Regular Multi-horizon Lee-Wick Black holes

based on [2308.12810] [2308.12251]

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17th Marcel Grossman Meting,

Pescara, 07-12 July 2024

Introduction: Why Higher-derivatives?

- GR, as a quantum field theory, is not perturbatively renormalizable
- Higher derivatives terms $R^2_{\mu\nu\alpha\beta}$, $R^2_{\mu\nu}$, R^2, $\Box R$ appear already when we consider semiclassical gravity

$$S_{\text{grav}} = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\Big[R + R_{\alpha\beta}F_1(\Box)R^{\alpha\beta} + RF_2(\Box)R\Big],$$

- Theories with 4 derivatives are renormalizable [Stelle 1977]
- Polynomial gravity with 6+ derivatives $\rightarrow$ super-renormalizable [Asorey, López, Shapiro, 1997]
- Problem of "ghosts" $\rightarrow$ Ostrogradsky instability, violation of unitarity at quatum level
- A number of proposals exists to reconcile renormalizability and unitarity [Bender, Mannheim, 2008; Modesto, Shapiro, 2016; Anselmi, Piva, 2017 ; Donoghue, Menezes, 2019]

  Nonlocal Gravity: [Krasnikov 1987; Kuz'min 1989; Tomboulis 1997; Modesto 2012; Biswas, Gerwick, Koivisto, Mazumdar, 2012]

Six derivative gravity with complex poles

Lee-Wick Action Modesto, Shapiro [1512.07600]; Modesto [1602.02421]

$$S_{LW} = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\,[R + G_{\mu\nu}\,F(\Box)\,R^{\mu\nu}]$$

$F_1(\Box) = -2F_2(\Box) = F(\Box) = \alpha_1 + \alpha_2\Box$,

$$D_{\mu\nu,\alpha\beta}(k) = \frac{1}{k^2 f(-k^2)}\left[P^{(2)}_{\mu\nu,\alpha\beta}(k) - \frac{1}{2}P^{(0-s)}_{\mu\nu,\alpha\beta}(k)\right] + \dots ,$$

$$\boxed{f(z) = 1 + F(z)\,z},$$

$f(z) = 1 + \alpha_1 z + \alpha_2 z^2 \rightarrow$ complex conjugate massive poles

$$\mu = a + ib\ , \quad \bar{\mu} = a - ib \qquad a, b > 0$$

$$\alpha_1 = -\frac{2\,\mathrm{Re}\,(\mu^2)}{|\mu|^4} = -\frac{2(a^2 - b^2)}{(a^2 + b^2)^2}\,, \qquad \alpha_2 = \frac{1}{|\mu|^4} = \frac{1}{(a^2 + b^2)^2}$$

$$\boxed{\alpha_2 \geqslant 0\,, \qquad -2\sqrt{\alpha_2} < \alpha_1 < 2\sqrt{\alpha_2}}\,.$$

Newtonian potential of a pointlike source

Newtonian limit for a pointlike source $\to$ linearized metric only depends on potential $\varphi(r)$

$$\mathrm{d}s^2 = -\left[1 + 2\varphi(r)\right]\mathrm{d}t^2 + \left[1 + 2r\varphi'(r)\right]\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2,$$

Modified Poisson equation Giacchini, Paula-Netto, [1806.05664],[2307.12357]

$$f(\Delta)\Delta\varphi = 4\pi G\rho, \qquad \rho(\vec{r}) = M\delta^{(3)}(\vec{r}),$$

Equivalently

$$\Delta\varphi = 4\pi G\rho_{\mathrm{eff}},$$

$$\rho_{\mathrm{eff}}(r) \equiv \frac{M}{2\pi^2 r}\int_0^\infty \mathrm{d}k\, \frac{k\sin(kr)}{f(-k^2)} = \frac{M(a^2+b^2)^2}{8\pi ab}\,\frac{e^{-ar}\sin(br)}{r}\,.$$

Bambi, Modesto, Wang [1611.03650]; Giacchini, Paula-Netto [1809.05907]; [2307.12357]

THE SMEARED δ-SOURCE APPROXIMATION

Effective energy-momentum tensor

$$\tilde{T}^{\mu}{}_{\nu} = \mathrm{diag}(-\rho_{\rm eff}, p_{r,{\rm eff}}, p_{\theta,{\rm eff}}, p_{\theta,{\rm eff}})$$

Effective field equations

$$G^{\mu}{}_{\nu} = 8\pi G\, \tilde{T}^{\mu}{}_{\nu},$$

These EoM approximate the e full EoMs

$$f(\Box)G^{\mu}{}_{\nu} + O(R^2_{\dots}) = 8\pi G\, T^{\mu}{}_{\nu}.$$

- The effective $\tilde{T}^{\mu}{}_{\nu}$ compensate for the truncation of $O(R^2_{\dots})$ in the full EoMs
- Impose the conservation $\nabla_{\mu}\tilde{T}^{\mu}{}_{\nu} = 0$.
- Effective pressure components p_r and $p_\theta \to$ determined by the field equations & conservation equation

Bambi, Modesto, Wang, [1611.03650]; [Nicolini, Smailagic, Spallucci,[hep-th/0507226]; Modesto, Moffat, Nicolini, [1010.0680]; Modesto, [1107.2403].

Static spherically symmetric solutions (1)

General line element:

$$\mathrm{d}s^2 = -A(r)\, e^{B(r)}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{A(r)} + r^2\mathrm{d}\Omega^2,$$

Field equations: $G^t{}_t$ and $G^r{}_r$ equations

$$\frac{A'}{r} + \frac{A-1}{r^2} = -8\pi G\rho_{\rm eff}\,, \quad \rightarrow \quad \textcolor{red}{A(r) \text{ only depends on } \rho_{\rm eff}(r)}$$

$$\frac{AB'}{r} = 8\pi G(\rho_{\rm eff} + p_r)\,,$$

Solution for $A(r)$

$$A(r) = 1 - \frac{2G\,m(r)}{r}\,, \qquad m(r) = 4\pi\int_0^r \mathrm{d}x\, x^2 \rho_{\rm eff}(x)$$

$$\boxed{m(r) = M - \frac{M}{2ab}e^{-ar}\Big\{C(r)\,\cos(br) + D(r)\,\sin(br)\Big\}},$$

$$C(r) = b\left[2a + (a^2+b^2)r\right]\,, \qquad D(r) = \left[a^2 - b^2 + a(a^2+b^2)r\right]\,.$$

Static spherically symmetric solutions (2)

From $A(r)$ the second equation can be solved for $B(r)$

$$\frac{AB'}{r} = 8\pi G(\rho_{\rm eff} + p_r)$$

${G^\theta}_\theta$ equation $\to$ conservation of ${\tilde{T}^\mu}_\nu \to p_\theta$
Fixing $p_r = p_r(\rho_{\rm eff})$ we can solve the system

1. $p_r(r) = -\rho_{\rm eff}(r) \to \boxed{B = 0}$, $g_{tt} = -g^{rr}$, Schwarzchild-like metric.
2. $p_r(r) = [A(r) - 1]\rho_{\rm eff}(r) \to B \neq 0$, NS-like

$$\boxed{B(r) = -\frac{GM(a^2 + b^2)}{ab} e^{-ar} \left[b\cos(br) + a\sin(br)\right]}$$

components ${R^{\alpha\beta}}_{\mu\nu}$ regular in $r = 0$, while $\Box R \sim O(r^{-1})$

Paula Netto, Giacchini, NB, Modesto [2308,12251] Giacchini, Paula Netto [2307.12357]

Oscillations of the mass function (2)

- Horizons $\to g_{rr}^{-1} = 0 \to A(r) = 0$

$$A(r) = 1 - GMaZ_q(y(r))$$

- Dimensionless function Z_q

$$Z_q(y) = 2\frac{q}{y} - \frac{1}{qy} e^{-\frac{y}{q}} \left\{ q[y + q(2 + qy)] \cos y - \left[q(q^2 - 1) - (q^2 + 1)y \right] \sin y \right\}.$$

- only depends on the parameters

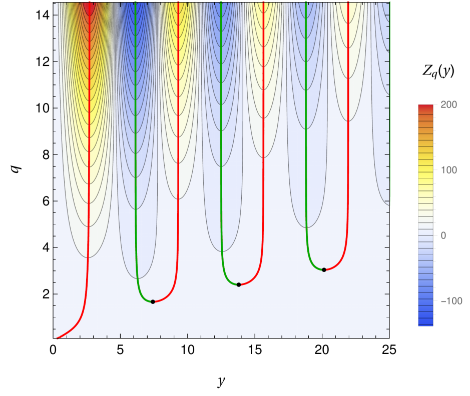
$$q \equiv \frac{b}{a} = \frac{\mathrm{Im}\,\mu}{\mathrm{Re}\,\mu}\,, \qquad y \equiv br = (\,\mathrm{Im}\,\mu)r\,,$$

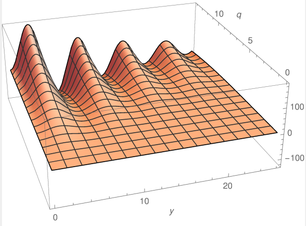
$$\lim_{r\to 0} A(r) = \lim_{r\to\infty} A(r) = 1$$

max num of horizons $N_H^{max} \to$ extrema of $Z_q \to$ increases with q

Structure of the Z_q

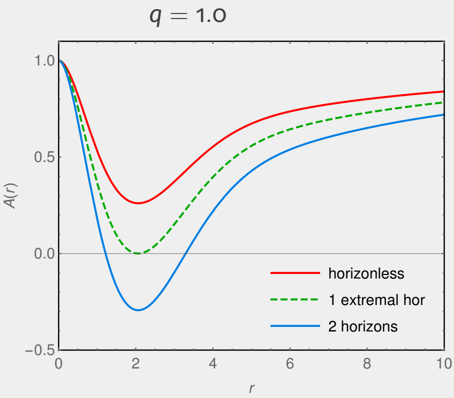
$$Z_q(y) = 2\frac{q}{y} - \frac{1}{qy}e^{-\frac{y}{q}}\left\{q[y + q(2 + qy)]\cos y - \left[q(q^2 - 1) - (q^2 + 1)y\right]\sin y\right\}.$$



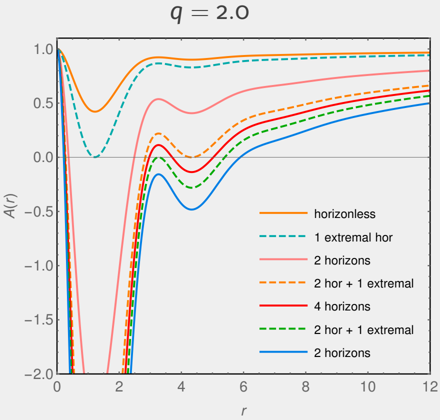


Horizons and mass gaps: two horizon case

$1.0 < q < 1.67$	
Mass gaps (in units of M_{P}^2/a)	Number of horizons
$M < 1.082$	0
$M > 1.082$	2



$1.67 < q < 2.40$	
Mass gaps (in units of M_{P}^2/a)	Number of horizons
$M < 0.345$	0
$0.345 < M < 2.024$	2
$2.024 < M < 2.593$	4
$M > 2.593$	2



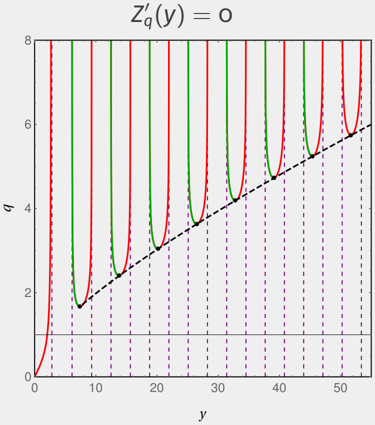
Horizons and mass gaps: the general case

Horizon structure is completely determine by $Z_q(y)$

- $y_i^{max} \approx (2i-1)\pi - \frac{1}{(2i-1)\pi}$, $\quad i \in \mathbb{N}$
- mass gaps $M_i(a,q) \approx M_p^2/(a\, Z_q(y_i^{max}))$
- interpolating function between "saddle points"
  $y(q) = -2q\, W_{-1}\left(-\frac{\sqrt{q}}{\sqrt{2}(q^2-1)}\right) \sim q \log q$

$N_{max} \sim q \log q$

$M_{max}(a,q) \sim \log q\, M_p^2/a$



NB, Giacchini, Paula Netto, Modesto, JCAP [2308.12810]

Black Hole Thermodynamics (1)

Horizon definition $A(r_{\rm H}) = 0 \to$ rewrite as

$$M(y_{\rm H}) = \frac{M_{\rm P}^2}{a\, Z_q(y_{\rm H})}\,.$$

Surface gravity and Hawking temperature

$$\kappa = \frac{1}{2} e^{\frac{1}{2}B(r_{\rm H})}\, A'(r_{\rm H})\,, \qquad\qquad T = \frac{\kappa}{2\pi}\,.$$

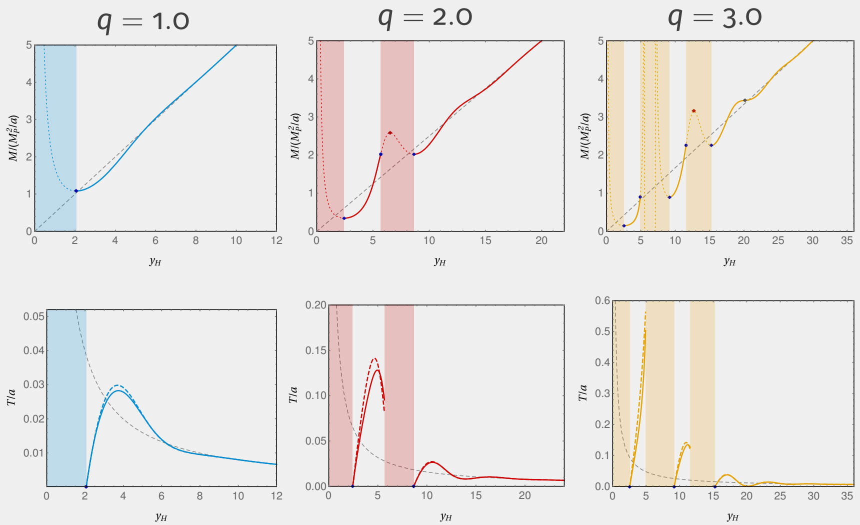
The Hawking temperature of the black hole in terms $y_{\rm H} = aqr_{\rm H}$

$$T(y_{\rm H}) = -\frac{aq}{4\pi}\frac{Z_q'(y_{\rm H})}{Z_q(y_{\rm H})}\, e^{\frac{1}{2}B_q(y_{\rm H})}.$$

For non-trivial shift function $B_q(y_{\rm H})$,

$$B_q(y_{\rm H}) = -\frac{1}{Z_q(y_{\rm H})}\frac{1+q^2}{q} e^{-\frac{y_{\rm H}}{q}}\,(q\cos y_{\rm H} + \sin y_{\rm H}).$$

Black Hole Thermodynamics (2)



Black Hole Thermodynamics (3)

- The minima of $M(y)$ (representing critical masses) are always zero temperature configuration
- in each of the allowed horizon position gap the black hole can undergo a process of evaporation
- Stefan Boltzman law

$$L = \sigma \mathcal{A} T^4 \,, \qquad \mathcal{A} = 4\pi r_{\mathrm{H}}^2$$

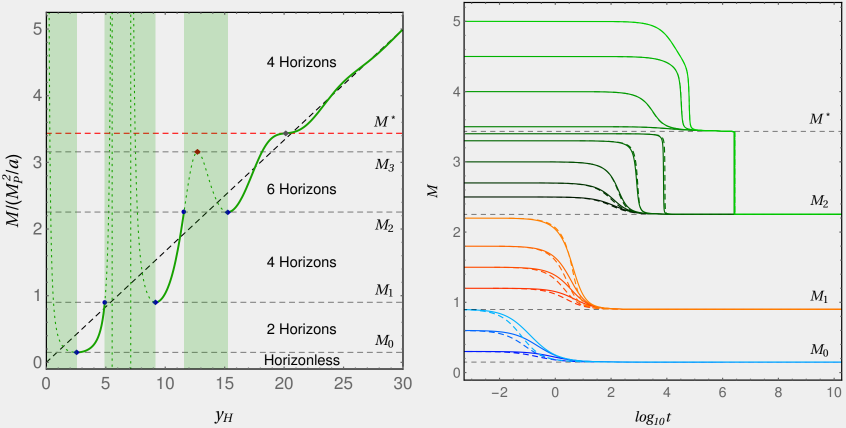
- Assuming all the luminosity L come from the loss of mass $\to$ we can estimate the evaporation time

$$\frac{\mathrm{d}M}{\mathrm{d}t} \sim -4\pi r_{\mathrm{H}}^2 T^4.$$

Estimate evaporation time

$$\Delta t \sim -(4\pi)^3 \frac{M_{\mathrm{P}}^2}{a^4 q^3} \int_{r_i}^{r_c} \mathrm{d}r' \frac{Z_q^2(aqr')}{r'^2 Z_q'^3(aqr')} \, e^{-2B(r')},$$

Black Hole Thermodynamics: example $q = 3.0$



- Time evolution of the total mass $M(t)$ (units $M_{\mathrm{P}} = 1$) during evaporation
- Numerical integration of Δt in the case $a = 1$, $q = 3$ for different values of the initial mass $M_i = M(t = 0)$.
- Solid lines $\rightarrow B(r) = 0$, dashed lines $\rightarrow$ non-trivial $B(r)$

- Lee Wick black holes $\rightarrow$ smeared delta source approximation.
  Two class of metrics: S-like, and NS like *to* both regular $r = 0$.
- Nontrivial horizon structure depending on the parameter $q = b/a \rightarrow$ characterize the oscillations of $m(r) \rightarrow$ for $q > 1.67$ multiple horizon and critical masses are allowed.
- Discrete number of critical masses $\{M_0, M_1, \ldots\} \rightarrow$ The smallest M_0 constitutes the usual mass gap for the black hole formation $\rightarrow$ multiple remnant regimes after evaporation.
- Non-Schwarzchild class of solution $\rightarrow B(r) \neq 0 \rightarrow$
  in agreement with the result found [Holdom 2002]
  and recent results on exact solutions Giacchini, Kolář, [2406.00997]

Thank you!