3r + 64\xi^2}$$

- Starobinsky inflation ($\alpha = 1$)

$$n_s \simeq 1 - \sqrt{\frac{r}{3}}$$

As soon as $\xi \neq 0$ and therefore the non-minimal coupling $\phi^2 R$ is turned on, the predictions of the two models differ

Summary

- Fundamental scale invariance as a solution to the flat potential problem for inflation
- Scale-invariant quadratic gravity: Noether's current conservation for single-field dynamics and vanishing entropy perturbations
- Promising numerical result: the model is competitive with Starobinsky

Summary

- Fundamental scale invariance as a solution to the flat potential problem for inflation
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- Promising numerical result: the model is competitive with Starobinsky

THANK YOU!