Recent work in Causal Set Theory

Yasaman K. Yazdi

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Quantum Gravity

What is the fundamental structure of spacetime?

- Essential features of spacetime
- Issues such as infinities
- Potential for quantum dynamics and answering open questions, e.g. in cosmology and black hole physics

Causal Set Theory

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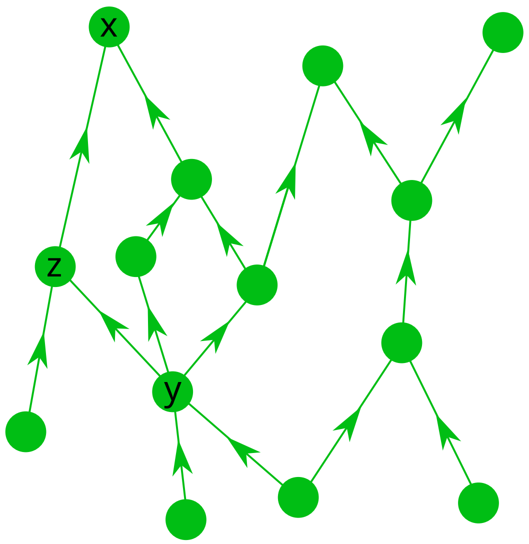

A **locally finite partially ordered set**.

- Essential features of spacetime: **causal structure**
- Issues such as infinities: **discreteness**
- Potential for quantum dynamics and answering open questions, e.g. in cosmology and black hole physics: **talks**

The set $\mathscr{C}$ is that of spacetime elements and the ordering relation $\preceq$ is the causal precedence relation.

Causal Set Theory

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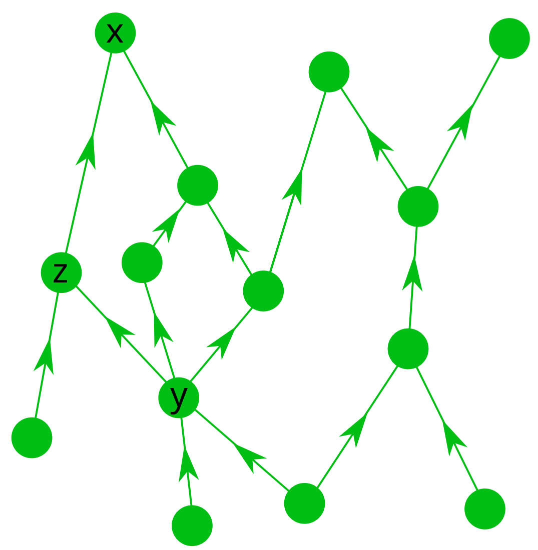

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The set $\mathscr{C}$ is that of spacetime elements and the ordering relation $\preceq$ is the causal precedence relation.

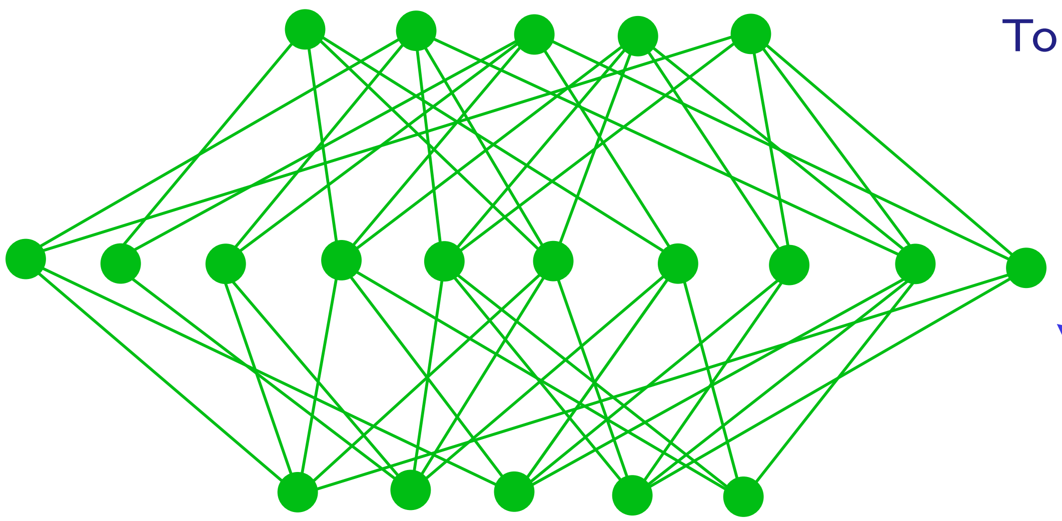
We know that we can recover all essential aspects of a continuous spacetime from a causal set [1].

Proper Distance
Marián Boguñá's talk

Symmetries
Christoph Minz's talk

[1] E. C. Zeeman, *Causality Implies the Lorentz Group*, J. Math. Phys. 5: 490-493 (1964). S. W. Hawking, A. R. King and P. J. McCarthy, *A New Topology for Curved Space-Time which Incorporates the Causal, Differential and Conformal Structures*, J. Math. Phys. 17: 174 (1976). D. Malament, *The Class of Continuous Timelike Curves Determines the Topology of Space-time*, J. Math. Phys 18: 1399 (1977)

Non-Manifoldlike Causal Sets



D. Kleitman and B. Rothschild, The Number of Finite Topologies, Proc. Amer. Math. Society, vol. 25, 1970.

A non-manifoldlike "KR order"

Symmetries

Christoph Minz's talk

A non-manifoldlike causal set may very well be the physical reality inside a black hole or in the very early universe, but macroscopically, we expect manifoldlike causal sets. **The dynamics must say how and why the numerous non-manifoldlike causal sets get suppressed.**

Path Integral Suppression

Steve Carlip's talk

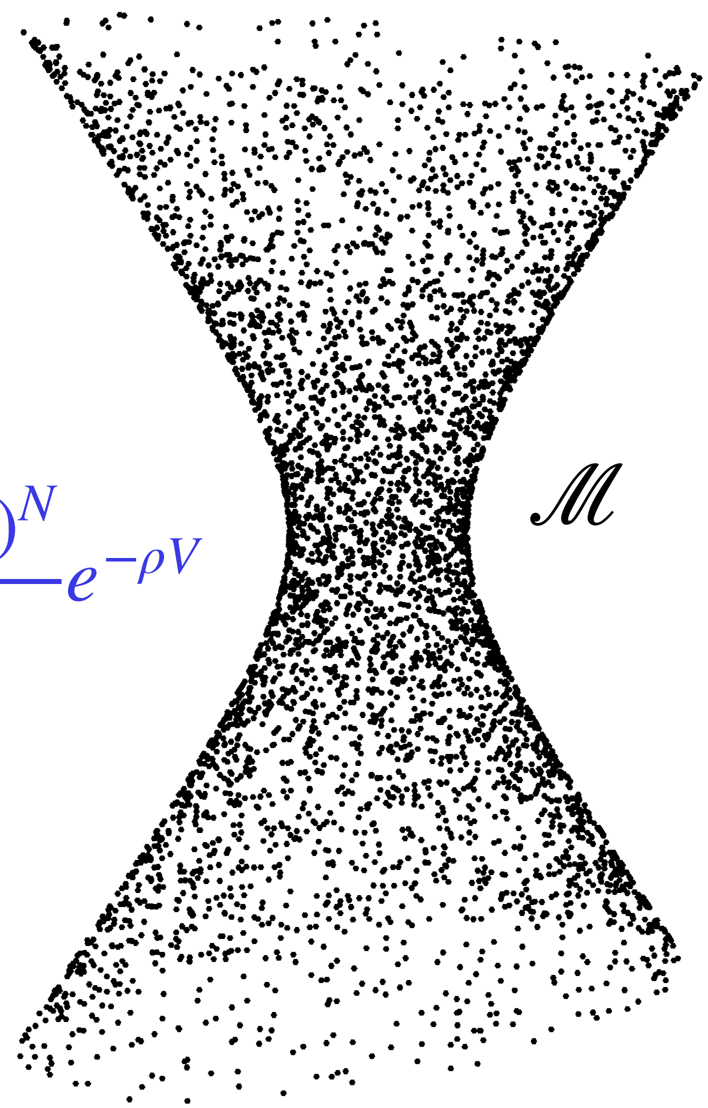
Manifoldlike Causal Sets

Causal sets that are approximated by continuum manifolds have a **number-volume correspondence** such that: the number of elements N within any arbitrary region with volume V is on average proportional to V. The **Poisson distribution** ensures this correspondence with minimal variance.

$$\langle N \rangle = \rho V$$
$$\delta N = \sqrt{\rho V}$$

If we want to study causal sets that resemble a certain spacetime, we can generate them by **sprinkling** points at random in $\mathscr{M}$ via a Poisson process such that

$$P(N) = \frac{(\rho V)^N}{N!} e^{-\rho V}$$



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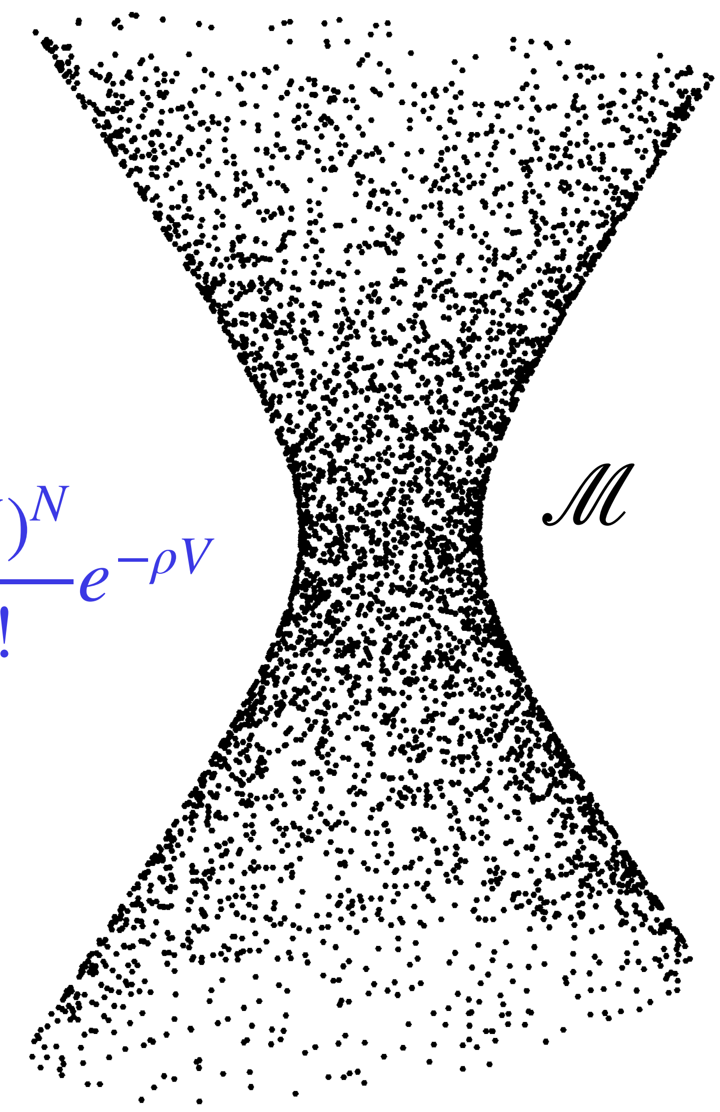

UV cutoff in units of length is $\left(\frac{V}{\langle N \rangle}\right)^{1/d} = \rho^{-1/d}$ **in any geometry**

Quantum Field Theory

Joshua Jones' talk

Fluctuations and Correlations in Causal Set Theory

arXiv:2407.03395

Heidar Moradi,[a] **Yasaman K. Yazdi,**[b,c] **and Miguel Zilhão**[d]

[a] *Physics and Astronomy, Division of Natural Sciences, University of Kent, Canterbury CT2 7NZ, United Kingdom*

[b] *School of Theoretical Physics, Dublin Institute for Advanced Studies, 10 Burlington Road, Dublin 4, Ireland.*

[c] *Theoretical Physics Group, Blackett Laboratory, Imperial College London, SW7 2AZ, United Kingdom*

[d] *Centre for Research and Development in Mathematics and Applications (CIDMA), Department of Mathematics, University of Aveiro, 3810-193 Aveiro, Portugal*

E-mail: h.moradi@kent.ac.uk, ykyazdi@stp.dias.ie, mzilhao@ua.pt

Abstract: We study the statistical fluctuations (such as the variance) of causal set quantities, with particular focus on the causal set action. To facilitate calculating such fluctuations, we develop tools to account for correlations between causal intervals with different cardinalities. We present a convenient decomposition of the fluctuations of the causal set action into contributions that depend on different kinds of correlations. This decomposition can be used in causal sets approximated by any spacetime manifold $\mathcal{M}$. Our work paves the way for investigating a number of interesting discreteness effects, such as certain aspects of the Everpresent Λ cosmological model.

Fluctuations and Correlations in Causal Set Theory

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Heidar Moradi,[a] Yasaman K. Yazdi,[b,c] and Miguel Zilhão[d]

Consider an ensemble $\mathrm{Sp}[\mathscr{M}]$ of causal sets produced by repeated Poisson sprinklings of a spacetime manifold $\mathscr{M}$: $\mathrm{Sp}[\mathscr{M}] = \{\mathscr{C}_1, \mathscr{C}_2, \ldots\}$.

Given a function $f : \mathrm{Sp}[\mathscr{M}] \longrightarrow \mathbb{R}$, its average over the ensemble of Poisson sprinklings is

$$\langle f \rangle \equiv \frac{1}{|\mathrm{Sp}[\mathscr{M}]|} \sum_{\mathscr{C} \in \mathrm{Sp}[\mathscr{M}]} f(\mathscr{C})$$

For example $\langle N \rangle = \rho V$

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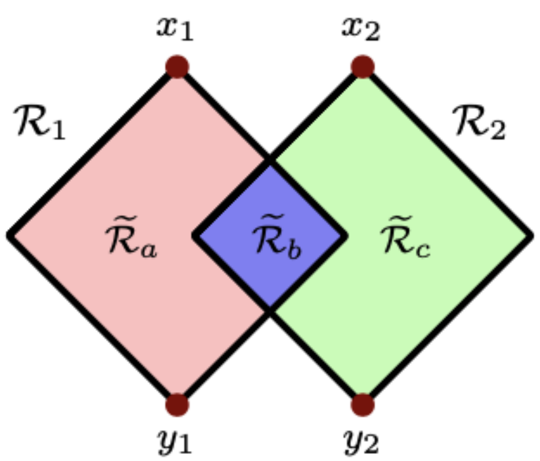
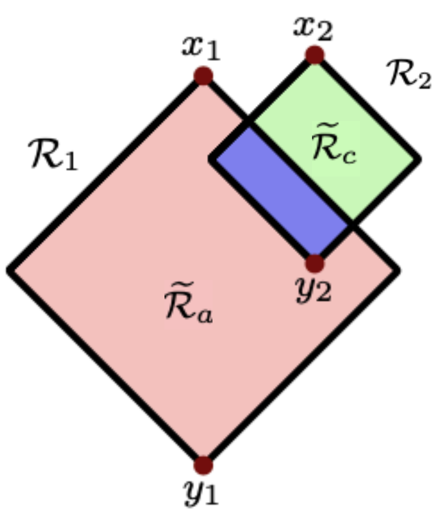
Fluctuations and Correlations in Causal Set Theory

Expressions for the probability to have the configurations below, where $\{x_1, y_1, x_2, y_2\} \in \mathscr{C}$, i elements in $\mathscr{R}_1$ and j elements in $\mathscr{R}_2$



$$\sum_{\substack{\alpha,\beta,\gamma\geq 0\\ \alpha+\beta=i\\ \beta+\gamma=j}} \frac{(\rho V_a)^\alpha(\rho V_b)^\beta(\rho V_c)^\gamma}{\alpha!\,\beta!\,\gamma!}\, e^{-\rho|V_1\cup V_2|}\rho^4 dV_{x_1}dV_{y_1}dV_{x_2}dV_{y_2},$$



$$\sum_{\substack{\alpha,\beta,\gamma\geq 0\\ \alpha+\beta+1=i\\ \beta+\gamma=j}} \frac{(\rho V_a)^\alpha(\rho V_b)^\beta(\rho V_c)^\gamma}{\alpha!\,\beta!\,\gamma!}\, e^{-\rho|V_1\cup V_2|}\,\rho^4 dV_{x_1}dV_{y_1}dV_{x_2}dV_{y_2},$$

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Fixed volume constraint on the path integral

$$\mathcal{Z}(V) \sim \int_{Vol(\mathscr{M})=V} \mathscr{D} g_{\mu\nu} \; e^{iS_G[g]}$$

Everpresent Λ

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$$\mathscr{Z}(V) \sim \int_{Vol(\mathscr{M})=V} \mathscr{D}g_{\mu\nu}\ e^{iS_G[g]} \xrightarrow{\text{Fourier Transform}} \mathscr{Z}(\Lambda) = \int dV\ e^{-i\Lambda V} \mathscr{Z}(V)$$

$$\mathscr{Z}(\Lambda) \sim \int \mathscr{D}g_{\mu\nu} \exp\left(iS_G[g] - i\Lambda \int d^4x \sqrt{-g} \right)$$

Therefore, spacetime volume V and Λ are quantum mechanically conjugate

$$\frac{\delta\Lambda}{8\pi G} \cdot \delta V \geq \frac{\hbar}{2}$$

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N-V correspondence according to Poisson distribution

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$$\delta\Lambda\ \delta V \sim 1 \xrightarrow[\langle\Lambda\rangle = 0]{\text{Assume}} \Lambda \sim \delta\Lambda \sim 1/\delta V = 1/\sqrt{V} \sim H^2 \sim 10^{-121}!$$

Sorkin, **“A Modified Sum-Over-Histories for Gravity”** reported in the article by Brill and Smolin: “Workshop on quantum gravity and new directions”, in Proceedings of the International Conference on Gravitation and Cosmology, Goa, India, 14–19 December **1987**, pp. 184–186, 1988.

Sorkin, **“Role of Time in the Sum-Over-Histories Framework for Gravity”,** International journal of theoretical physics 33 (1994), no. 3 523–534. The text of a talk at the conference, ”The History of Modern Gauge Theories,” in Logan, Utah, in July **1987**.

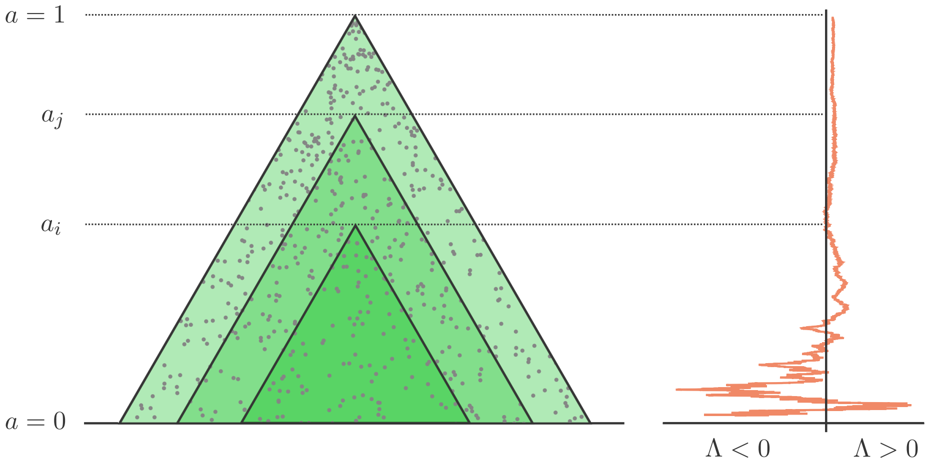
Das, Nasiri, and YKY, **“Aspects of Everpresent Λ (I): A Fluctuating Cosmological Constant from Spacetime Discreteness”**, **2023**, *JCAP* 10 (2023) 047, arXiv:2304.03819.

Das, Nasiri, and YKY, **“Aspects of Everpresent Λ (II): Cosmological Tests of Current Models”**, **2023**, arXiv:2307.13743.

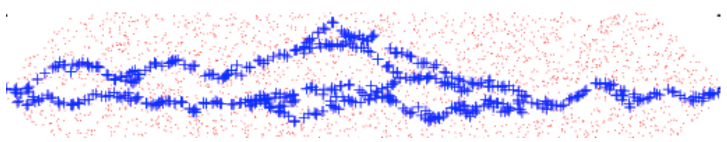
- Phenomenological models can produce universes with reasonable values of H_0 and can fit **supernova data**
- Everpresent Λ is a **dynamical** dark energy model, as is increasingly favored by observations (e.g. DESI). Evidence for **sign change** in dark energy would a smoking gun for it.
- Current models struggle to fit **CMB** to the level of ΛCDM
- Ongoing and Future work: **early universe**, **tensions**, and **more refined phenomenological models**

Potential to Answer Some Open Questions in Cosmology and Black Hole Physics

- Everpresent Λ, the cosmological constant puzzle, and the Hubble tension



- Brownian motion, dark matter and the S_8 tension



Swerves

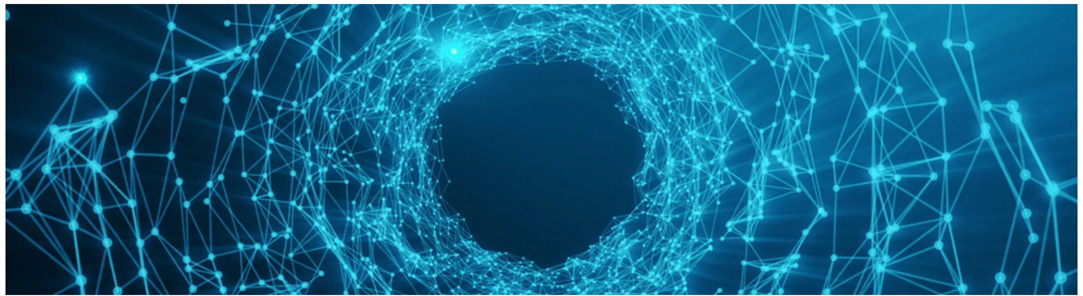
Arad Nasiri's talk

- The state of the universe, and imprints of discreteness in the primordial power spectrum

Quantum Field Vacuum and Entropy

Joshua Jones' talk

- Black Hole Thermodynamics



Thank You