# Magnon-antimagnon pair production by magnetic field inhomogeneities and the bosonic Klein effect*

T. C. **Adorno**[a], S. P. Gavrilov[b,c], D. M. Gitman[d,e]

[a] Xi'an Jiaotong-Liverpool University (China); [b] Tomsk State University (Russia);
[c] Herzen State Pedagogical University (Russia); [d] P. N. Lebedev Physical Institute (Russia);
[e] University of São Paulo (Brazil)

***Seventeenth Marcel Grossmann Meeting***

Pescara, Italy, 7 – 12 July, 2024

*Based on the manuscript by **T.C.A**, S.P. Gavrilov, D.M. Gitman in *Phys. Rev. B* **110**, 014410 (2024)

tiago.adorno@xjtlu.edu.cn

# Contents

- Magnons: What are they?
- Effective field theory (EFT) model for low-energy magnons — scalar QED (sQED) with external fields
- Strong-field sQED, vacuum instability
- Magnon-antimagnon pair production from the vacuum by external magnetic fields
- Final comments

# Magnons

Spin waves (SW): collective excitations of magnetic moments in ordered magnets. Magnons are quantized SW, corresponding to quasi-particles that obey Bose-Einstein statistics*. *Quantum magnonics* is an emerging area of research due to its importance to quantum information and computation*.

Magnons propagate in distinct magnetic materials: Ferromagnetic, ferrimagnetic, and antiferromagnetic. Magnonic excitations in antiferromagnetics exhibit linear dispersion relations (relativistic-like) and a continuum effective field theory (EFT) description of low energy excitations is possible†.

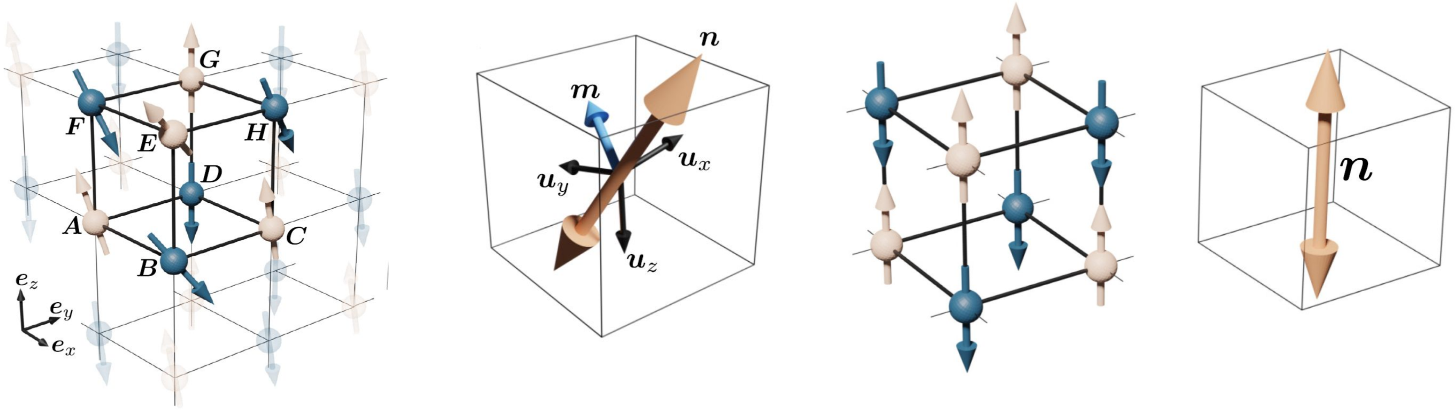
* H. Y. Yuan *et.al.*, *Physics Reports* **965**, 1 (2022). † M. Hongo *et.al.*, *Phys. Rev. B* **104**, 134403 (2022).

# Magnons

Antiferromagnetic materials: spin systems on a cubic lattice with SO(3) symmetry-breaking interactions, described by the Hamiltonian,

$$\hat{H} = \sum_{n} \sum_{i=1}^{d} J \delta^{ab} \hat{s}_a^n \hat{s}_b^{n+i} - \sum_{n} \left[ \mu B^a(\mathbf{r}_n) \hat{s}_a^n + C^{ab} \hat{s}_a^n \hat{s}_b^n \right], \; J > 0, \; [\hat{s}_a^n, \hat{s}_b^n] = i\epsilon_{ab}^c \hat{s}_c^n \,,$$

where $C^{ab} \hat{s}_a^n \hat{s}_b^n$ is the single-ion anisotropic interaction.




* Picture extracted from O. Pylypovskyi *et.al.*, Phys. Rev. B **103**, 134413 (2021).

# Magnons

The magnetic state of a spin octamer is characterized the Néel vector (staggered magnetic moment), $\mathbf{n} = (n_x, n_y, n_z),\ n^a n_a = 1$. In the long wavelength regime, the SO(3) gauge invariant effective Lagrangian in the quadratic approximation can be expressed as*

$$\mathcal{L} = \frac{f_t^2}{2}(D_0 n^a)^2 - \frac{f_s^2}{2}(\partial_i n^a)^2 + rC^{ab} n_a n_b\,, \quad (1)$$

$$D_0 n^a = \partial_t n^a - \epsilon^a_{bc} n^b \mu B^c$$

where $f_t,\ f_s,\ r$ are low-energy parameters. Realizing the Néel vector in terms of a complex scalar field $\Phi(X)$

$$\mathbf{n} = \left(\frac{\Phi + \Phi^*}{\sqrt{2}}, \frac{\Phi - \Phi^*}{\sqrt{2}i}, \sqrt{1 - \Phi^*\Phi}\right)$$

* M. Hongo *et.al.*, *Phys. Rev. B* **104**, 134403 (2022).

# Effective field theory of magnons

The Lagrangian (1) acquires the form

$$\mathcal{L}^{(2)} = f_t^2 \left( D_0\Phi^* D_0\Phi - \Delta^2\Phi^*\Phi \right) - f_s^2 \delta^{ij}\partial_i\Phi^*\partial_j\Phi \,,$$
$$D_0\Phi = (\partial_0 + iU)\,\Phi \,,\;\; D_0\Phi^* = (\partial_0 - iU)\,\Phi^* \,,$$

where $U(X) = \mu B(X)$ and $rC^{ab} = f_t^2 \Delta^2 \delta^{a3}\delta^{b3}/2$. Here

- The ratio $v_s = f_s/f_t$ plays the role of the speed of light;
- The energy gap $\Delta$ plays the role of mass (For antiferromagnetic* $MnF_2$, $\begin{cases} \Delta \sim 1\mathrm{meV}, \\ v_s \sim 60\mathrm{m/s} \end{cases}$)

The corresponding relativistic wave equation is a Klein-Gordon like equation for $\Phi(X)$:

$$\left( D_0^2 - v_s^2 \boldsymbol{\nabla}^2 + \Delta^2 \right) \Phi(X) = 0 \,, \boldsymbol{\nabla} = \partial_i\partial_i \,.$$

*Bayrakci *et.al.*, *Science* **312**, 1926 (2006).

# Formulation based on strong-field QED

External fields: time-independent *steplike* magnetic fields:

$$\mathbf{B}(X) = (0, 0, B(x)),\ \partial_x B(x) \le 0,\ \forall x \in (-\infty, +\infty),\ B(-\infty) > B(+\infty)\,.$$

Complete sets of solutions of the KG equation have the form:

$$\Phi_m(X) = \exp\left(-i\varepsilon t + i\mathbf{p}_\perp \mathbf{r}_\perp\right)\varphi_m(x)\,,\ \mathbf{r}_\perp = (0, y, z)\,,\ m = (\varepsilon, \mathbf{p}_\perp)\,,$$

$$\left\{v_s^2 \partial_x^2 + [\varepsilon - U(x)]^2 - \pi_\perp^2\right\}\varphi_m(x) = 0,\ U(x) = \mu B(x),\ \pi_\perp = \sqrt{v_s^2 \mathbf{p}_\perp^2 + \Delta^2}\,.$$

where the scalar field obeys the eigenvalue equations

$$\hat{p}_0 \Phi_m(X) = \varepsilon \Phi_m(X),\ \hat{p}_y \Phi_m(X) = p_y \Phi_m(X),\ \hat{p}_z \Phi_m(X) = p_z \Phi_m(X).$$

The total potential energy experienced by a magnon with positive $\mu$ is

$$\delta U = U_\mathrm{L} - U_\mathrm{R} > 0,\ U_\mathrm{L} = U(-\infty),\ U_\mathrm{R} = U(\infty).$$

# Formulation based on strong-field QED

The field is **homogeneous** at “left” $S_{\mathrm{L}} = (-\infty, x_{\mathrm{L}}]$ and “right” $S_{\mathrm{R}} = [x_{\mathrm{R}}, \infty)$ regions: solutions have well-defined asymptotic forms,

$$
\begin{aligned}
{}_{\zeta}\phi_m(x) &= {}_{\zeta}\mathcal{N} e^{i\zeta|p^{\mathrm{L}}|(x-x_{\mathrm{L}})}, \; x \in S_{\mathrm{L}}, \\
{}^{\zeta}\phi_m(x) &= {}^{\zeta}\mathcal{N} e^{i\zeta|p^{\mathrm{R}}|(x-x_{\mathrm{R}})}, \; x \in S_{\mathrm{R}}, \; \zeta = \mathrm{sgn}(p^{\mathrm{L/R}}) = \pm 1 \, .
\end{aligned}
$$

with **real** asymptotic momenta $|p^{\mathrm{L/R}}| = \frac{\zeta}{v_s}\sqrt{[\pi_0(\mathrm{L/R})]^2 - \pi_\perp^2}$ provided

$$
[\pi_0(\mathrm{L/R})]^2 > \pi_\perp^2, \; \pi_0(\mathrm{L/R}) = \varepsilon - U(x_{\mathrm{L/R}}), \; \pi_\perp^2 = v_s^2 \mathbf{p}_\perp^2 + \Delta^2 \, .
$$

The normalization constants ${}_{\zeta}\mathcal{N}$, ${}^{\zeta}\mathcal{N}$ are calculated *via* the inner product

$$
\int \left\{ \Phi^* (i\partial_0 - U)\Phi' + \Phi \left[ (i\partial_0 - U)\Phi' \right]^* \right\} dt d\mathbf{r}_\perp \, .
$$

# Formulation based on strong-field QED

Solutions are subjected to the normalization conditions

$$\left( {}_{\zeta}\Phi_m,\ {}_{\zeta'}\Phi_{m'} \right)_x = \zeta\delta_{\zeta,\zeta'}\delta_{m,m'}\,,\ \left( {}^{\zeta}\Phi_m,\ {}^{\zeta'}\Phi_{m'} \right)_x = \zeta\delta_{\zeta,\zeta'}\delta_{m,m'}.$$

They are complete and orthonormal

$$\begin{aligned} {}^{\zeta}\Phi_m(X) &= {}_{+}\Phi_m(X)\, g\left({}_{+}\middle|{}^{\zeta}\right) - {}_{-}\Phi_m(X)\, g\left({}_{-}\middle|{}^{\zeta}\right), \\ {}_{\zeta}\Phi_m(X) &= {}^{+}\Phi_m(X)\, g\left({}^{+}\middle|{}_{\zeta}\right) - {}^{-}\Phi_m(X)\, g\left({}^{-}\middle|{}_{\zeta}\right), \end{aligned}$$

where $\left( {}_{\zeta}\phi_m,\ {}^{\zeta'}\phi_{m'} \right)_x = \delta_{mm'},\ g\left({}_{\zeta}\middle|{}^{\zeta'}\right),\ g\left({}^{\zeta'}\middle|{}_{\zeta}\right) = g\left({}_{\zeta}\middle|{}^{\zeta'}\right)^*$. Inner product on t-const. plane:

$$(\Phi,\Phi') = \frac{1}{v_s^2}\int_{V_\perp} d\mathbf{r}_\perp \int_{-K^{(\mathrm{L})}}^{K^{(\mathrm{R})}} \Psi^\dagger(X)\,\sigma_1\Psi(X)\,dx,\ \ \Psi(X) = \begin{pmatrix} i\partial_0 - U(x) \\ 1 \end{pmatrix}\Phi(X),$$

where

$$K^{(\mathrm{L})} - |x_{\mathrm{L}}|\,,\, K^{(\mathrm{R})} - x_{\mathrm{R}} \gg x_{\mathrm{R}} - x_{\mathrm{L}},$$

# Canonical quantization

The conditions $[\pi_0(\mathrm{L/R})]^2 > \pi_\perp^2$ introduce restrictions on the quantum numbers. For *critical* fields $\delta U = U_\mathrm{L} - U_\mathrm{R} > 2\Delta$, the manifold of quantum numbers divides into **5** ranges*. Particle creation occurs **only** in the **Klein zone**† :

$$\Omega_3 = \{U_\mathrm{R} + \pi_\perp \leq \varepsilon \leq U_\mathrm{L} - \pi_\perp,\ \pi_\perp \leq \delta U/2\},\ \pi_\perp = \sqrt{\Delta^2 + v_s^2 \mathbf{p}_\perp^2}\,.$$

The quantization is realized using exact solutions of the KG Eq. classified as **particle /antiparticle** states and **incoming/outgoing** states. Solutions are classified as follows†:

| | |
|---|---|
| IN-solutions: ${}_-\Phi_{m_3}(X)\,,\ {}^-\Phi_{m_3}(X)$ | OUT-solutions: ${}_+\Phi_{m_3}(X)\,,\ {}^+\Phi_{m_3}(X)$ |

Then, we can introduce IN and OUT vacua and sets of creation/annihilation operators:

| | |
|---|---|
| IN-set: ${}_-a_{m_3}(\mathrm{in})\,,\ {}^-b_{m_3}(\mathrm{in})$ | OUT-set: ${}_+a_{m_3}(\mathrm{out})\,,\ {}^+b_{m_3}(\mathrm{out})$ |

*S. P. Gavrilov, D. M. Gitman, Phys. Rev. D **87**, (2016). † **T. C. Adorno**, S. P. Gavrilov, D. M. Gitman, https://arxiv.org/abs/2310.20035

# Canonical quantization

The operators annihilate the corresponding vacua and obey the anticom. relations:

$$ {}_{-}a_{m_3}(\mathrm{in})\,|0,\mathrm{in}\rangle = {}^{-}b_{m_3}(\mathrm{in})\,|0,\mathrm{in}\rangle = 0\,,\ {}_{+}a_{m_3}(\mathrm{out})\,|0,\mathrm{out}\rangle = {}^{+}b_{m_3}(\mathrm{out})\,|0,\mathrm{out}\rangle = 0 $$

$$ \left[{}_{-}a_{m_3}(\mathrm{in})\,,\ {}_{-}a^{\dagger}_{m'_3}(\mathrm{in})\right]_{+} = \left[{}^{-}b_{m_3}(\mathrm{in})\,,\ {}^{-}b^{\dagger}_{m'_3}(\mathrm{in})\right]_{+} = \delta_{m_3 m'_3} $$

$$ \left[{}_{+}a_{m_3}(\mathrm{out})\,,\ {}_{+}a^{\dagger}_{m'_3}(\mathrm{out})\right]_{+} = \left[{}^{+}b_{m_3}(\mathrm{out})\,,\ {}^{+}b^{\dagger}_{m'_3}(\mathrm{out})\right]_{+} = \delta_{m_3 m'_3} $$

With the aid of the operators, the quantization of the KG field reads:

$$ \begin{aligned} \hat{\Phi}_3(X) &= \sum_{m\in\Omega_3} \mathcal{M}_m^{-1/2}\left[\,{}_{-}a_m(\mathrm{in})\ {}_{-}\Phi_m(X) + {}^{-}b^{\dagger}_m(\mathrm{in})\ {}^{-}\Phi_m(X)\right], \\ &= \sum_{m\in\Omega_3} \mathcal{M}_m^{-1/2}\left[\,{}_{+}a_m(\mathrm{out})\ {}_{-}\Phi_m(X) + {}^{+}b^{\dagger}_m(\mathrm{out})\ {}^{-}\Phi_m(X)\right], \end{aligned} $$

where $\mathcal{M}_m = 2\frac{\mathcal{T}}{T}\left|g\left({}_{+}|{}^{-}\right)\right|^2\,,\quad m\in\Omega_3$

# Canonical quantization

Using the orthogonality between solutions we can establish the Bogoliubov transform.:

$$ {}_{-}a_m(\text{in}) = g\left({}^{+}|_{-}\right)^{-1}\left[g\left({}^{+}|_{+}\right)\ {}_{+}a_m(\text{out}) + \ {}^{+}b_m^{\dagger}(\text{out})\right] $$

$$ {}^{-}b_m^{\dagger}(\text{in}) = g\left({}_{+}|^{-}\right)^{-1}\left[\ {}_{+}a_m(\text{out}) + g\left({}_{+}|^{+}\right)\ {}^{+}b_m^{\dagger}(\text{out})\right] $$

Using these relations, we can calculate the mean number of OUT particles created from the IN vacuum, total numbers, and vacuum-vacuum transition probabilities:

$$ N_m^{\text{cr}} = \left\langle 0, \text{in}\right|\ {}_{+}a_m^{\dagger}(\text{out})\ {}_{+}a_m(\text{out})\left|0, \text{in}\right\rangle = \left|g\left({}_{+}|^{-}\right)\right|^{-2} $$

$$ N^{\text{cr}} = \sum_{m\in\Omega_3} N_m^{\text{cr}} = \frac{TV_{\perp}}{(2\pi)^3}\int_{U_{\text{R}}+\pi_{\perp}}^{U_{\text{L}}-\pi_{\perp}} d\varepsilon \int d\mathbf{p}_{\perp} N_m^{\text{cr}} $$

$$ P_v = \left|\langle 0, \text{out}|0, \text{in}\rangle\right|^2 = \exp\left[\sum_{m\in\Omega_3} \ln\left(1 + N_m^{\text{cr}}\right)^{-1}\right] $$

# Exactly-solvable external fields

**(I) L-constant step** →

$$B\left(x\right)=\begin{cases} B'L/2\,, & x\in S_{\mathrm{L}}=\left(-\infty,-L/2\right]\,, \\ -B'x\,, & x\in S_{\mathrm{int}}=\left(-L/2,L/2\right)\,, \\ -B'L/2 & x\in S_{\mathrm{R}}=\left[L/2,+\infty\right)\,, \end{cases}$$

**(II) Sauter-like step** →

$$B\left(x\right)=-B'L_{\mathrm{S}}\tanh\left(x/L_{\mathrm{S}}\right)$$

**(III) Exponential step** →

$$B\left(x\right)=B'\begin{cases} k_1^{-1}\left(1-e^{k_1x}\right)\,, & x\in\left(-\infty,0\right] \\ k_2^{-1}\left(e^{-k_2x}-1\right)\,, & x\in\left(0,+\infty\right) \end{cases}$$

**(IV) Inverse-square step** →

$$B\left(x\right)=B'\begin{cases} \varrho_1\left[1-\left(1-x/\varrho_1\right)^{-1}\right]\,, & x\in\left(-\infty,0\right] \\ \varrho_2\left[\left(1+x/\varrho_2\right)^{-1}-1\right]\,, & x\in\left(0,+\infty\right) \end{cases}$$

## (I) L-constant step

$$N_m^{\mathrm{cr}} = \frac{8e^{-\pi\lambda/2}}{\sqrt{\xi_1^2-\lambda}\sqrt{\xi_2^2-\lambda}} \left| f_1^{(-)}(\xi_2)\, f_2^{(-)}(\xi_1) - f_2^{(-)}(\xi_2)\, f_1^{(-)}(\xi_1) \right|^{-2}$$

$$f_1^{(\pm)}(\xi) = \left(1 \pm \frac{i}{\sqrt{\xi^2-\lambda}}\frac{d}{d\xi}\right) D_{-\nu-1}\left[\pm(1+i)\xi\right],\ f_2^{(\pm)}(\xi) = \left(1 \pm \frac{i}{\sqrt{\xi^2-\lambda}}\frac{d}{d\xi}\right) D_{\nu}\left[\pm(1-i)\xi\right]$$

$$\xi(x) = \frac{\varepsilon + \mu B' x}{\sqrt{v_s \mu B'}}, \quad \lambda = \frac{\pi_\perp^2}{v_s \mu B'}$$

"*Gradually*-varying" field configuration: $\sqrt{\frac{|\mu B'|}{v_s}} L \gg \max\left(1, \frac{\Delta^2}{v_s \mu B'}\right),$

$$N_m^{\mathrm{cr}} = \exp(-\pi\lambda)\left[1 + O\left(|\xi_1|^{-3}\right) + O\left(\xi_2^{-3}\right)\right]$$

"*Sharply*-varying" field configuration: $\delta U L / v_s \ll 1$

$$N_m^{\mathrm{cr}} \approx \frac{4\left|p^{\mathrm{L}}\right|\left|p^{\mathrm{R}}\right|}{\left|\left|p^{\mathrm{L}}\right| - \left|p^{\mathrm{R}}\right| + i\sigma\right|^2},\ \sigma = \left[\left|p^{\mathrm{L}}\right|\left|p^{\mathrm{R}}\right| + (i+\lambda)\frac{|\mu B'|}{v_s}\right] L\,.$$

## (II) Sauter-like step

$$N_m^{\mathrm{cr}} = \frac{\sinh\left(\pi L_{\mathrm{S}}\left|p^{\mathrm{R}}\right|\right)\sinh\left(\pi L_{\mathrm{S}}\left|p^{\mathrm{L}}\right|\right)}{\sinh^2\left[\pi L_{\mathrm{S}}\left(\left|p^{\mathrm{R}}\right|-\left|p^{\mathrm{L}}\right|\right)/2\right]+\cosh^2\left(\frac{\pi}{2}\sqrt{\left(L_{\mathrm{S}}\delta U/v_s\right)^2-1}\right)}.$$

"*Gradually*-varying" field configuration: $\delta U L_s/v_s \gg 1$

$$N_m^{\mathrm{cr}} \approx e^{-\pi\tau}, \quad \tau = L_{\mathrm{S}}\left(2\left|\mu B'\right|L_{\mathrm{S}}/v_s - \left|p^{\mathrm{R}}\right| - \left|p^{\mathrm{L}}\right|\right).$$

"*Sharply*-varying" field configuration: $\delta U L_{\mathrm{S}}/v_s \ll 1$

$$N_m^{\mathrm{cr}} \approx \frac{4\left|p^{\mathrm{L}}\right|\left|p^{\mathrm{R}}\right|}{\left(\frac{\delta U^2 L_{\mathrm{S}}}{2v_s^2}\right)^2+\left(\left|p^{\mathrm{L}}\right|-\left|p^{\mathrm{R}}\right|\right)^2}.$$

# (III) Exponential step

$$N_m^{\mathrm{cr}} = \frac{4\left|p^{\mathrm{L}}\right|\left|p^{\mathrm{R}}\right|}{\exp\left[-\pi\left(k_1^{-1}\left|p^{\mathrm{L}}\right| - k_2^{-1}\left|p^{\mathrm{R}}\right|\right)\right]}\left|\left(k_1 h_1 y_1^2 \frac{d}{d\eta_1} y_2^1 + k_2 h_2 y_2^1 \frac{d}{d\eta_2} y_1^2\right)\Big|_{x=0}\right|^{-2} .$$

$$y_1^j(\eta_j) = e^{-\eta_j/2}\eta_j^{\nu_j}\Phi(a_j, c_j; \eta_j), \ y_2^j(\eta_j) = e^{\eta_j/2}\eta_j^{-\nu_j}\Phi(1-a_j, 2-c_j; -\eta_j)$$

$$\eta_1 = ih_1 e^{k_1 x}, \ \eta_2 = ih_2 e^{-k_2 x}, \ h_j = 2\mu B'/k_j^2 v_s .$$

"*Gradually*-varying" field configuration: $\min(h_1, h_2) \gg \max\left\{1, \Delta^2/v_s\mu B'\right\}, \ k_1/k_2 = \mathrm{fixed}$

$$N_m^{\mathrm{cr}} \approx \begin{cases} \exp\left\{-\frac{2\pi}{k_1}\left[\left|\pi_0(\mathrm{L})\right| - \left|p^{\mathrm{L}}\right|\right]\right\}, & 0 \le \varepsilon < U_{\mathrm{L}} - \pi_\perp , \\ \exp\left\{-\frac{2\pi}{k_2}\left[\pi_0(\mathrm{R}) - \left|p^{\mathrm{R}}\right|\right]\right\}, & U_{\mathrm{R}} + \pi_\perp \le \varepsilon < 0 . \end{cases}$$

"*Sharply*-varying" field configuration: $U_{\mathrm{L}}/k_1 \ll 1, \ \ |U_{\mathrm{R}}|/k_2 \ll 1$

$$N_m^{\mathrm{cr}} \approx \frac{4\left|p^{\mathrm{L}}\right|\left|p^{\mathrm{R}}\right|}{\left(\left|p^{\mathrm{L}}\right| - \left|p^{\mathrm{R}}\right|\right)^2 + b^2}, \ b = \frac{2U_{\mathrm{L}}}{k_1}\left[\frac{U_{\mathrm{L}}}{4} + \left|\pi_0(\mathrm{L})\right|\right] + \frac{2|U_{\mathrm{R}}|}{k_2}\left[\frac{|U_{\mathrm{R}}|}{4} + \pi_0(\mathrm{R})\right] .$$

# (IV) Inverse-square step

$$N_m^{\mathrm{cr}} = \left|p^{\mathrm{L}}\right| \left|p^{\mathrm{R}}\right| \left| \left[ \left|p^{\mathrm{L}}\right| w_1^2(z_2) \frac{d}{dz_1} w_2^1(z_1) + w_2^1(z_1) \left|p^{\mathrm{R}}\right| \frac{d}{dz_2} w_1^2(z_2) \right] \Big|_{x=0} \right|^{-2} .$$

$$w_1^j(z_j) = e^{-i\pi\kappa_j/2} W_{\kappa_j,\mu_j}(z_j) , \; w_2^j(z_j) = e^{-i\pi\kappa_j/2} W_{-\kappa_j,\mu_j}\left(e^{-i\pi} z_j\right) ,$$

$$z_1(x) = 2i \left|p^{\mathrm{L}}\right| \varrho_1 (1 - x/\varrho_1) , \; z_2(x) = 2i \left|p^{\mathrm{R}}\right| \varrho_2 (1 + x/\varrho_2) ,$$

$$\kappa_1 = -i \frac{\mu B' \varrho_j^2}{v_s} \frac{\pi_0(\mathrm{L})/v_s}{\left|p^{\mathrm{L}}\right|} , \; \kappa_2 = i \frac{\mu B' \varrho_2^2}{v_s} \frac{\pi_0(\mathrm{R})/v_s}{\left|p^{\mathrm{R}}\right|} , \; \mu_j = (-1)^j \sqrt{\frac{1}{4} - \left(\frac{\mu B' \varrho_j^2}{v_s}\right)^2} ,$$

"*Gradually*-varying" field configuration: $\min\left(U_{\mathrm{L}}\varrho_1, \left|U_{\mathrm{R}}\right| \varrho_2\right) \gg \max\left\{1, \Delta^2/v_s \mu B'\right\} , \; \varrho_1/\varrho_2 = \text{fixed}$

$$N_m^{\mathrm{cr}} \approx \begin{cases} \exp\left(2\pi\omega_1^+\right) , & 0 \le \varepsilon < U_{\mathrm{L}} - \pi_\perp , \\ \exp\left(2\pi\omega_2^-\right) , & U_{\mathrm{R}} + \pi_\perp \le \varepsilon < 0 . \end{cases} \quad \omega_j^\pm = \mp(-1)^j i \left(\kappa_j \pm \mu_j\right)$$

"*Sharply*-varying" field configuration: $\max\left(U_{\mathrm{L}}\varrho_1/v_s, \left|U_{\mathrm{R}}\right| \varrho_2/v_s\right) \ll 1, \; \varrho_1/\varrho_2, \; \text{fixed} .$

$$N_n^{\mathrm{cr}} \approx \frac{4\left|p^{\mathrm{L}}\right| \left|p^{\mathrm{R}}\right|}{\left(\left|p^{\mathrm{L}}\right| - \left|p^{\mathrm{R}}\right|\right)^2 + d^2} , \; d = \frac{\pi_0(\mathrm{L})}{U_{\mathrm{L}}} \left|p^{\mathrm{L}}\right|^2 \varrho_1 + \frac{\pi_0(\mathrm{R})}{U_{\mathrm{R}}} \left|p^{\mathrm{R}}\right|^2 \varrho_2 .$$

# Total quantities

In the gradually-varying regime, total numbers can be presented in a universal form*

$$N^{\mathrm{cr}} \approx \frac{V_{\perp}T}{(2\pi)^3}\int_{x_{\mathrm{L}}}^{x_{\mathrm{R}}} dx U'(x)^2 \exp\left[-\pi\frac{\Delta^2}{v_s U'(x)}\right] .$$

$$N_m^{\mathrm{cr}} \approx V_{\perp}T r^{\mathrm{cr}}\frac{\delta U}{|\mu B'|}\begin{cases} 1\,, & \text{for } L\text{-constant step} \\ \tilde{\delta}/2\,, & \text{for Sauter-like step} \\ G\left(2, \pi\frac{\Delta^2}{v_s|\mu B'|}\right), & \text{for exponential step} \\ \frac{1}{2}G\left(\frac{3}{2}, \pi\frac{\Delta^2}{v_s|\mu B'|}\right), & \text{for inverse-square step} \end{cases}$$

$$\tilde{\delta} = \sqrt{\pi}\Psi\left(\frac{1}{2}, -2; \pi\frac{\Delta^2}{v_s|\mu B'|}\right), \quad G(\alpha, z) = e^x x^\alpha \Gamma(-\alpha, x) .$$

$$P_{\mathrm{v}} \approx \exp\left\{-\frac{V_{\perp}T}{(2\pi)^3}\sum_{l=1}^{\infty}(-1)^{l-1}\int_{x_{\mathrm{L}}}^{x_{\mathrm{R}}} dx\frac{U'(x)^2}{l^2}\exp\left[-\pi\frac{l\Delta^2}{v_s U'(x)}\right]\right\} .$$

*See also S. P. Gavrilov, D. M. Gitman, S. Shishmarev, *Phys. Rev. D* **99**, 116014 (2019)

# Summary

- Effective field theory (EFT) models for low-energy magnons — scalar QED (sQED) with external fields
- Quantized the EFT model rigorously, in the framework of QED with inhomogeneous external fields
- Computed pertinent quantities characteryzing magnon-antimagnon pair production from the vacuum

# THANK YOU

## VISIT US

WWW.XJTLU.EDU.CN

## FOLLOW US

@XJTLU

Xi'an Jiaotong-Liverpool University
西交利物浦大学

tiago.adorno@xjtlu.edu.cn

# Funding*

- Research conference fund – RCF
- Research development fund – RDF

VISIT US

WWW.XJTLU.EDU.CN

FOLLOW US

@XJTLU

Xi'an Jiaotong-Liverpool University
西交利物浦大学

* This talk was funded by the XJTLU Research Conference Fund, SMP – RCF-20240528001 (**T. C. Adorno**) tiago.adorno@xjtlu.edu.cn