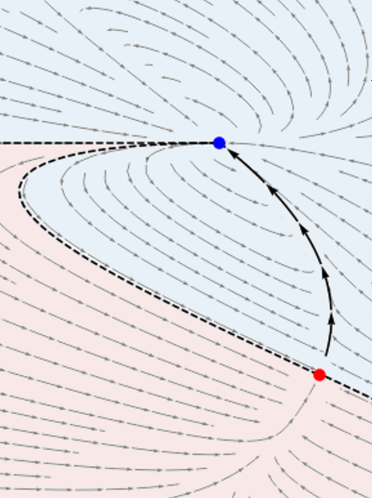
17th Marcel Grossmann Meeting - Black holes in alternative theories of gravity session

Pescara, 12th of July 2024



Black holes at a crossroads during the late stages of evaporation in quadratic gravity

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in collaboration with Alfio Bonanno

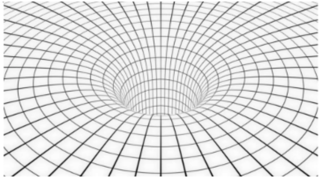
SISSA

INFN Istituto Nazionale di Fisica Nucleare Sezione di Trieste

Introduction:

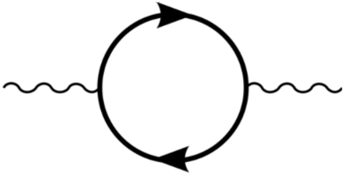
Black hole evaporation and quadratic gravity

Why black hole evaporation? - Semiclassical gravity



Classical curved spacetime

$+$



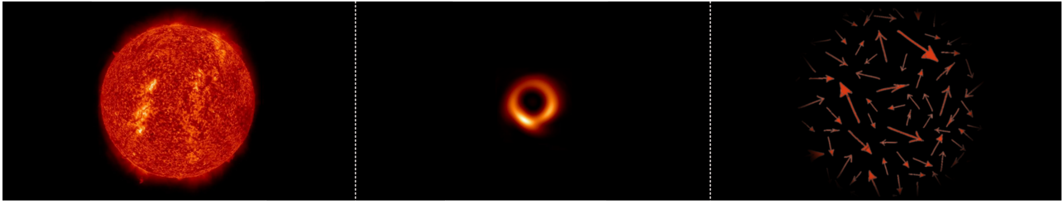
Quantum Field Theory

$\Downarrow$

Black hole evaporation

Fundamental requirement: $E \ll E_{Quantum \;\; Gravity}$

Why black hole evaporation? - Information paradox



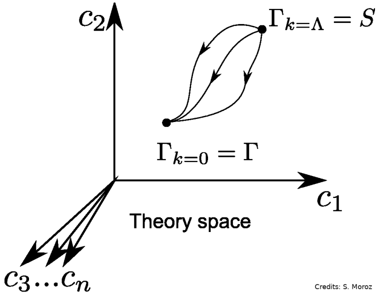
Information is accessible | Information is not accessible | Information is lost

Final stages of evaporation $\Longrightarrow$ $T_{BH} \to \infty$ $\Longrightarrow$ $E \sim E_{Quantum\ Gravity}$?

Solution: quantum corrections for gravity?

Why quadratic gravity? - Wilsonian approach

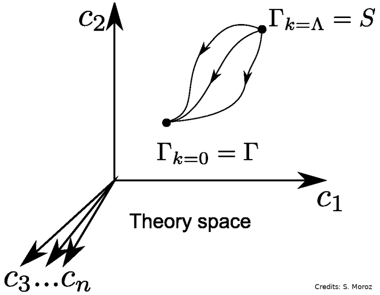
Non-renormalizable theory $\implies$ effective field theory at low energies



$$\mathcal{I}_{eff} = \int d^4x\sqrt{-g}\left[E^4 c_1 + E^2 c_2 R + c_3 R^2 + c_4 R^{\mu\nu}R_{\mu\nu} + c_5 R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} + \frac{c_6}{E^2}R^3 + \dots\right]$$

Why quadratic gravity? - Wilsonian approach

Non-renormalizable theory $\Longrightarrow$ effective field theory at low energies



$$\mathcal{I}_{eff} = \int d^4x\sqrt{-g}\left[\underbrace{E^4 c_1 + E^2 c_2 R}_{GR} + c_3 R^2 + c_4 R^{\mu\nu}R_{\mu\nu} + c_5 R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} + \frac{c_6}{E^2}R^3 + \ldots\right]$$

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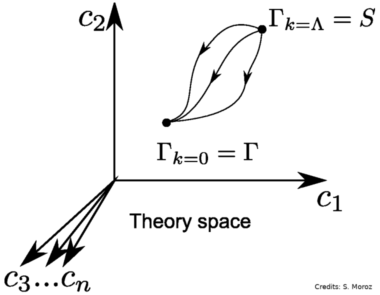

$$\mathcal{I}_{eff} = \int d^4x\sqrt{-g}\left[E^4 c_1 + \underbrace{E^2 c_2 R + c_3 R^2 + c_4 R^{\mu\nu} R_{\mu\nu} + c_5 R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}}_{\text{Quadratic Gravity}} + \frac{c_6}{E^2} R^3 + \ldots\right]$$

Quadratic gravity: a classical model for quantum corrections

$$\mathcal{I}_{QG} = \int d^4x\sqrt{-g}\left[\gamma\, R + \beta\, R^2 - \alpha\, C^{\mu\nu\rho\sigma}\, C_{\mu\nu\rho\sigma} + \chi\, \mathcal{G}\right] \begin{cases} S = 2, \;\; m = 0 \\ S = 0, \;\; m_0^2 = \gamma/6\beta \\ S = 2, \;\; m_2^2 = \gamma/2\alpha \end{cases}$$

PRO: general, IR limit of fundamental theories, renormalizable
K. Stelle (1977), B. Zwiebach (1985)

CON: negative energy states $\implies$ non-unitary theory

Fundamental assumption!!!

Classical solutions as first quantum corrections

Quadratic gravity: a classical model for quantum corrections

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Classical solutions as first quantum corrections

Black holes in quadratic gravity:

Old and new solutions

Symmetries and weak field limit

Staticity, spherical symmetry:

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

Asymptotic flatness (isolated objects):
K. Stelle (1978), A. Bonanno and S.S. (2019)

$$h(r) \sim 1 - \frac{2\,M}{r} + 2\,S_2^- \frac{\mathrm{e}^{-m_2\,r}}{r}$$

$$f(r) \sim 1 - \frac{2\,M}{r} + S_2^- \frac{\mathrm{e}^{-m_2\,r}}{r}(1 + m_2\,r)$$

Total (ADM) mass: M, Yukawa charge: S_2^-

Event horizon: internal boundary

Series expansion around horizon radius r_H:
A. Perkins et al. (2015)

$$h(r) = h_1 (r - r_H) + \sum_{n=2}^{\infty} h_n (r - r_H)^n$$

$$f(r) = f_1 (r - r_H) + \sum_{n=2}^{\infty} f_n (r - r_H)^n$$

Hawking: $T_{BH} = \frac{1}{4\pi}\sqrt{h_1 f_1}$, Wald: $S_{BH} = 16\pi^2\gamma \left(r_H^2 + \frac{2}{m_2^2} (1 - f_1 r_H) \right)$

Singularity: behaviour close to the origin

Series expansion around origin:
A. Perkins et al. (2015)

$$h(r) = r^t \sum_{n=0}^{\infty} h_{t+n} r^n \qquad t = \lim_{r\to 0} \frac{\mathrm{d}\log(h(r))}{\mathrm{d}\log(r)}$$

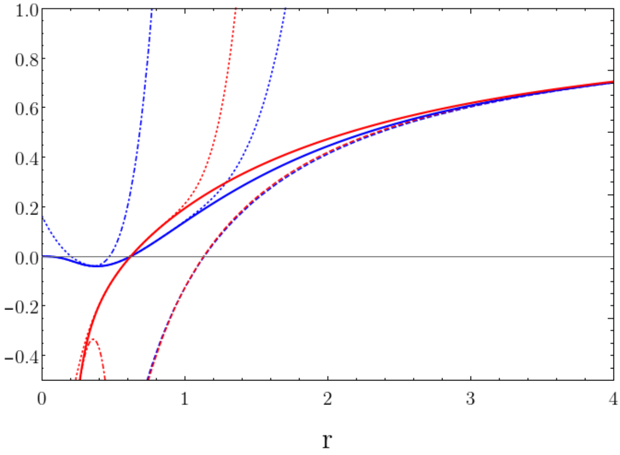
$$\Longrightarrow$$

$$f(r) = r^s \sum_{n=0}^{\infty} f_{s+n} r^n \qquad s = \lim_{r\to 0} \frac{\mathrm{d}\log(f(r))}{\mathrm{d}\log(r)}$$

Divergent metric: $t = -1$, $s = -1$,

Vanishing metric: $t = 2$, $s = -2$

Numerical methods: shooting method



$h(r) = 1 - \frac{2M}{r} + 2S_2^- \frac{e^{-m_2 r}}{r}$

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$h(r) = h_1(r - r_H) + h_2(r - r_H)^2 + \ldots$

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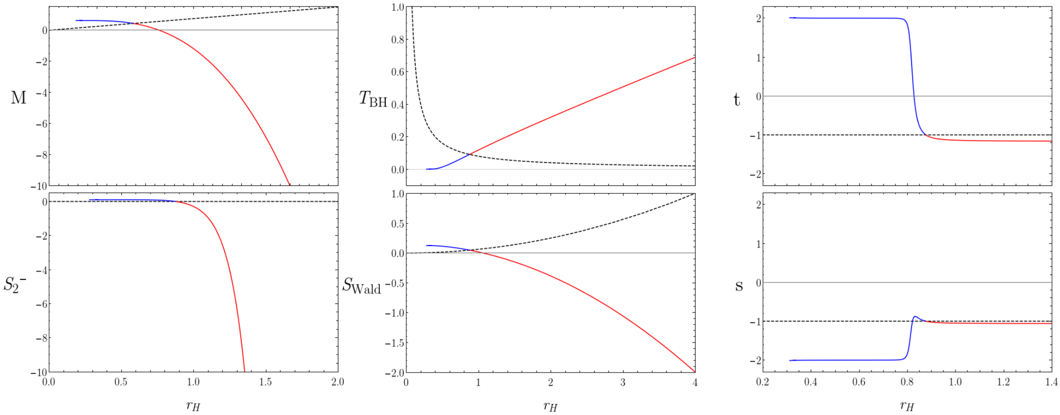
$h(r)$

$f(r)$

$h(r) = h_t r^t + h_{t+1} r^{t+1} + \ldots$

$f(r) = f_s r^s + f_{s+1} r^{s+1} + \ldots$

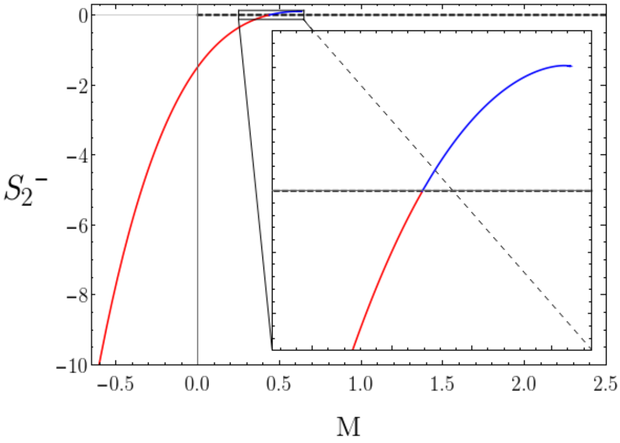
Properties of black holes in quadratic gravtity



Black hole crossroads:

Possible directions for evaporation

What crossroads? - Schwarzschild/non-Schwarzschild black holes



$$M_{tr} \sim 4.8\sqrt{\alpha}10^{-38}M_{\odot}$$

$$r_{H,tr} \sim 1.4\sqrt{\alpha}10^{-37}km$$

$$T_{BH,tr} \sim 1.3\,\alpha^{-1/2}10^{30}K$$

$\alpha < 10^{60}$ B. Giacchini (2016)

$$M_{tr} < 4.8 \cdot 10^{-8}M_{\odot}$$

$$r_{H,tr} < 1.4 \cdot 10^{-7}km$$

$$T_{BH,tr} > 1.3\,K$$

Exploring possibilities: dynamical stability

Metric perturbation:

$$g_{\mu\nu} = \bar{g}_{\mu\nu}(r) + \epsilon\,\delta g_{\mu\nu}(r,t) = \bar{g}_{\mu\nu} + \epsilon\,(h_{\mu\nu} + s_{\mu\nu} + t_{\mu\nu})$$

Reducing the degrees of freedom: $t_{\mu\nu} \to t_{\mu\nu}(\varphi)$
A. Held and J. Zhang (2023)

Regge-Wheeler-Zerilli-like equation:

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} - \frac{\mathrm{d}^2}{\mathrm{d}r^{*2}}\right)\varphi + V(r^*)\varphi = 0$$

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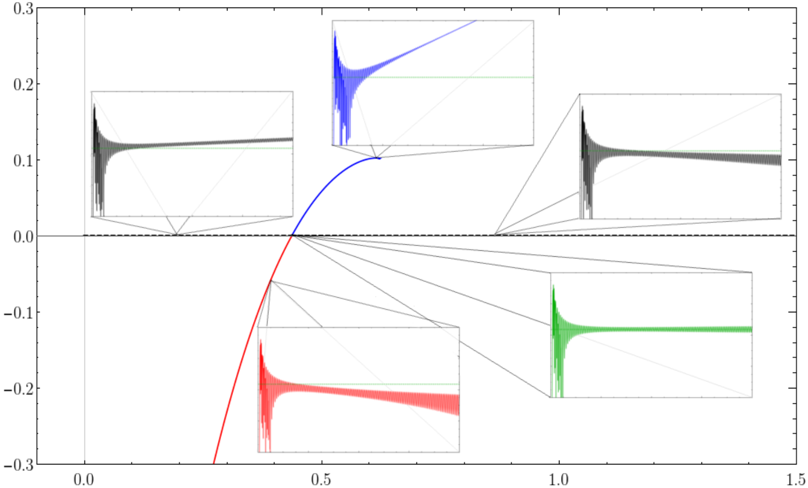
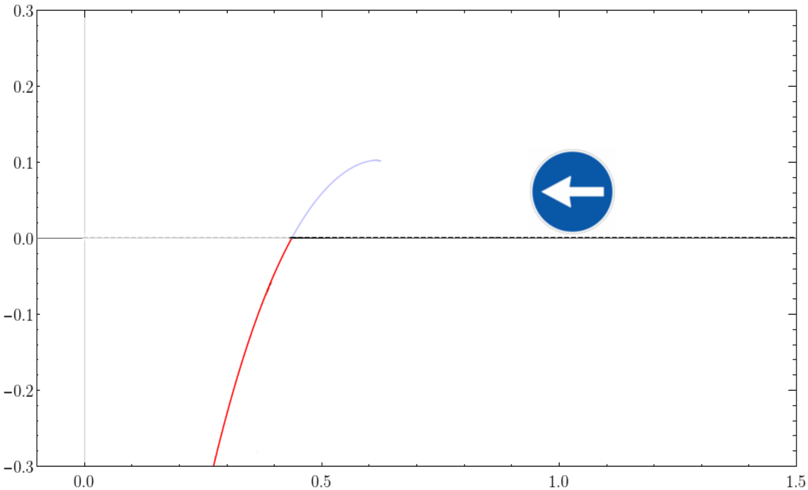
Exploring possibilities: stability at the crossroads



First direction: standard evaporation



Standard evaporation:

- adiabatic approximation

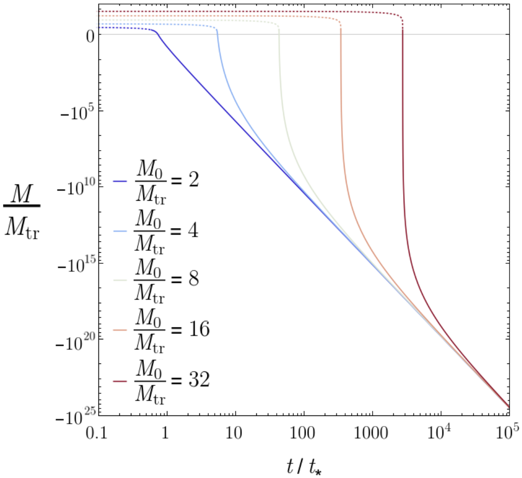
  $\implies$ requires stability

- emission of standard particles

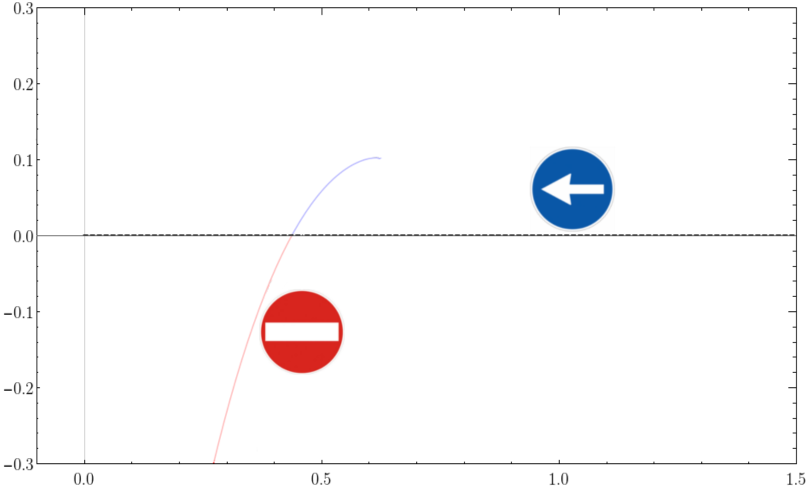
  $$\implies \frac{\mathrm{d}M(t)}{\mathrm{d}t} \sim -\sigma T_{BH}^2$$

$\implies$ evaporate in Yukawa attractive BHs

$$M(t_{now}) \sim -10^{206} M_\odot \text{ !!!!!}$$



Second direction: standard evolution



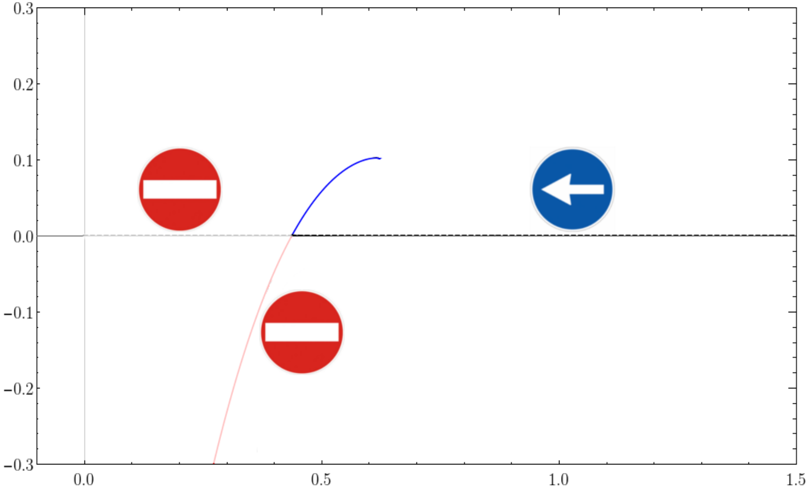
Large Schwarzschild black hole $\implies$ small Schwarzschild black hole

$$\Box\psi_{\mu\nu} + 2R_{\mu\rho\nu\sigma}\psi^{\mu\nu} = m_2^2\psi_{\mu\nu} \qquad \implies \qquad \psi_{tt} \sim e^{i\omega t}\frac{e^{-\sqrt{m_2^2-\omega^2}\,r}}{r}$$

$r_H = r_{tr} \implies \omega = 0 \implies$ unsuppressed perturbation $\psi_{tt} \sim \frac{e^{-m_2 r}}{r}$

$\implies$ transition into non-Schwarzschild black hole! (Evidence at non-linear level)
W. E. East and N. Siemonsen (2023)

Third direction: ghost instability



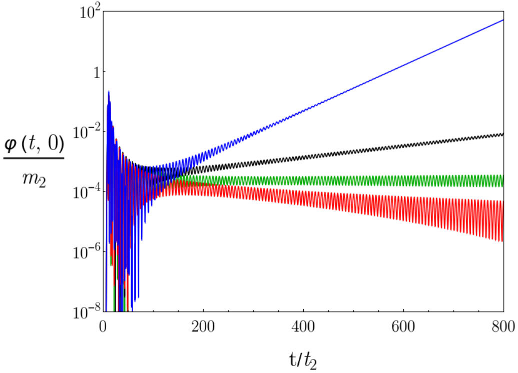
Ghost phase transition: strength of instability

Greater instability $\Longrightarrow$ faster growth

$$\psi_{\mu\nu} \sim \mathrm{e}^{\eta t}\,\frac{\delta M}{r} + \mathrm{e}^{\lambda t}\,\delta S\,\frac{\mathrm{e}^{-m_2 r}}{r}$$

$$\lambda > \eta$$

$\Longrightarrow$ higher probability of transition into Yukawa repulsive black holes?



Following the third direction: unstable evaporation

No adiabatic expansion $\implies$ full time-dependent evolution?

Asymptotic flatness (still isolated objects):

$$f(r,t) \sim 1 - \frac{2\,M(t)}{r} + \frac{1}{r}\int \mathrm{d}r\, r^2 \int \mathrm{d}s\mathrm{d}\tau\, G_{(\Box - m_2^2)}(r,t,s,\tau)\,\mathcal{C}\left(T_{\mu\nu}\right)$$

$M(t)$ is the time-dependent ADM (and Misner-Sharp, and Hawking-Hayward) mass

$$\left(\partial_t^2 + m_2^2\right)\partial_t M(t) = \frac{1}{8\alpha}\lim_{r\to\infty} r^2\, T_{tr}(r,t)$$

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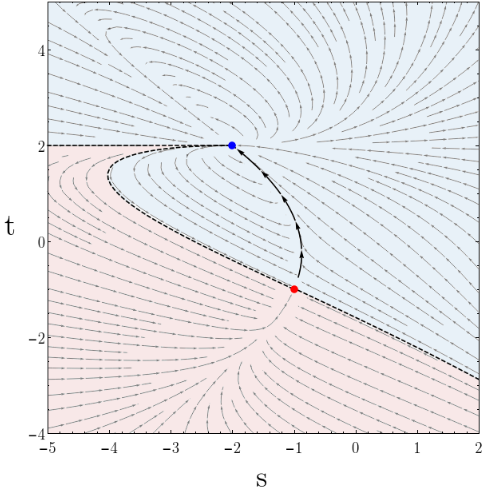
Stability in instabilities: nature of the singularity

Equations for t and s in $x = -\log(r)$:

$$\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{1}{2}\left(t\,s + 4s + t^2 + 2t + 4\right)$$

$$\frac{\mathrm{d}s}{\mathrm{d}x} = \frac{1}{2(t-2)}\left(2s^2 t - s^2 + st^2 + 8s - t^3 + 3t^2 + 8\right)$$



Unstable evaporation:

- exponential growth of perturbations

$$\implies \quad T_{tr} \propto e^{\nu\, t}$$

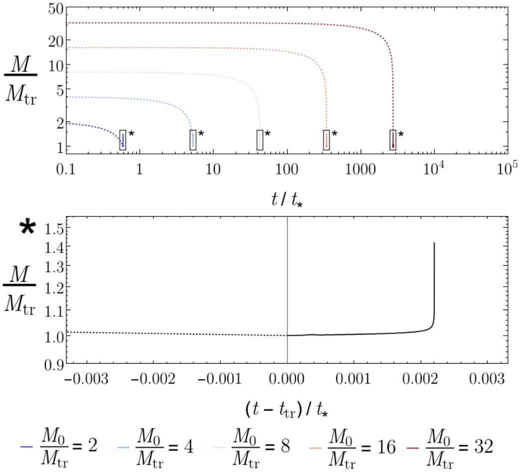
- emission of ghost particles

$$\implies \quad T_{tr} > 0$$

Assumption!

Endpoint is a static black hole with $r_H \to 0$

$$\implies t_e - t_{tr} \sim 10^{-45}\sqrt{\alpha}\, s < 10^{-15} s$$



Conclusions

Simple and conservative approach:

Information paradox: semiclassical gravity breaks down at high energies

$\Longrightarrow$ inclusion of first order quantum corrections to gravity

Many strong (but sensible) assumptions:

- classical solutions of quadratic gravity as first-order quantum corrections
- Schwarzschild $\Longrightarrow$ Yukawa repulsive phase transition
- no large deviations from static solutions

Conclusions

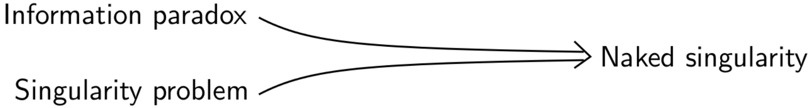
Consequences: U-turn for black hole evaporation

General relativity:

- mass and entropy decrease $\to 0$
- temperature increases $\to \infty$
- endpoint is flat space

Quadratic gravity:

- mass and entropy increase $\to M_f$, S_f
- temperature decreases $\to 0$
- endpoint is a naked singularity



What to expect?

Hope you will tell me, thank's for the attention!

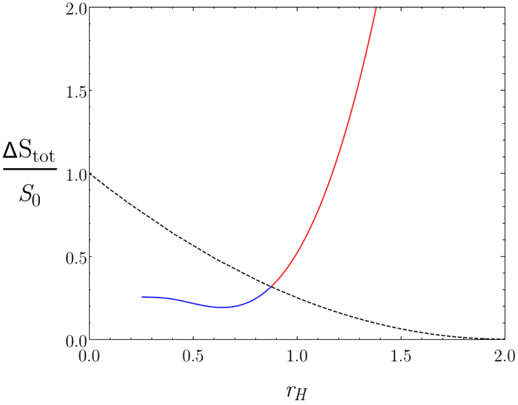
Exploring possibilities: thermodynamical spontaneity

Total thermodynamical entropy:

$$\delta S_{tot} = -\frac{\delta M}{T_{BH}} + \frac{\delta M}{T_U}$$

$$\Delta S_{tot} = -\Delta S_{BH} + \frac{\Delta M}{T_U}$$

$$\frac{\Delta S_{tot}}{S_0} = 1 - \frac{1}{T_U S_0}(M - T_U S_{BH})$$



A no-(scalar) hair theorem and ghosts

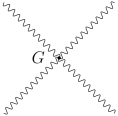
$C_{\mu\nu\rho\sigma}$ is traceless $\Longrightarrow$ trace of vacuum e.o.m. is $\left(\Box - m_0^2\right) R = 0$

$$\left\{\begin{array}{l}\text{staticity}\\ \text{asymptotic flatness}\\ \text{presence of event horizon}\end{array}\right. \Longrightarrow R = 0 \text{ in all spacetime}$$

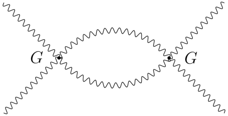
W. Nelson (2010), A. Perkins et al. (2015)

R^2 term is irrelevant $\Longrightarrow$ $C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma}$ term is crucial (ghosts!)

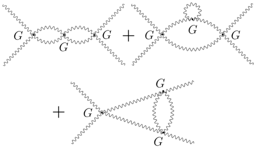
Why quadratic gravity? - Perturbative approach



$$\int d^4x\sqrt{-g}\, R$$

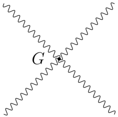


$$\int d^4x\sqrt{-g}\ \left[a\, R^2 + b\, R^{\mu\nu} R_{\mu\nu} + c\, R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}\right]$$

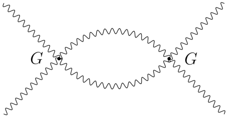


$$\int d^4x\sqrt{-g}\ \left[d\, R^{\mu\nu\rho\sigma} R_{\rho\sigma}{}^{\gamma\delta} R_{\gamma\delta\mu\nu} + e\, R^{\mu\nu} R_{\nu}{}^{\rho} R_{\rho\mu} + ...\right]$$

Why quadratic gravity? - Perturbative approach



$$\int d^4x\sqrt{-g}\, R \quad \Big\} \; GR$$



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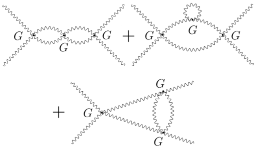

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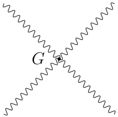
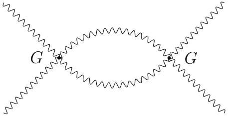
Why quadratic gravity? - Perturbative approach



$$\int d^4x\sqrt{-g}\, R$$



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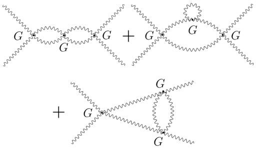
} *Quadratic Gravity*



$$\int d^4x\sqrt{-g}\,\left[d\, R^{\mu\nu\rho\sigma} R_{\rho\sigma}{}^{\gamma\delta} R_{\gamma\delta\mu\nu} + e\, R^{\mu\nu} R_{\nu}{}^{\rho} R_{\rho\mu} + \ldots\right]$$