Black hole thermodynamics and boundary terms

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Motivation

- Gauss-Bonnet boundary term in 4D affects black hole entropy
- shift by a universal (topology-determined) constant
- the same applies to any Lovelock density in its critical dimension
- scalar-tensor theories allow for more drastic changes in thermodynamics
- both entropy and temperature seem to be modified

M. Gadioux, H. S. Reall, Phys. Rev. D 108 (2023)

Scalar-tensor Einstein-Gauss-Bonnet gravity

- our results apply to theories with $\phi\mathcal{G}$ term in the Lagrangian and shift symmetry
- a convenient example

$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi}\left[R - 2\Lambda + \alpha\left(\phi\mathcal{G} + 4G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 4\left(\partial\phi\right)^2\nabla_\alpha\nabla^\alpha\phi + 2\left(\partial\phi\right)^4\right)\right]$$

- $\mathcal{G}$ is the Gauss-Bonnet invariant $\mathcal{G} = R_{\alpha\beta\lambda\rho}R^{\alpha\beta\lambda\rho} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2$
- in 4 dimensions $\mathcal{G} = \nabla_\mu\mathcal{G}^\mu$ ⟹ the action is invariant under a shift of ϕ by a constant $\phi \to \phi + C$ (up to a total divergence term)
- the theory has analytical static, spherically symmetric black hole solutions

H. Lu and Y. Pang, Phys. Lett. B 809 (2020); R. A. Hennigar, D. Kubiznak, R. B. Mann and C. Pollack, JHEP 2020 (2020), S.-W. Wei and Y.-X. Liu, Phys. Rev. D 101 (2020)

The entropy conundrum

- Wald entropy naively equals $S_{\mathrm{W}} = \mathcal{A}/4 + 4\pi\alpha\phi\,(r_+) = \mathcal{A}/4 + 2\pi\alpha \ln\left(\mathcal{A}/L^2\right)$

  breaks shift symmetry!

- Noether current is not shift-invariant (symplectic current is)

- reason: shift symmetry up to a total divergence term $\phi \to \phi + C$

  $$\mathcal{L} \to \mathcal{L} + \nabla_\mu\left[\left(\alpha\sqrt{-g}/16\pi\right) C\mathcal{G}^\mu\right]$$

- on shell (from the scalar field EoM)

$$\mathcal{G}^\mu = 8G^{\mu\nu}\nabla_\nu\phi - 8\nabla^\nu\nabla_\nu\phi\nabla^\mu\phi + 8\nabla^\nu\phi\nabla_\nu\nabla^\mu\phi + 8\nabla_\nu\phi\nabla^\nu\phi\nabla^\mu\phi \equiv \mathcal{S}^\mu$$

- add a total derivative $\mathcal{L}_{\mathrm{shift}} = -\nabla_\mu\left[\alpha\left(\sqrt{-g}/16\pi\right)\phi\mathcal{S}^\mu\right]$ to the Lagrangian
  - restores exact shift invariance on shell

Brown-York approach to thermodynamics

- Euclidean grandcanonical ensemble
- the partition function approximated by logarithm of the classical action
- static, spherically symmetric, Euclidean black hole

$$\mathrm{d}s^2 = b^2(y)\,\mathrm{d}\tau^2 + a^2(y)\,\mathrm{d}y^2 + r^2(y)\,\mathrm{d}\Omega_2$$

- τ is 2π-periodic, $y \in [0,1]$ with $r(y=0) = r_+$, $r(y=1) = r_\mathrm{b}$ (York boundary)
- regularity of the horizon: $b(0) = 0$, $(r'/a)|_{y=0} = 0$, $(b'/a)|_{y=0} = 1$
- inverse black hole temperature $\beta = 2\pi b(1)$

H. W. Braden, J. D. Brown, B. F. Whiting, and J. W. York, Phys. Rev. D 42 (1990);
ML, D. Kubizňák, R. A. Hennigar, JHEP 2023 (2023)

Brown-York approach

- action $I_{\rm E} = -\int \left(\mathcal{L}_{\phi\rm GB} + \mathcal{L}_{\rm shift}\right) \mathrm{d}^4x + (1/8\pi) \oint \left(K - K_0 + \mathcal{K}_{\phi\rm GB}\right) \sqrt{h}\mathrm{d}^3x$

GHY surface term

- we impose regularity conditions and constraint equations
- EoM's not imposed ⟹ still off shell
- we obtain an explicit symmetry reduced action $I_{\rm E}\left(r_+\right)$
- looking for stationary points w.r.t. r_+ fixes at infinity $\beta = (2\pi/\kappa) / \left(1 + 2\alpha/r_+^2\right)$
- we recover Bekenstein entropy $S = \mathcal{A}/4$ ⟹ shift symmetry preserved

Modified temperature?

- the Hawking temperature appears to be $T_{\mathrm{H}} = (\kappa/2\pi)\left(1 + 2\alpha/r_+^2\right)$
- but the Hawking effect is kinematic, how can the temperature be modified

  A: propagation speed of gravitons in scalar-tensor theories does not equal c
  the Hawking effect can thus be modified

  B: there is a "screening mechanism" affecting propagation of the emitted particles

  C: thermodynamic black hole temperature distinct from the Hawking one

K. Hajian, S. Liberati, M. M. Sheikh-Jabbari, M. H. Vahidinia, Phys. Lett. B 812 (2020)

Take home message

- the “standard” way to do black hole thermodynamics cannot be blindly trusted
- scalar-tensor Einstein-Gauss-Bonnet gravity illustrates this:

1) Gauss-Bonnet term in 4D is a surface term that affects entropy
2) the black hole temperature appears to be modified

Thank you!