# Hierarchical inference of cosmological and astrophysical population properties from gravitational wave observations and galaxy catalogs


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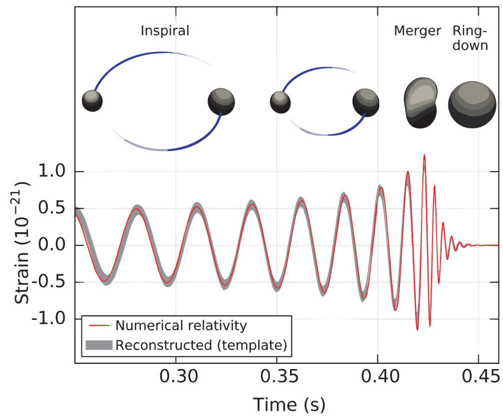
# GW AS A NEW COSMOLOGICAL PROBE

GW from compact binaries coalescences (CBC), e.g BBH, BNS, BH-NS mergers, are ***standard sirens*** (Schutz, 1986; Holz et al. 2005; Moresco et al. 2022).

From Abbott et al. 2016




$$h(t) = \frac{\mathcal{M}_{\rm det}^{5/3} f_{\rm det}^{2/3}(t)}{d_L} F(\text{angles}) \cos\left(\Phi(t)\right)$$

$\Longrightarrow h(t) \propto 1/d_L$: possible to measure the luminosity distance to the source without any intermediate distance ladder calibrator (only GR has been assumed).

- Possible to constrain $H_0$, the integrated $H(z)$ or modified GW propagation through the distance-redshift relation

$$d_L(z) = \frac{(1+z)}{H_0} \int_0^z \frac{c\mathrm{d}z}{E(z; \lambda_{\rm cosmo})}$$

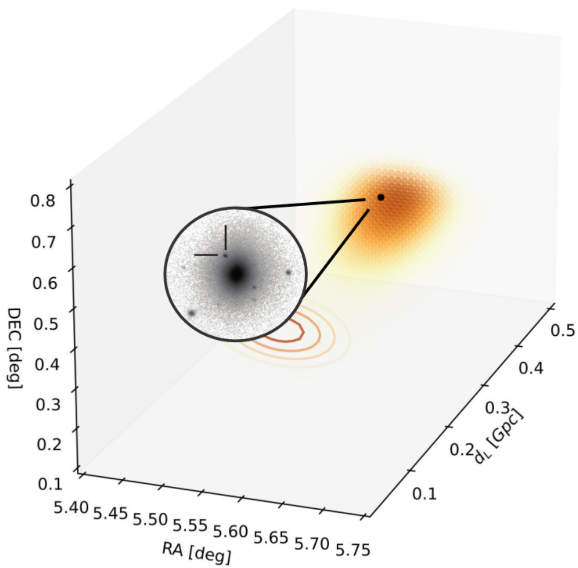
if the redshift of the source is known.

- Binary chirp mass and redshift are completely degenerate: $\mathcal{M}_{\rm det} = (1+z)\mathcal{M}_{\rm src} \Longrightarrow$ **the redshift cannot be measured from GW data alone**.

# Inferring the redshift - Bright Sirens

Mainly three ways:

1. by detecting an EM counterpart (**bright sirens**) (Holz et al. 2005; Nissanke et al. 2010);
2. by using information from a catalog of possible hosts, usually a galaxy catalog (**statistical dark sirens**);
3. by exploiting features in the source-frame mass distribution (**spectral dark sirens**).




**Pros**:

⇒ Very precise.

⇒ With dozens of brights sirens it could be possible to obtain a few % measurement of $H_0$ (Chen et al. 2024).

**Cons**:

⇒ Difficult to constrain other cosmological parameters.

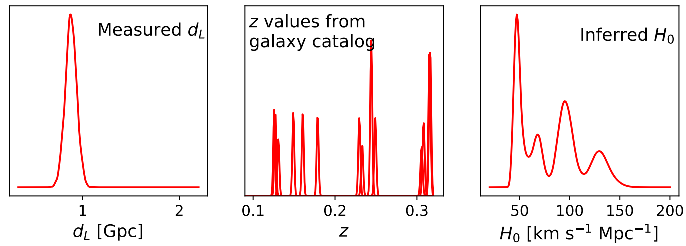
⇒ Need at least one neutron star.

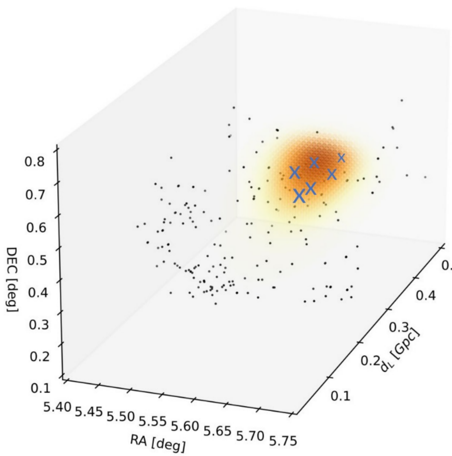
⇒ Only **GW170817** (Abbott et al. 2017) out of ∼ 200 events so far.

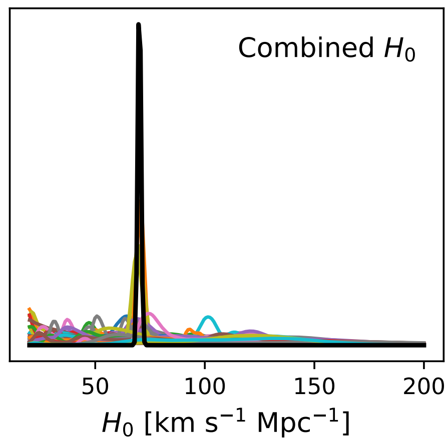
# Inferring the redshift - Statistical Dark Sirens

Mainly three ways:

1. by detecting an EM counterpart (**bright sirens**);
2. by using information from a catalog of possible hosts, usually a galaxy catalog (**statistical dark sirens**) (Schutz, 1986; Gair et al. 2023);
3. by exploiting features in the source-frame mass distribution (**spectral dark sirens**).



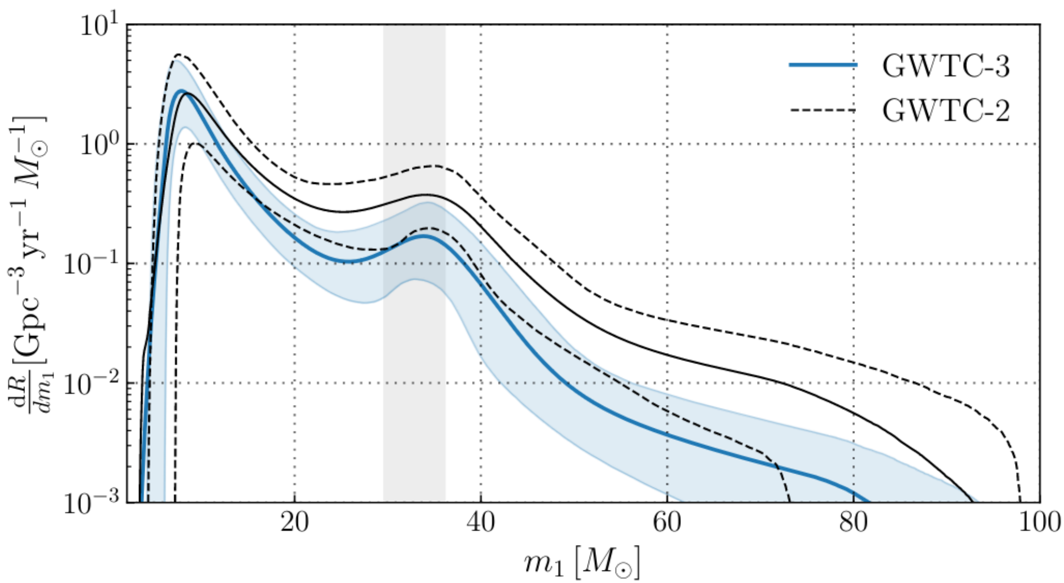
## Inferring the redshift - Spectral Dark Sirens

Mainly three ways:

1. by detecting an EM counterpart (**bright sirens**);
2. by using information from a catalog of possible hosts, usually a galaxy catalog (**statistical dark sirens**);
3. by exploiting features in the source-frame mass distribution (**spectral dark sirens**) (Chernoff et al. 1993; Del Pozzo, 2012; Ezquiaga et al. 2022).

$$m_{i,\mathrm{det}} = (1+z)\, m_{i,\mathrm{src}} = \frac{d_L H_0}{\int_0^z \frac{c\,dz}{E(z;\lambda_{\mathrm{cosmo}})}}\, m_{i,\mathrm{src}}$$

- $m_{i\,\mathrm{det}}$ and $d_L$ are observed. If $m_{i\,\mathrm{src}}$ is known, it is possible to constrain $H_0$ and $\lambda_c$.
- Presence of features in the source-frame mass distribution can be used to break the degeneracy at the statistical level.




From Abbott et al. 2023

# Combining Dark Sirens methods

It is natural to seek a **unified Dark Sirens method**:

1. both the redshift distribution obtained from a galaxy catalog and the mass distribution describes population properties in the source frame ("population priors");
2. the galaxy catalog method relies on assumptions about the population model, and can therefore be interpreted as an extension of the spectral sirens method;
3. it is further important to marginalize over population parameters to get robust results.

Mainly three codes that unifies the dark sirens methods and jointly infer cosmological and population parameters:

- ICAROGW 2.0 (Mastrogiovanni et al. 2024)
- GWCOSMO 2.0 (Gray et al. 2023)
- CHIMERA (Borghi et al. 2024)

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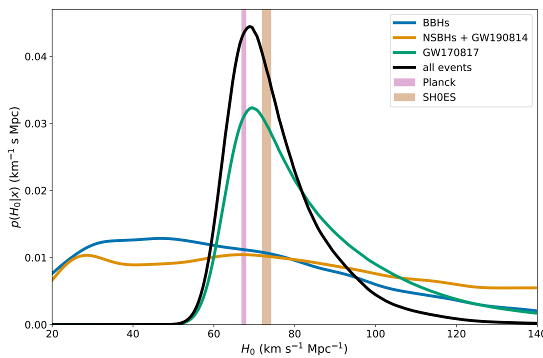
**Cosmology and Astrophysics with Standard Sirens and Galaxy Catalogs in View of Future Gravitational Wave Observations**

Nicola Borghi[1,2], Michele Mancarella[3,4], Michele Moresco[1,2], Matteo Tagliazucchi[1,2], Francesco Iacovelli[5,6], Andrea Cimatti[1,7], and Michele Maggiore[5,6]

From Gray et al. 2023

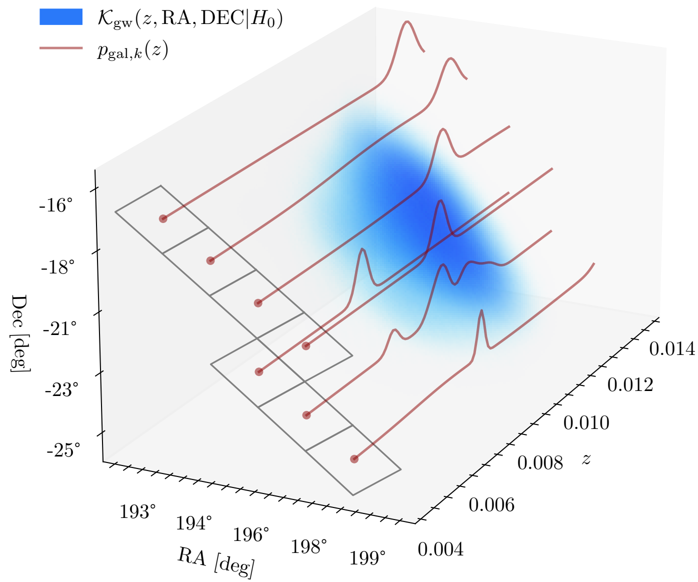


## The CHIMERA pipeline

CHIMERA (**C**ombined **H**ierarchical **I**nference **M**odel for **E**lectromagnetic and g**R**avitational-wave **A**nalysis): a novel Python code for the joint inference of cosmology and population properties of GW sources from GW data and galaxy catalogs. Borghi et al. 2024

**Workflow:**

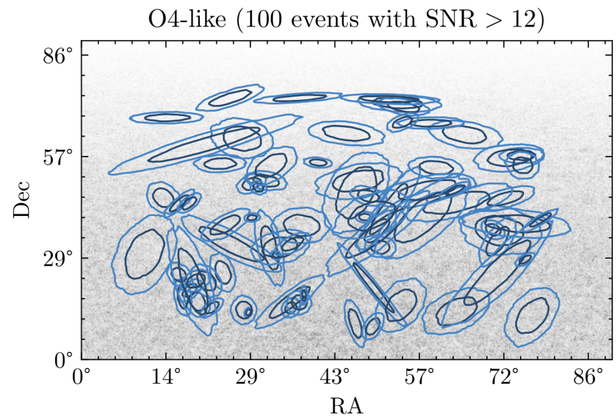
1. Pre-computation of integration redshift grids and catalog probability given the cosmological priors.
2. For each GW event:
   - **pixelization** of the localisation area;
   - 3D **KDE** estimate of the GW probability, weighted by the mass distribution, within the localization volume;
   - pixel-by-pixel **integration in redshift** of the KDE times the probability of having an host galaxy and sum of all the integrals;
3. Multiplication of the posterior of each event.
4. Monte Carlo approximation of the selection bias.

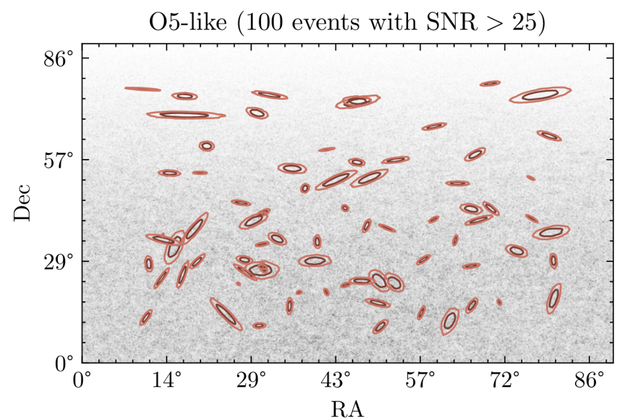



https://github.com/CosmoStatGW/CHIMERA

# Mock Catalogs

- **Mock galaxy catalog**: **luminosity-complete** subsample of the **MICE** Grand Challenge light-cone simulation (Fosalba et al. 2015) with a **uniform in comoving volume** density distribution.
  - ⇒ Cut in luminosity corresponds to $\log_{10}(M/\mathrm{M}_\odot) > 10.5$;
- **Mock GW events** drawn from fiducial population distributions:
  - ⇒ Cosmology: flat ΛCDM with $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_{m,0} = 0.25$.
  - ⇒ Mass distribution: Power Law + Gaussian peak
  - ⇒ Rate evolution: Madau-like




Fisher matrix-based computation of SNR and posterior samples using `GWFAST` (Iacovelli et al. 2022) and two configurations:

⇒ **O4-like:** L1, H1, Virgo, KAGRA at O4 design sensitivity.

⇒ **O5-like:** as before + LIGO India at O5 design sensitivity.

**Cut in SNR** to select the 100 best events over 1 year of observation.

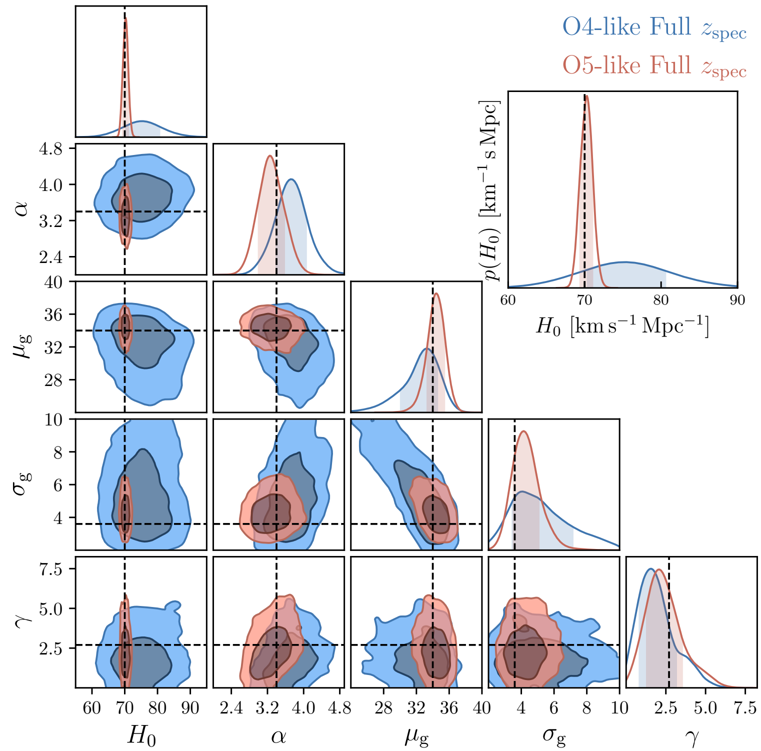
# O4 vs O5

Three MCMC analyses using the EMCEE sampler (Foreman-Mackey et al. 2013) for each GW catalog (6 runs in total):

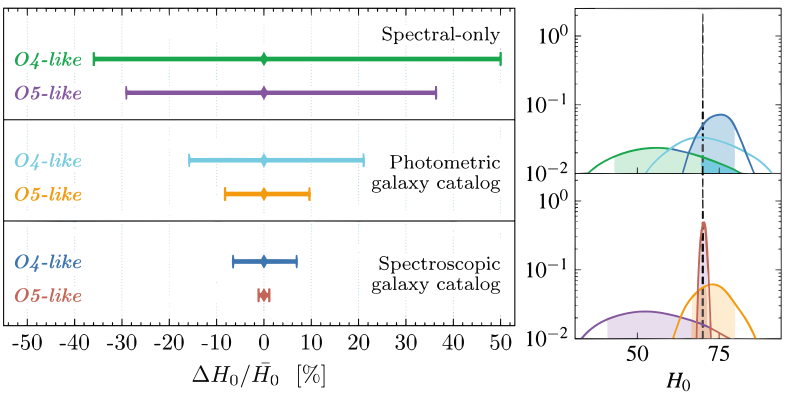
1. without the galaxy catalog (pure spectral sirens);
2. using the galaxy catalog with "photometric" errors, $\sigma_z/(1+z) = 0.05$;
3. using the galaxy catalog with "spectroscopic" errors, $\sigma_z/(1+z) = 0.001$.

The better sensitivity of O5 and the cut at higher SNR imply:

- $\Rightarrow$ better localization of GW events and so a more precise measurement of $H_0$;
- $\Rightarrow$ improved constraints on population parameter.



# Constraints on $H_0$

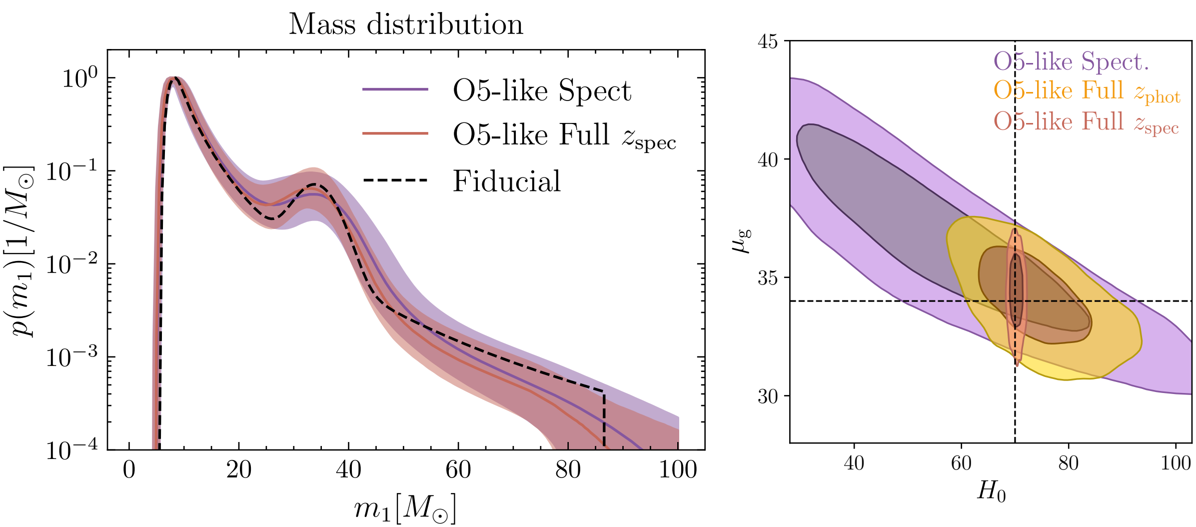



| | Spectral | Full $z_{\rm phot}$ | Full $z_{\rm spec}$ |
|---|---|---|---|
| O4-like | $64^{+32}_{-23}$ (43 %) | $76^{+16}_{-12}$ (18 %) | $75.3^{+5.2}_{-4.9}$ (7 %) |
| O5-like | $55^{+20}_{-16}$ (32 %) | $73.3^{+7.2}_{-6.3}$ (9 %) | $70.24^{+0.76}_{-0.81}$ (1 %) |

- Possible to achieve **1% accuracy** on $H_0$ in the O5 configuration with just 100 GW events and a complete spectroscopic galaxy catalog.
- In O5, using a photometric catalog degrades the accuracy to 9%.
- Constraint obtained with O4+spectroscopic catalog (7%) better than those with O5+photometric catalog.
- Spectral sirens cases not competitive with such a number of events (43%, 32%).
- Results robust since obtained while marginalising over population parameters.

# Constraints on population parameters

Constraints on population parameters dominated by the number of events considered, but including galaxies helps in reducing the correlation between $H_0$ and some population parameters.



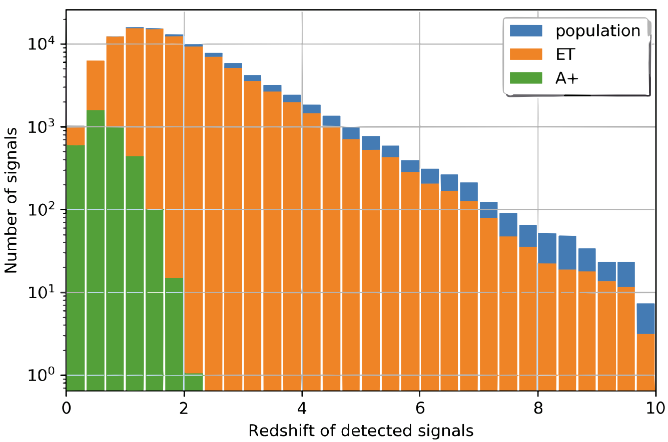
## TOWARDS 3G DETECTORS

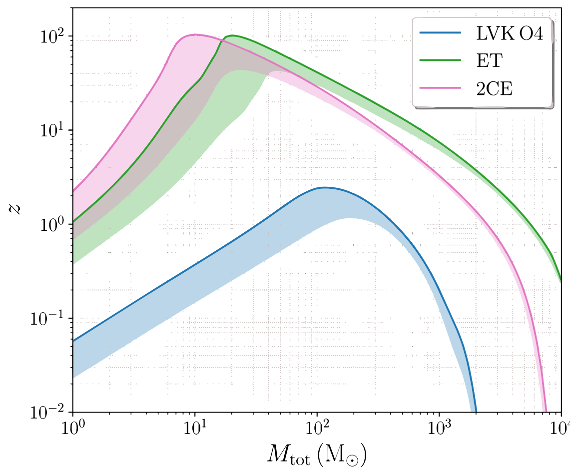
With current data, $\sim$ **90 GW events**, it requires **$10^5$ CPU hours** for a complete population fit.

**3G GW observatories** (ET, CE, LISA ...) will detect $\sim$ **$10^5$ events per year**.

It will be possible to reconstruct population properties, constrain the expansion history, and test modify gravity.

The **computational cost scales linearly** with the number of GW events: it's crucial to improve the pipelines to accommodate the large amount of future data.



# Improving CHIMERA

**Main bottleneck**: KDE evaluation ($\sim 82\%$ of total time spent)

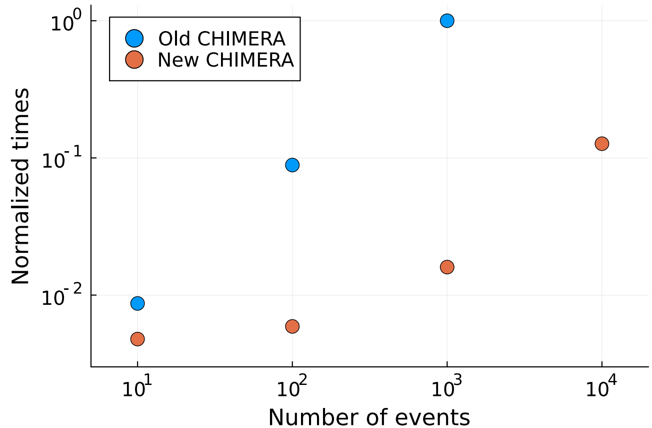
$$t_{\mathrm{KDE}} \sim \mathcal{O}\left(N_{\mathrm{events}} \times N_{\mathrm{pix,event}} \times N_{\mathrm{samples}} \times N_{\mathrm{z-grid}}\right)$$

**Improvements:**

- 3D KDE factorization:

$$p_{\mathrm{gw}}(z, \mathrm{RA}, \mathrm{DEC}|\lambda) = p_{\mathrm{gw}}(\mathrm{RA}, \mathrm{DEC})\, p_{\mathrm{gw}}(z|\lambda)$$

- KDE with binned data.
- Alternative density estimate algorithms (histogram, ASH, ...).
- Porting on GPU mass, rate and cosmology functions (to be optimised).



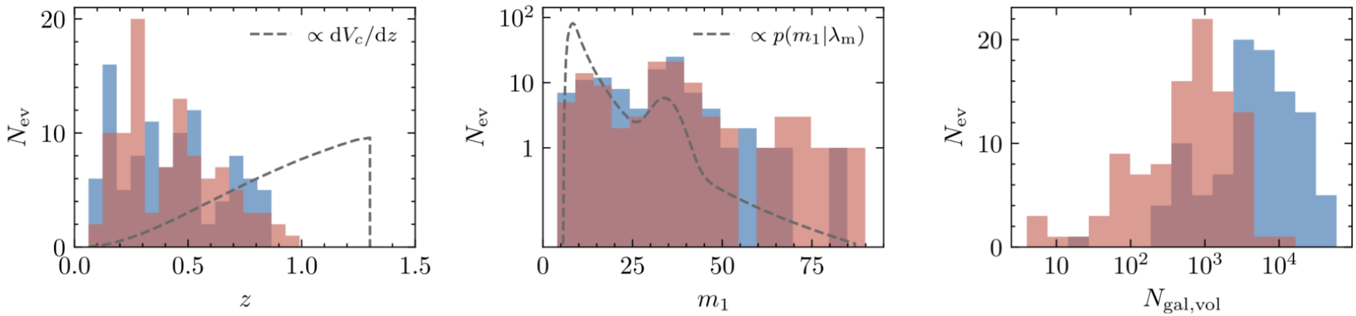
## Summary and ongoing projects

We developed a new code able to jointly constrain cosmological and population parameters from GW data with or without galaxy catalogs.
Different ongoing projects using `CHIMERA`:

1. Improving the pipeline and better porting on GPUs for a flagship use case of the ICSC - National Centre for HPC
2. Mock Data Challenge between the various codes of hierarchical inference
3. Extension of `CHIMERA` to include modified gravity propagation (M. Fiebig, MSc student @ University of Bologna)
4. Extension of `CHIMERA` to real GW data and galaxy catalogs (G. Cuomo, MSc student @ University of Bologna)
5. Explore the synergies between galaxy evolution and GW astronomy (N. Borghi, post-doc @ University of Bologna, main code developer)

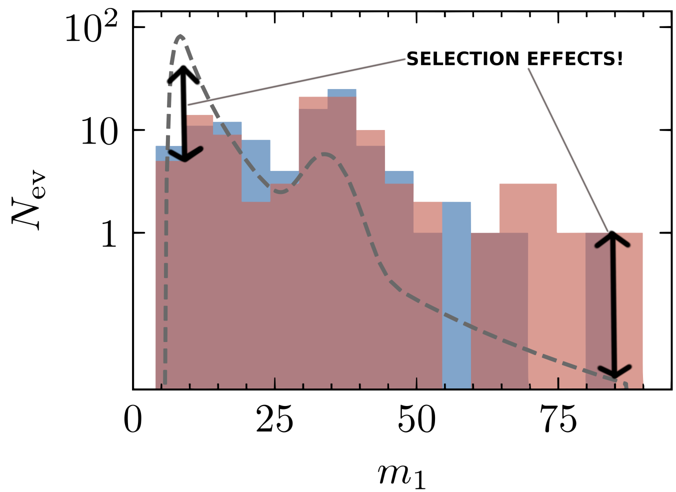
**Thank You for Your Attention!**

## Backup - catalogs properties




**Selection effects**: more massive BBH are more likely to be detected as they produce louder GW signals.

To reconstruct the true underlying population distributions it is necessary to correct this bias.

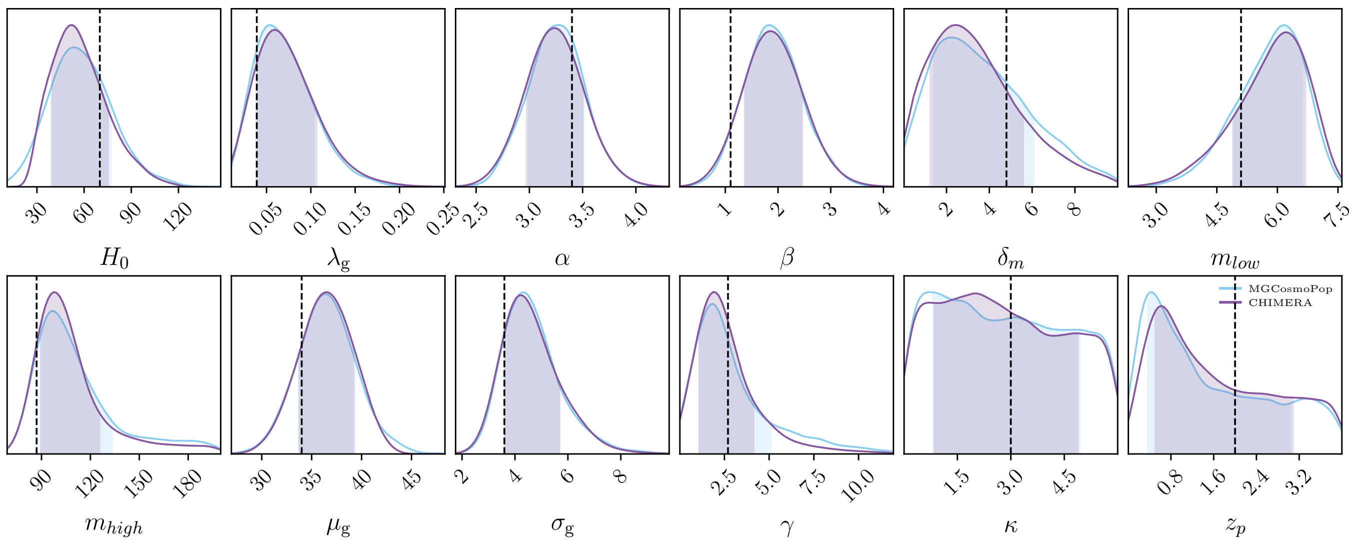


## Backup - MCMC parameters and priors

| Parameter | Description | Fiducial Value | Prior |
|---|---|---|---|
| | **Cosmology (flat $\Lambda$CDM)** | | |
| $H_0$ | Hubble constant [km/s/Mpc] | 70.0 | $\mathcal{U}(10.0, 200.0)$ |
| $\Omega_{\mathrm{m},0}$ | Matter energy density | 0.25 | Fixed |
| | **Rate evolution (Madau-like)** | | |
| $\gamma$ | slope at $z < z_p$ | 2.7 | $\mathcal{U}(0.0, 12.0)$ |
| $\kappa$ | slope at $z > z_p$ | 3 | $\mathcal{U}(0.0, 6.0)$ |
| $z_{\mathrm{p}}$ | peak redshift | 2 | $\mathcal{U}(0.0, 4.0)$ |
| | **Mass distribution (PowerLaw+Peak)** | | |
| $\alpha$ | (primary) slope of the power law | 3.4 | $\mathcal{U}(1.5, 12.0)$ |
| $\beta$ | (secondary) slope of the power law | 1.1 | $\mathcal{U}(-4.0, 12.0)$ |
| $\delta_m$ | (primary) smoothing parameter [$M_\odot$] | 4.8 | $\mathcal{U}(0.01, 10.0)$ |
| $m_{\mathrm{low}}$ | lower value [$M_\odot$] | 5.1 | $\mathcal{U}(2.0, 50.0)$ |
| $m_{\mathrm{high}}$ | upper value [$M_\odot$] | 87.0 | $\mathcal{U}(50.0, 200.0)$ |
| $\mu_{\mathrm{g}}$ | (primary): Gaussian component mean [$M_\odot$] | 34.0 | $\mathcal{U}(2.0, 50.0)$ |
| $\sigma_{\mathrm{g}}$ | (primary): Gaussian component std. dev. [$M_\odot$] | 3.6 | $\mathcal{U}(0.4, 10.0)$ |
| $\lambda_{\mathrm{g}}$ | (primary): fraction of the Gaussian component | 0.039 | $\mathcal{U}(0.01, 0.99)$ |

# Backup - CHIMERA validation

We **validate the `CHIMERA` code** against `MGCosmoPop` (Mancarella et al. 2022) in a pure spectral sirens analysis of the O5-like catalog.



# Backup - Population priors I

$p_{\mathrm{pop}}(\boldsymbol{\theta}|\boldsymbol{\lambda})$ is the **population prior** and can be written as:

$$p_{\mathrm{pop}}(\boldsymbol{\theta}|\boldsymbol{\lambda}) = p(m_1, m_2|\lambda_{\mathrm{m}}) \frac{p_{\mathrm{gal}}(z, \hat{\Omega}|\lambda_{\mathrm{c}})\, p_{\mathrm{rate}}(z|\lambda_{\mathrm{r}})}{\int \mathrm{d}z\, \mathrm{d}\hat{\Omega}\, p_{\mathrm{gal}}(z, \hat{\Omega}|\lambda_{\mathrm{c}})\, p_{\mathrm{rate}}(z|\lambda_{\mathrm{r}})}$$

- $\Rightarrow$ $p(m_1, m_2|\lambda_{\mathrm{m}})$ is the probability of having $m_1, m_2$ given a mass distributions;
- $\Rightarrow$ $p_{\mathrm{rate}}(z|\lambda_{\mathrm{r}}) \propto \psi(z; \lambda_{\mathrm{rate}})/(1+z)$ is the probability of having a merger at redshift $z$;
- $\Rightarrow$ $p_{\mathrm{gal}}(z, \hat{\Omega}|\lambda_{\mathrm{c}})$ is the probability that there is a galaxy (*host*) at $(z, \hat{\Omega})$ and is constructed from a galaxy catalog and takes into account the **completeness**

**Rate evolution**: Madau

$$\psi(z; \lambda_{\mathrm{rate}}) = \frac{(1+z)^{\gamma}}{1 + \left(\frac{1+z}{1+z_p}\right)^{\gamma+\kappa}},$$

# Backup - Statistical Framework

The dark (+ bright) sirens methods can be combined within a **Hierarchical Bayesian Framework** to infer joint constraints on population and cosmology:

$$\boxed{\mathcal{L}\left(\{\boldsymbol{d}_i\}_{i=1}^{N_{\rm obs}} \middle| \boldsymbol{\lambda}\right) \propto \frac{1}{\xi(\boldsymbol{\lambda})^{N_{\rm obs}}} \times \prod_{i=1}^{N_{\rm obs}} \int \mathrm{d}\boldsymbol{\theta} \, \frac{p_{\rm GW}(\boldsymbol{\theta}(\boldsymbol{\theta}^{\rm D}, \lambda_{\rm cosmo})|\boldsymbol{d}_i)}{\pi(\boldsymbol{\theta}^{\rm D}) \det \left|\frac{\mathrm{d}\boldsymbol{\theta}^{\rm D}}{\mathrm{d}\boldsymbol{\theta}}\right|} \, p_{\rm pop}(\boldsymbol{\theta}|\boldsymbol{\lambda})}$$

Two sets of parameters

1. **event-level parameters** in detector- or source-frame: $\boldsymbol{\theta}^{\rm D} = \{m_1^{\rm D},\, m_2^{\rm D},\, d_L,\, \hat{\Omega},\, \ldots\}$, $\boldsymbol{\theta} = \{m_1,\, m_2,\, z,\, \hat{\Omega},\, \ldots\}$;
2. **population-level hyper-parameters**: $\boldsymbol{\lambda} = \{\lambda_{\rm cosmo},\, \lambda_{\rm mass},\, \lambda_{\rm rate}\} \implies$ what we want to constrain from GW data $\boldsymbol{d} = \{\boldsymbol{d}_i\}_{i=1}^{N_{\rm obs}}$.

Key ingredients:

$\Rightarrow$ $p_{\rm GW}(\boldsymbol{\theta}(\boldsymbol{\theta}^{\rm D}, \lambda_{\rm cosmo})|\boldsymbol{d}_i)$ depends on the GW event and **measurement uncertainties**;

$\Rightarrow$ $\xi(\boldsymbol{\lambda})$ corrects the bias due to **selection effects**;

$\Rightarrow$ $p_{\rm pop}(\boldsymbol{\theta}|\boldsymbol{\lambda})$ is the **population prior**

$$p_{\rm pop}(\boldsymbol{\theta}|\boldsymbol{\lambda}) = p(m_1, m_2|\lambda_{\rm mass}) \, \frac{p_{\rm gal}(z, \hat{\Omega}|\lambda_{\rm cosmo}) \, p_{\rm rate}(z|\lambda_{\rm rate})}{\int \mathrm{d}z \, \mathrm{d}\hat{\Omega} \, p_{\rm gal}(z, \hat{\Omega}|\lambda_{\rm cosmo}) \, p_{\rm rate}(z|\lambda_{\rm rate})}.$$

# Backup - Population priors II

**Mass distribution:** power law + Gaussian peak

$$p(m_1, m_2|\lambda_{\rm mass}) = p(m_1|\lambda_{\rm mass})\, p(m_2|m_1, \lambda_{\rm mass})\,,$$

where probability of the primary BH mass is given by

$$p(m_1|\lambda_{\rm mass}) \propto \left[(1-\lambda_{\rm p})\,\mathcal{P}(m_1) + \lambda_{\rm p}\,\mathcal{G}(m_1)\right]\,\mathcal{S}(m_1)\,.$$

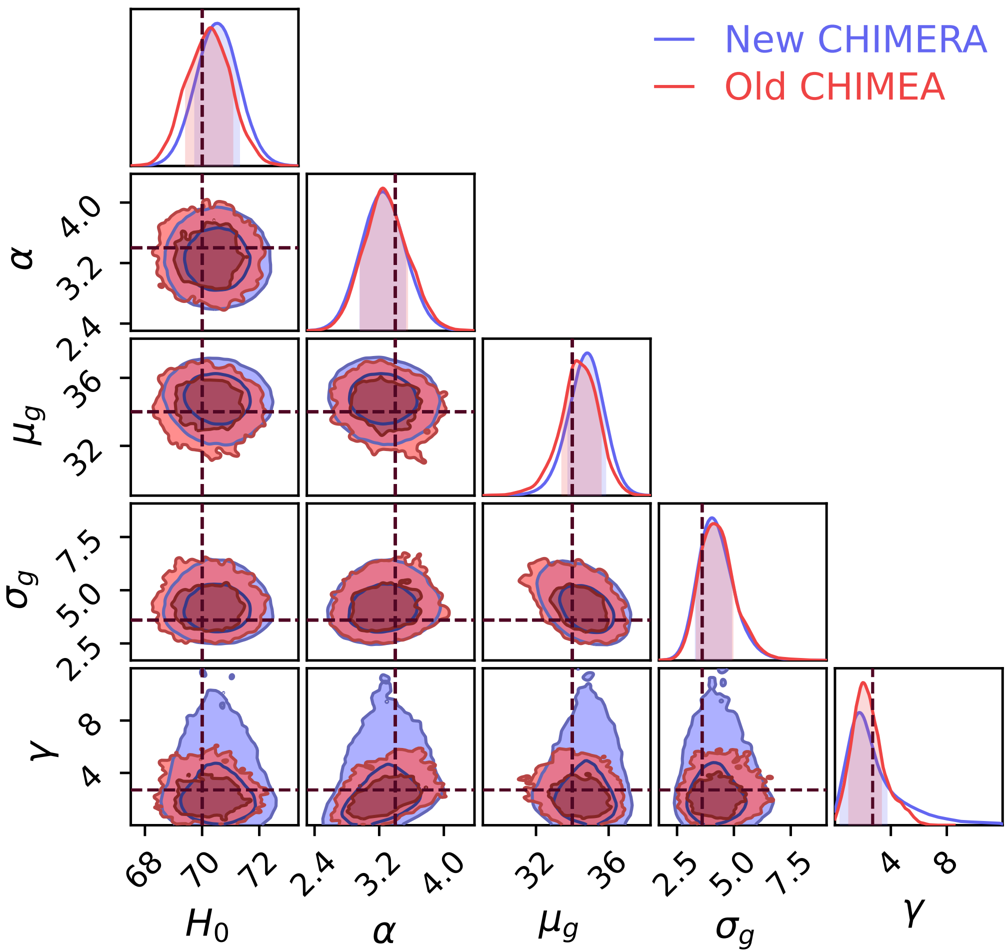
Here, $\mathcal{P}(m_1) \propto m_1^{-\alpha}$ is a power-law truncated in the domain $m_1 \in [m_{\rm low}, m_{\rm high}]$, $\mathcal{G}(m) \propto \mathcal{N}(\mu_{\rm g}; \sigma_{\rm g}^2)$ is the Gaussian component, and $\mathcal{S}(m_1) \in [0, 1]$ is a smoothing piece-wise function defined as Abbott et al. 2021:

$$S\left(m_1 \mid m_{\rm low}, \delta_m\right) = \begin{cases} 0 & \left(m_1 < m_{\rm low}\right) \\ \left[f\left(m_1 - m_{\rm low}, \delta_m\right) + 1\right]^{-1} & \left(m_{\rm low} \leqslant m_1 < m_{\rm low} + \delta_m\right)\,, \\ 1 & \left(m_1 \geqslant m_{\rm low} + \delta_m\right) \end{cases} \quad \text{with} \quad f\left(m', \delta_m\right) = \exp\left(\frac{\delta_m}{m'} + \frac{\delta_m}{m' - \delta_m}\right)$$

The secondary BH mass is modeled by a power-law with an index $\beta$ in the domain $m \in [m_{\rm low}, m_1]$.

$$p(m_2|m_1, \lambda_{\rm mass}) \propto \begin{cases} m_2^{-\beta} & (m_{\rm low} \leqslant m_2 \leqslant m_1) \\ 0 & \text{otherwise} \end{cases}$$

# Backup - Validation of the New CHIMERA



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