Black hole explosions in loop quantum gravity

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Cipriani, FF, Wilson Ewing, arXiv:2404.04192

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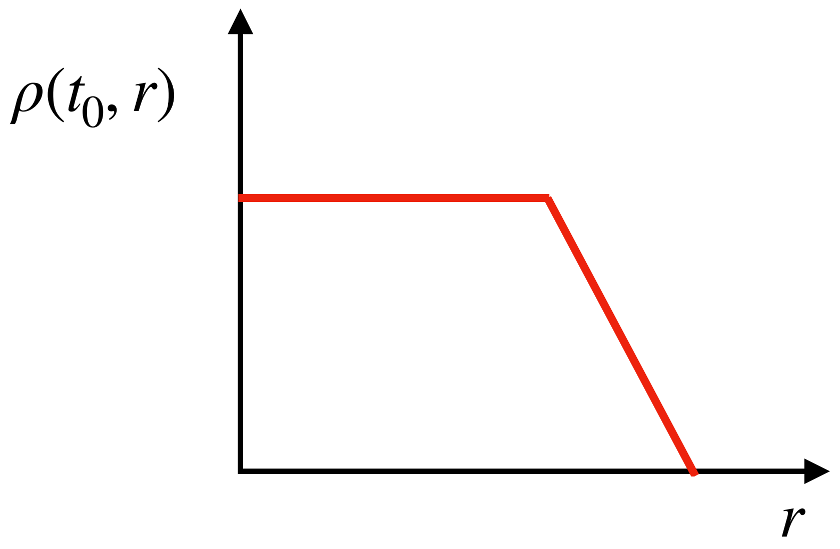
Introduction

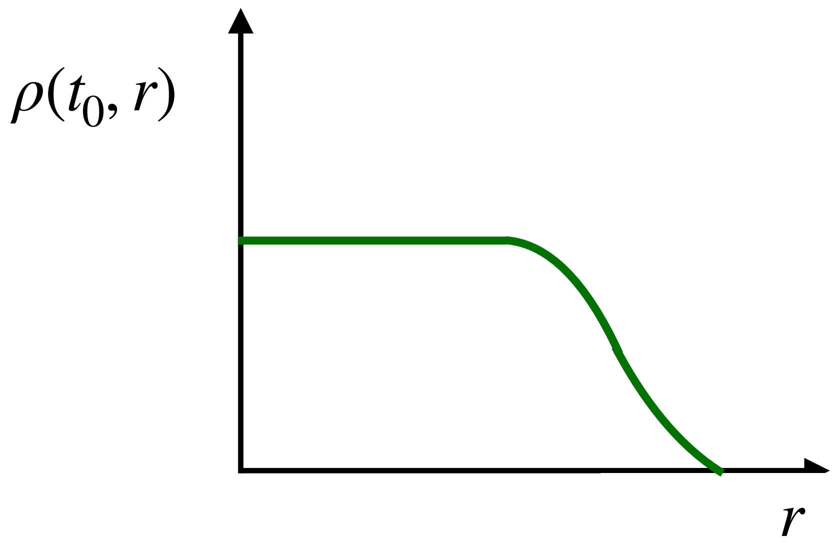
The quantum Oppenheimer-Snyder model in loop quantum gravity predicts three main phases: collapse, bounce and expansion.

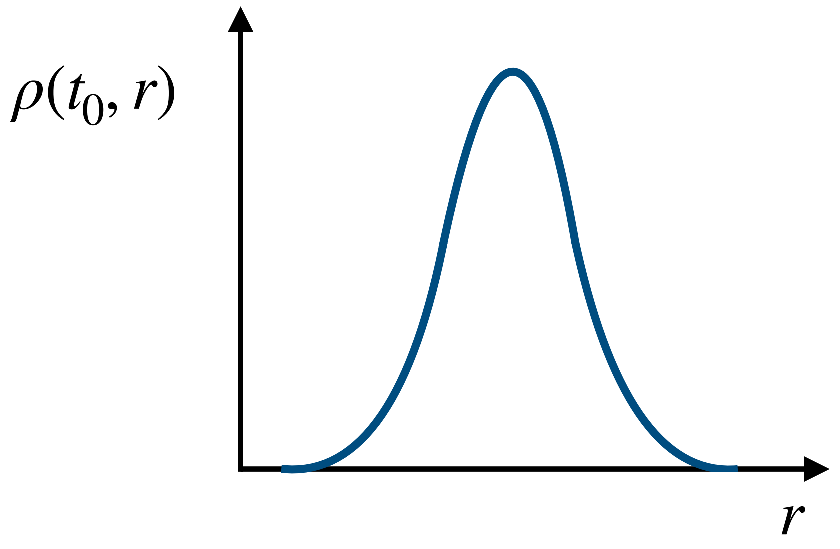
The dynamics is symmetric around the bounce point $\left(\rho_{bounce} = \dfrac{3}{8\pi G\gamma^2\Delta} \equiv \rho_c\right)$.

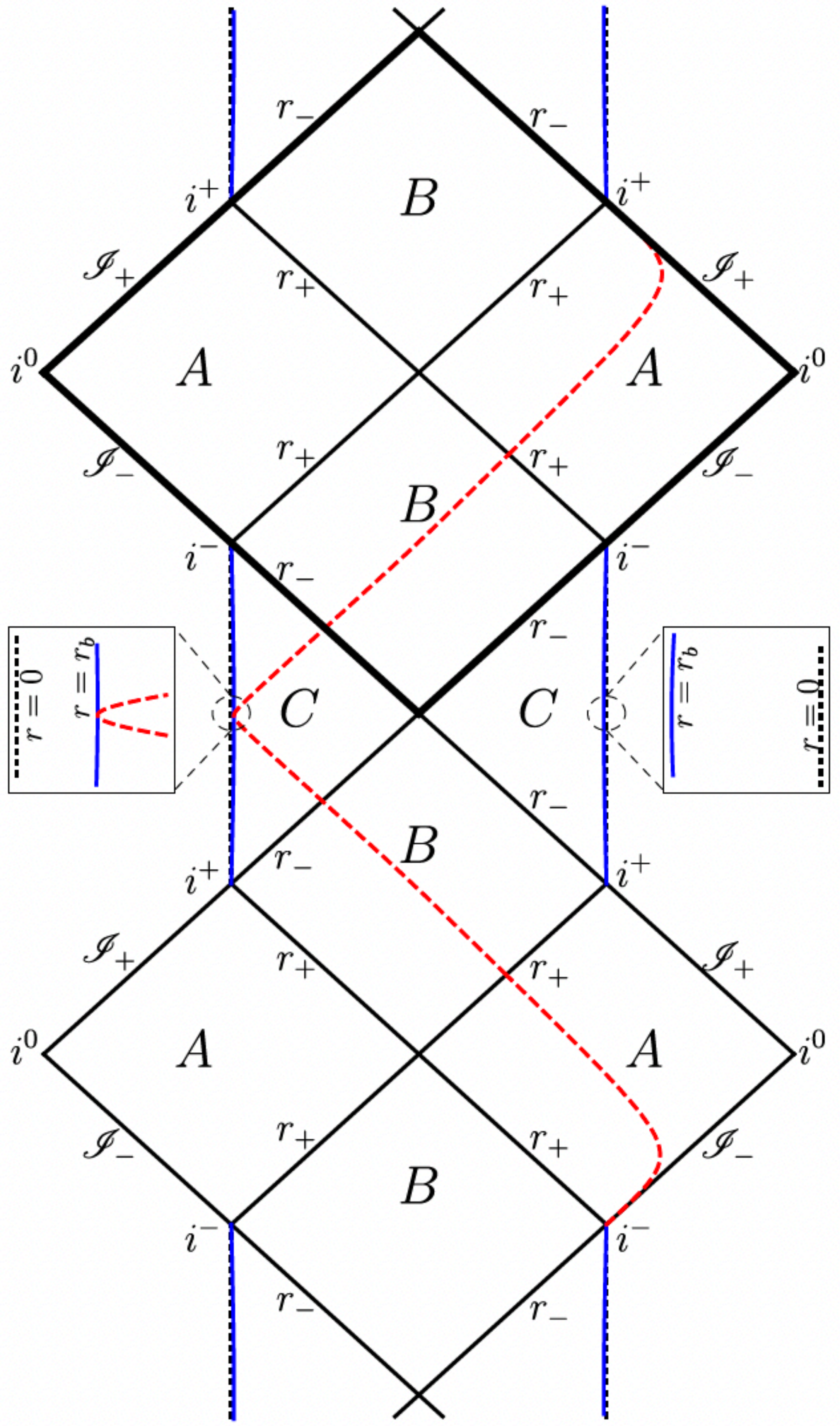
[Lewandowski, Ma, Yang, Zhang, 2023; Giesel, Liu, Singh, Weigl, 2023; FF, Rovelli, Soltani, 2023]

- Is the OS model a good prototype?
- How does the picture change if we consider continuous initial energy density profiles?









[Lewandowski, Ma, Yang, Zhang, 2023]

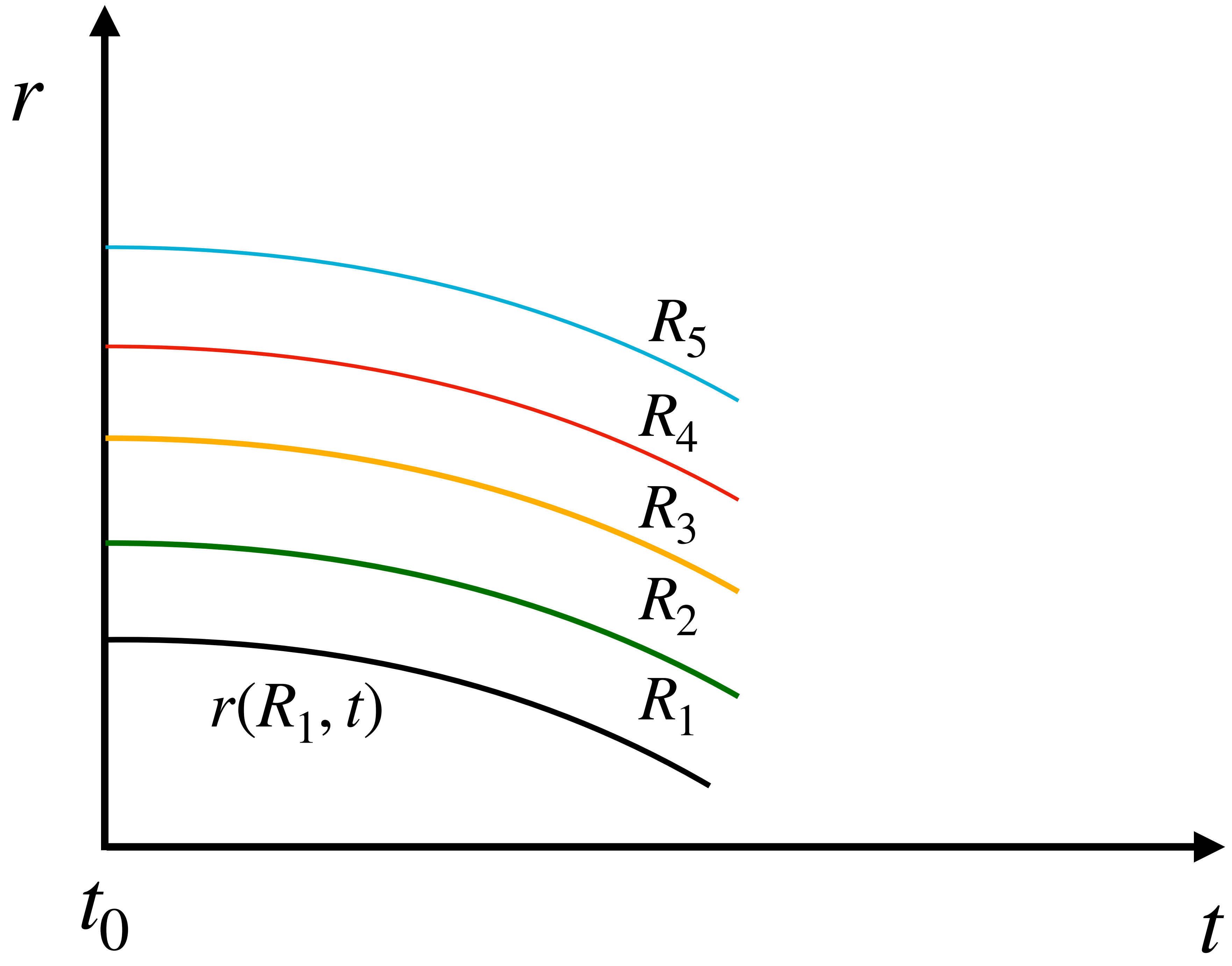
Lemaître-Tolman-Bondi metric

The effective metric describing star collapse in LTB coordinates in the marginally bound case reads:

$$ds^2 = -dt^2 + \left[\partial_R r(R,t)\right]^2 dR^2 + r(R,t)^2 d\Omega^2$$

This metric describes both the matter and vacuum region of the space-time.

Interpretation: we can imagine to divide the spatial part of the manifold in spherical shells parametrized by the radial coordinate R. The solution $r(R,t)$ of the EOMs is the areal radius r of the shell R at time t.



Effective dynamics in LTB coordinates

The LTB effective equations for spherically symmetric dust collapse (marginally bound case):

$$\left(\frac{\dot{r}}{r}\right)^2 = \frac{2Gm}{r^3}\left(1 - \frac{2\Delta Gm}{r^3}\right),$$

[Giesel, Liu, Singh, Weigl, 2023]

where Δ is the area gap in LQG: $\Delta \sim l_P^2$ and $m(R)$ is the mass function fixed by the initial energy density profile.

The general solution of the EOM is:

$$r(R,t) = [2Gm(R)]^{1/3}\left[\frac{9}{4}\left(t - \alpha(R)\right)^2 + \Delta\right]^{1/3}.$$

$\Longrightarrow$ Each shell R should bounce at: $t = \alpha(R)$. The solution seems to be symmetric around the bounce point.

But: LTB equations break down when the solution develops *shell crossing singularities*.

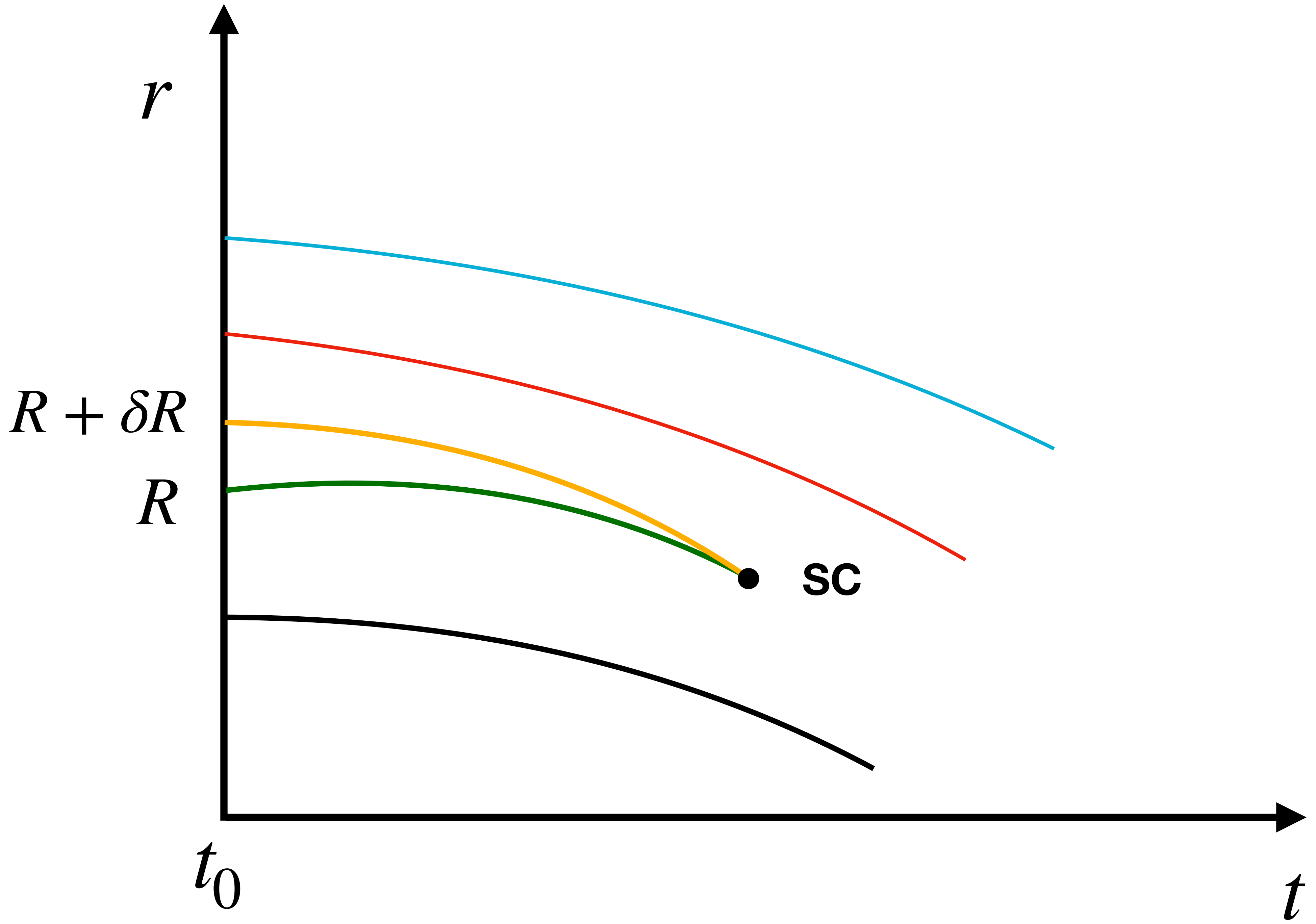
Shell-crossing singularities in LTB space-times

The dust energy density is given by: $$\rho(R,t) = \frac{\partial_R m(R)}{4\pi r^2 \partial_R r(R,t)}\,.$$

- If $\partial_R r(R,t) = 0$ for some shell R at some time t a shell-crossing (SC) arises (coordinate singularity).

- Moreover, if for the same R: $\partial_R m(R) \neq 0$ (matter region)

$$\Rightarrow \quad \rho(R,t) = +\infty, \quad R_{\mu\nu}g^{\mu\nu} = +\infty.$$



A shell-crossing singularity (SCS) forms: it is a physical *weak* singularity.

In classical GR, many initial configurations develop SCS, but one can choose suitable initial profiles that don't develop such singularities *[Hellaby, Lake, 1984]*; the same holds in LQC *[Singh, 2009]*.

Can SCS be avoided also in the effective star collapse dynamics?

Shell-crossing singularity theorem

Theorem: *"for the marginally bound case, a shell-crossing singularity will necessary form at some R if the initial energy density profile is non-negative, continuous, of compact support and for which $m(R)$ is not everywhere zero."* *[FF, Husain, Wilson-Ewing, 2024]*

Additionally, if $\partial_R \rho(R, t_0) \leq 0$, the time at which a shell-crossing singularity forms will be:

$$t_{bounce}(R) < t_{SCS}(R) < t_{bounce}(R) + \frac{2}{3}\sqrt{\Delta}$$

Gravitational shockwave/SCS identification

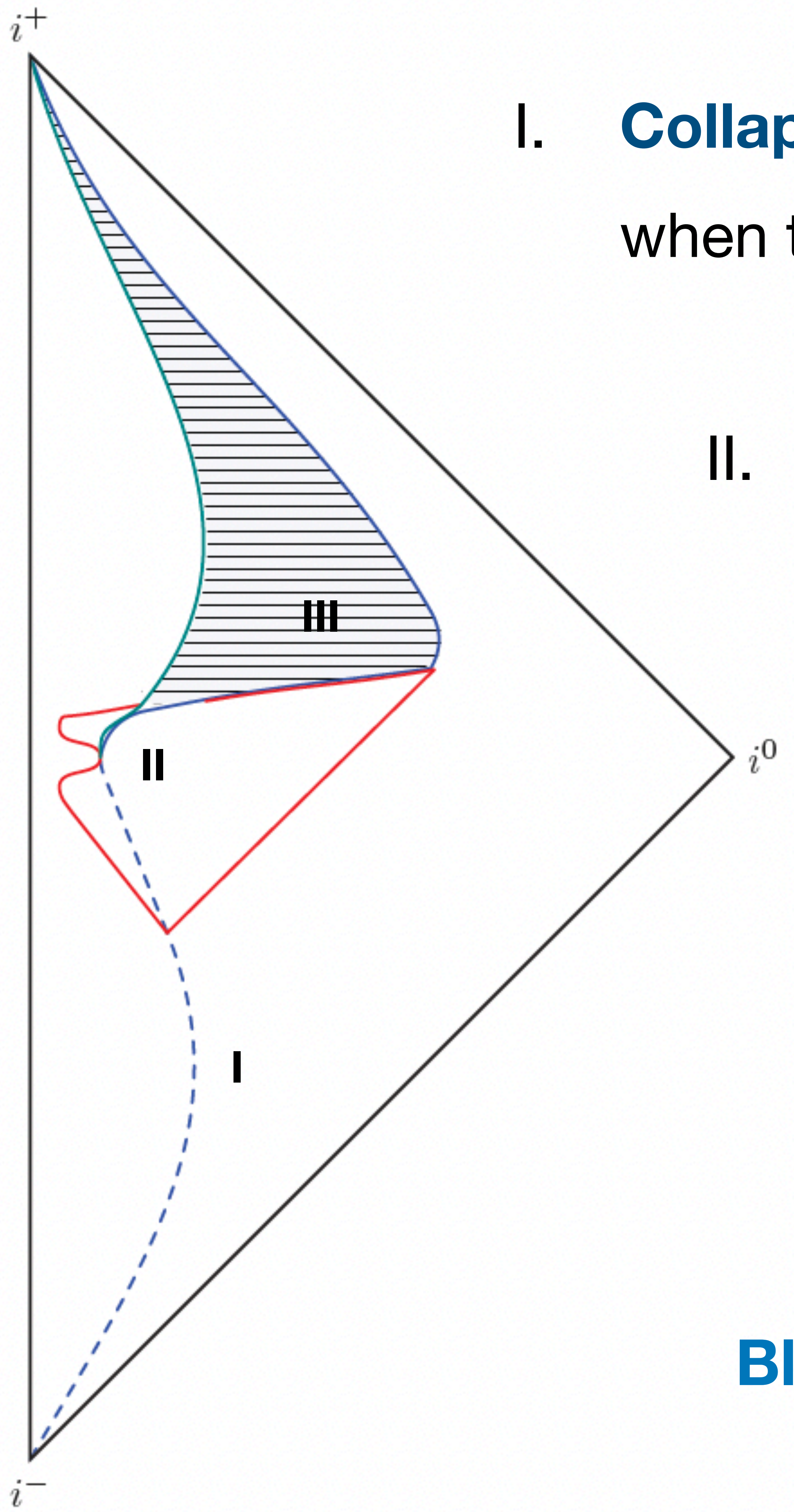
LTB coordinates cannot be used to study the dynamics when shell crossing singularities form.

However:

LTB coordinates		**Painlevé-Gullstrand coordinates**
Shell-crossing singularity in decoupled ODEs	$\longleftrightarrow$	Characteristic crossing in the PDE/physical discontinuity in the gravitational field

The dynamics beyond characteristic crossing in a PDE can be studied by using the integral form of the equations (this is commonly done to study shockwaves in fluid dynamics for example, but also for SCS in classical GR *[Nolan, 2003]*).

Penrose diagram and black hole life-time



I. **Collapse phase**: the energy density of the star progressively increases and its volume decreases; when the horizon forms the star becomes as a black hole.

II. **Bouncing phase**: when the energy density of the core becomes planckian, the shells of the core bounce and crush the collapsing shells of the tail.

→ A *shell-crossing singularity* forms, together with a *discontinuity* in the gravitational field.

III. **Shockwave phase**: the whole matter content of the star rapidly concentrates in a thin shell moving outward together with the discontinuity. When the shell reaches the outer horizon, the black hole explodes.

Black hole life-time:

$$T \sim \frac{8\pi M^2}{3m_p}$$

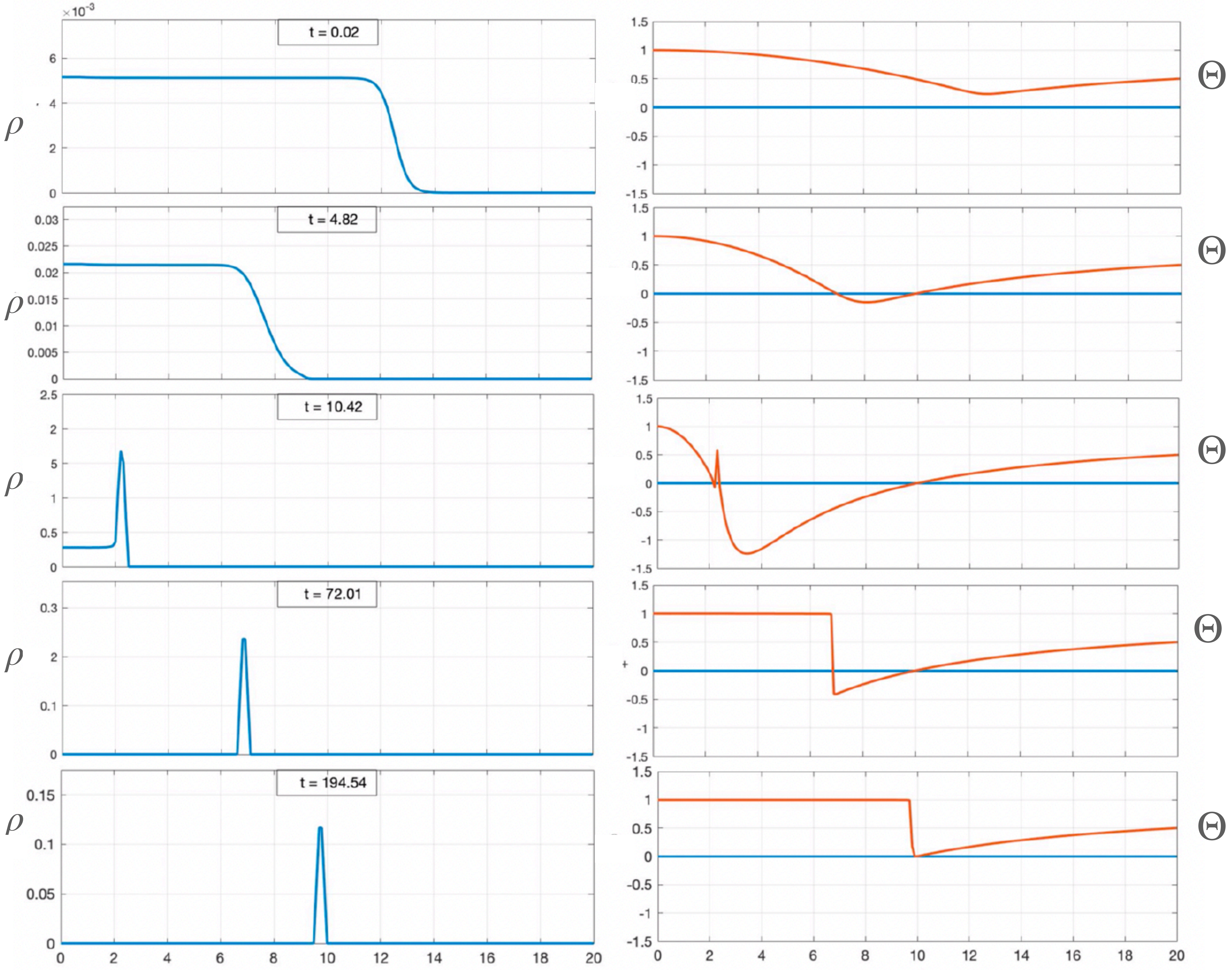
[Husain, Kelly, Santacruz, Wilson-Ewing, 2022]

Numerical simulation

Initial profile:

$$\rho(r, t_0) = C\left(1 - \tanh\frac{r - r_0}{\sigma}\right)$$

$$C \propto m_{tot} = 5, \quad r_0 = 13, \quad \sigma = 1.1$$



Beyond marginally bound configurations

The LTB equations in the most general case ($\varepsilon(R) \neq 0$):

$$\left(\frac{\dot{r}}{r}\right)^2 = \left(\frac{2Gm}{r^3} + \frac{\varepsilon}{r^2}\right)\left[1 - \Delta\left(\frac{2Gm}{r^3} + \frac{\varepsilon}{r^2}\right)\right],$$

For the initial profile:

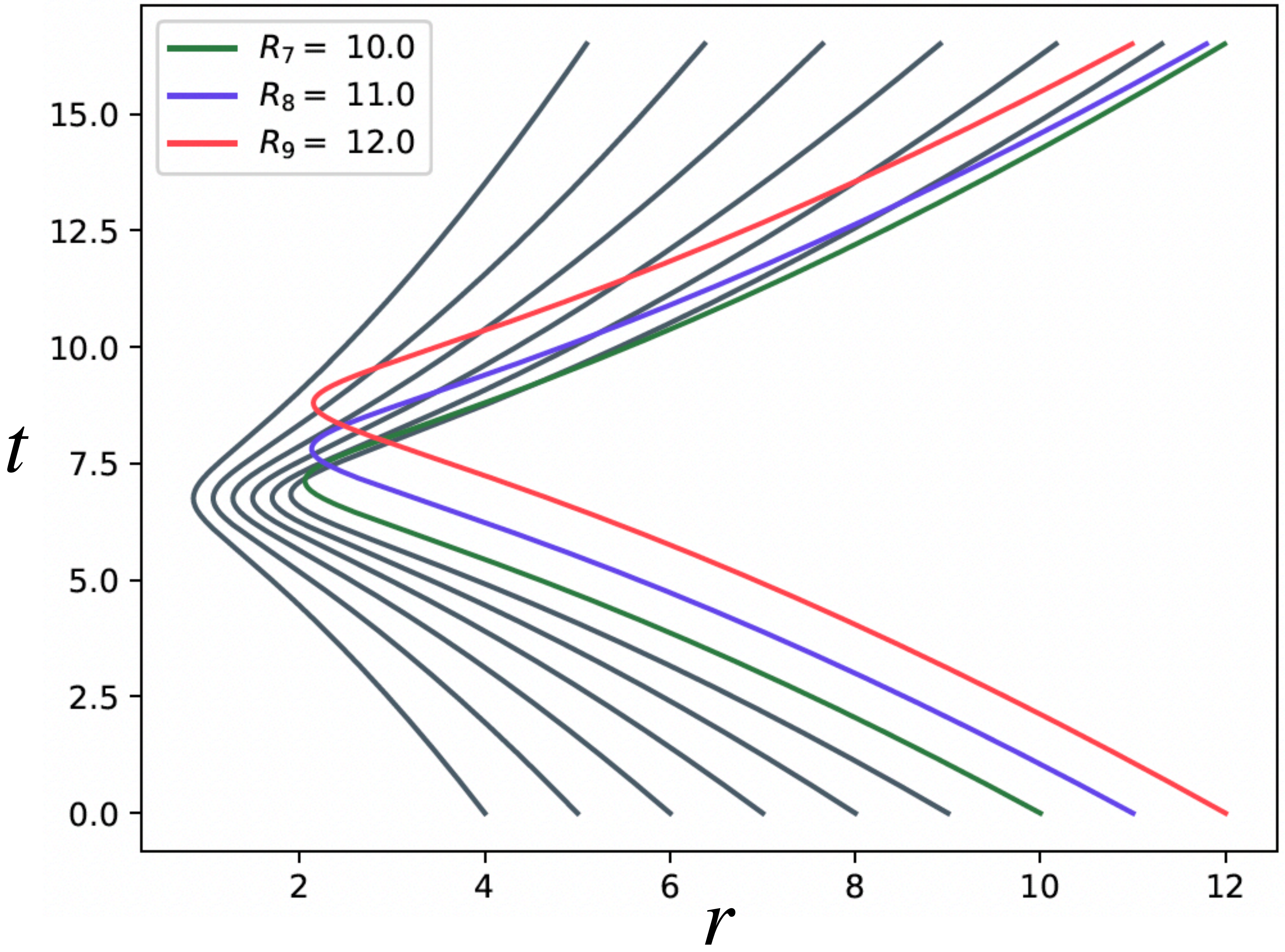
$$\rho(R, t_0) = C\left(1 - \tanh\frac{R - R_0}{\sigma}\right),$$

$$\varepsilon(R, t_0) = \begin{cases} -\alpha\frac{R^2}{R_0^2}, & \text{for } R < R_0 \\ -\alpha, & \text{for } R \geq R_0 \end{cases}$$

And parameters:

$$C \propto m_{tot} = 5, \quad R_0 = 10, \quad \sigma = 1.1, \quad \alpha = 0.01$$

[Cipriani, FF, Wilson-Ewing, 2024]



Conclusions

Shell-crossing singularities arise in effective LQG dust collapse for each continuous non-negative profile with compact support, including profiles arbitrarily close to OS.

To study the dynamics after SCS formation one has to seek for the weak solution ,which is dominated in the post-bounce phase by an outgoing propagating shell-crossing singularity (thin shell of matter)/discontinuity in the gravitational field.

Black holes die through an 'explosion' in a time $T \sim \frac{8\pi M^2}{3m_p}$, which for macroscopic black holes is much shorter than the Page time (avoiding completely the information loss paradox).

Thank you for your attention!