Thermodynamics sheds light on the nature of dark matter galactic halos

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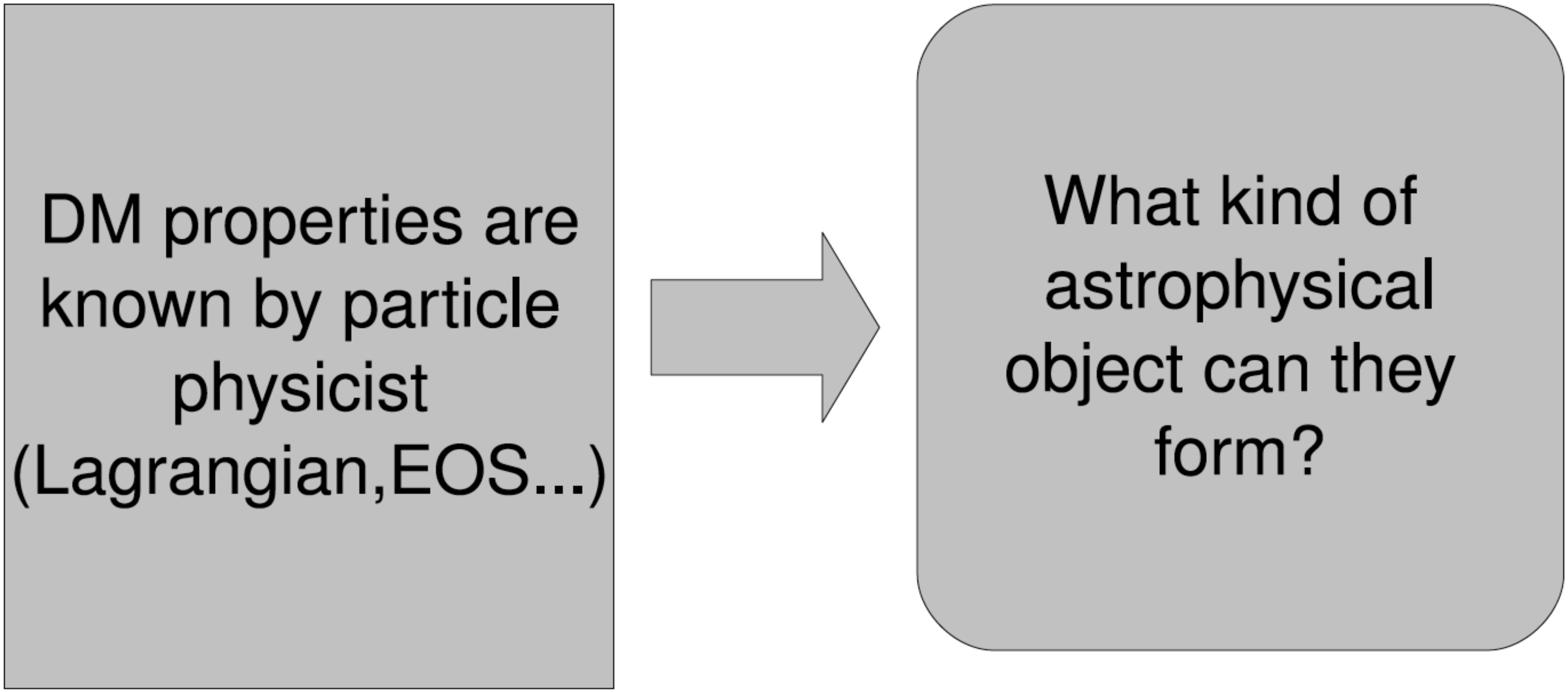
Pescara, Italia. July 7-12,2024.

What is dark matter made of?

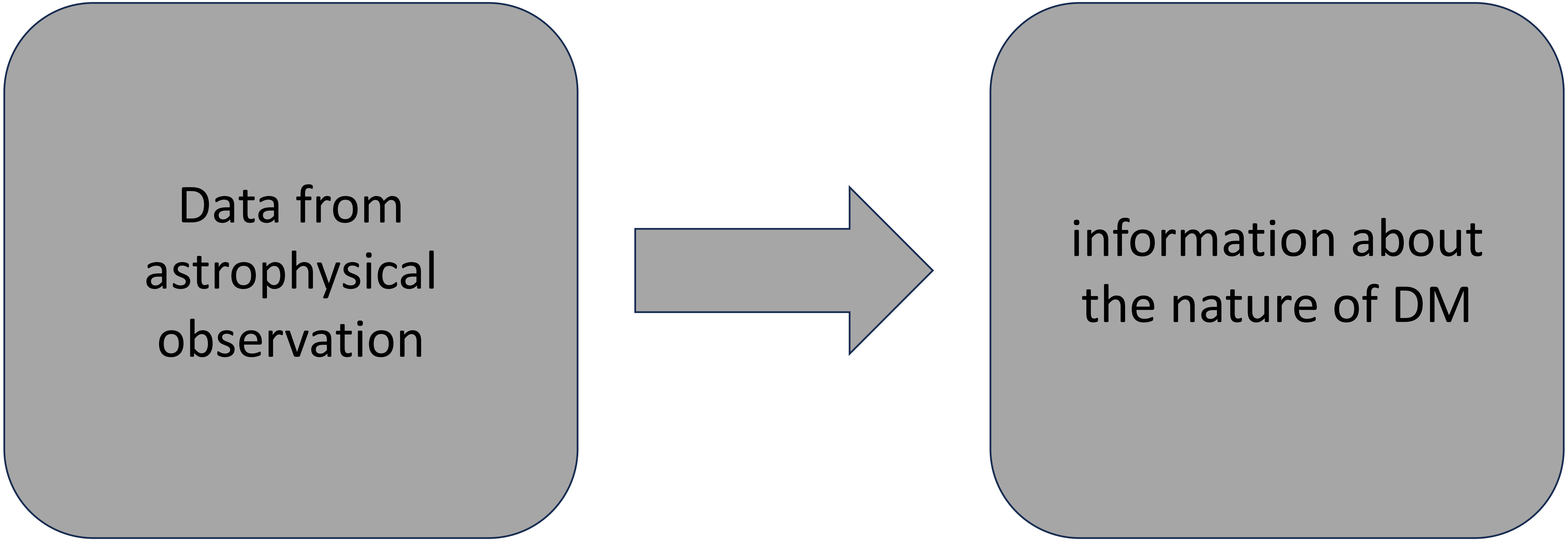
DM candidate	**I.** Ωh^2	**II.** Cold	**III.** Neutral	**IV.** BBN	**V.** Stars	**VI.** Self	**VII.** Direct	**VIII.** γ-rays	**IX.** Astro	**X.** Probed	**Result**
SM Neutrinos	$\times$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	–	–	$\checkmark$	$\times$
Sterile Neutrinos	$\sim$	$\sim$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$!	$\checkmark$	$\sim$
Neutralino	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$!	$\checkmark$!	$\checkmark$!	$\checkmark$	$\checkmark$
Gravitino	$\checkmark$	$\checkmark$	$\checkmark$	$\sim$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\sim$
Gravitino (broken R-parity)	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
Sneutrino $\tilde{\nu}_L$	$\sim$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\checkmark$!	$\checkmark$!	$\checkmark$	$\times$
Sneutrino $\tilde{\nu}_R$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$!	$\checkmark$!	$\checkmark$!	$\checkmark$	$\checkmark$
Axino	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
SUSY Q-balls	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\sim$	–	$\checkmark$!	$\checkmark$	$\checkmark$	$\checkmark$	$\sim$
B^1 **UED**	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$!	$\checkmark$!	$\checkmark$!	$\checkmark$	$\checkmark$
First level graviton UED	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\times$	$\checkmark$	$\times^a$
Axion	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$!	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
Heavy photon (Little Higgs)	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$!	$\checkmark$!	$\checkmark$	$\checkmark$
Inert Higgs model	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$!	–	$\checkmark$	$\checkmark$
Champs	$\checkmark$	$\checkmark$	$\times$	$\checkmark$	$\times$	–	–	–	–	$\checkmark$	$\times$
Wimpzillas	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\sim$	$\sim$

[M. Taoso, G. Bertone and Masiero, JCAP 2008]

The fundamental physicist way...



A phenomelogical way...

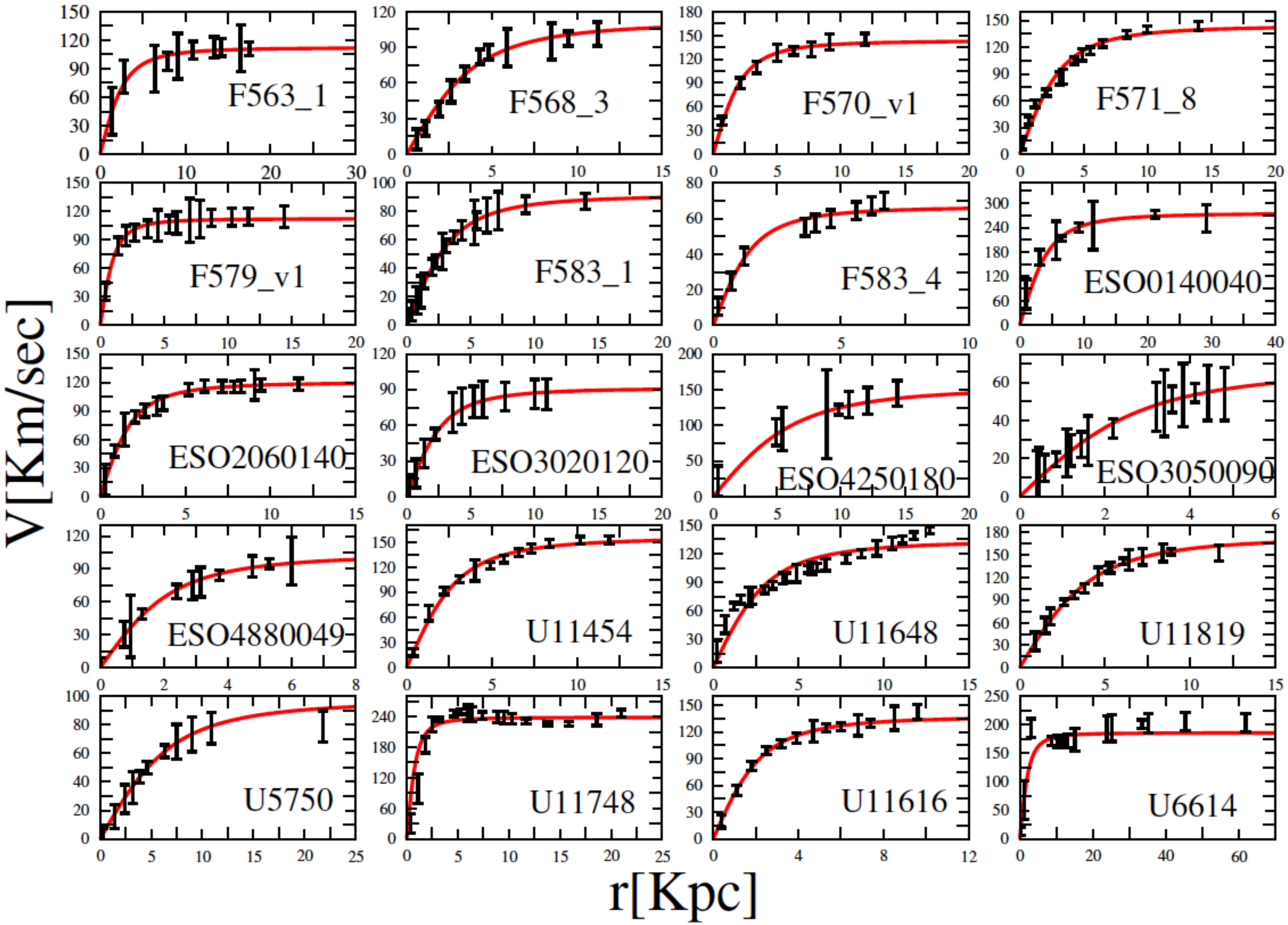


What information can be extracted at galactic scales?

Look at the rotational curve of galaxies

The data:

LSB Galaxies



Start with a spherically symmetric space-time

$$ds^2 = -e^{2\Phi/c^2}\, c^2\, dt^2 + \frac{dr^2}{1 - \frac{2\,G\,m}{c^2\,r}} + r^2\, d\Omega^2,$$

One of the geometric potentials is determined by observations:

$$\frac{\Phi'}{c^2} = \frac{\beta^2}{r},$$

Use normalized $\rho = \rho_0\, \bar{\rho}$, $p = p_0\, \bar{p}$, $m = M_0\, n$, $r = R_0\, x$. Einstein's equations and the conservation equation read:

$$\begin{aligned} n' &= 3\,x^2\,\bar{\rho} \\ \left(1 - q\,\frac{2\,n}{x}\right)\frac{\beta^2}{x} - q\,\frac{n}{x^2} &= 3\,q\,x\,\bar{p}, \\ \bar{p}' + (\bar{p} + \bar{\rho})\,\frac{\beta^2}{x} &= 0, \end{aligned}$$

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$$\bar p' + S(x)\,\bar p = T(x),$$

with

$$\begin{aligned} S(x) &= -\frac{2\,\beta^2\left(1 + \beta^2 - 2\,\beta^4 - x\,\beta^{2\prime}\right)}{(1+\beta^2)\,(1+2\,\beta^2)\,x}, \\ T(x) &= -\frac{\beta^2\left(\beta^2 + 2\,\beta^4 + x\,\beta^{2\prime}\right)}{3\,(1+\beta^2)\,(1+2\,\beta^2)\,x^3\,q}. \end{aligned}$$

For the Newtonian case:

$$\begin{aligned} n' &= 3\,x^2\,\bar{\rho} \\ -q\,\frac{n}{x^2} &= 3\,q\,x\,\bar{p}, \\ \bar{p}' + \bar{\rho}\,\frac{\beta^2}{x} &= 0, \end{aligned}$$

And we only need β^2:

$$\beta^2(x) = \beta_0^2\,\frac{x^2}{x^2 + rc^2}$$

Persic-Salucci-Stel (1996) rotational curve velocity profile

From Persic-Salucci-Stel (1996) rotational curve velocity profile an EOS for DM is obtained

$$n = \frac{{\beta_0}^2}{q} \frac{x^3}{(x^2 + a^2)}$$

$$\bar{\rho} = \frac{{\beta_0}^2}{3\,q} \frac{\left(x^2 + 3\,a^2\right)}{(x^2 + a^2)^2},$$

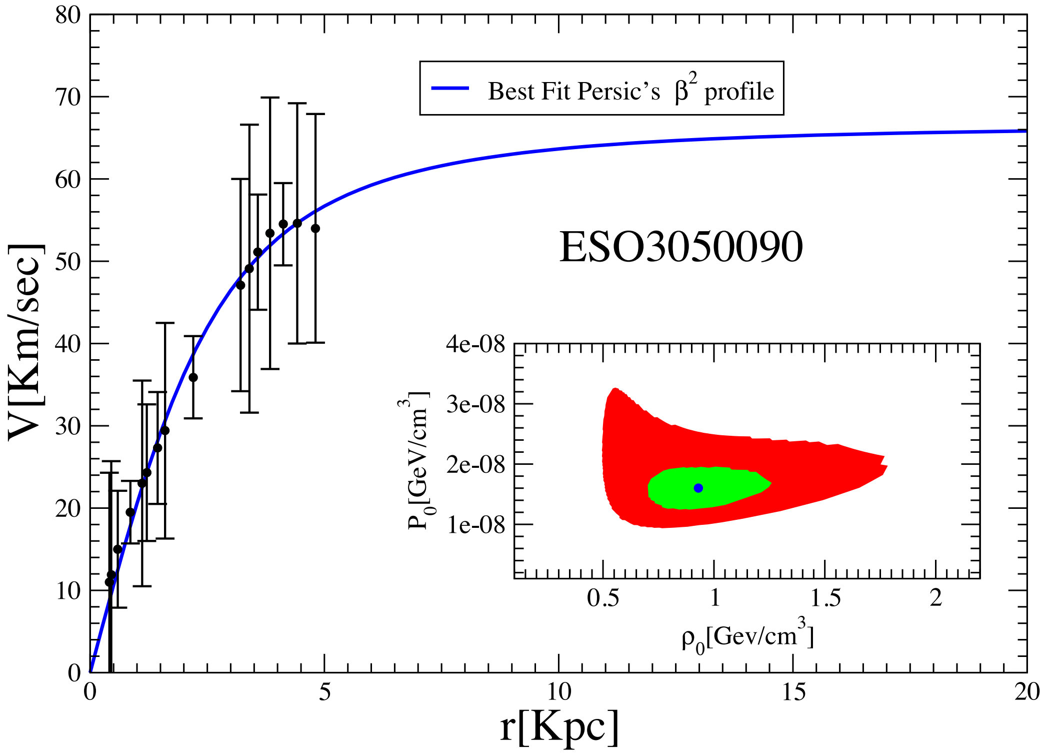
$$\bar{p} = \frac{{\beta_0}^4}{6\,q} \frac{\left(x^2 + 2\,a^2\right)}{(x^2 + a^2)^2},$$

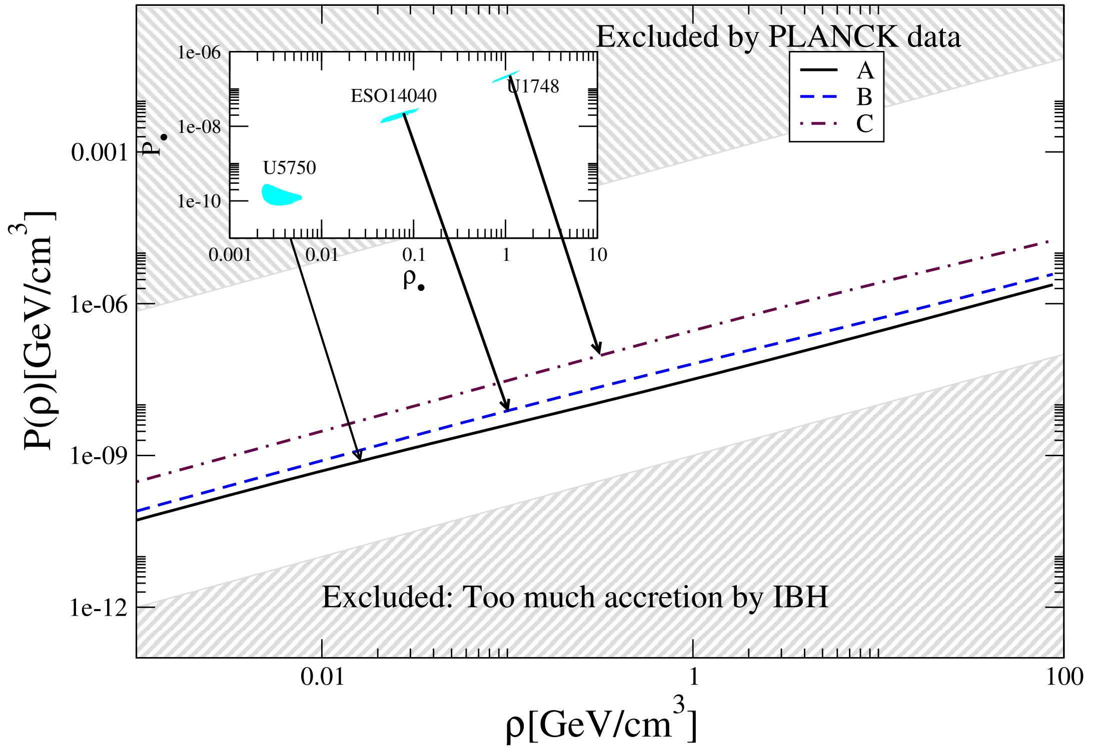
$$a^2 = \frac{3\,\bar{p}_0}{q\,\bar{\rho}_0^2}, \quad \beta_0^2 = \frac{3\,\bar{p}_0}{\bar{\rho}_0}.$$

$$\bar{p}(\bar{\rho}) = \bar{p}_0 \left(\frac{3}{4} \left(\frac{\bar{\rho}}{\bar{\rho}_0} \right) - \frac{1}{16} \left(1 - \sqrt{1 + 24 \left(\frac{\bar{\rho}}{\bar{\rho}_0} \right)} \right) \right)$$

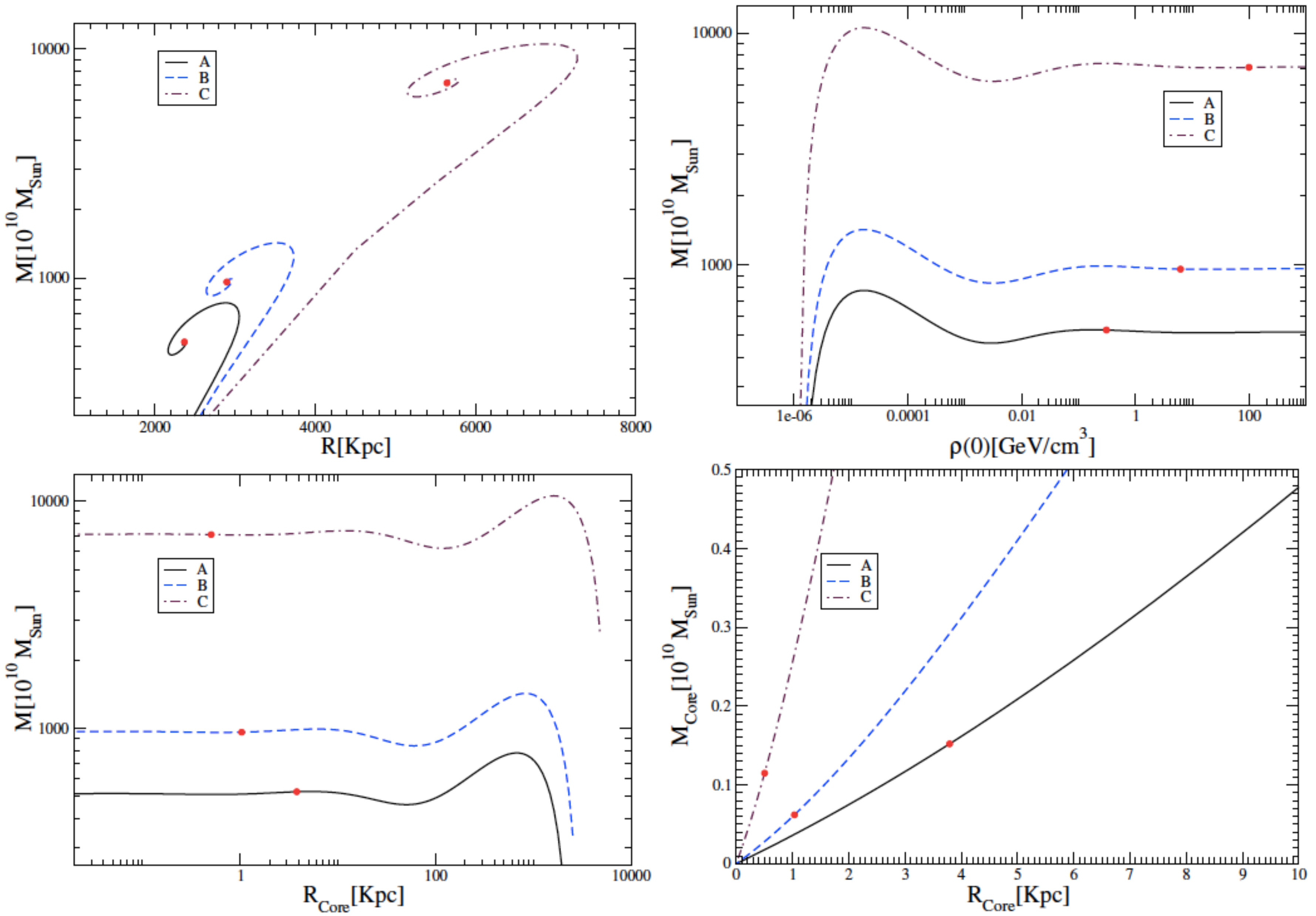
It is a good moment to test the pressureless hypothesis

Label	Galaxy	$\beta_0(10^4)$	a	$\bar{\rho}_\bullet$	$\bar{p}_\bullet$
A	U5750	3.23	7.75	3.63×10^{-3}	1.26×10^{-10}
B	ESO2060140	4.00	2.16	7.17×10^{-2}	3.82×10^{-9}
C	U11748	7.94	1.07	1.15	2.42×10^{-7}

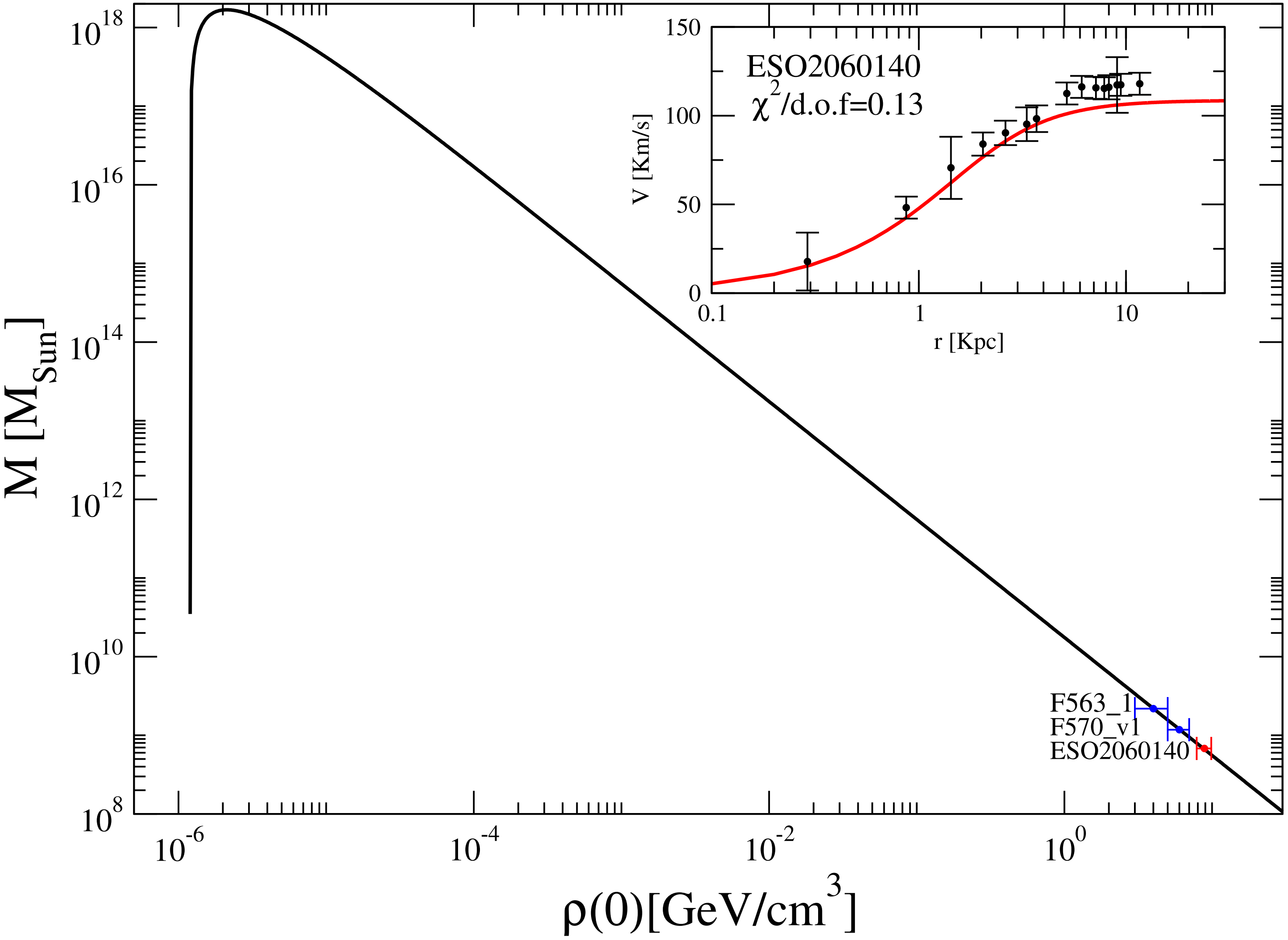




Full properties for the halos:



There is a problem:



Prefered values are in the unstable branch of the possible configurations

A further analysis for galactic dark matter halos with pressure
A Aceña, J Barranco, A Bernal, E López, M Llerena
arXiv preprint arXiv:2112.05865
Accepted for publication in ApJ

Thermodynamics sheds light on the nature of dark matter galactic halos

A. Aceña,[1] J. Barranco,[2] A. Bernal,[2] and E. López[3]

Let us star with the Burkert density profile

$$\rho(r) = \frac{\rho_0 r_0^3}{(r + r_0)(r^2 + r_0^2)}$$

Mass profile can be directly computed and hydrostatic equilibrium demands:

$$\frac{dp}{dr} = -\frac{GM\rho}{r^2}$$

This gives us the pressure

Thermal equilibrium demands:

$$\frac{dT}{dr} = -\frac{GM\rho T}{r^2 p}\nabla$$

The quantity ∇ depends on the thermal processes that the gas undergoes. Since we do not have any *a priori* information on energy production and transport within the DM halo, we choose to postulate the equation of state,

Here, an important difference: given an equation of state

$$T(\rho, p)$$

we can get the temperature profile

$$T(r).$$

1. *Ideal gas:* The equation of state is

$$T = \frac{m}{k}\frac{p}{\rho},$$

2. *Fermi gas:* The equation of state takes the parametric form

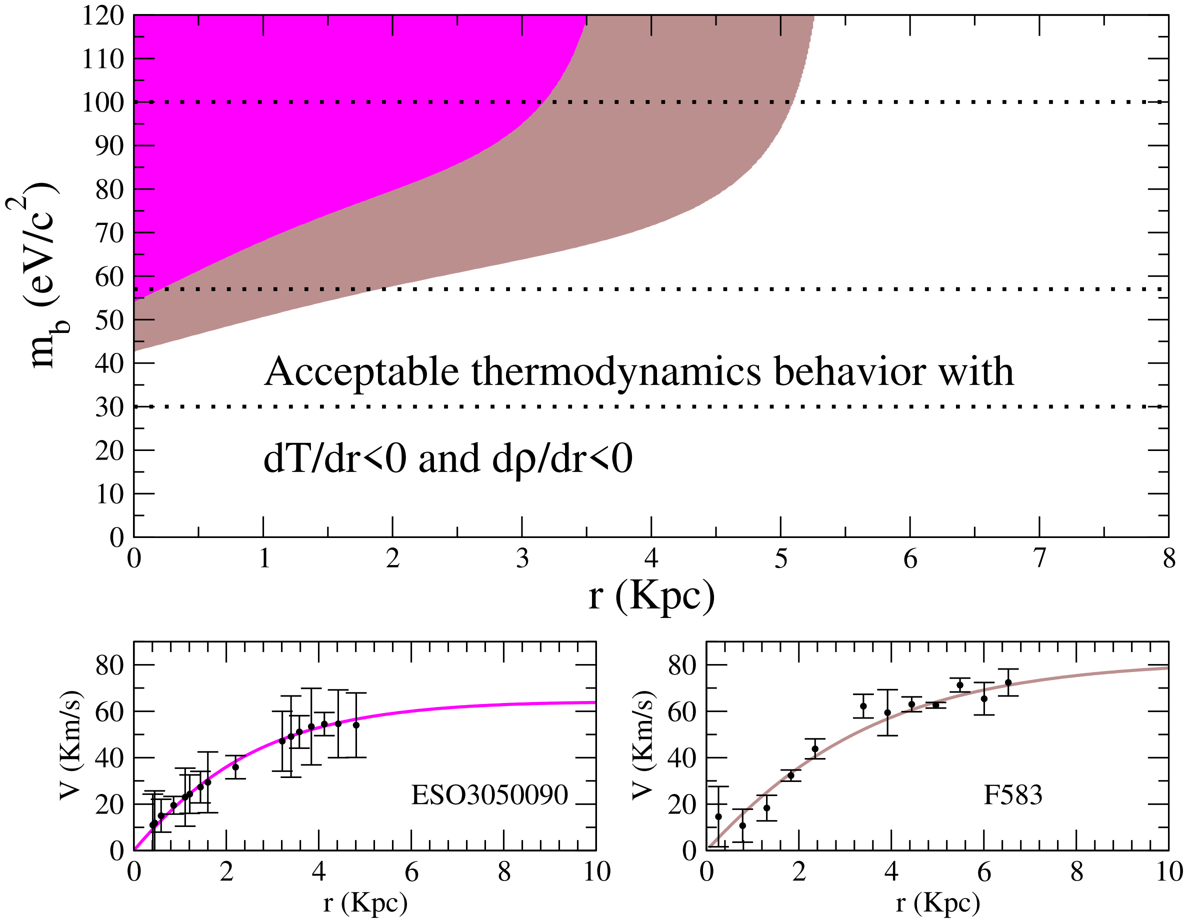
$$T = \frac{m}{k}\frac{p}{\rho}\frac{f_{3/2}(z)}{f_{5/2}(z)}, \qquad \frac{f_{3/2}^{5}(z)}{f_{5/2}^{3}(z)} = \frac{h^6}{8\pi^3 g^2 m^8}\frac{\rho^5}{p^3},$$

3. *Bose gas:* We need to distinguish whether Bose-Einstein condensation is occurring or not. If there is no condensation,

$$T = \frac{m}{k}\frac{p}{\rho}\frac{g_{3/2}(z)}{g_{5/2}(z)}, \qquad \frac{g_{3/2}^5(z)}{g_{5/2}^3(z)} = \frac{h^6}{8\pi^3 m^8}\frac{\rho^5}{p^3},$$

Condensation occurs if $T < T_C$

$$T_C = \frac{h^2 \rho^{\frac{2}{3}}}{2\pi k m^{\frac{5}{3}} \zeta^{\frac{2}{3}}\left(\frac{3}{2}\right)},$$



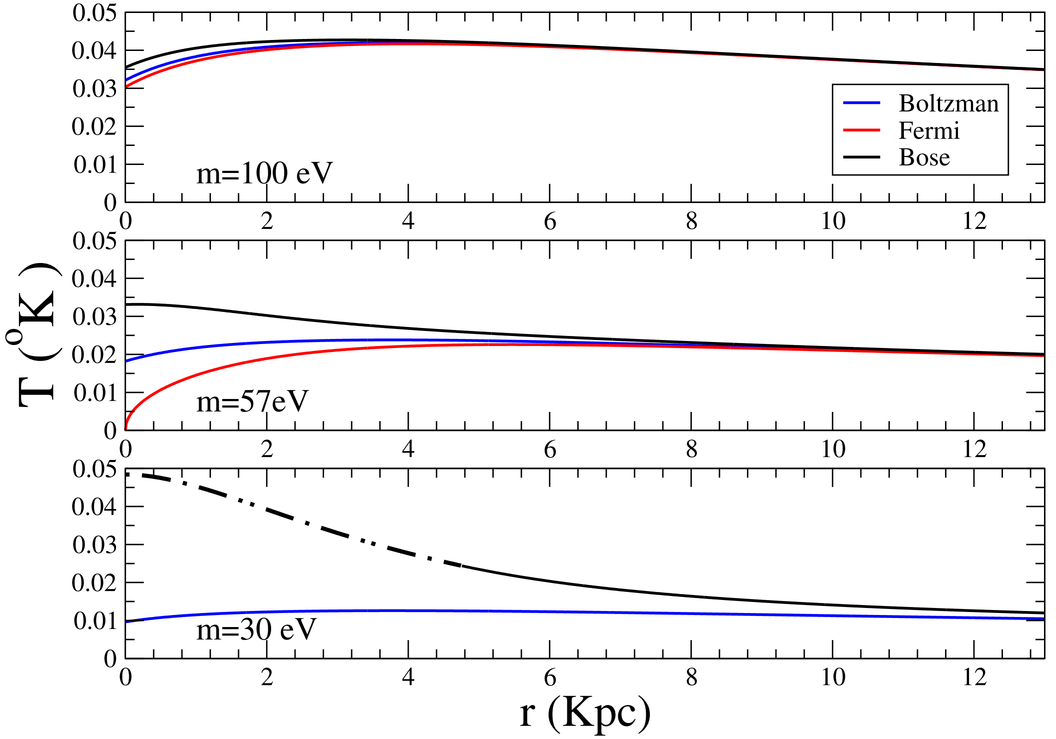


FIG. 1. Temperature profiles for the parameters $\rho_0 = 2.34\,(GeV/c^2)/cm^3$, $r_0 = 3.37\,kpc$, corresponding to the galaxy ESO3050090.

$$m_{max} = 43\,eV/c^2.$$

Other phenomenological relations:

$$\rho_0 \; r_0 = 141^{+82}_{-52} \, M_\odot/pc^2 .$$

Mon. Not. R. Astron. Soc. **397,** 1169–1176 (2009) doi:10.1111/j.1365-2966.2009.15004.x

A constant dark matter halo surface density in galaxies

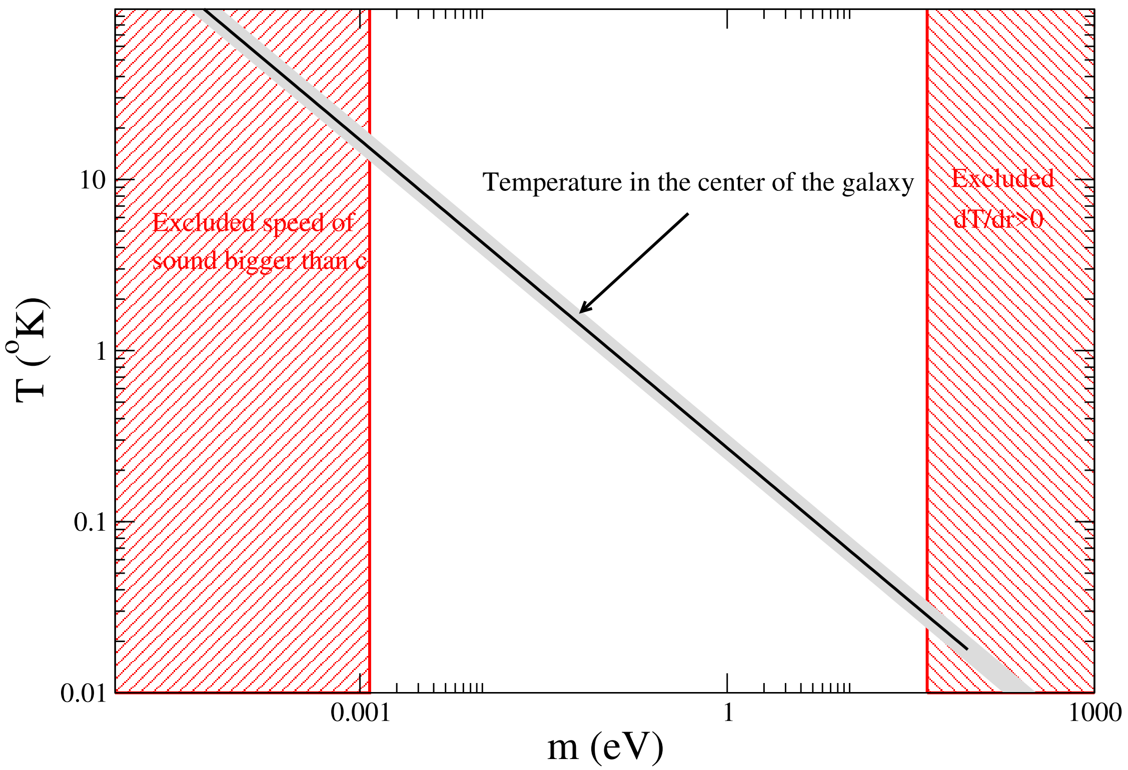
F. Donato,[1★] G. Gentile,[2,3] P. Salucci,[4] C. Frigerio Martins,[5] M. I. Wilkinson,[6]
G. Gilmore,[7] E. K. Grebel,[8] A. Koch[9] and R. Wyse[10]

But in our case:

$$a^2 = \frac{3\,\bar{p}_0}{q\,\bar{\rho}_0^2}\,, \quad \beta_0^2 = \frac{3\,\bar{p}_0}{\bar{\rho}_0}\,.$$

$$p_0 = 0.83 \times G(\rho_0 r_0)^2 = 4.8 \times 10^{-11} \, \frac{g}{cm\, s^2}$$

$$T_0 = \frac{1}{k}\left(\frac{h^6 p_0^2}{8\pi^3 m^3 \zeta^2\left(\frac{5}{2}\right)}\right)^{\frac{1}{5}} = \frac{0.27\,K}{(m[eV/c^2])^{\frac{3}{5}}}.$$



$$v_s^2 = \frac{5\zeta\left(\frac{5}{2}\right)kT}{3\zeta\left(\frac{3}{2}\right)m} = \left(\frac{5^5 h^6 \zeta^3\left(\frac{5}{2}\right)p^2}{2^3 3^5 \pi^3 \zeta^5\left(\frac{3}{2}\right)m^8}\right)^{\frac{1}{5}}$$

$$m_{min} = \left(\frac{5^5 \zeta^3\left(\frac{5}{2}\right)h^6 p_0^2}{2^3 3^5 \pi^3 \zeta^5\left(\frac{3}{2}\right)c^{10}}\right)^{\frac{1}{8}} = 1.2\times 10^{-3}\,eV/c^2.$$

Conclusions

- The rotational velocity curve can give the strongest constraints on the dark matter pressure
- If temperature is neglected, the known dark matter profiles leads to dark matter equation fo state that might not be dynamically stable
- Introducing the temperature, and working the other way around of postulating that DM is either a ideal gas, a Fermi gas or a Bose gas the temperature profile can be obtained
- Only if DM is a Bose gas, with a particle mass below 43 eV the temperatire profile is physically reasonable.
- To avoid superluminial sound of speed within the Bose condensate, the boson’s mass should be biger than 1.2e-3 eV