On the Maximum Mass and Stability of Differentially Rotating Neutron Stars

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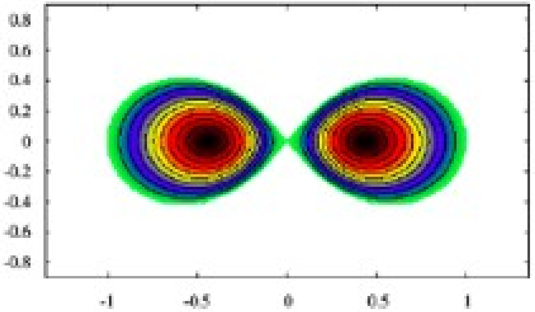

in collaboration with:

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INNOVATIVE ECONOMY
NATIONAL COHESION STRATEGY

FNP
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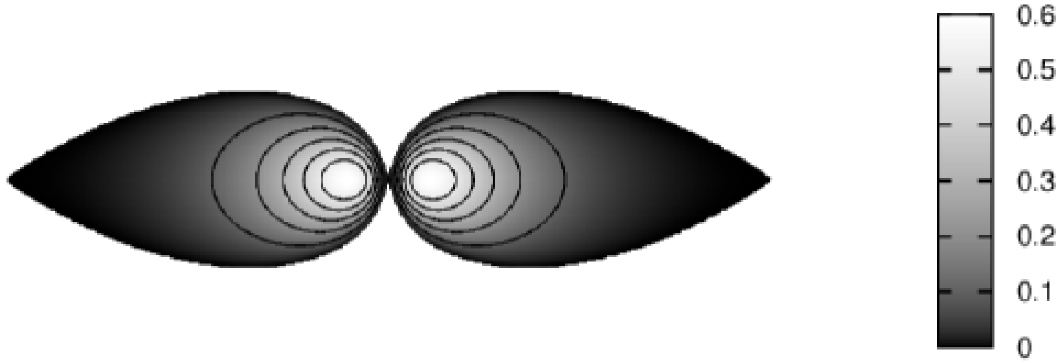
FlatStar- a highly accurate and stable numerical code for differentially rotating neutron stars

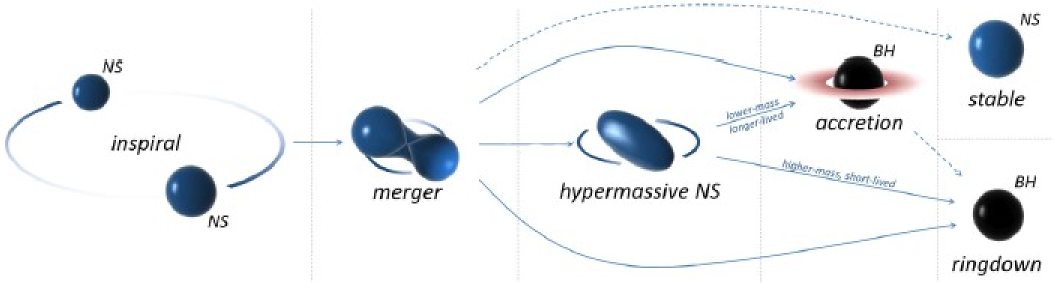


An example of an extremal configuration that other codes are far from being able to obtain (Ansorg, Gondek-Rosinska, Villain, 2009, Gondek-Rosinska et al. 2017). Results obtained by using the Newton-Raphson spectral code *FlatStar* for differentially rotating neutron stars.

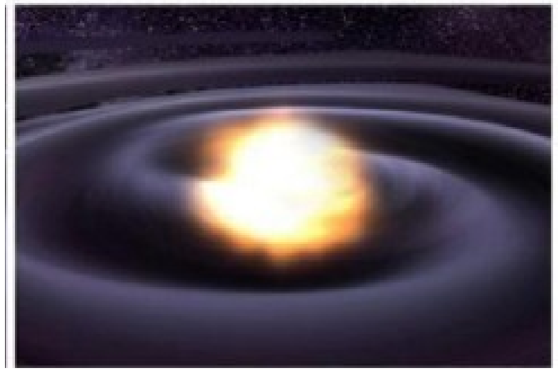
Motivation

Differential rotation plays important role in:

- core collapse $\rightarrow$ a protoneutron star
- a remnant of NS-NS merger



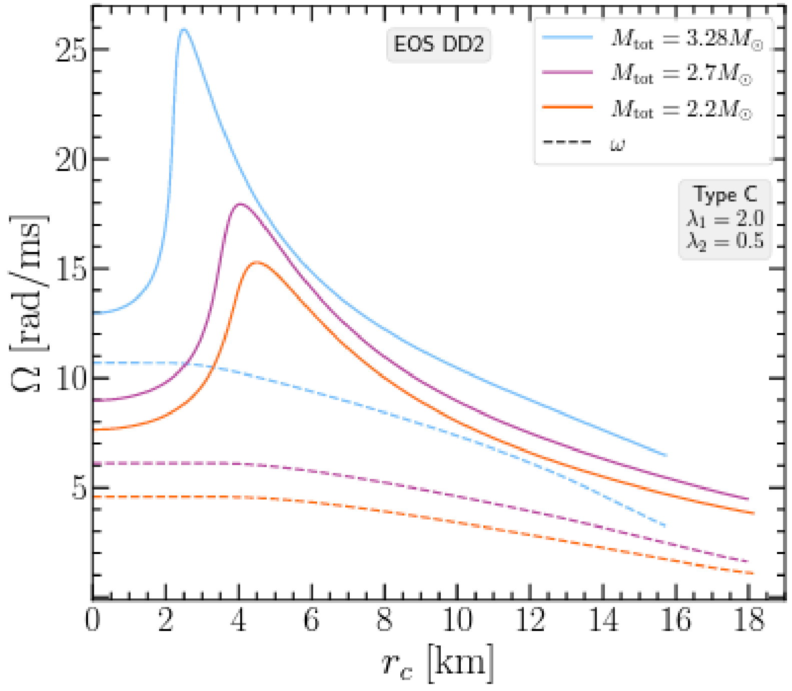
(Bartos, Brady, Marka 2013)

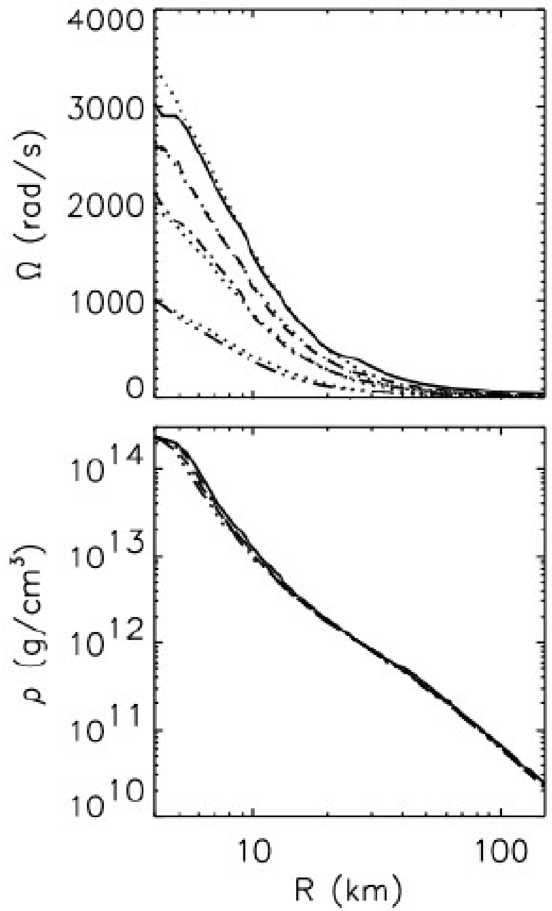


- The maximum mass of rotating neutron stars
- Are the most massive HMNS dynamically stable?
- Where is the stability limit?
- What can we learn from the dynamics of HMNS?

Angular velocity profiles -differentially rotating neutron stars

BNS remnant (e.g. Iosif & Stergioulas 2022) vs Core collapse





Equatorial profiles of angular velocity (top), and density (bottom) of PNSs from axisymmetric simulations of stellar core collapse (Dimmelmeier *et al.* 2002). Models with different amounts of angular momentum of the iron core, namely, $|T/W| = 0.9\%$ (solid), 0.5% (dashes), 0.25% (dash-dot), and 0.05% (dash-3 dots). The dotted lines on the upper panel are fits to the simple law with $R_0^2 = 50$ km^2.
Fit done by J.A. Pons, in Villain *et al.* 2004.

Equilibrium models of differentially rotating NS and SQS

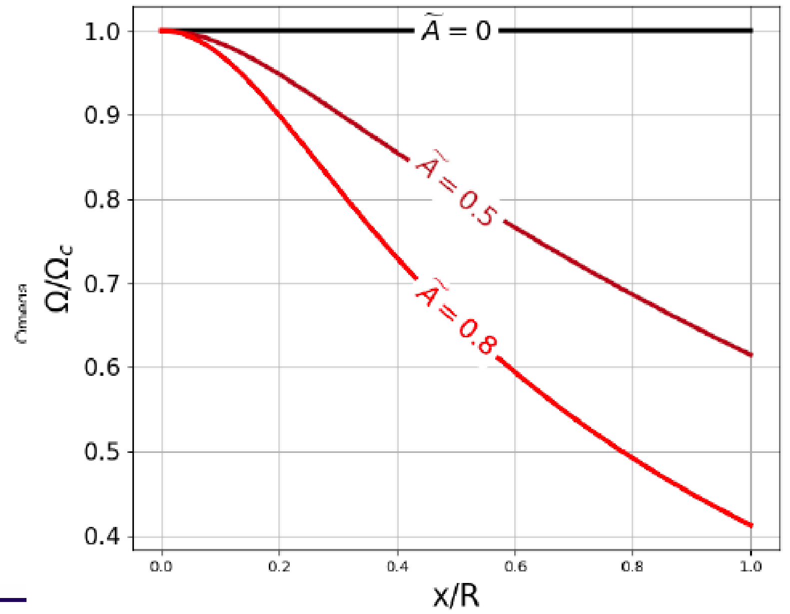
Astrophysically motivated (consistent with the core collapse results) a simple differential rotation law proposed by Komatsu, Eriguchi, Hachisu [1989]:

$$F(\Omega) = u^t u_\phi = \mathbf{R_0^2}(\Omega_c - \Omega)$$

R_0 lenght describing the degree of differential rotation ($r = R_0$: $\Omega = \Omega_c/2$, Ω_c is the 'angular velocity on the axis'. We use $A^{-1} = \tilde{A} = R_e/R_0$ (R_e - equatorial radius, for **uniform rotation** $R_0 -> \infty$**,** $\tilde{A} -> 0$**)**.

NS - a polytropic EOS: $P = K\rho^\Gamma$
with $\Gamma = 1.5, 1.8, 2, 2.5, 3$
SQS - MIT Bag model

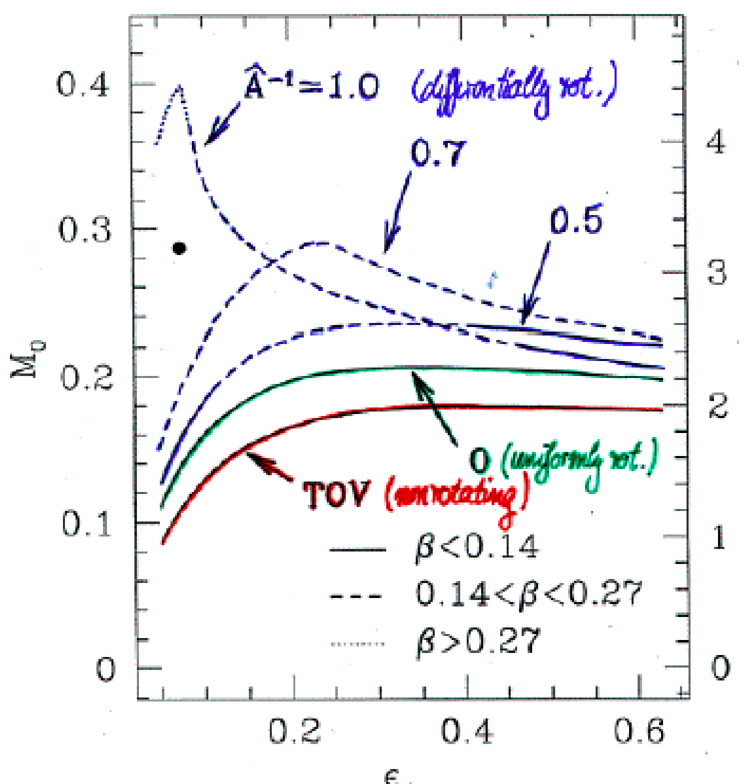
Relativistic multidomain spectral code Flatstar–> **axisymmetric and stationary** solutions for broad ranges of stellar densities and degree of differential rotation $0.01 < \tilde{A} < 1.5$



The effect of the rotation on the maximum mass of NS

The effect of **rigid** rotation on the M_{max}:
NS up to 20% (e.g. Cook et al. 1994)
SS $\sim$ 44% (Gourgoulhon et al. 1999, Gondek-Rosinska et al. 2000)

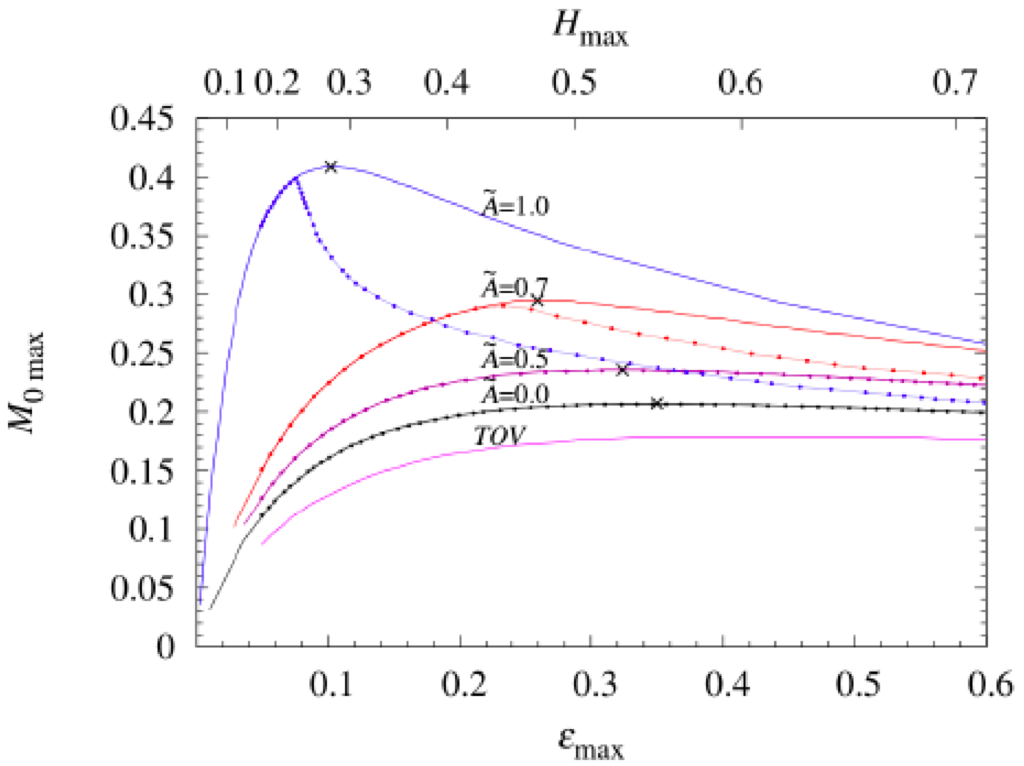
The effect of **differential** rotation on the M_{max} (Baumgarte et al. 2000, Morrison et al. 2004) depends on the degree of differential rotation $\tilde{A} = A^{-1}$



Differential rotation significantly increases the maximum allowed mass of NS and may temporarily stabilize the remnant of BNS merger- delayed collapse. The outcomes - no, delayed, or prompt collapse depend on the mass and A^{-1}. GW observations of coalescing BNS or a core collapse may be able to distinguish these outcomes.

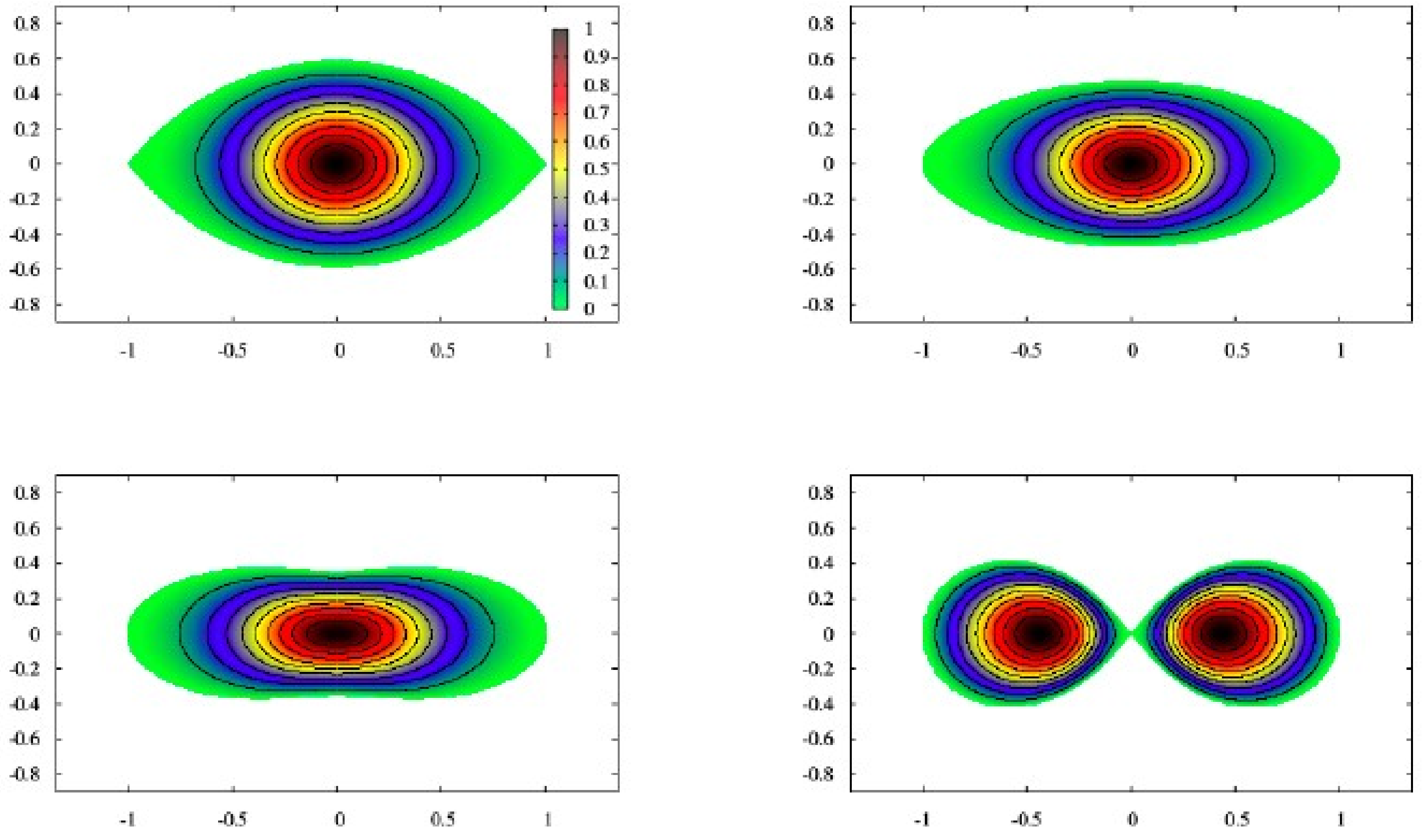
The maximum mass of differentially rotating NS - comparison

Highly accurate and stable code (Ansorg, Gondek-Rosińska, Villain, 2009, Gondek-Rosinska et al. 2017) allows to construct relativistic models of differentially rotating NS for broad ranges of degree of differential rotation, maximal densities and r_{ratio} from 1 to zero.

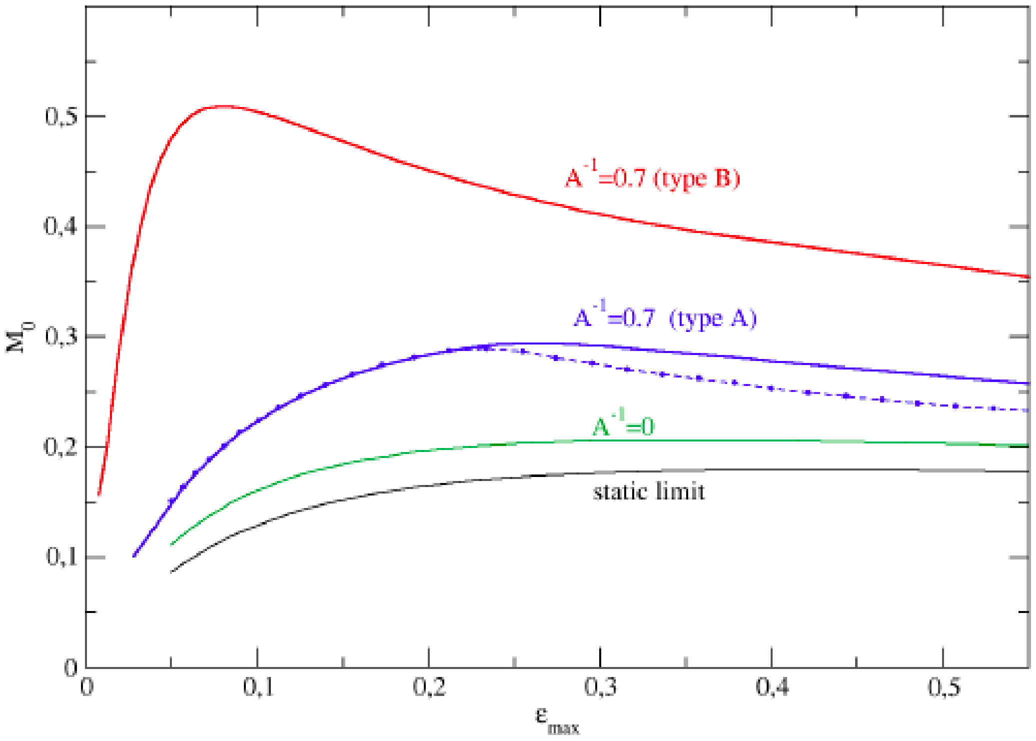


Solid lines - our results (Gondek-Rosinska et al,apJ, 2017) dotted lines- Baumgarte, Shapiro, Shibata [2000] Good agreement for modest degree of differential rotation and/or small densities.

Neutron Stars with the maximum allowed mass for $\tilde{A} = 0,\ 0.5,\ 0.7$ and 1

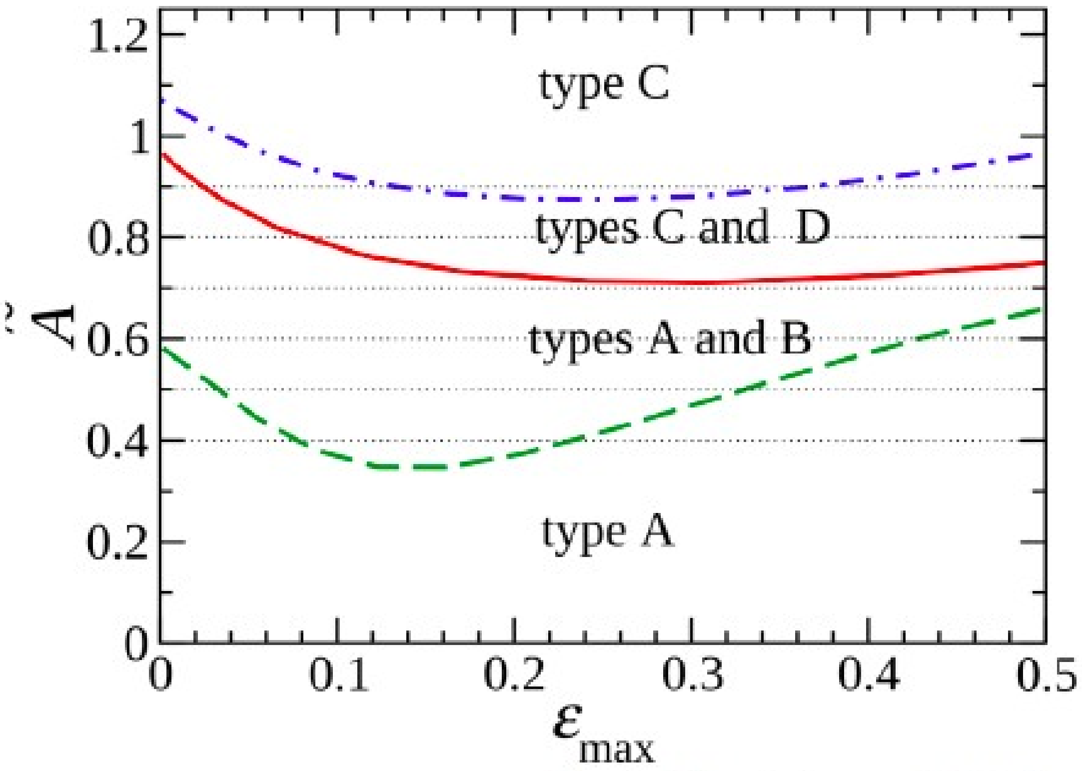


The maximum mass for $\tilde{A} = 0.7$, types A and B

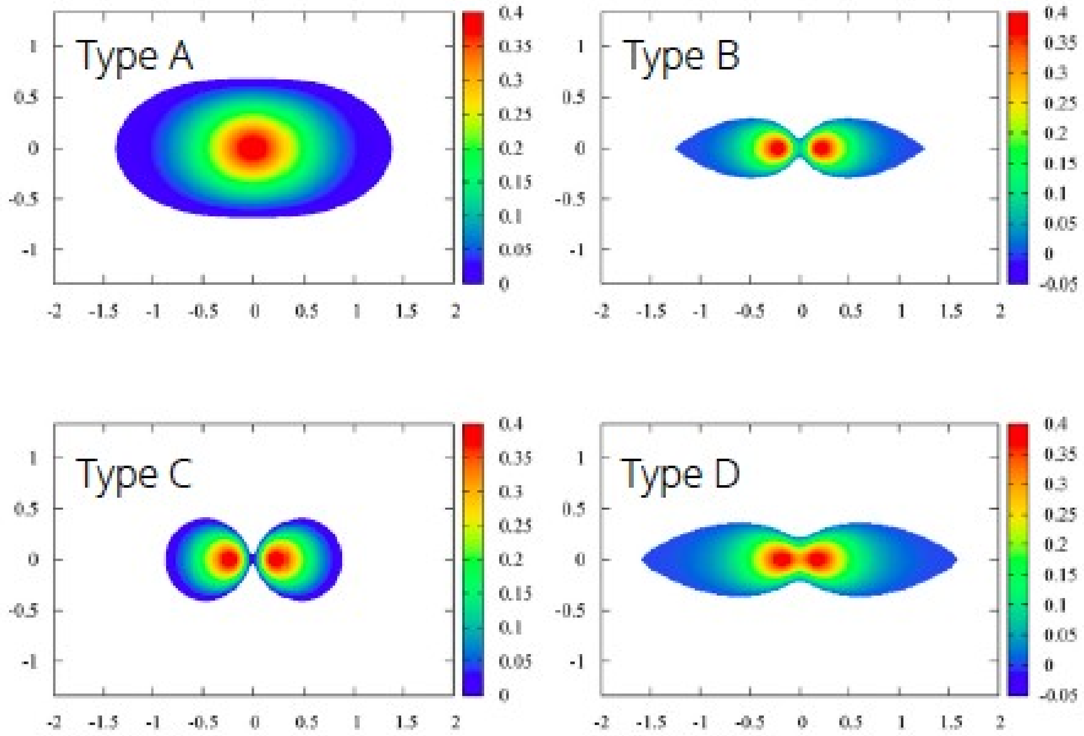


Maximum mass of new type B is much higher than maximum mass for type A with the same degree of differential rotation (eg. for polytrope with $\Gamma = 2.0$).

Different types of solutions

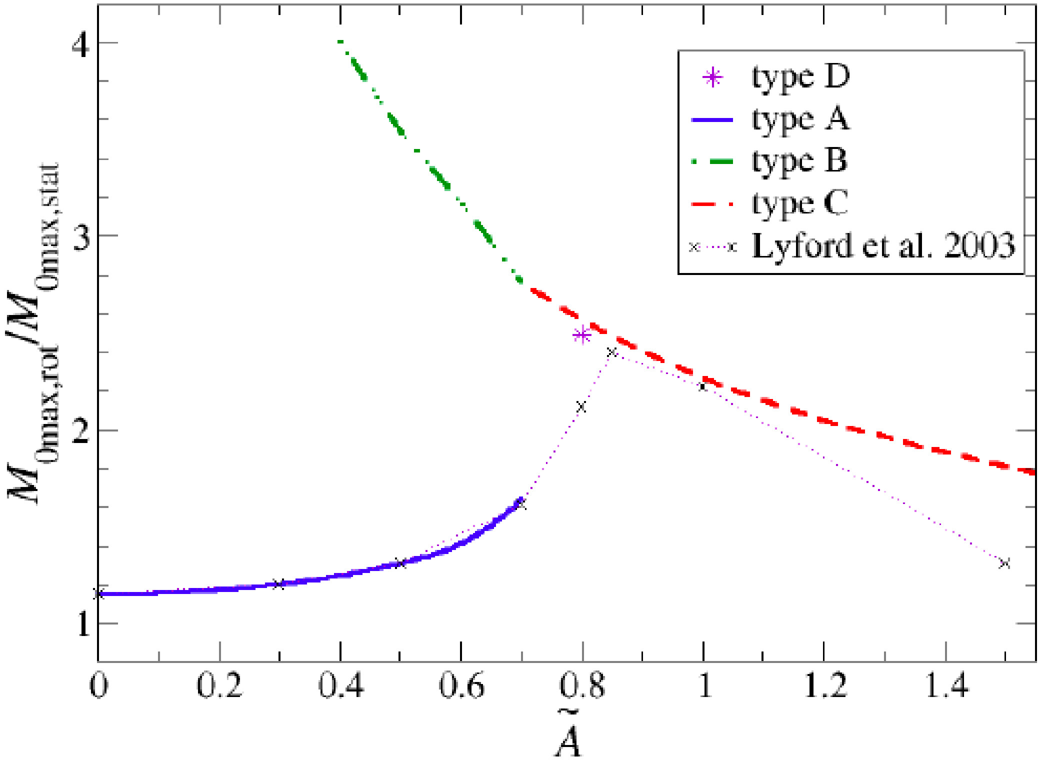


Gondek-Rosinska et al. 2017



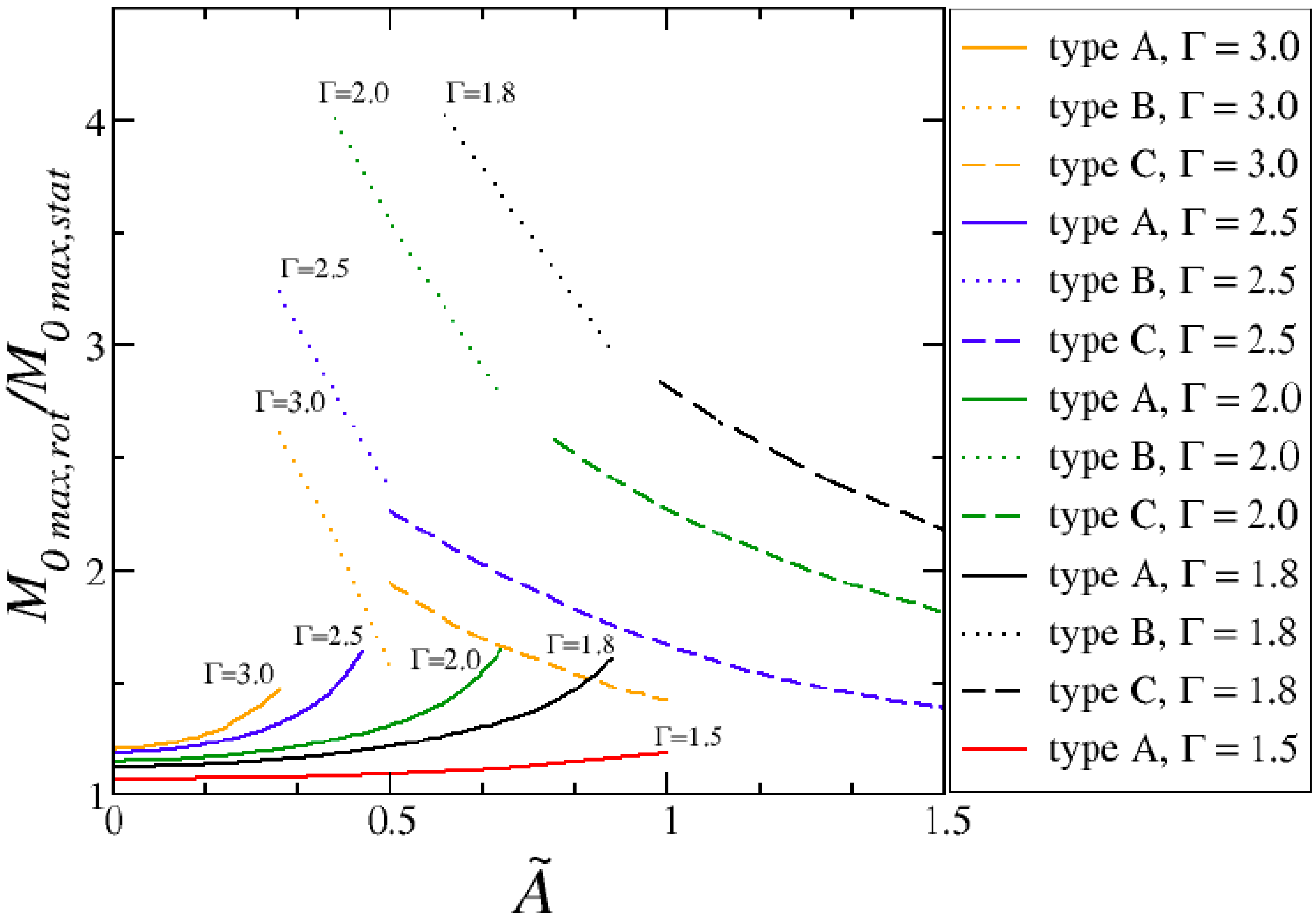
Studzinska et al. (2016)

The maximum allowed mass of differentially rotating NS

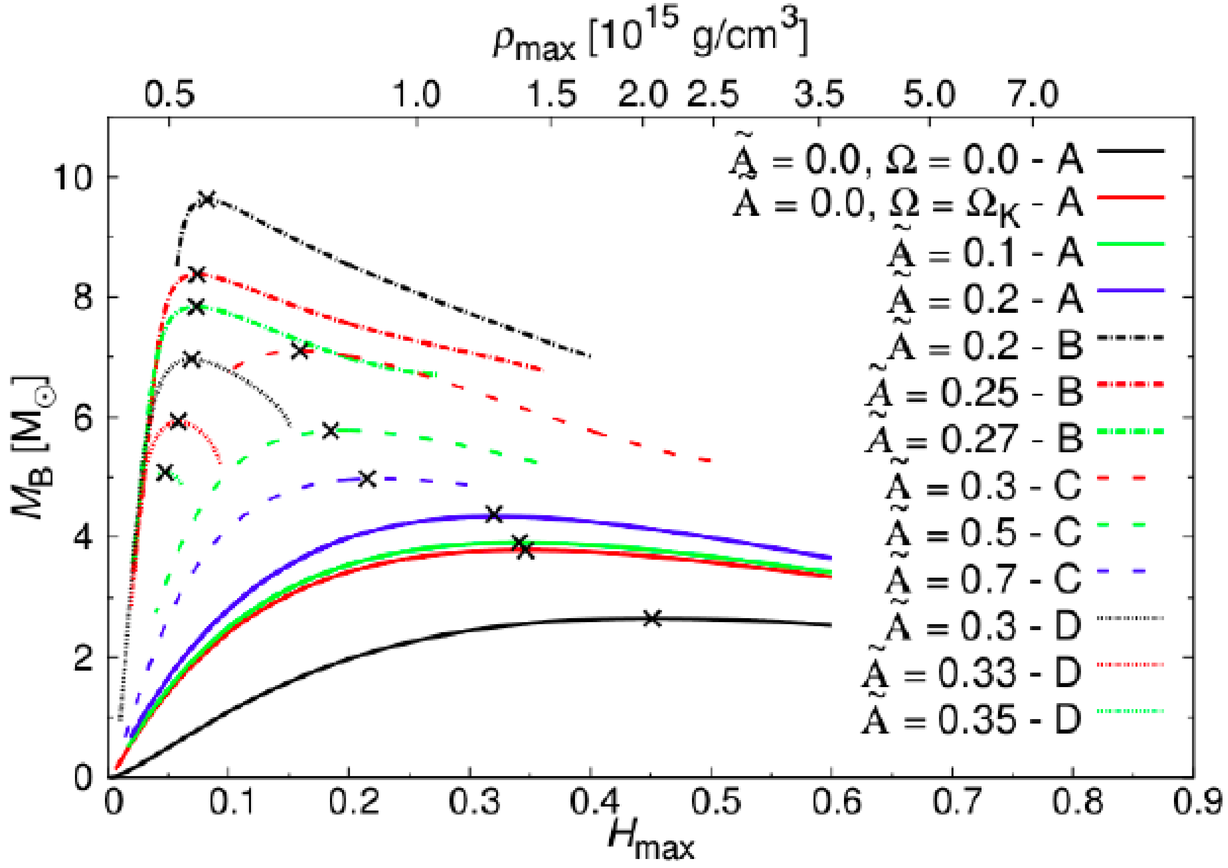


The maximum mass of differentially rotating neutron stars for given equation of state depends on the degree of differential rotation and on a type of solution (classified as A,B,C,D).

The effect of EOS on the maximum allowed mass

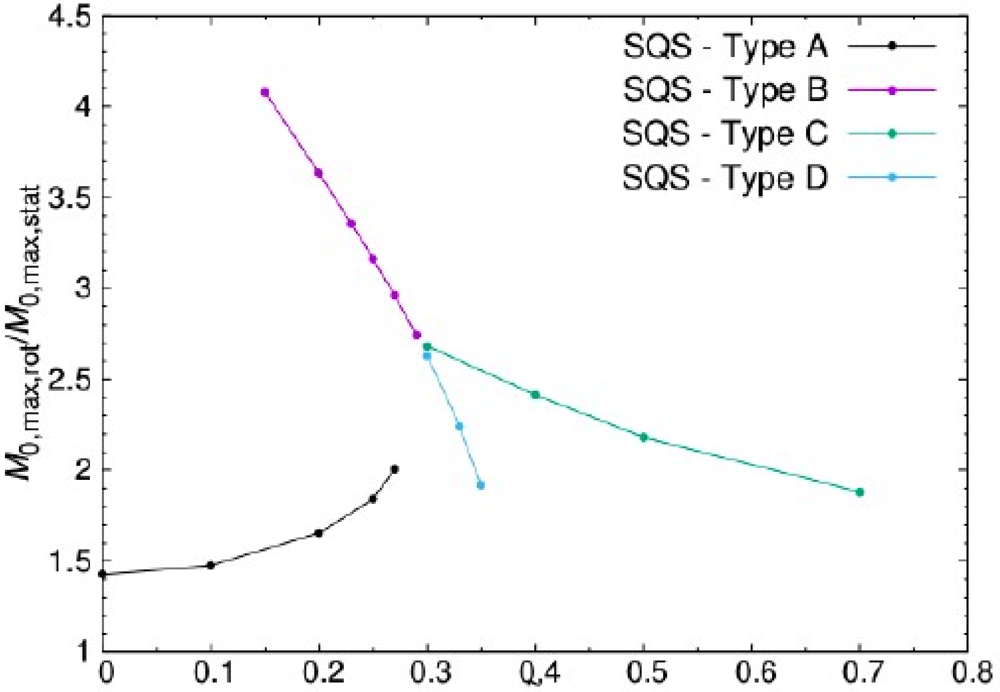


The maximum allowed mass for differentially rotating Strange Stars



Szkudlarek,Gondek-Rosinska,Villain, Ansorg, 2019, ApJ

The maximum mass of differentially rotating strange stars



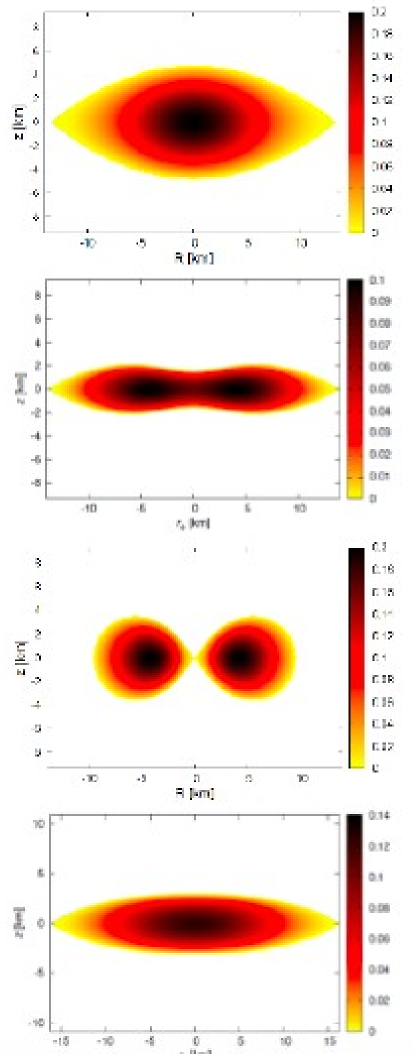
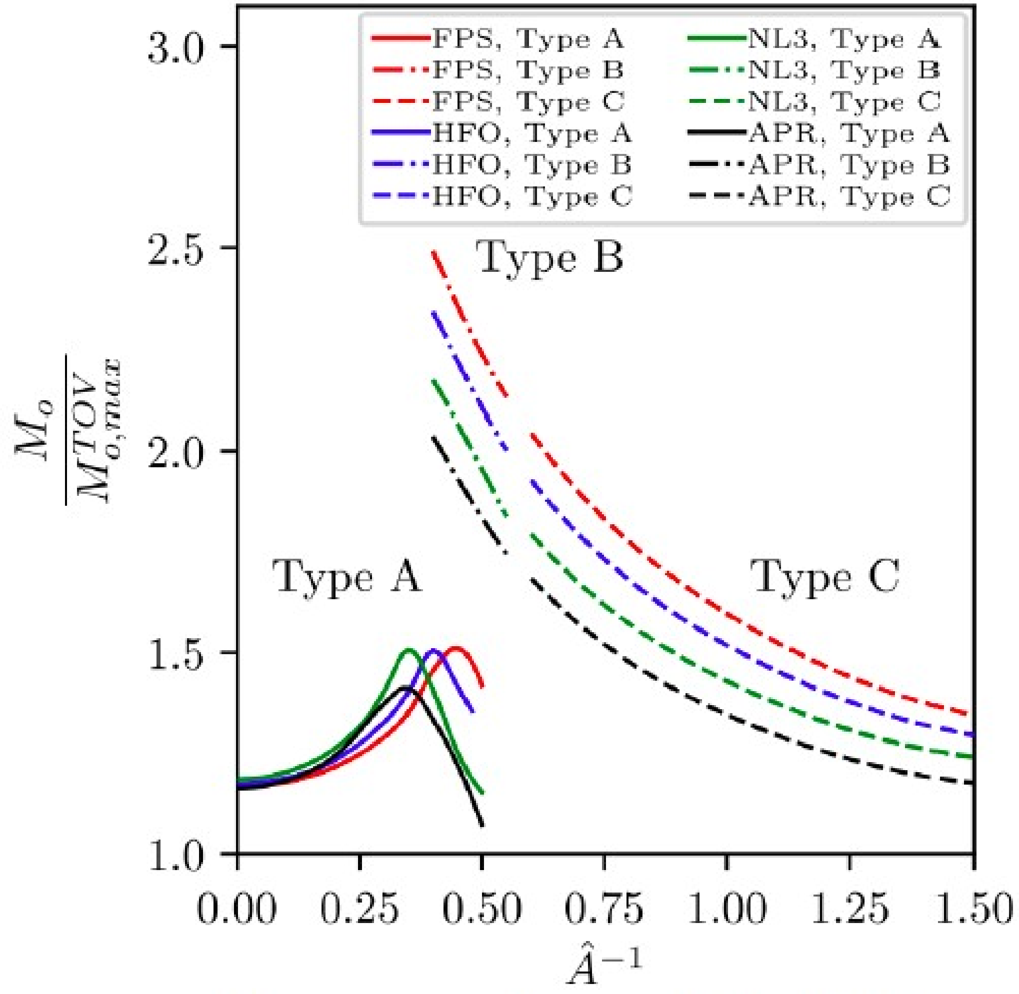
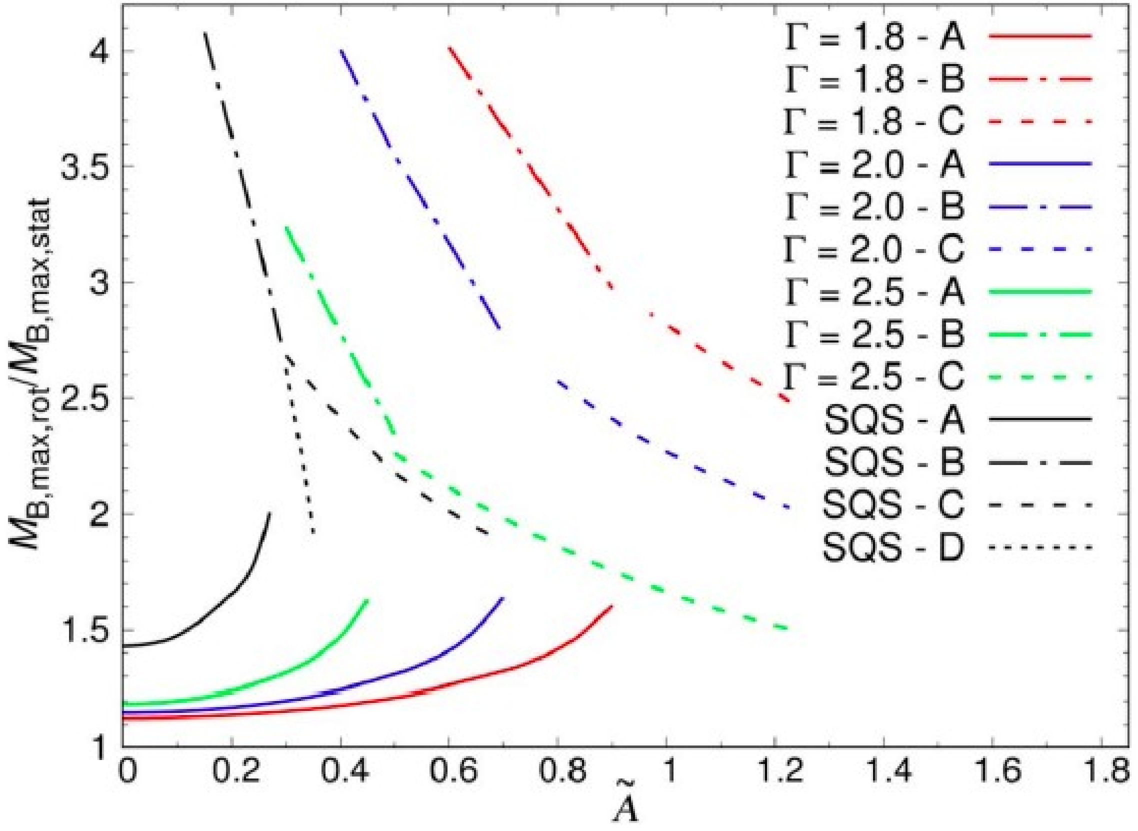


Figure: Maximum baryonic mass normalised by the maximum mass in the non-rotating case as a function of the degree of differential rotation $\tilde{A}$.The maximum mass depends on the degree of differential rotation **and the type of the solution**.

Confirmation of the universal relations - realistic EOS of NS



Max. mass for realistic EOS
(Espino & Paschalidis 2019)

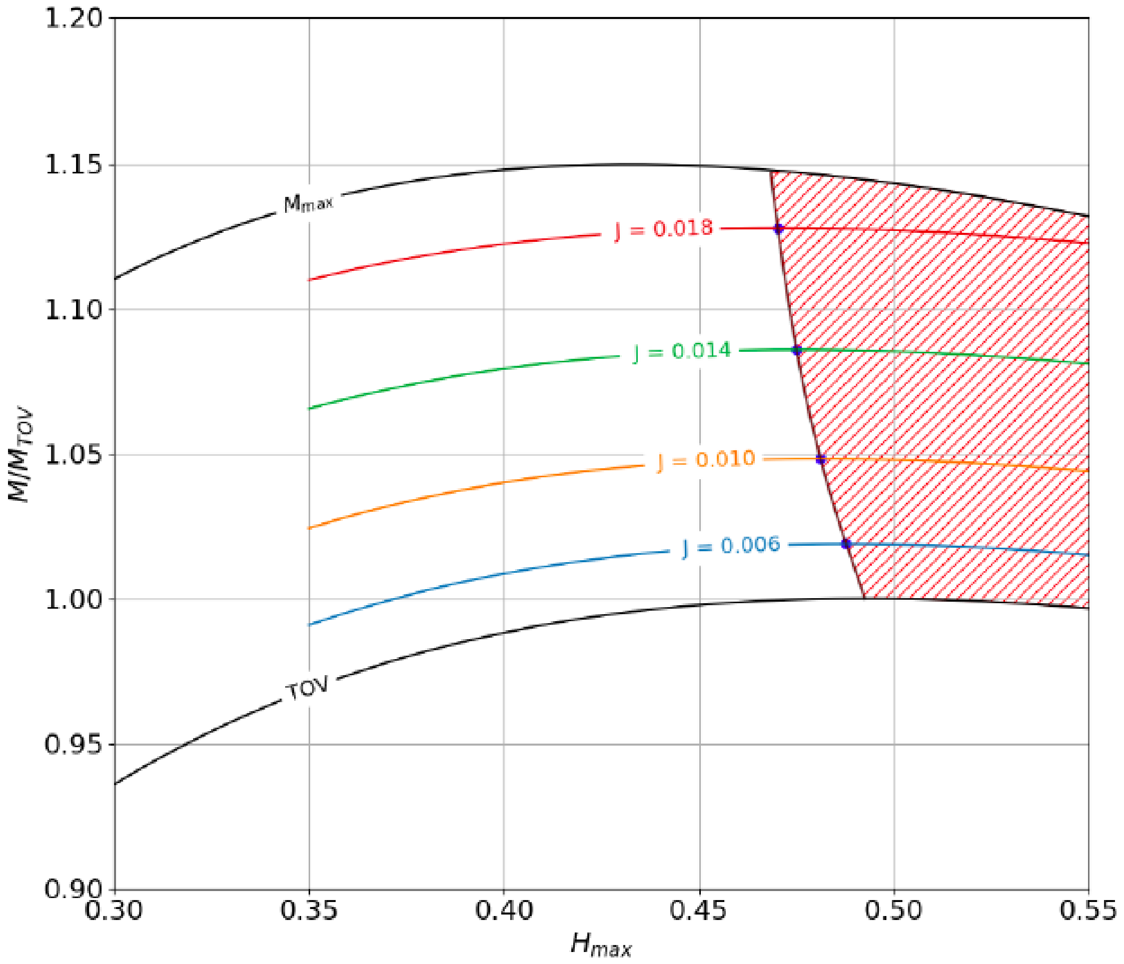


Max. mass for polytropes and strange stars
(Szkudlarek et al. 2019)

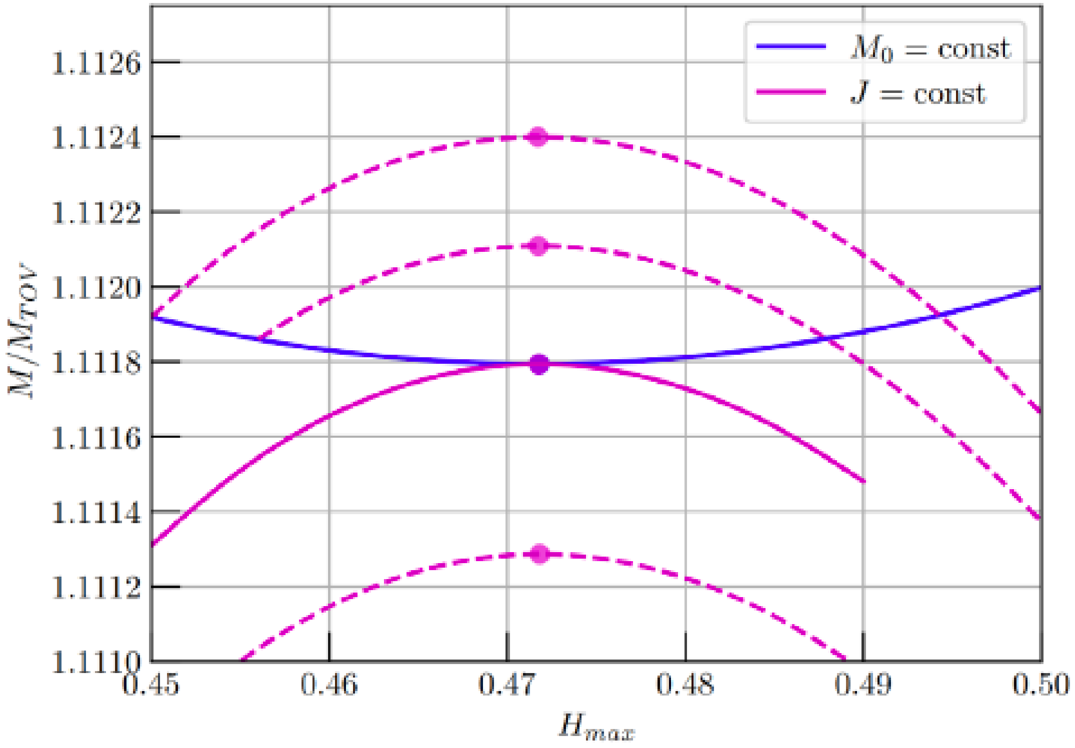
Are massive, differentially rotating neutron stars stable against prompt collapse to BH?

Turning point criterion for rigid rotation

- Turning points are at **maximum** of mass on J=const sequences
- **Sufficient** criterion of dynamical instability for **rigid** rotation (Friedman, Ipser, Sorkin 1988)



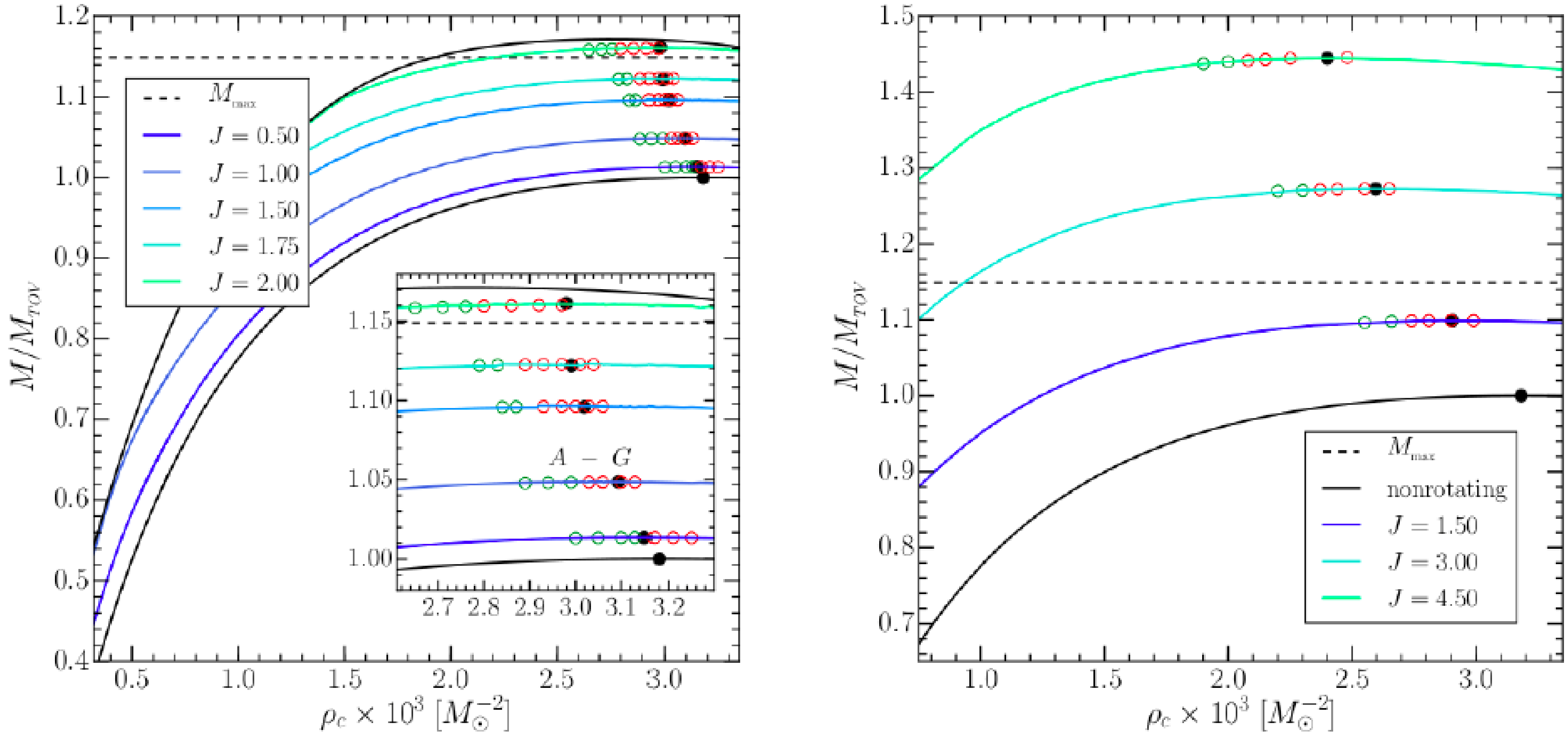
Turning point criterion (rigid rotation)



Rigid rotation:

- Maximum of **gravitational mass** on sequences of fixed **angular momentum** marks the onset of instability

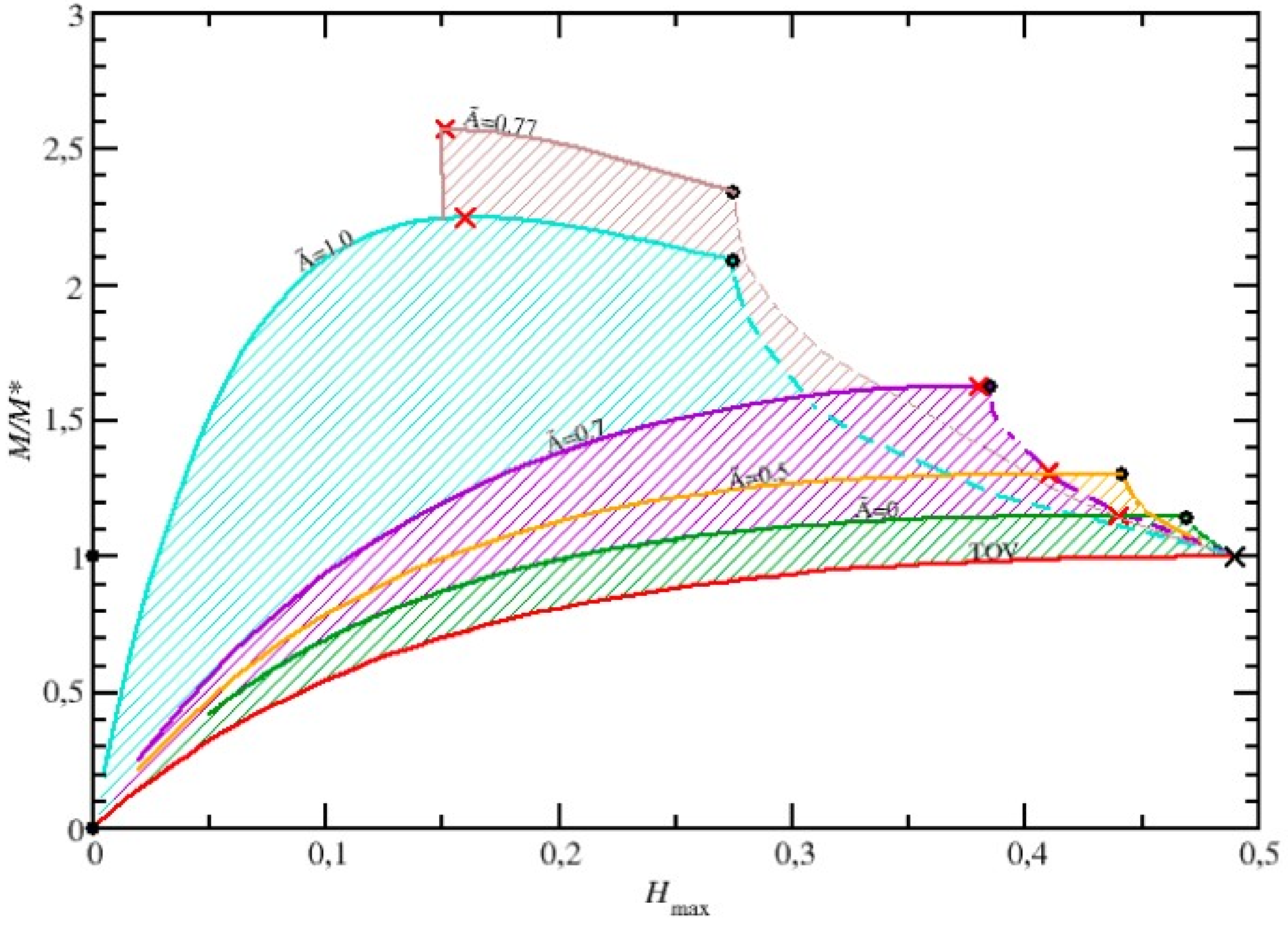
Turning points and differential rotation



Stability determined by hydrodynamical simulations - criterion is still sufficient
(Weih, Most, Rezzolla 2017)

Are hypermassive neutron stars stable to prompt collaps?

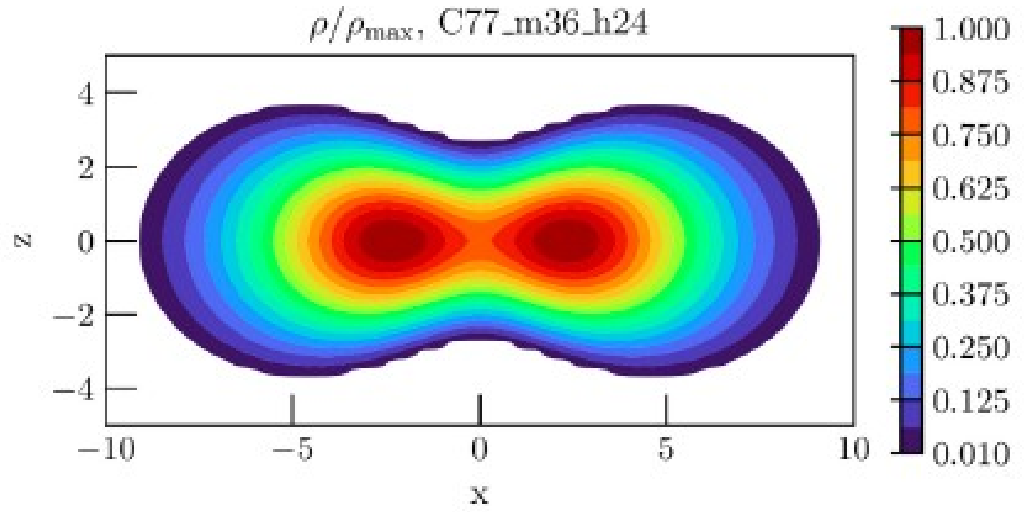
The solution space of differentially rotating NS (only type A and C) if we assume the turning point criteria (Gondek-Rosinska, Szewczyk et al. 2024, in prep)



The most massive configurations are estimated to be **stable** by turning-point criterion

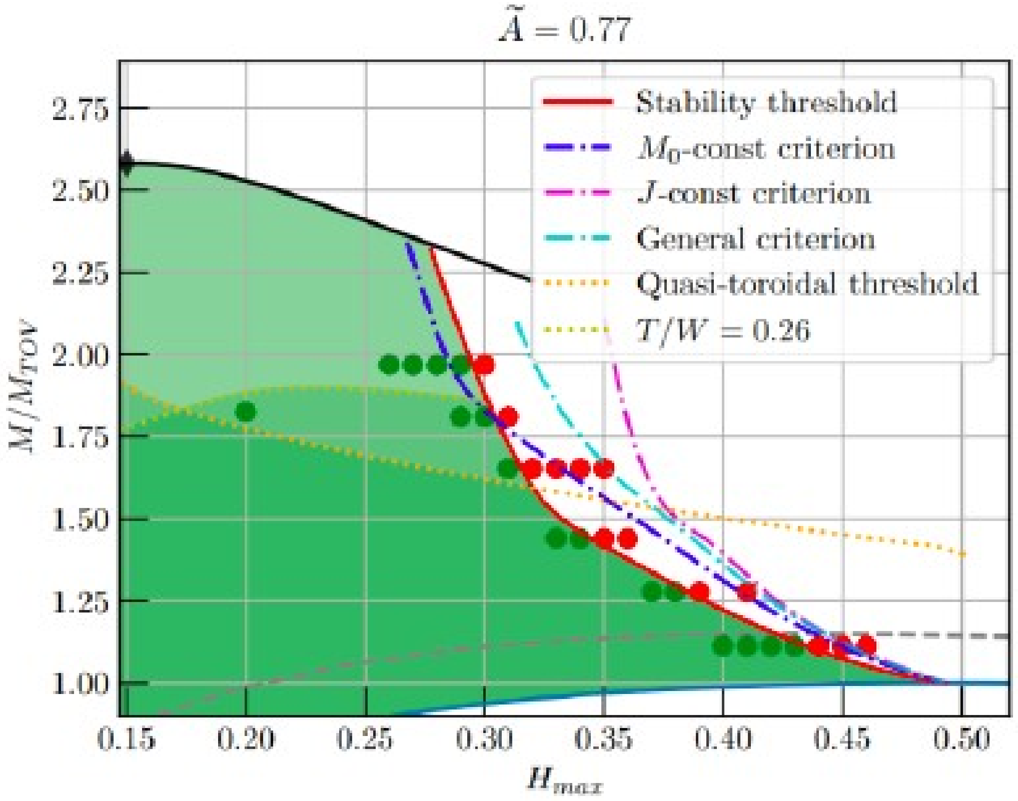
2D simulations: numerical scheme

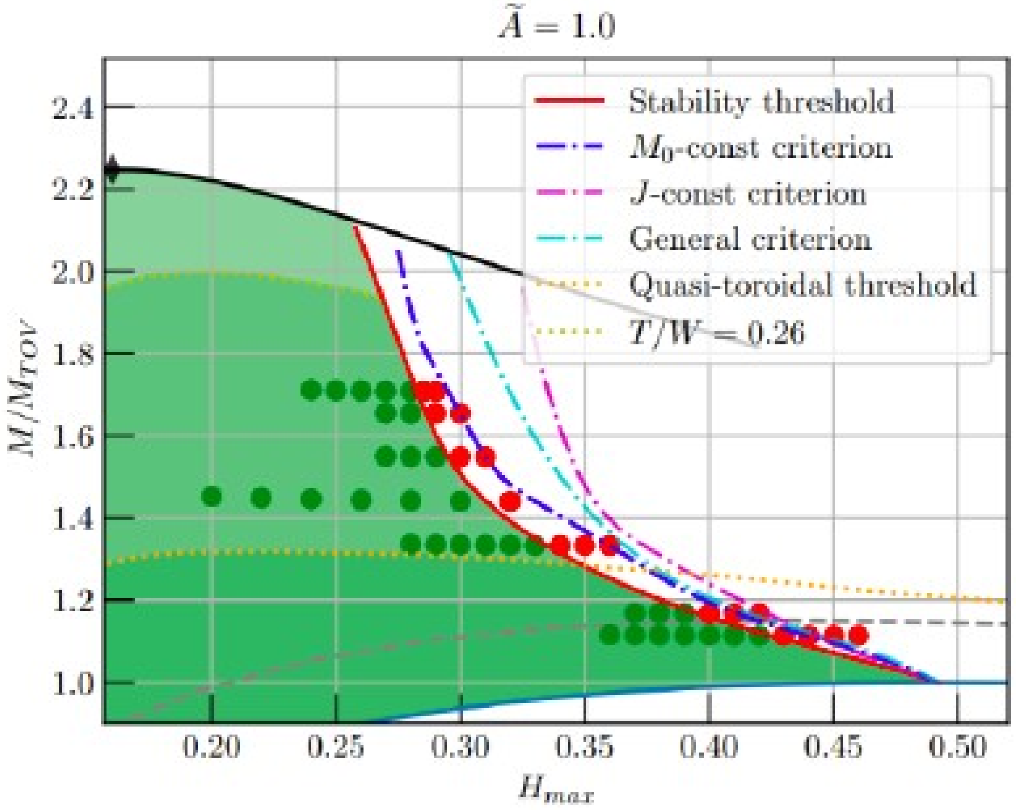
- Initial data calculated by FlatStar
- CoCoNuT code (relativistic hydrodynamics, dynamical space-time evolution, Dimmelmeier et al. 2002)
- Axial symmetry
- CFC approximation
- Additional radial perturbations
- 10ms length



Initial data calculated by FlatStar (meridional cut)

Quasi-radial instability for quasi-toroidal configurations





Szewczyk, Gondek-Rosińska, Cerda-Duran (in prep)

Summary

Using highly accurate code based on spectral method (Ansorg, Gondek-Rosińska, Villain [2009]) we have calculated relativistic models of axisymmetric rotating NS for broad ranges of degree of differential rotation

- The maximum mass of differentially rotating NS for given EOS depends on the degree of differential rotation and on **a type of solution (classified as A,B,C,D)**
- We have found new types of solutions B and D (existing for modest degree of differential rotation and $r_p/r_e \lesssim 0.3$), for both NS and SQS, which were not considered in previous works based on other algorithms due to complexity of the problem and numerical limitations (Gondek-Rosinska et al. 2017, Studzinska et al. 2016, Szkudlarek et al. 2019)
- Recently confirmed by other groups, e.g.Espino & Paschalidis 2019, Zhou et al. 2019..
- Existence of four types of solutions is a **universal behavior** for differentially rotating compact stars described by the Komatsu et

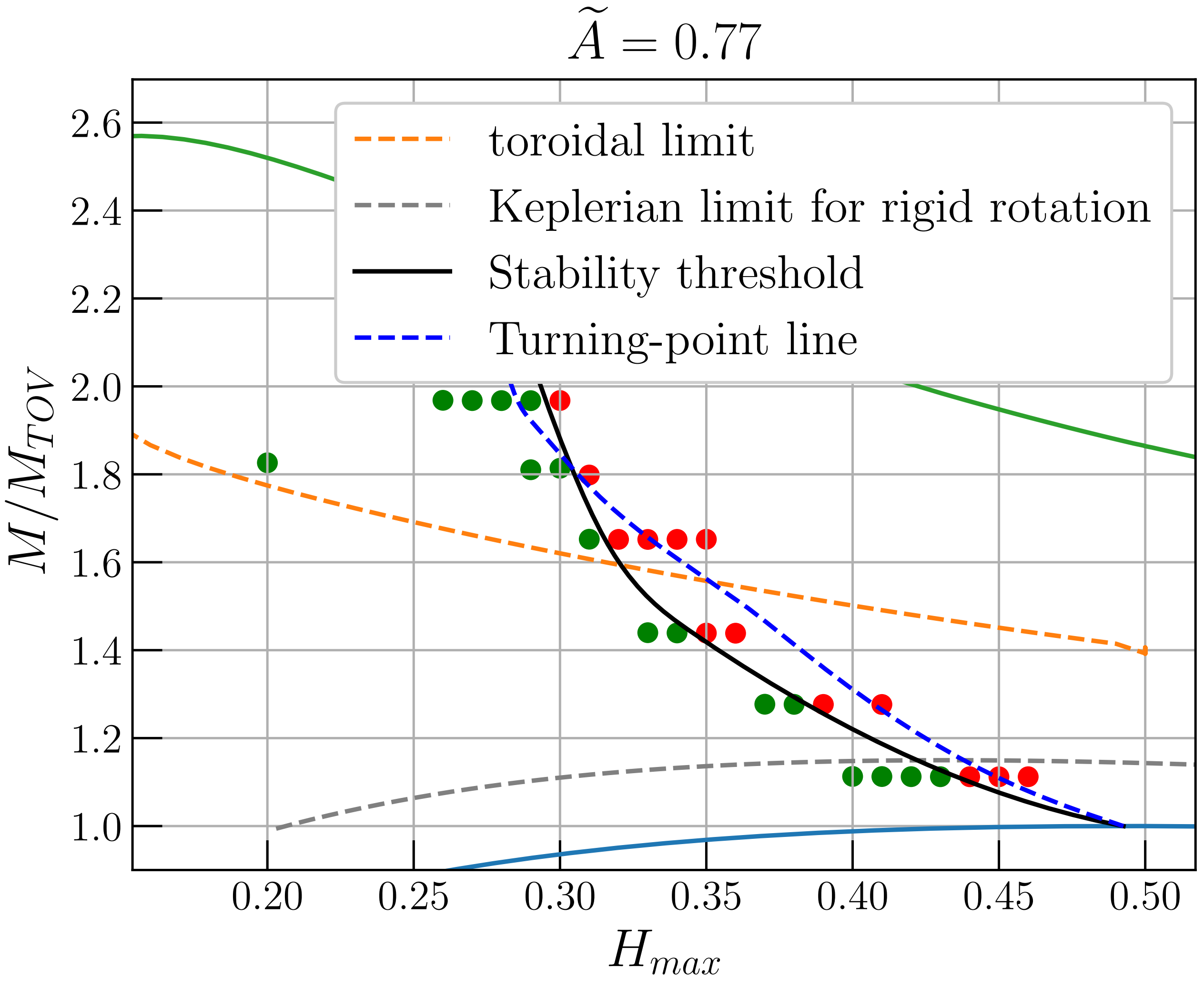
Summary

- Differential rotation significantly increases up to 4 times the maximum allowed mass of static NS and may temporarily stabilize a new born protoneutron star or a remnant of binary NS merger. The highest increase of mass is obtained for the newly found type of differentially rotating NS, type B
- The highest increase of mass is obtained for the newly found types of differentially rotating NS for the modest degree of rotation.
- Gravitational waves observations of coalescing binary NS or a core collapse may be able to distinguish the outcomes; prompt collapse, delayed collapse, a stable NS

- Massive NS can be stabilized by **differential rotation**
- The most massive configurations can be estimated to be dynamically stable by the **turning-point criterion**
- Maximum mass for stationary solution is ~4M_TOV
- We found stable configurations with M=2M_TOV
- Need a check with full-GR simulation (no CFC) and 3D simulation (non-radial modes)
- Potential source of gravitational waves at collapse (Giacomazzo et al 2011)

Are toroidal hypermassive neutron stars stable ?

different kind of instabilities should be taken into account e.g. Espino et al 2019, Shibata et. al. 2003, Watt et al. 2005, Yoshida et al 2019



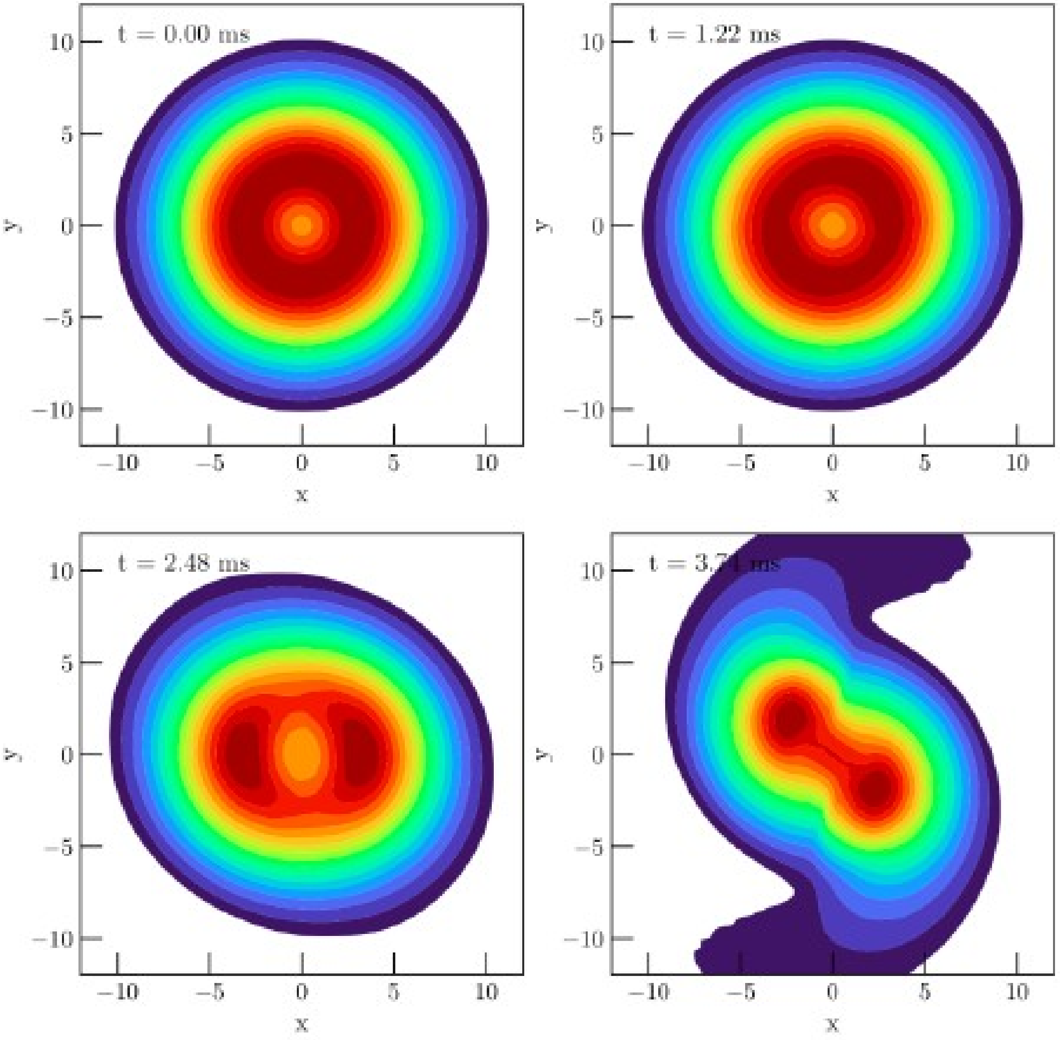
Non-axisymmetrical instabilities

- 3D simulations (GRHydro code)
- Full GR
- Limit of dynamical bar-mode instability (Shibata et al. 2000)
- Low-T/W instabilities
- 20 ms of evolution with moderate resolution
- Azimuthal mode decomposition:

$$A_m \sim \int \rho e^{im\phi} \mathrm{d}V$$

See e.g. by Espino et al. 2019

Growth of m=2 mode (bar-mode)

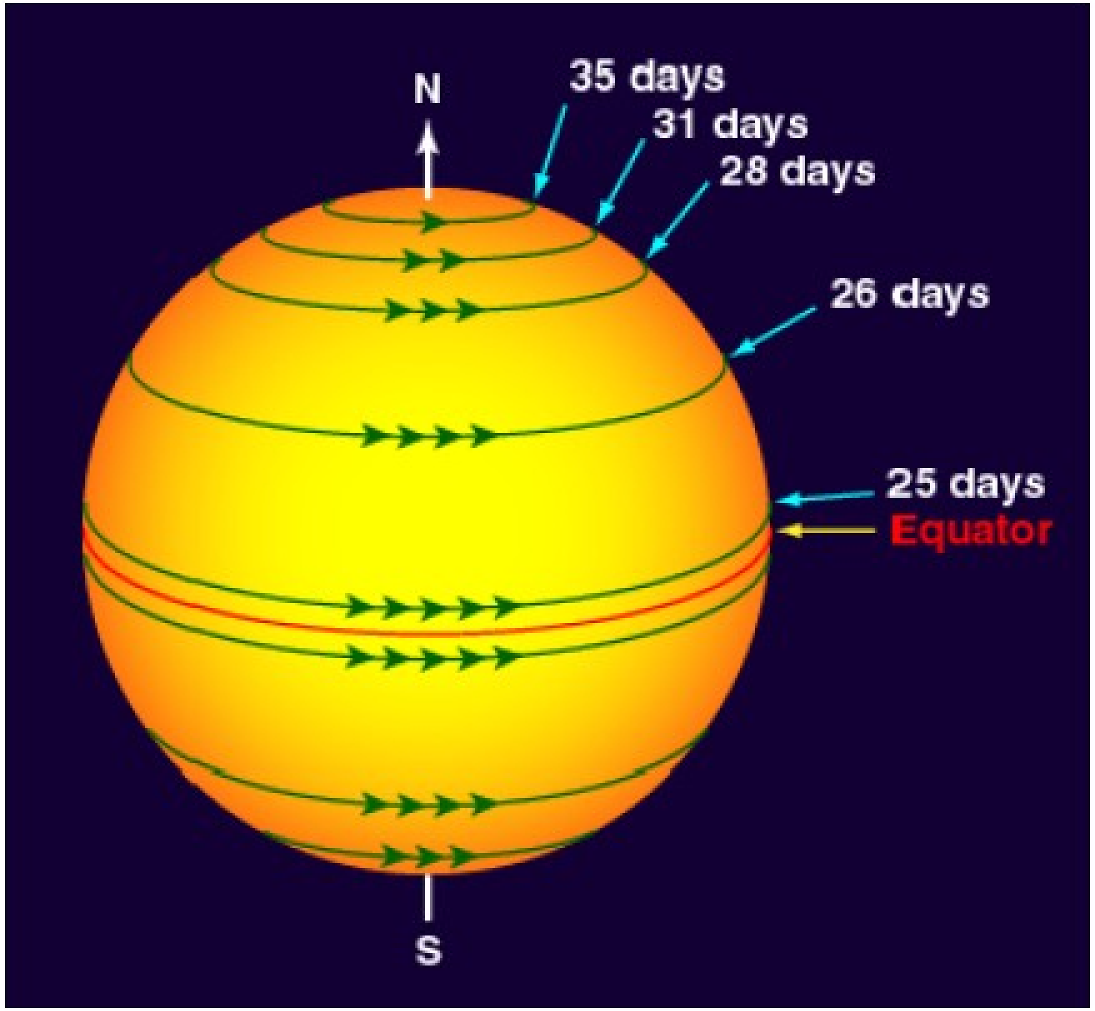


For T/W > 0.26 NS becomes dynamically unstable, leading to collapse to BH

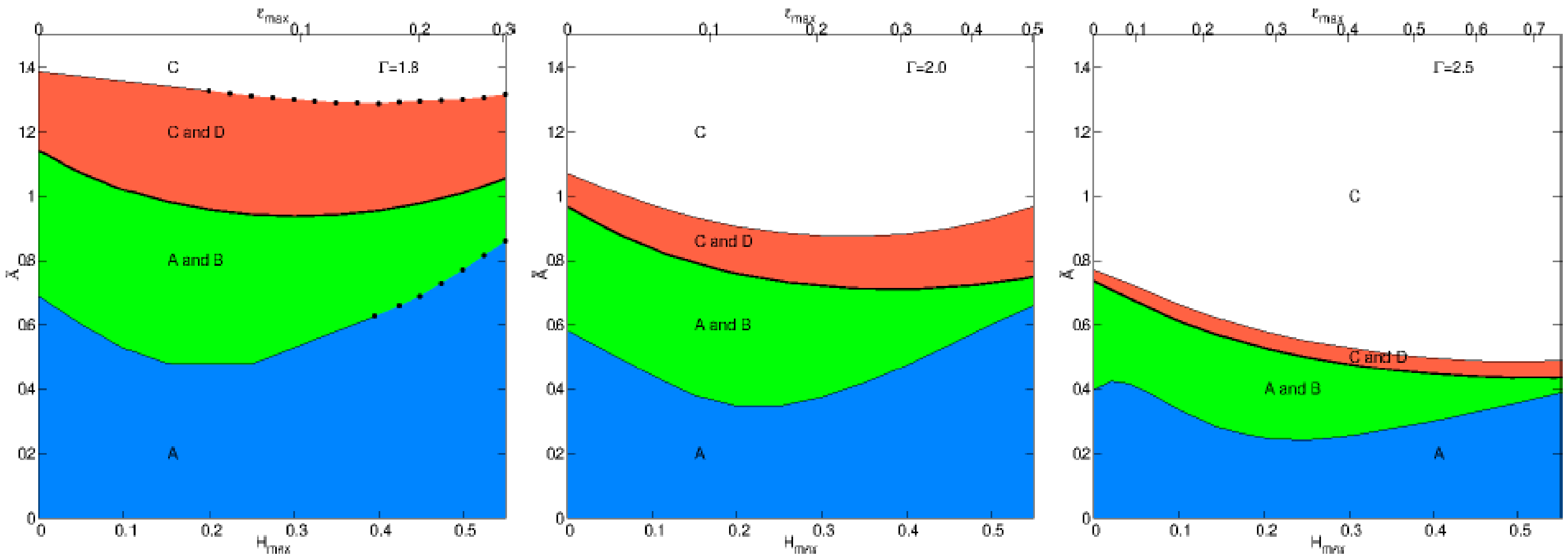
<-- Evolution of density profile in equatorial plane with nonaxisymmetrical perurbations.

Differential rotation is common – example - Sun

Differential rotation $\Omega = \Omega(r, \theta)$

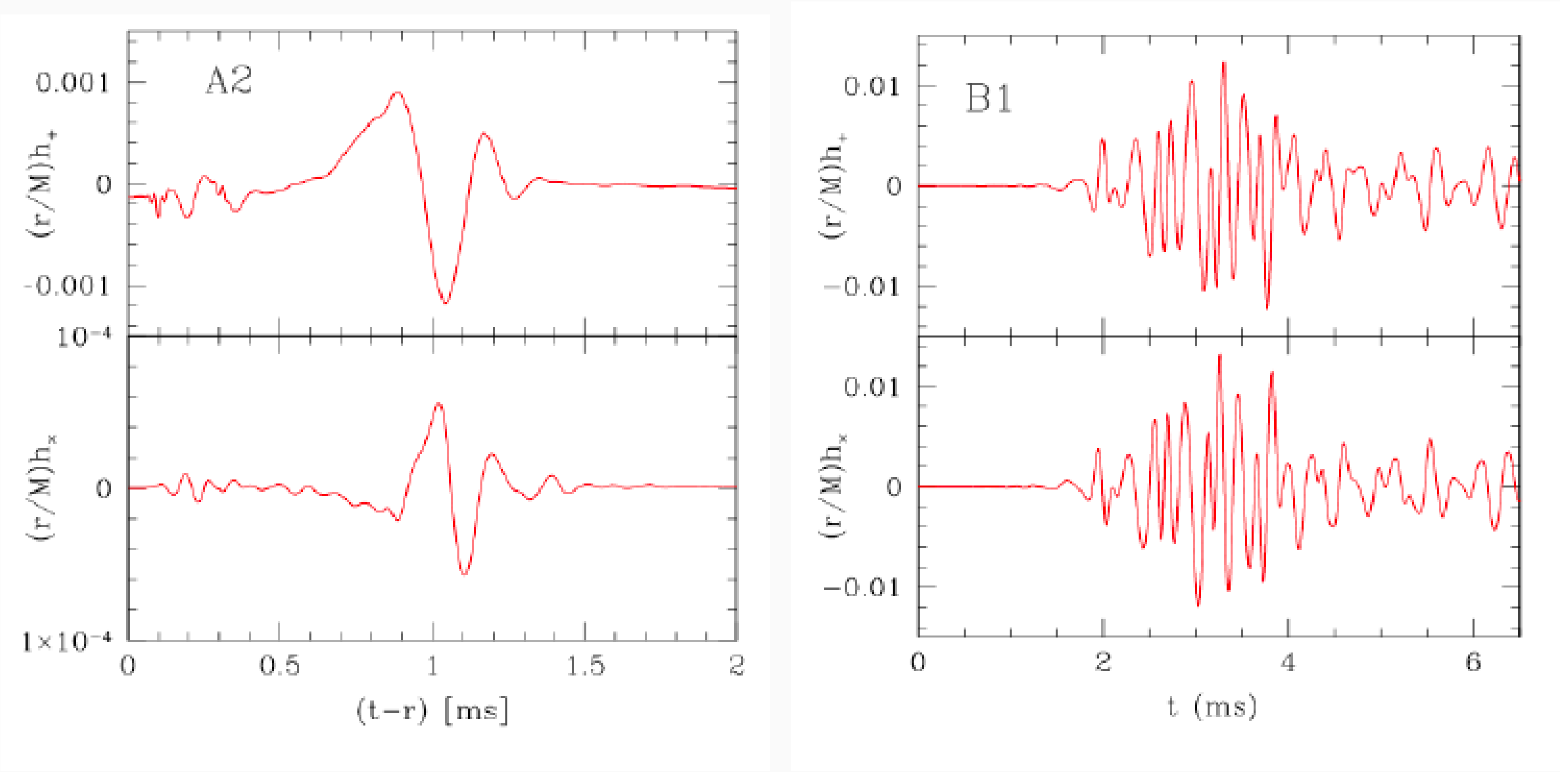


The effects of EOS on types for diffentially rotating NS



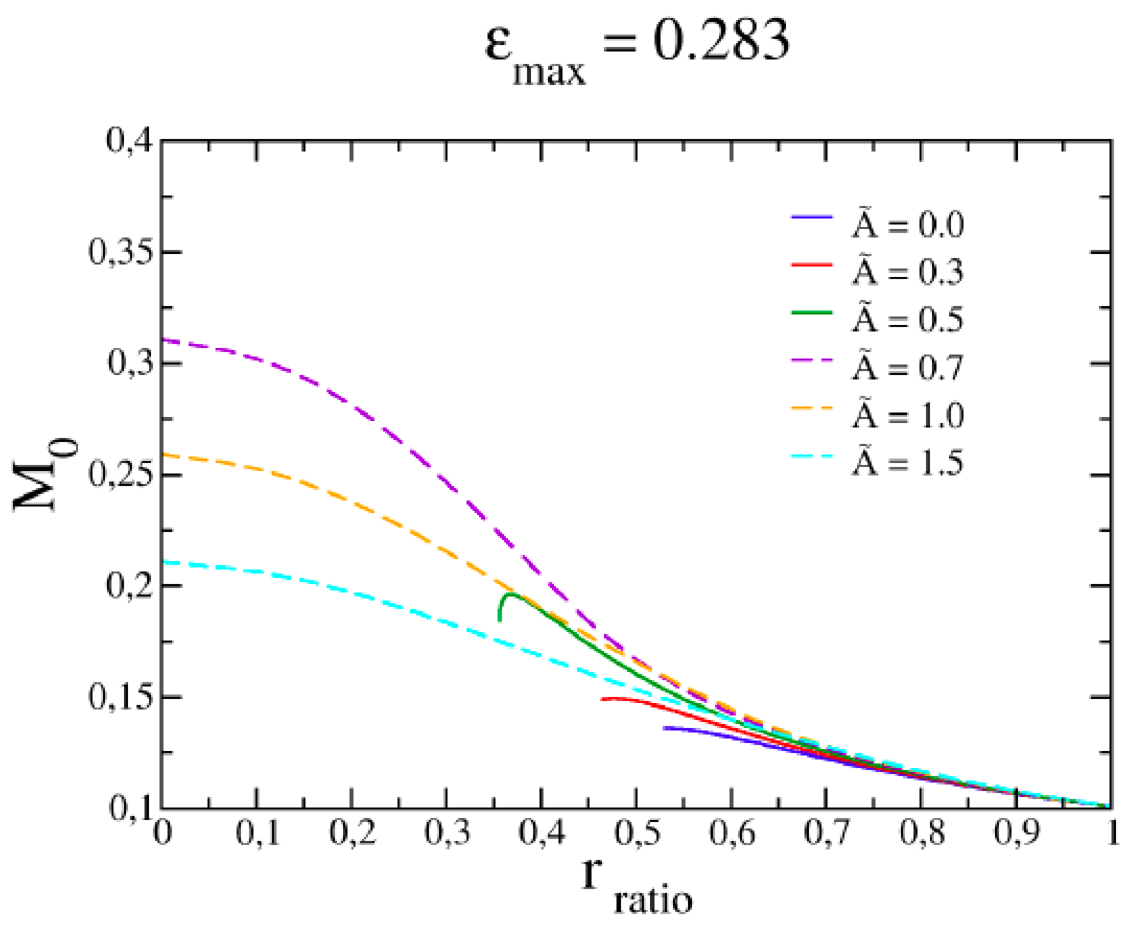
The stiffer EOS is the larger region of type C configurations and narrower for types A,B and D

GW signal during collapse



Gravitational-wave amplitudes h_+ and h_x for two collapses, left: $J/M^2 < 1$, right: $J/M^2 > 1$ (Giacomazzo et al 2011)

Differentially rotating NS - type A and C



Type A (solid lines)- the maximum mass is obtained close to the mass-shed limit, while for **Type C** (dashed lines) for toroidal configurations $r_{\rm ratio} = r_p/r_e = 0$. Existence of **separatrix** $A_{\rm crit}$