

# Black hole thermodynamics and boundary terms

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Seventeenth Marcel Grossmann Meeting, Pescara, July 8, 2024

# Motivation

- Gauss-Bonnet boundary term in 4D affects black hole entropy
- shift by a universal (topology-determined) constant
- the same applies to any Lovelock density in its critical dimension
- scalar-tensor theories allow for more drastic changes in thermodynamics
- both entropy and temperature seem to be modified

# Scalar-tensor Einstein-Gauss-Bonnet gravity

- our results apply to theories with  $\phi\mathcal{G}$  term in the Lagrangian and shift symmetry
- a convenient example
$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi} \left[ R - 2\Lambda + \alpha \left( \phi\mathcal{G} + 4G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + 4(\partial\phi)^2 \nabla_\alpha \nabla^\alpha \phi + 2(\partial\phi)^4 \right) \right]$$
- $\mathcal{G}$  is the Gauss-Bonnet invariant  $\mathcal{G} = R_{\alpha\beta\lambda\rho} R^{\alpha\beta\lambda\rho} - 4R_{\alpha\beta} R^{\alpha\beta} + R^2$
- in 4 dimensions  $\mathcal{G} = \nabla_\mu \mathcal{G}^\mu$   the action is invariant under a shift of  $\phi$  by a constant  $\phi \rightarrow \phi + C$  (up to a total divergence term)
- the theory has analytical static, spherically symmetric black hole solutions

H. Lu and Y. Pang, Phys. Lett. B 809 (2020); R. A. Hennigar, D. Kubiznak, R. B. Mann and C. Pollack, JHEP 2020 (2020), S.-W. Wei and Y.-X. Liu, Phys. Rev. D 101 (2020)

# The entropy conundrum

➤ Wald entropy naively equals  $S_W = \mathcal{A}/4 + 4\pi\alpha\phi(r_+) = \mathcal{A}/4 + 2\pi\alpha \ln(\mathcal{A}/L^2)$

breaks shift symmetry!

➤ Noether current is not shift-invariant (symplectic current is)

➤ reason: shift symmetry up to a total divergence term  $\phi \rightarrow \phi + C$

$$\mathcal{L} \rightarrow \mathcal{L} + \nabla_\mu [(\alpha\sqrt{-g}/16\pi) C\mathcal{G}^\mu]$$

➤ on shell (from the scalar field EoM)

$$\mathcal{G}^\mu = 8G^{\mu\nu}\nabla_\nu\phi - 8\nabla^\nu\nabla_\nu\phi\nabla^\mu\phi + 8\nabla^\nu\phi\nabla_\nu\nabla^\mu\phi + 8\nabla_\nu\phi\nabla^\nu\phi\nabla^\mu\phi \equiv \mathcal{S}^\mu$$

➤ add a total derivative  $\mathcal{L}_{\text{shift}} = -\nabla_\mu [\alpha(\sqrt{-g}/16\pi)\phi\mathcal{S}^\mu]$  to the Lagrangian

- restores exact shift invariance on shell

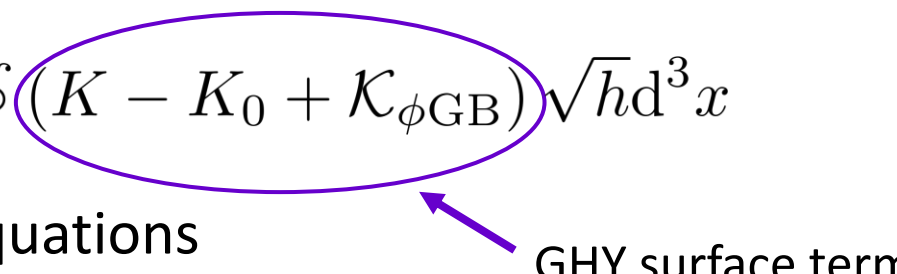


# Brown-York approach to thermodynamics

- Euclidean grandcanonical ensemble
- the partition function approximated by logarithm of the classical action
- static, spherically symmetric, Euclidean black hole

$$ds^2 = b^2(y) d\tau^2 + a^2(y) dy^2 + r^2(y) d\Omega_2$$

- $\tau$  is  $2\pi$ -periodic,  $y \in [0, 1]$  with  $r(y=0) = r_+$ ,  $r(y=1) = r_b$  (York boundary)
- regularity of the horizon:  $b(0) = 0$ ,  $(r'/a)|_{y=0} = 0$ ,  $(b'/a)|_{y=0} = 1$
- inverse black hole temperature  $\beta = 2\pi b(1)$

# Brown-York approach

- action  $I_E = - \int (\mathcal{L}_{\phi_{\text{GB}}} + \mathcal{L}_{\text{shift}}) d^4x + (1/8\pi) \oint (K - K_0 + \mathcal{K}_{\phi_{\text{GB}}}) \sqrt{h} d^3x$   

- we impose regularity conditions and constraint equations
- EoM's not imposed  still off shell
- we obtain an explicit symmetry reduced action  $I_E(r_+)$
- looking for stationary points w.r.t.  $r_+$  fixes at infinity  $\beta = (2\pi/\kappa) / (1 + 2\alpha/r_+^2)$
- we recover Bekenstein entropy  $S = \mathcal{A}/4$   shift symmetry preserved

# Modified temperature?

- the Hawking temperature appears to be  $T_{\text{H}} = (\kappa/2\pi) (1 + 2\alpha/r_+^2)$
- but the Hawking effect is **kinematic**, how can the temperature be modified
  - A:** propagation speed of gravitons in scalar-tensor theories does not equal  $c$   
the Hawking effect can thus be modified
  - B:** there is a “screening mechanism” affecting propagation of the emitted particles
  - C:** thermodynamic black hole temperature distinct from the Hawking one

## Take home message

- the “standard” way to do black hole thermodynamics cannot be blindly trusted
- scalar-tensor Einstein-Gauss-Bonnet gravity illustrates this:
  - 1) Gauss-Bonnet term in 4D is a surface term that affects entropy
  - 2) the black hole temperature appears to be modified



Thank you!