# Viability of loop quantum cosmology at the level of bispectrum

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Based on:

Roshna K, V. Sreenath, JCAP 08 (2023) 014

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# Standard model of Cosmology

- Standard model of cosmology namely $\Lambda$CDM model [1], models our Universe just using six parameters.
- The tiny perturbations generated in the early universe leads to anisotropies in the CMB which in turn lead to the large-scale structure.
- Inflation [2], due to its simplicity, provides the most popular explanation for the origin of these perturbations.
- Current observations by Planck are consistent with the predictions of slow roll inflation $i.e$ Gaussian and nearly scale invariant primordial perturbations.

[1] Planck Collaboration, A & A 641, A6, (2020).
[2] Planck Collaboration, A & A 641, A10, (2020).

# Primordial non-Gaussianity

- Inflation also predicts small deviations from Gaussiantiy in the primordial perturbations, known as primordial non-Gaussianity.
- The amplitude of non-Gaussianity is quantified by dimensionless quantity $f_{\rm NL}$.
- Observations by Planck [3] point towards a small $f_{\rm NL}$ which is consistent with that generated in the slow roll inflation.

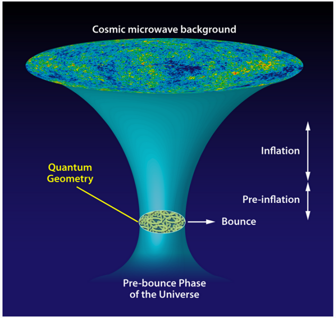
$$\boxed{\mathfrak{f}_{\rm NL} = \mathcal{O}(10^{-2})(\text{slow roll inflation})}$$

The degrees of non-Gaussianities produced by standard inflation differ significantly from those generated by other mechanisms. In this talk, we will examine the non-Gaussianity generated from Loop Quantum Cosmology (LQC) and its imprints in CMB.

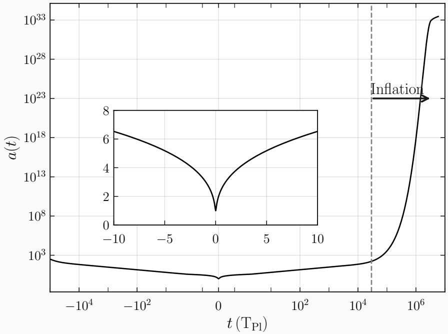
[3] Planck Collaboration, A & A 641, A9, (2020) .

# Loop quantum cosmology (LQC)

Loop Quantum Cosmology[4] is a simple extension of the standard $\Lambda$CDM model with a cosmic bounce before the inflationary phase.




Picture credits: P. Singh, Physics 5, 142 (2012).



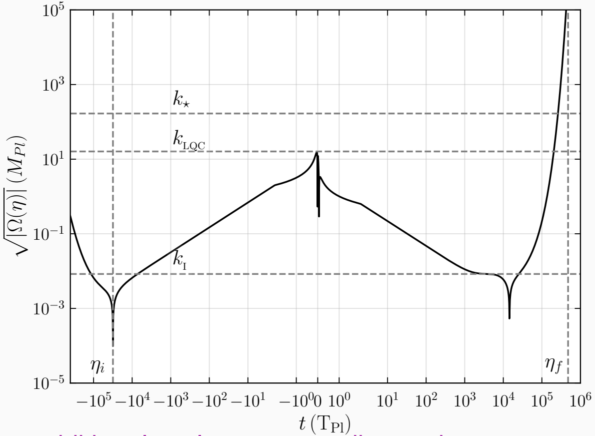

Effective equations describing background:

$$H^2 = \frac{8\,\pi\,G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho\left(1 - \frac{4\,\rho}{\rho_c}\right) - 4\,\pi G\,P\,(1 - 2\,\frac{\rho}{\rho_c}).$$

[4] A. Ashtekar, T. Pawlowski, P. Singh, Phys. Rev. Lett., 96, 141301,(2006).

# Perturbations in LQC

Perturbations are treated as test fields living on the background described by effective equations [5].




- The evolution of the scalar perturbation in terms of Mukhanov-Sasaki variable $v_k = a\,\varphi_k$, is given by,

$$v_k'' + \left(k^2 + \Omega(\eta)\right) v_k = 0.$$

The effect of the bounce is to introduce an additional scale corresponding to the curvature at the bounce named as $k_{\rm LQC}$.

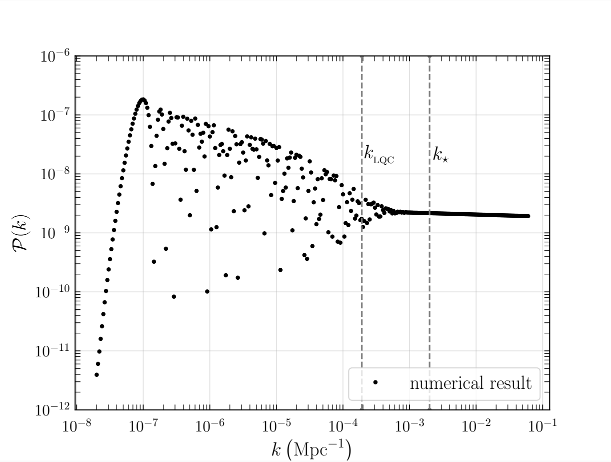
$$\boxed{k_{\rm LQC} = a(\eta_B)\sqrt{R_B/6} \approx a(\eta_B)\sqrt{\kappa\rho_{\rm B}}}$$

[5] I. Agullo, A. Ashtekar, W. Nelson, Phys. Rev. Lett., 109, 251301, (2012).

# Primordial power spectrum in LQC

Perturbations are quantified using correlation functions.

The power spectrum of curvature perturbation $\mathcal{R}$ is:

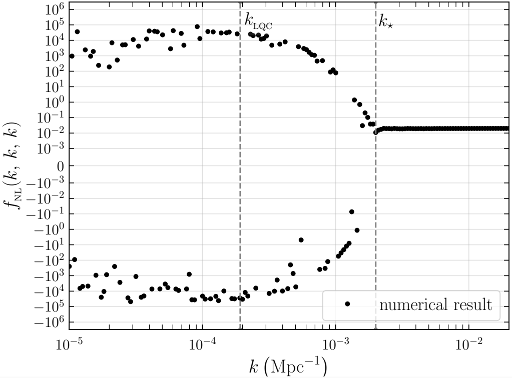



- The scalar power spectrum generated in LQC is almost scale invariant for $k > k_{\rm LQC}$.
- The effect from bounce appear for $k \lesssim k_{\rm LQC}$.

# Primordial bispectra in LQC

- Non-Gaussianity is quantified by dimensionless quantity $f_{_{\rm NL}}(k_1, k_2, k_3)$ which is related to scalar bispectrum [6] $B_{_{\mathcal{R}}}(k_1,\, k_2,\, k_3)$ through the relation,

$$f_{_{\rm NL}}(k_1, k_2, k_3) \equiv -\frac{5}{6}(2\,\pi^2)^{-2}\left(\frac{\mathcal{P}_{\mathcal{R}}(k_1)}{k_1^3}\,\frac{\mathcal{P}_{\mathcal{R}}(k_2)}{k_2^3} + \frac{\mathcal{P}_{\mathcal{R}}(k_2)}{k_2^3}\,\frac{\mathcal{P}_{\mathcal{R}}(k_3)}{k_3^3} + \frac{\mathcal{P}_{\mathcal{R}}(k_3)}{k_3^3}\,\frac{\mathcal{P}_{\mathcal{R}}(k_1)}{k_1^3}\right)^{-1} \times B_{\mathcal{R}}(k_1, k_2, k_3).$$



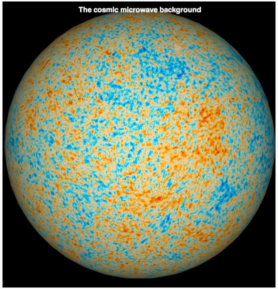

$\boxed{|f_{_{\rm NL}}| \approx 10^4}$

Highly non-Gaussian in nature

[6] I. Agullo, B. Bolliet, V. Sreenath, Phys. Rev. D 97, 6, 066021, (2018).

# CMB bispectrum

Primordial perturbations leave their imprints in the CMB radiation as temperature fluctuations $(T)$ and as electric $(E)$ and magnetic polarizations $(B)$ modes.




Picture credits: Planck Collaboration.

The primordial curvature perturbation[7] $\mathcal{R}$ is imprinted on the CMB multipoles $a_{lm}$ by a convolution with transfer functions $\Delta_l(k)$ given as,

$$a^X_{lm} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3} \Delta^X_l(k)\,\mathcal{R}\,(k) Y_{lm}\,(\hat{k}).$$

[7] See, for instance, E. Komatsu, D. N. Spergel, Phys. Rev. D, 63:063002, (2001), J. R. Fergusson, E. P. S. Shellard, Phys. Phys. Rev. D, 76:083523, (2007).

# Primordial correlations to CMB anisotropies

Perturbations are quantified using correlation functions.

Primordial power spectrum ($\mathcal{P}_{\mathcal{R}}(k)$)

$\Downarrow$

Angular power spectrum ($C_\ell$)

Primordial bispectrum ($B_{\mathcal{R}}(k_1, k_2, k_3)$)

$\Downarrow$

$??$

The quantity is known as CMB reduced bispectrum $b_{l_1 l_2 l_3}$. In particular, we investigate the imprints of primordial bispectrum generated in LQC in the CMB reduced bispectrum.

# CMB bispectrum

CMB bispectrum $B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3}$:

$$
\begin{aligned}
\langle a^{\mathsf{X}}_{\ell_1 m_1} a^{\mathsf{Y}}_{\ell_2 m_2} a^{\mathsf{Z}}_{\ell_3 m_3} \rangle \quad &= \quad (4\pi)^3 \, (-i)^{\ell_1+\ell_2+\ell_3} \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3} \int \frac{\mathrm{d}^3 k_2}{(2\pi)^3} \int \frac{\mathrm{d}^3 k_3}{(2\pi)^3} \Delta^{\mathsf{X}}_{\ell_1} \Delta^{\mathsf{Y}}_{\ell_2} \Delta^{\mathsf{Z}}_{\ell_3} \\
&\times \quad \langle \mathcal{R}_{k_1} \mathcal{R}_{k_2} \mathcal{R}_{k_3} \rangle \, Y_{\ell_1 m_1}(\hat{k}_1) \, Y_{\ell_2 m_2}(\hat{k}_2) \, Y_{\ell_3 m_3}(\hat{k}_3).
\end{aligned}
$$

Reduced bispectrum $b_{l_1 l_2 l_3}$:

$$
\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = \mathcal{G}^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} b_{l_1 l_2 l_3}.
$$

$\mathcal{G}^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3}$ is known as the gaunt factor which imposes the following condition on multipoles

- $l_1, l_2, l_3$ should satisfy the triangle condition $i.e. \mid l_1 - l_2 \mid \leq l_3 \leq l_1 + l_2$.
- $l_1 + l_2 + l_3$ is even.

# CMB bispectrum

In terms of primordial bispectrum $B_{\mathcal{R}}(k_1, k_2, k_3)$ the reduced CMB bispectrum $b^{X\,Y\,Z}_{l_1 l_2 l_3}$ is :

$$b^{X\,Y\,Z}_{l_1 l_2 l_3} = \left(\frac{2}{\pi}\right)^3 B_0 \int_0^\infty x^2\,dx \int_0^\infty dk_1 \int_0^\infty dk_2 \int_0^\infty dk_3\,(k_1 k_2 k_3)^2 B_{\mathcal{R}}(k_1, k_2, k_3)$$
$$\Delta^X_{l_1}(k_1)\,\Delta^Y_{l_2}(k_2)\,\Delta^Z_{l_3}(k_3) j_{l_1}(k_1 x)\,j_{l_2}(k_2 x)\,j_{l_3}(k_3 x),$$

where $X, Y, Z$ corresponds to temperature or polarization.

- The calculation involves four integrals, three over wavenumbers and one over $x$ variable, which is computationally expensive.
- This calculation can, however, be simplified, essentially just to two integrals, if we use the analytical template and separable property of the primordial bispectrum.

# Analytical template of $f_{\rm NL}(k_1, k_2, k_3)$

In region $k >> k_{\rm LQC}$ (scale invariant part) :

The shape of bispectrum of wavenumbers $k >> k_{\rm LQC}$ is approximated by using the local template:

$$B_{\mathcal{R}}^{\rm local}(k_1, k_2, k_3) = -\frac{6}{5}(2\pi^2)^2 \mathfrak{f}_{\rm NL}^{\rm local}\left(\frac{\widetilde{\mathcal{P}}_{\mathcal{R}}(k_1)}{k_1^3}\frac{\widetilde{\mathcal{P}}_{\mathcal{R}}(k_2)}{k_2^3} + \frac{\widetilde{\mathcal{P}}_{\mathcal{R}}(k_2)}{k_2^3}\frac{\widetilde{\mathcal{P}}_{\mathcal{R}}(k_3)}{k_3^3} + \frac{\widetilde{\mathcal{P}}_{\mathcal{R}}(k_3)}{k_3^3}\frac{\widetilde{\mathcal{P}}_{\mathcal{R}}(k_1)}{k_1^3}\right),$$

where $\widetilde{\mathcal{P}}_{\mathcal{R}}(k) = A_s(k/k_\star)^{n_s-1}$ and we fix $\mathfrak{f}_{\rm NL}^{\rm local} = 10^{-2}$.

## Analytical template of $f_{\rm NL}(k_1, k_2, k_3)$

In region $k \lesssim k_{\rm LQC}$ (scale dependent and oscillatory) :

The bispectrum of curvature perturbation for wavenumbers $k \lesssim k_{\rm LQC}$ is approximated by the template:

$$B_{\mathcal{R}}^{\rm bounce}(k_1, k_2, k_3) = -\frac{6}{5}\,(2\pi^2)^2\, \mathfrak{f}_{\rm NL}^{\rm bounce} \left(\frac{\mathcal{P}_{\mathcal{R}}(k_1)}{k_1^3}\,\frac{\mathcal{P}_{\mathcal{R}}(k_1)}{k_1^3}\,\frac{\mathcal{P}_{\mathcal{R}}(k_1)}{k_1^3}\right)^{1/2} \mathrm{e}^{-0.647\,\frac{k_1+k_2+k_3}{k_{\rm LQC}}} \times\, \sin\left(\frac{k_1 + k_2 + k_3}{k_I}\right).$$

# Analytical template of $f_{\rm NL}(k_1, k_2, k_3)$

In region $k \lesssim k_{\rm LQC}$ (scale dependent and oscillatory) :

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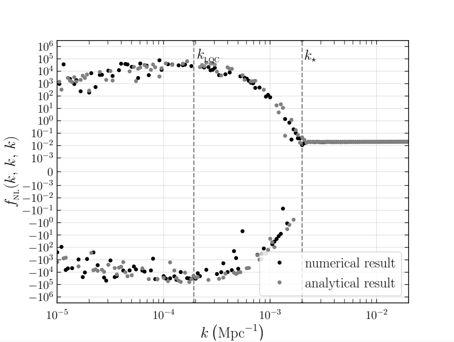
The complete template for bispectrum of curvature perturbation generated in LQC is

$$\boxed{B_{\mathcal{R}}^{\rm LQC}(k_1, k_2, k_3) = B_{\mathcal{R}}^{\rm local}(k_1, k_2, k_3) + B_{\mathcal{R}}^{\rm bounce}(k_1, k_2, k_3)\,.}$$

# Analytical template of $f_{\rm NL}(k_1, k_2, k_3)$

Analytical template of the shape of non-Gaussianity generated in LQC is modelled as:

$$f_{\rm NL}(k_1,k_2,k_3) \approx \mathfrak{f}_{\rm NL}^{\rm bounce}\left(\frac{\mathcal{P}_{\mathcal{R}}(k_1)}{k_1^3}\,\frac{\mathcal{P}_{\mathcal{R}}(k_2)}{k_2^3}\,\frac{\mathcal{P}_{\mathcal{R}}(k_3)}{k_3^3}\right)^{1/2} \mathrm{e}^{-0.647\,\frac{k_1+k_2+k_3}{k_{\rm LQC}}} \times \sin\left(\frac{k_1+k_2+k_3}{k_I}\right) + \mathfrak{f}_{\rm NL}^{\rm local}$$




The template of bispectrum qualitatively captures the essential features of the primordial non-Gaussianity generated in LQC.

# CMB bispectrum in separable [8] form for local template

The shape of the bispectrum for the local template:

$$B^{\rm local}_{\mathcal{R}}(k_1,k_2,k_3) = -\,\frac{6}{5}\,(2\pi^2)^2\,\mathfrak{f}^{\rm local}_{\rm NL}\left(\frac{\widetilde{\mathcal{P}}_{\mathcal{R}}(k_1)}{k_1^3}\,\frac{\widetilde{\mathcal{P}}_{\mathcal{R}}(k_2)}{k_2^3}+\frac{\widetilde{\mathcal{P}}_{\mathcal{R}}(k_2)}{k_2^3}\,\frac{\widetilde{\mathcal{P}}_{\mathcal{R}}(k_3)}{k_3^3}+\frac{\widetilde{\mathcal{P}}_{\mathcal{R}}(k_3)}{k_3^3}\,\frac{\widetilde{\mathcal{P}}_{\mathcal{R}}(k_1)}{k_1^3}\right),$$

where $\widetilde{\mathcal{P}}_{\mathcal{R}}(k) = A_s(k/k_\star)^{n_s-1}$.

## The reduced bispectrum of local template:

$$\begin{aligned}
b^{\rm local}_{\ell_1\ell_2\ell_3} &= -\left(\frac{2}{\pi}\right)^3\frac{6}{5}\,(2\pi^2)^2\,\mathfrak{f}^{\rm local}_{\rm NL}\int_0^\infty {\rm d}x\,x^2\left[E^X_{\ell_1}(x)\,E^Y_{\ell_2}(x)\,G^Z_{\ell_3}(x)+G^X_{\ell_1}(x)\,E^Y_{\ell_2}(x)\,E^Z_{\ell_3}(x)\right.\\
&+\left.E^X_{\ell_1}(x)\,G^Y_{\ell_2}(x)\,E^Z_{\ell_3}(x)\right]
\end{aligned}$$

where,

$$\begin{aligned}
E^X_\ell(x) &= \int_0^\infty dk\,\Delta^{\rm X}_\ell(k)\,j_\ell(kx)\,k^{-1}\,\widetilde{\mathcal{P}}_{\mathcal{R}}(k_1),\\
G^X_\ell(x) &= \int_0^\infty dk\,\Delta^{\rm X}_\ell(k)\,j_\ell(kx)\,k^2.
\end{aligned}$$

[8] See, for instance, E. Komatsu, D. N. Spergel, Phys. Rev. D, 63:063002, (2001), R. Durrer, The Cosmic Microwave Background, Cambridge University Press, 12, (2020).

# CMB bispectrum in separable form for bounce

The bispectrum of $\mathcal{R}$ in the regime $k \lesssim k_{\rm LQC}$ can be approximated using the following template:

$$B_{\mathcal{R}}^{\rm bounce}(k_1, k_2, k_3) \;=\; -\frac{6}{5}\,(2\pi^2)^2\, \mathfrak{f}_{\rm NL}^{\rm bounce} \left(\frac{\mathcal{P}_{\mathcal{R}}(k_1)}{k_1^3}\,\frac{\mathcal{P}_{\mathcal{R}}(k_2)}{k_2^3}\,\frac{\mathcal{P}_{\mathcal{R}}(k_3)}{k_3^3}\right)^{1/2} \mathrm{e}^{-0.647\,\frac{k_1+k_2+k_3}{k_{\rm LQC}}} \times\, \sin\left(\frac{k_1 + k_2 + k_3}{k_I}\right).$$

The reduced bispectrum of CMB in the regime $k \lesssim k_{\rm LQC}$:

$$\begin{aligned} b_{\ell_1\ell_2\ell_3}^{\rm bounce} \;=\;& -\left(\frac{2}{\pi}\right)^3 \frac{6}{5}\,(2\pi^2)^2\, \mathfrak{f}_{\rm NL}^{\rm bounce} \int_0^\infty dx\, x^2 \Big[A_{\ell_1}^{X}(x)\, B_{\ell_2}^{Y}(x)\, B_{\ell_3}^{Z}(x) \\ &+ B_{\ell_1}^{X}(x)\, A_{\ell_2}^{Y}(x) B_{\ell_3}^{Z}(x) + B_{\ell_1}^{X}(x)\, B_{\ell_2}^{Y}(x)\, A_{\ell_3}^{Z}(x) - A_{\ell_1}^{X}(x)\, A_{\ell_2}^{Y}(x)\, A_{\ell_3}^{Z}(x)\Big], \end{aligned}$$

where,

$$\begin{aligned} A_{\ell}^{X}(x) \;&=\; \int_0^\infty dk\, \Delta_\ell^{\rm X}(k)\, j_\ell(kx)\, \sqrt{(\mathcal{P}_{\mathcal{R}}(k)k}\, \mathrm{e}^{-0.647\,\frac{k}{k_{\rm LQC}}}\, \sin\Big(\frac{k}{k_{_I}}\Big), \\ B_{\ell}^{X}(x) \;&=\; \int_0^\infty dk\, \Delta_\ell^{\rm X}(k)\, j_\ell(kx)\, \sqrt{\mathcal{P}_{\mathcal{R}}(k)k}\, \mathrm{e}^{-0.647\,\frac{k}{k_{\rm LQC}}}\, \cos\Big(\frac{k}{k_{_I}}\Big). \end{aligned}$$

# Reduced bispectrum in LQC

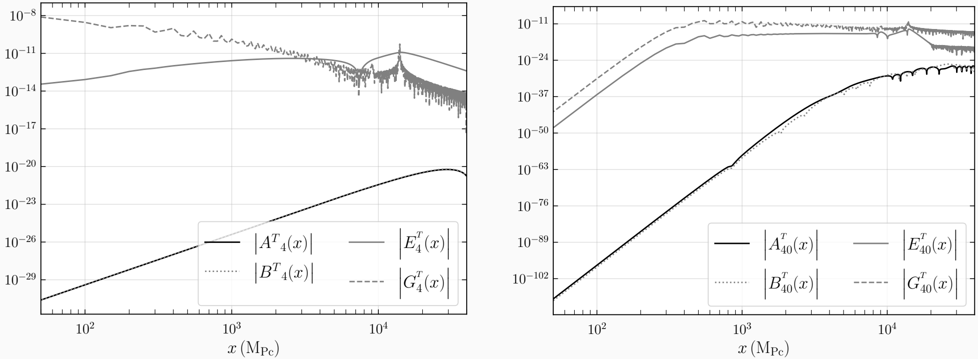
The reduced bispectrum in LQC:

$$
\begin{aligned}
b^{\rm local}_{\ell_1\ell_2\ell_3} &= -\left(\frac{2}{\pi}\right)^3 \frac{6}{5}\,(2\pi^2)^2\, \mathfrak{f}^{\rm local}_{\rm NL} \int_0^\infty {\rm d}x\, x^2 \Big[E^X_{\ell_1}(x)\, E^Y_{\ell_2}(x)\, G^Z_{\ell_3}(x) + G^X_{\ell_1}(x)\, E^Y_{\ell_2}(x)\, E^Z_{\ell_3}(x) \\
&+ E^X_{\ell_1}(x)\, G^Y_{\ell_2}(x)\, E^Z_{\ell_3}(x)\Big],
\end{aligned}
$$

$$
\begin{aligned}
b^{\rm bounce}_{\ell_1\ell_2\ell_3} &= -\left(\frac{2}{\pi}\right)^3 \frac{6}{5}\,(2\pi^2)^2\, \mathfrak{f}^{\rm bounce}_{\rm NL} \int_0^\infty dx\, x^2 \Big[A^X_{\ell_1}(x)\, B^Y_{\ell_2}(x)\, B^Z_{\ell_3}(x) \\
&+ B^X_{\ell_1}(x)\, A^Y_{\ell_2}(x) B^Z_{\ell_3}(x) + B^X_{\ell_1}(x)\, B^Y_{\ell_2}(x)\, A^Z_{\ell_3}(x) - A^X_{\ell_1}(x)\, A^Y_{\ell_2}(x)\, A^Z_{\ell_3}(x)\Big],
\end{aligned}
$$

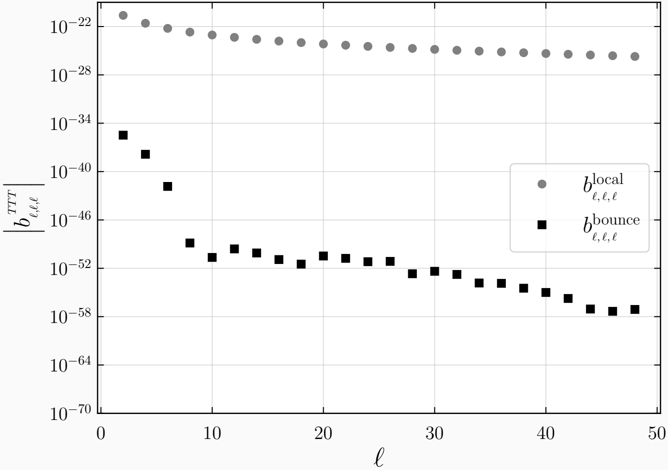
$$
\boxed{b^{\rm LQC}_{\ell_1\ell_2\ell_3} = b^{\rm bounce}_{\ell_1\ell_2\ell_3} + b^{\rm local}_{\ell_1\ell_2\ell_3}.}
$$

# Behaviour of functions $A_\ell^X(x), B_\ell^X(x), E_\ell^X(x), G_\ell^X(x)$




The behaviour of functions $A_\ell^X(x), B_\ell^X(x), E_\ell^X(x), G_\ell^X(x)$ with $x$ for multipoles $\ell = 4$ and $40$. The contributions from the local part of the bispectrum are clearly dominant compared to those arising from the bounce part.

# Reduced bispectrum in LQC

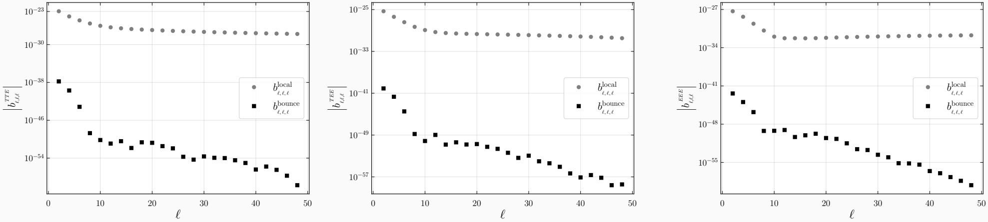



Note that $b^{\text{local}}_{\ell_1\ell_2\ell_3} >> b^{\text{bounce}}_{\ell_1\ell_2\ell_3}$. Hence,

$$\boxed{b^{\text{LQC}}_{\ell_1\ell_2\ell_3} = b^{\text{bounce}}_{\ell_1\ell_2\ell_3} + b^{\text{local}}_{\ell_1\ell_2\ell_3} \approx b^{\text{local}}_{\ell_1\ell_2\ell_3}}$$

# Reduced bispectrum in LQC

The plots of reduced bispectra $b^{\mathrm{TTE}}_{\ell_1,\ell_2,\ell_3}$, $b^{\mathrm{TEE}}_{\ell_1,\ell_2,\ell_3}$ and $b^{\mathrm{EEE}}_{\ell_1,\ell_2,\ell_3}$. Note that $b^{\mathrm{local}}_{\ell_1\ell_2\ell_3} >> b^{\mathrm{bounce}}_{\ell_1\ell_2\ell_3}$.




$$\boxed{b^{\mathrm{LQC}}_{\ell_1\ell_2\ell_3} = b^{\mathrm{bounce}}_{\ell_1\ell_2\ell_3} + b^{\mathrm{local}}_{\ell_1\ell_2\ell_3} \approx b^{\mathrm{local}}_{\ell_1\ell_2\ell_3}}$$

The reduced bispectrum generated in LQC is dominated by the local part. This, in turn, implies that the reduced bispectrum generated in LQC is similar to that generated in slow roll inflation.

# Summary

- The primordial perturbations generated in LQC is highly non-Gaussian in nature, yet our computation of reduced bispectra of temperature and electric polarization are similar to that of slow roll inflation. Hence, we conclude that the primordial non-Gaussianity generated in LQC is compatible with the constraints from Planck [9].

[9] Planck Collaboration, A & A, 641, A9 (2020).

# Discussion

- Our result could be compared with a non-oscillatory template [10] where they found that in the absence of oscillations, contributions from bounce are significant enough to be observed by Planck.
- Our calculation shows that the reduced bispectra generated in LQC is similar to that generated in slow roll inflation. This is because of the highly oscillatory nature of the primordial bispectrum. The reduced bispectra involves integrals over wavenumbers which average over these oscillations.
- Our result, thus highlights the fact that a small reduced bispectra need not necessarily imply the absence of primordial non-Gaussianity. Hence, it would also be interesting to look for any other measurable imprints of such oscillatory and scale dependent primordial non-Gaussianity.

[10] P. C. M. Delgado, R. Durrer, N. Pinto-Neto, JCAP, 11, 24, (2021), B. van Tent, P. C. M. Delgado, R. Durrer, Phys. Rev. Lett. 130, 191002, (2023

# Thank You!

# Summary and Discussion

- Our result can also be compared with a purely oscillatory template considered by Planck [11] and M. Munchmeyer et al. [12] They considered oscillations on the scale of $\mathcal{O}\left(10^{-3} - 10^{-2}\right)$ leading to a multiple periodicity of about $\ell \simeq 14 - 140$. In contrast, our oscillation scale of $k_{\rm I} = 10^{-7}\,{\rm Mpc}^{-1}$, results in periodicity smaller than one. We believe this is the reason why the reduced bispectrum corresponding to the highly oscillatory part of our template is negligible.

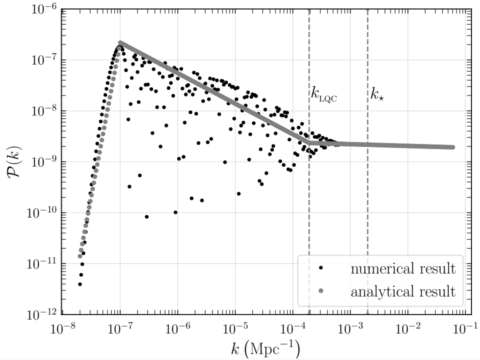
[11] Planck Collaboration, A & A, 641, A9 (2020).

[12] M. Munchmeyer, F. Bouchet, M.G.Jackson, B. Wandelt, A & A 570, A94 (2014).

# Analytical template of primordial power spectra

Analytical template of the power spectrum[13]:

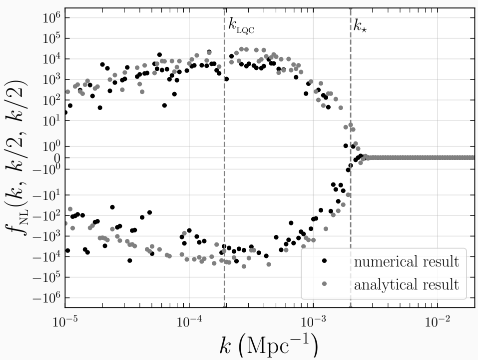
$$\mathcal{P}_{\mathcal{R}}(k) = A_s \begin{cases} \left(\frac{k}{k_{\rm I}}\right)^6 \left(\frac{k_{\rm I}}{k_{\rm LQC}}\right)^{-0.6} & if \quad k \leq k_{\rm I}, \\ \left(\frac{k}{k_{\rm LQC}}\right)^{-0.6} & if \quad k_{\rm I} < k \leq k_{\rm LQC}, \\ \left(\frac{k}{k_{\rm LQC}}\right)^{(n_s-1)} & if \quad k > k_{\rm LQC}. \end{cases}$$

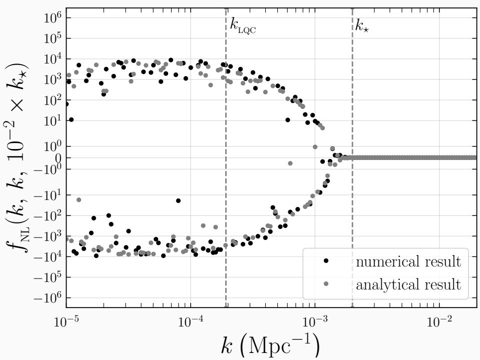



- The scalar power spectrum generated in LQC is almost scale invariant for $k >> k_{\rm LQC}$.
- The effect from bounce appear for $k << k_{\rm LQC}$.

[13] I. Agullo, D. Kranas, V. Sreenath, Class. Quant. Grav. 38, 6,065010, (2021).

# Different configurations



# Analytical template of $f_{\rm NL}(k_1, k_2, k_3)$

In region $k \lesssim k_{\rm LQC}$ (scale dependent and oscillatory) :

The bispectrum of curvature perturbation in terms of $v_k(\eta)$ is

$$\begin{aligned} B_{\mathcal{R}}(k_1, k_2, k_3) &= \frac{v_{k_1}(\eta_f)}{z(\eta_f)} \frac{v_{k_2}(\eta_f)}{z(\eta_f)} \frac{v_{k_3}(\eta_f)}{z(\eta_f)} \int_{\eta_i}^{\eta_f} {\rm d}\eta'\, g(\eta')\, \mathcal{F}(v_{k_1}(\eta'),\, v_{k_2}(\eta')\, v_{k_3}(\eta')) \\ &\quad + \text{complex conjugate.} \end{aligned}$$

We approximate the mode $v_k(\eta')$ by ${\rm e}^{-i\,k\,\eta'}$,

$$\begin{aligned} \mathcal{I} &= \int_{-\eta_0}^{\eta_0} {\rm d}\eta'\, g(\eta') {\rm e}^{i\,(k_1 + k_2 + k_3)\eta'}, \\ &= \int_{-\infty}^{\infty} {\rm d}\eta'\, g(\eta') {\rm e}^{i\,(k_1 + k_2 + k_3)\eta'}\, W(|\eta' - \eta_0|). \end{aligned}$$

The real part of the pole would lead to an oscillatory behaviour and the imaginary part will lead to an exponential behaviour.

# Analytical template of $f_{\rm NL}(k_1, k_2, k_3)$

## (a) Imaginary part of the pole:

An analytical approximation for the scale factor[14] valid close to the bounce is given by:

$$a(t) = a_B \left(1 + 3\,\kappa\,\rho_c\,t^2\right)^{1/6}.$$

The pole of is at $t_p = i/\sqrt{(3\,\kappa\,\rho_c)}$ and in conformal time $\eta_p = i\,\frac{0.647}{k_{\rm LQC}}$.

The scale dependence of the integral due to the imaginary pole can be approximated to be $\mathrm{e}^{i\,(k_1+k_2+k_3)\,\eta_p} = \mathrm{e}^{-0.647\,\frac{k_1+k_2+k_3}{k_{\rm LQC}}}$.

## (b) Real part of the pole:

The real poles could lead to an oscillatory behaviour and is modelled as $\frac{\sin(k_1+k_2+k_3)}{k_I}$.

[14] B. Bolliet, J. Grain, C. Stahl, L. Linsefors, and A. Barrau, Phys. Rev. D 91, 084035 (2015).

## Analytical template of $f_{\rm NL}(k_1, k_2, k_3)$

As $v_k(\eta_f)/z(\eta_f) = \mathcal{R}_k(\eta_f)$, we substitute $\frac{v_k(\eta_f)}{z(\eta_f)} \approx \sqrt{\frac{\mathcal{P}_{\mathcal{R}}(k)}{k^3}}$ and hence the bispectrum of curvature perturbations in the regime $k \lesssim k_{\rm LQC}$ can be approximated using template:

$$\begin{aligned} B_{\mathcal{R}}^{\rm bounce}(k_1, k_2, k_3) \quad &= \quad -\frac{6}{5}\,(2\pi^2)^2\, \mathfrak{f}_{\rm NL}^{\rm bounce} \left(\frac{\mathcal{P}_{\mathcal{R}}(k_1)}{k_1^3}\,\frac{\mathcal{P}_{\mathcal{R}}(k_2)}{k_2^3}\,\frac{\mathcal{P}_{\mathcal{R}}(k_3)}{k_3^3}\right)^{1/2} {\rm e}^{-0.647\,\frac{k_1+k_2+k_3}{k_{\rm LQC}}} \\ &\quad \times\, \sin\left(\frac{k_1+k_2+k_3}{k_I}\right), \end{aligned}$$

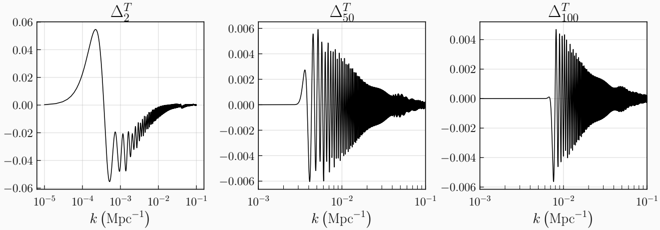
where we fix $\mathfrak{f}_{\rm NL}^{\rm bounce} = 1\,{\rm M_{Pl}}^{-3/2}$.

The complete template for bispectrum of curvature perturbation generated in LQC is

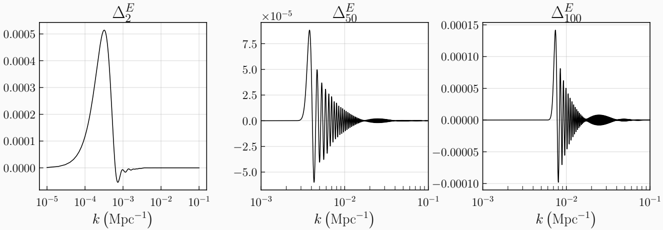
$$\boxed{B_{\mathcal{R}}^{\rm LQC}(k_1, k_2, k_3) = B_{\mathcal{R}}^{\rm local}(k_1, k_2, k_3) + B_{\mathcal{R}}^{\rm bounce}(k_1, k_2, k_3)\,.}$$

# Transfer functions

- The transfer functions are obtained from [15]CLASS,
  - Temperature transfer functions:




  - Electric polarization transfer functions:




[15] D. Blas, J. Lesgourgues, T. Tram, CLASS II: Approximation schemes, JCAP, 1107, 034, (2011).