Seventeenth Marcel Grossman Meeting

Monday, July 08, 2024

Hairy black holes in extended Einstein-Maxwell-scalar theories with magnetic charge and kinetic couplings

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Phys. Rev. D (in press), arXiv:2403.17484 [gr-qc]

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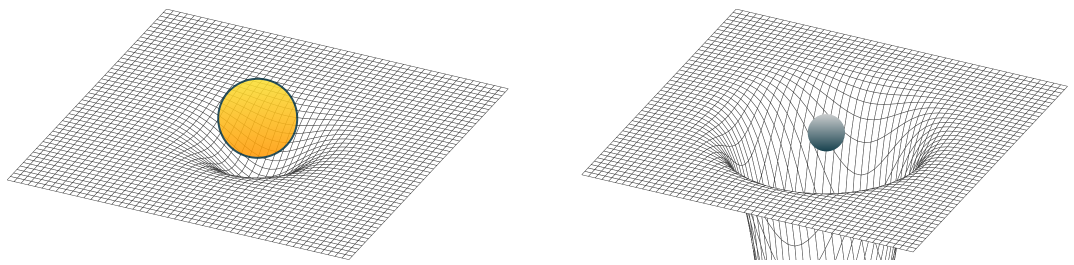
1. Introduction

background

◆ tests of General Relativity

GR describes weak gravity
with high precision.

GR's accuracy in strong field regime
has not yet been clarified well enough.



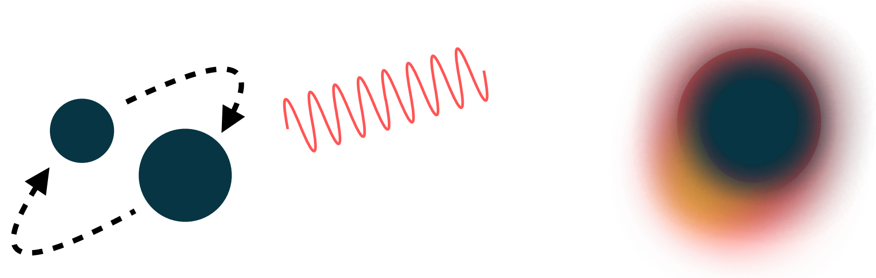
weak field regime
(e.g., solar system)

strong field regime
(e.g., black holes)

◆ observations of black holes

Observations of strong field regime
such as BH have significantly advanced.

(e.g., LVK Collaborations, EHT Collaboration)



Gravitational Waves

Black Hole shadow

◆ no hair theorem (without matter, stationary, and asymptotically flat)

Einstein-Maxwell theory ⇨ Kerr-Newman BH

\+ canonical scalar field ⇨ $\phi = \mathrm{const.}$ ⇨ Kerr-Newman BH

⇨ Can we find hairy BH solutions with scalar field minimally coupled to gravity?

1. Introduction

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu,$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\mathcal{E}^{\mu\nu\alpha\beta}F_{\alpha\beta},$$

previous research

◆ hairy solutions from dilatonic / axionic couplings

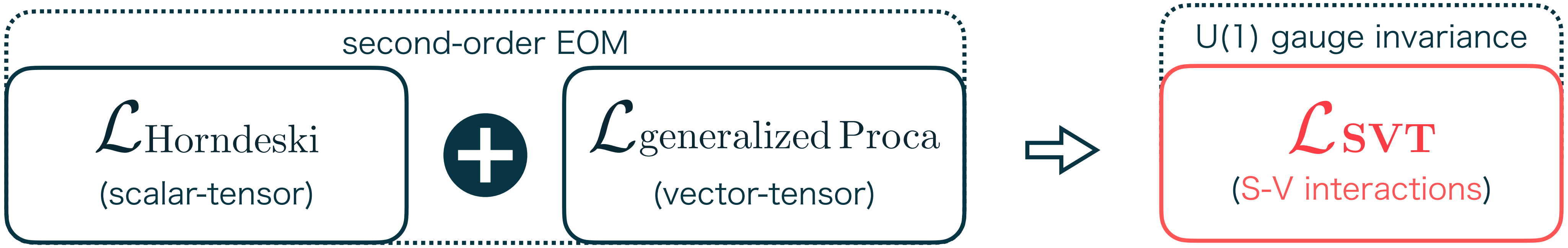
$$\mathrm{e}^{-2\phi}F_{\mu\nu}F^{\mu\nu}, \quad \phi F_{\mu\nu}\tilde{F}^{\mu\nu} \quad \Rightarrow \quad ds^2 = ds^2_{(\mathrm{GR})} + ds^2_{(\phi)}$$

scalar-vector couplings

BH could obtain “scalar hair”

◆ scalar-vector-tensor theories

Such interactions can be generalized requiring EOM to be up to second order.



second-order EOM

$\mathcal{L}_{\mathrm{Horndeski}}$ (scalar-tensor)

$+$

$\mathcal{L}_{\mathrm{generalized\ Proca}}$ (vector-tensor)

$\Rightarrow$

U(1) gauge invariance

$\mathcal{L}_{\mathrm{SVT}}$ (S-V interactions)

◆ S-V interactions in context of the SVT theories

$$\mathcal{L}_{\mathrm{SVT}} = f_2(\boxed{\phi}, \partial_\mu\phi, \boxed{F_{\mu\nu}}, \boxed{\tilde{F}_{\mu\nu}}) + \cdots \quad \Rightarrow$$

dilatonic / axionic coupling

✓ hairy solutions

1. Introduction

objectives

What will happen when including the coupling with $\partial_\mu\phi$?

$$\mathcal{L}_{\rm SVT} = f_2(\boxed{\phi}, \boxed{\partial_\mu\phi}, \boxed{F_{\mu\nu}}, \boxed{\tilde{F}_{\mu\nu}}) + \cdots$$

dilatonic/axionic coupling ⇨ ✓ hairy solutions

derivarive coupling ⇨ ?

The derivative interactions can be constructed as follows :

$$\boxed{\partial_\mu\phi\,\partial^\nu\phi\,F_{\nu\alpha}F^{\mu\alpha},\ \partial_\mu\phi\,\partial^\mu\phi\,F_{\alpha\beta}F^{\alpha\beta},\ \cdots}$$

- **Can we find new hairy BH solutions arising from the derivative interactions?**
- **How to distinguish the scalar hair originated from different interactions?**

2. Setup

general action & ansatz

◆ The second-order SVT interaction with U(1) gauge invariance

$$\boxed{\mathcal{L}^2_{\rm SVT} = f_2(\phi, X, F, \tilde{F}, Y)}$$

$$X = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi, \quad F = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad \tilde{F} = -\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}, \quad F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu, \quad \tilde{F}^{\mu\nu} = \frac{1}{2}\mathcal{E}^{\mu\nu\alpha\beta}F_{\alpha\beta}, \quad Y = \nabla_\mu\phi\nabla^\nu\phi F^{\mu\alpha}F_{\nu\alpha},$$

($\partial_\mu\phi\,\partial^\nu\phi\tilde{F}^{\mu\alpha}\tilde{F}_{\nu\alpha}$, $\partial_\mu\phi\,\partial^\nu\phi F^{\mu\alpha}\tilde{F}_{\nu\alpha}$ can be expressed in terms of the above scalar quantities)

◆ static and spherically symmetric spacetime

⇨ $$\boxed{ds^2 = -f(r)dt^2 + h^{-1}(r)dr^2 + r^2d\Omega^2, \quad d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2,}$$

event horizon is defined as, $r = r_h$ s.t. $f(r_h) = h(r_h) = 0$

◆ field ansatz

⇨ $$\phi = \phi(r), \quad A_\mu = (V(r), 0, 0, -P\cos\theta),$$ P : magnetic charge

2. Setup

$$X=-\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi,\quad F=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu},\quad \tilde{F}=-\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu},\quad Y=\nabla_\mu\phi\nabla^\nu\phi F^{\mu\alpha}F_{\nu\alpha},$$

constructing extended EMS theories

$$\mathcal{L}^2_{\rm SVT}=f_2(\phi,X,F,\tilde{F},Y)$$

$$'=\partial/\partial r$$

◆ background values of the scalar quantities

$$X=-\frac{h\,\phi'^2}{2},\quad F=\frac{h\,V'^2}{2f}-\frac{P^2}{2r^4},\quad \tilde{F}=\frac{PV'}{r^2}\sqrt{\frac{h}{f}},\quad Y=-\frac{h^2\phi'^2V'^2}{f}$$

In the presence of magnetic charge, $\tilde{F}\neq 0,\ Y\neq 4XF$

◆ previous research & our focus

dilatonic/axionic coupling $g_1(\phi)F,\ g_2(\phi)\tilde{F}$ ⇨ ✓ hairy solutions

derivative coupling $\bar{g}_3(\phi,X)Y$ ⇨ ?

◆ extended Einstein-Maxwell-scalar theories

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[\underbrace{\frac{M_{\rm pl}^2}{2}R}_{\text{GR}}+\underbrace{F+X}_{\text{kinetic terms}}+\underbrace{g_1(\phi,X)F+g_2(\phi,X)\tilde{F}+\bar{g}_3(\phi,X)Y}_{\text{interacting terms}}\right]$$

We normalize $\bar{g}_3(\phi,X)=g_3(\phi,X)/(4X)$ without loss of generality

If g_1, g_2, g_3 have zero/positive powers of X, the Lagrangian is regular at the horizon

3. Asymptotic solutions

◆ field equations

variational principle ⇨ equations of motion $E_f\,, E_h, E_V\,, E_\phi$

◆ boundary conditions

$r \simeq r_h$

regular at horizon

$r \to \infty$

asymptotically flat at spatial infinity

r

◆ assumption for the couplings

The couplings work as corrections to the Reissner-Nordström solution :

$$f_{\rm RN} = h_{\rm RN} = 1 - \frac{2M}{r} + \frac{Q^2 + P^2}{2M_{\rm pl}^2 r^2}$$

M : mass
Q : electric charge
P : magnetic charge

The RN metric can be written in the form

$$f_{\rm RN} = \left(1 - \frac{r_h}{r}\right)\left(1 - \mu\frac{r_h}{r}\right)$$

r_h : outer horizon

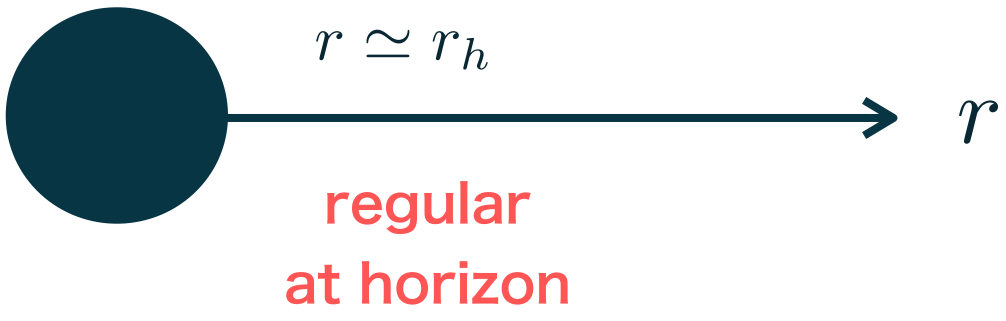
$$\mu = 2M/r_h - 1 \quad (0 < \mu < 1)$$

3.A. Asymptotic solutions (around the horizon)

◆ **series expansion of the variables**

$$f = \sum_{i=1}^{\infty} f_i (r - r_h)^i, \quad h = \sum_{i=1}^{\infty} h_i (r - r_h)^i,$$

$$V = V_0 + \sum_{i=1}^{\infty} V_i (r - r_h)^i, \quad \phi = \phi_0 + \sum_{i=1}^{\infty} \phi_i (r - r_h)^i,$$



$r \simeq r_h$

r

regular
at horizon

⇨ E_f, E_h, E_V, E_ϕ ⇨ solve for f_i, h_i, V_i, ϕ_i iteratively

⇨ $f(r) = f_1 (r - r_h) + \boldsymbol{f_2} (r - r_h)^2 + \cdots$

RN

RN + coupling

$$f_2 = f_{2,RN} + \boldsymbol{\Phi_{1,\phi}} \phi_1 / C \qquad \phi_i \propto \cdots \propto \phi_2 \propto \phi_1 \propto \boldsymbol{\Phi_{1,\phi}}$$

$$\Phi_{1,\phi} = \Phi_{1,\phi}(g_{1,\phi}, g_{2,\phi}, g_{3,\phi})$$

⇨ The metric is modified from the order of $(r - r_h)^2$

The ϕ-dependence in g_i is essential for the scalar hair

3.B. Asymptotic solutions (at large distances)

◆ **series expansion of the variables**

$$f = 1 + \sum_{i=1}^{\infty} \frac{\tilde{f}_i}{r^i}, \quad h = 1 + \sum_{i=1}^{\infty} \frac{\tilde{h}_i}{r^i},$$

$$V = V_\infty + \sum_{i=1}^{\infty} \frac{\tilde{V}_i}{r^i}, \quad \phi = \phi_\infty + \sum_{i=1}^{\infty} \frac{\tilde{\phi}_i}{r^i}.$$

asymptotically flat at spatial infinity

⇨ E_f, E_h, E_V, E_ϕ ⇨ solve for $\tilde{f}_i, \tilde{h}_i, \tilde{V}_i, \tilde{\phi}_i$ iteratively

⇨ $$f(r) = 1 + \underbrace{\frac{\tilde{f}_1}{r}}_{\text{RN}} + \underbrace{\frac{\boldsymbol{\tilde{f}_2}}{r^2}}_{\text{RN + coupling}} + \cdots \qquad h(r) = 1 + \underbrace{\frac{\tilde{h}_1}{r}}_{\text{RN}} + \underbrace{\frac{\boldsymbol{\tilde{h}_2}}{r^2}}_{\text{RN + coupling}} + \cdots$$

$$\tilde{h}_2 - \tilde{f}_2 = \tilde{\phi}_1^2 / \tilde{C} \qquad \phi(r) = \phi_\infty + \frac{\tilde{\phi}_1}{r} + \cdots$$

⇨ The couplings start to appear in f, h at the order of $1/r^2$

The scalar charge $\tilde{\phi}_1$ triggers the deviation from RN

4. Concrete models

◆ hairy solutions from $\bar{g}_3 Y$

$$\left(\mathcal{L}_{\rm int} = g_1(\phi, X)F + g_2(\phi, X)\tilde{F} + \frac{g_3(\phi, X)}{4X}Y\right)$$

$$\boxed{g_i(\phi, X)\ = c_i\phi/M_{\rm pl}}$$

Previous research showed that

$g_1 F,\ g_2\tilde{F}$ ⇨ scalar hair

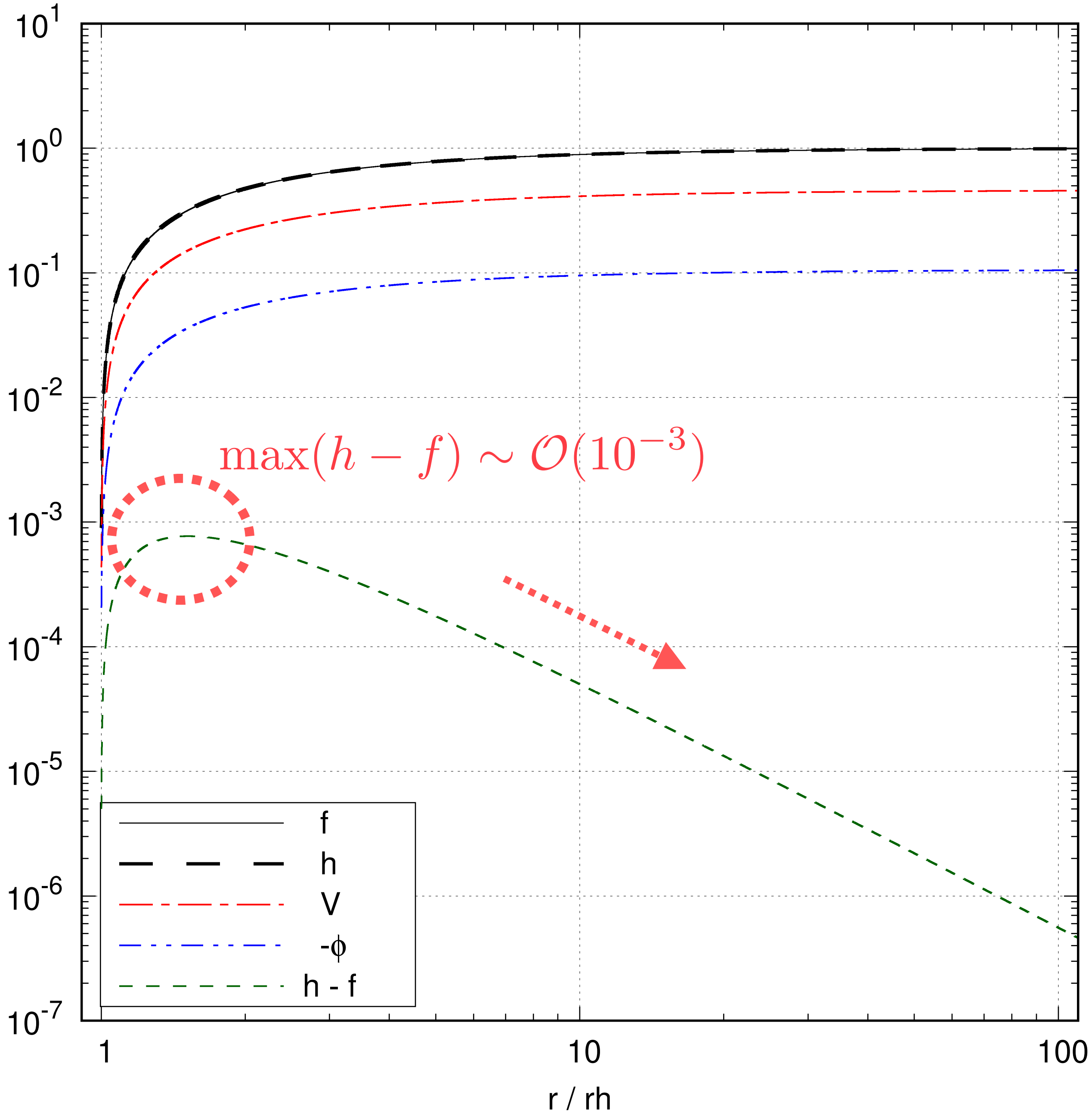
To examine whether

$\bar{g}_3 Y$ ⇨ new solutions ?

, we focus on the case :

$$\boxed{g_3 = c_3\phi/M_{\rm pl},\ g_1 = g_2 = 0}$$

⇨ New type of BH solutions with scalar hair arising from the derivative coupling



For the initial condition, we used the asymptotic solutions near the horizon with $c_3 = 1, \mu = 0.1, P = 0.1M_{\rm pl}r_h, \phi_0 = -1.0\times 10^{-4}M_{\rm pl}, V_0 = 0$ at $r = 1.001r_h$

4. Concrete models

◆ large coupling limit (analytic)

$$\left(\mathcal{L}_{\rm int} = g_1(\phi, X)F + g_2(\phi, X)\tilde{F} + \frac{g_3(\phi, X)}{4X}Y\right)$$

$$\Phi_{1,\phi} = \Phi_{1,\phi}(g_{1,\phi}, g_{2,\phi}, g_{3,\phi})$$

$$g_i(\phi, X) = c_i\phi/M_{\rm pl}$$

Deviation from the RN solution (around the horizon) :

$$|h - f| = \left|\frac{\Phi_{1,\phi}^2}{2M_{\rm pl}^2 r_h^2\,(1-\mu)(2r_h^4 - \Phi_{1,X})}\right|(r - r_h)^2 + \cdots$$

To distinguish effects of the couplings, consider large coupling limit : $|c_i| \gg 1, \;\; d_i = 0$

$$\frac{|h - f|}{(r - r_h)^2} \simeq \begin{cases} \left|\dfrac{P^2 c_1 (2\mu M_{\rm pl}^3 r_h^2 - P^2 c_1 \phi_0)}{(1-\mu)\phi_0 M_{\rm pl}^4 r_h^6} - \dfrac{\mu(\mu M_{\rm pl}^2 r_h^2 + 2P^2)}{(1-\mu)\phi_0^2 r_h^4}\right| + \mathcal{O}\left(\dfrac{1}{c_1}\right) & (|c_1| \gg \{|c_2|, |c_3|\}) \\ & (c_1\phi_0 > 0) \\ \dfrac{P^2 c_2^2 (2\mu M_{\rm pl}^2 r_h^2 - P^2)}{(1-\mu) M_{\rm pl}^4 r_h^6} & (|c_2| \gg \{|c_3|, |c_1|\}) \\ \left|-\dfrac{(2\mu M_{\rm pl}^2 r_h^2 - P^2)^2}{4(1-\mu)\phi_0^2 M_{\rm pl}^2 r_h^6} + \dfrac{(2\mu M_{\rm pl}^2 r_h^2 - P^2)^2}{2(1-\mu) c_3 \phi_0^3 M_{\rm pl} r_h^6}\right| + \mathcal{O}\left(\dfrac{1}{c_3^2}\right) & (|c_3| \gg \{|c_1|, |c_2|\}) \end{cases}$$

⇨ dominant contribution will switch from c_1 to c_1^2

⇨ |h - f| monotonically increases in proportion to c_2^2

⇨ |h - f| saturates in the large limit of c_3

4. Concrete models

◆ large coupling limit (numerical)

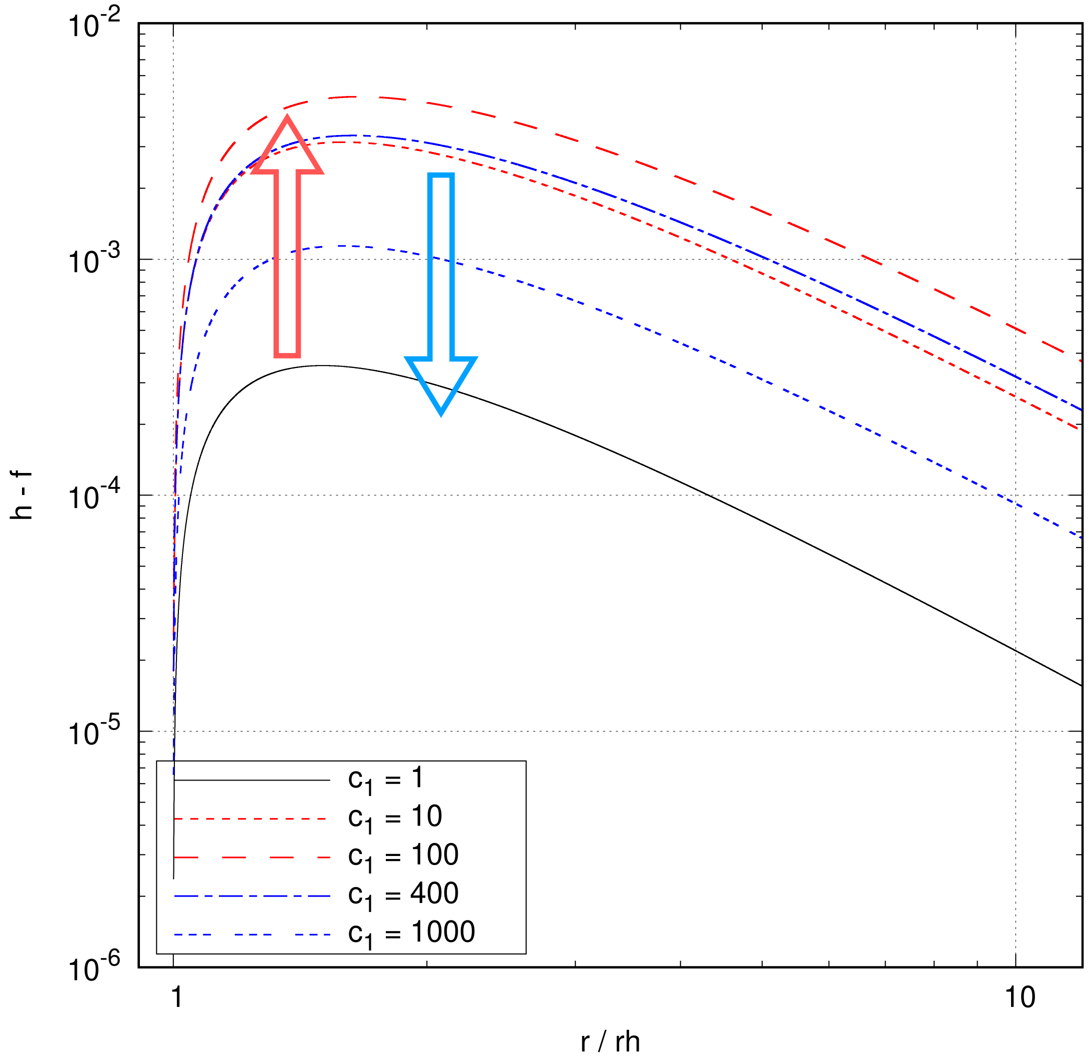
$$\left(\mathcal{L}_{\rm int} = g_1(\phi,X)F + g_2(\phi,X)\tilde{F} + \frac{g_3(\phi,X)}{4X}Y\right)$$

$$g_i(\phi,X) = c_i\phi/M_{\rm pl}$$

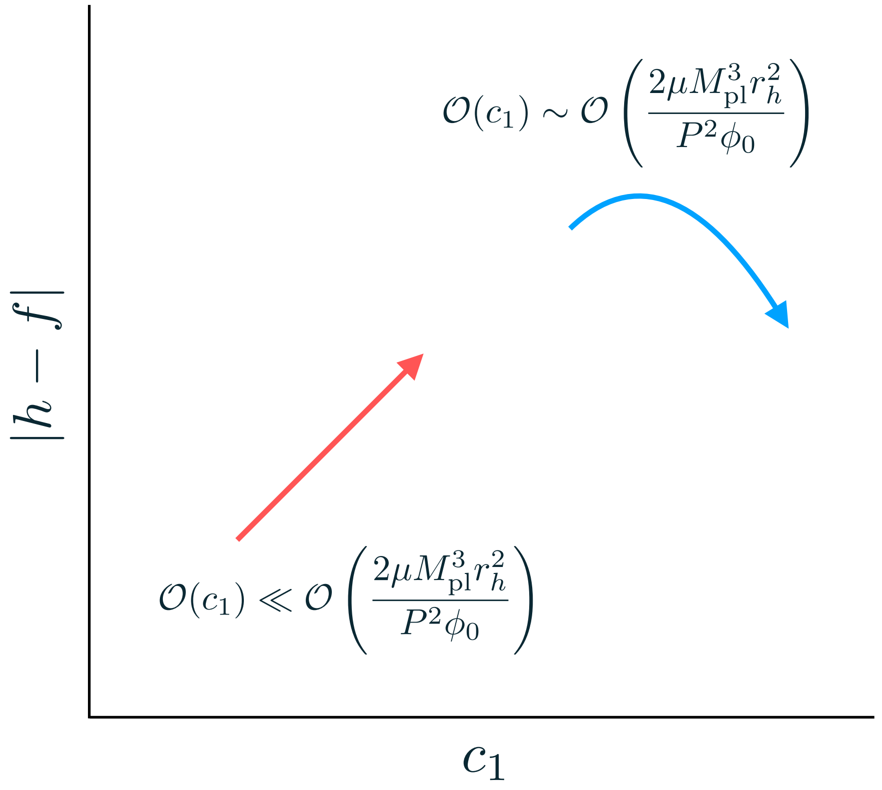
Analytic solution :

$$\frac{|h-f|}{(r-r_h)^2} \simeq \left| \frac{P^2 c_1 (2\mu M_{\rm pl}^3 r_h^2 - P^2 c_1 \phi_0)}{(1-\mu)\phi_0 M_{\rm pl}^4 r_h^6} - \frac{\mu(\mu M_{\rm pl}^2 r_h^2 + 2P^2)}{(1-\mu)\phi_0^2 r_h^4} \right| + \mathcal{O}\left(\frac{1}{c_1}\right)$$

Numerical solution :



For the initial condition, we used the asymptotic solutions with $\mu = 0.1, P = 0.01 M_{\rm pl} r_h, \phi_0 = 0.5 M_{\rm pl}, V_0 = 0$ at $r = 1.001 r_h$



$$\mathcal{O}(c_1) \sim \mathcal{O}\left(\frac{2\mu M_{\rm pl}^3 r_h^2}{P^2\phi_0}\right)$$

$$\mathcal{O}(c_1) \ll \mathcal{O}\left(\frac{2\mu M_{\rm pl}^3 r_h^2}{P^2\phi_0}\right)$$

There is a threshold at which dominant contribution switches

4. Concrete models

◆ large coupling limit (numerical)

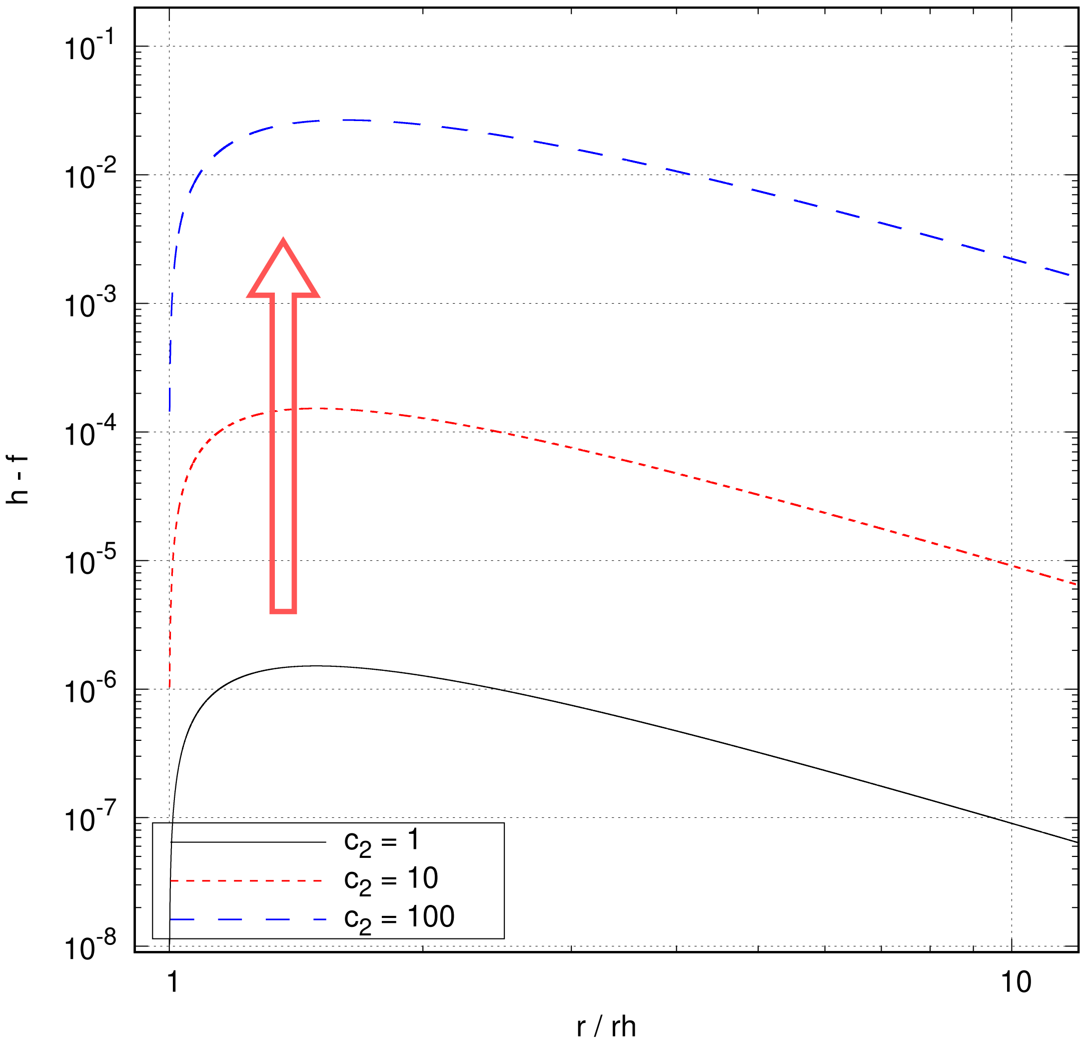
$$\left(\mathcal{L}_{\rm int} = g_1(\phi,X)F + g_2(\phi,X)\tilde{F} + \frac{g_3(\phi,X)}{4X}Y\right)$$

$$g_i(\phi,X) \;= c_i\phi/M_{\rm pl}$$

Analytic solution :

$$\frac{|h-f|}{(r-r_h)^2} \simeq \frac{P^2\boldsymbol{c_2^2}(2\mu M_{\rm pl}^2 r_h^2 - P^2)}{(1-\mu)M_{\rm pl}^4 r_h^6}$$

Numerical solution :



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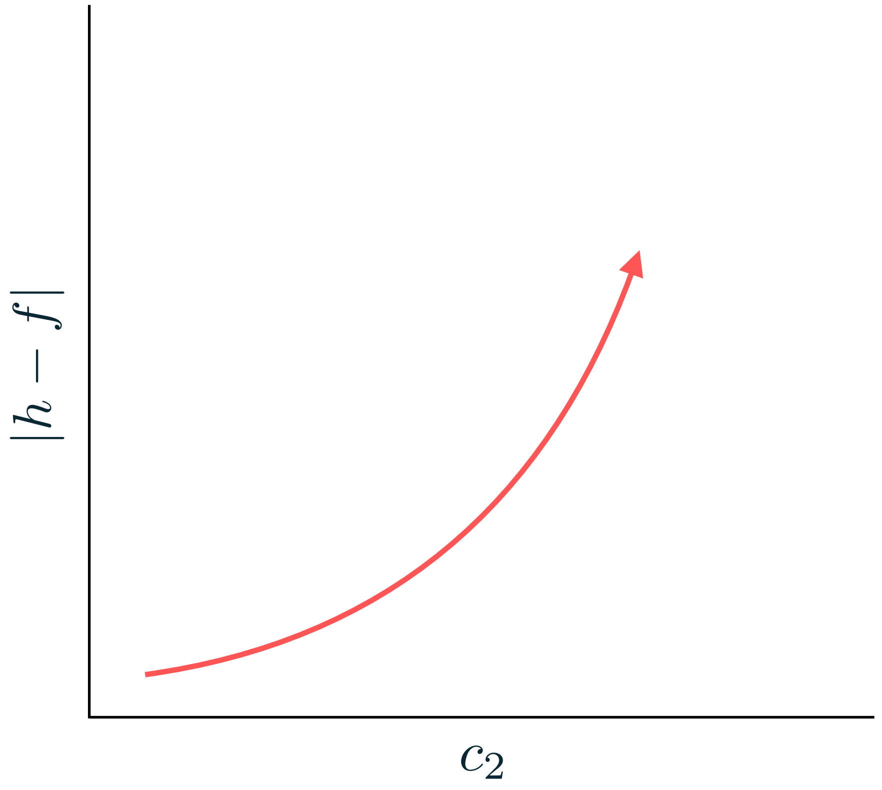

$|h-f|$ increases
in proportion to c_2^2

4. Concrete models

◆ large coupling limit (numerical)

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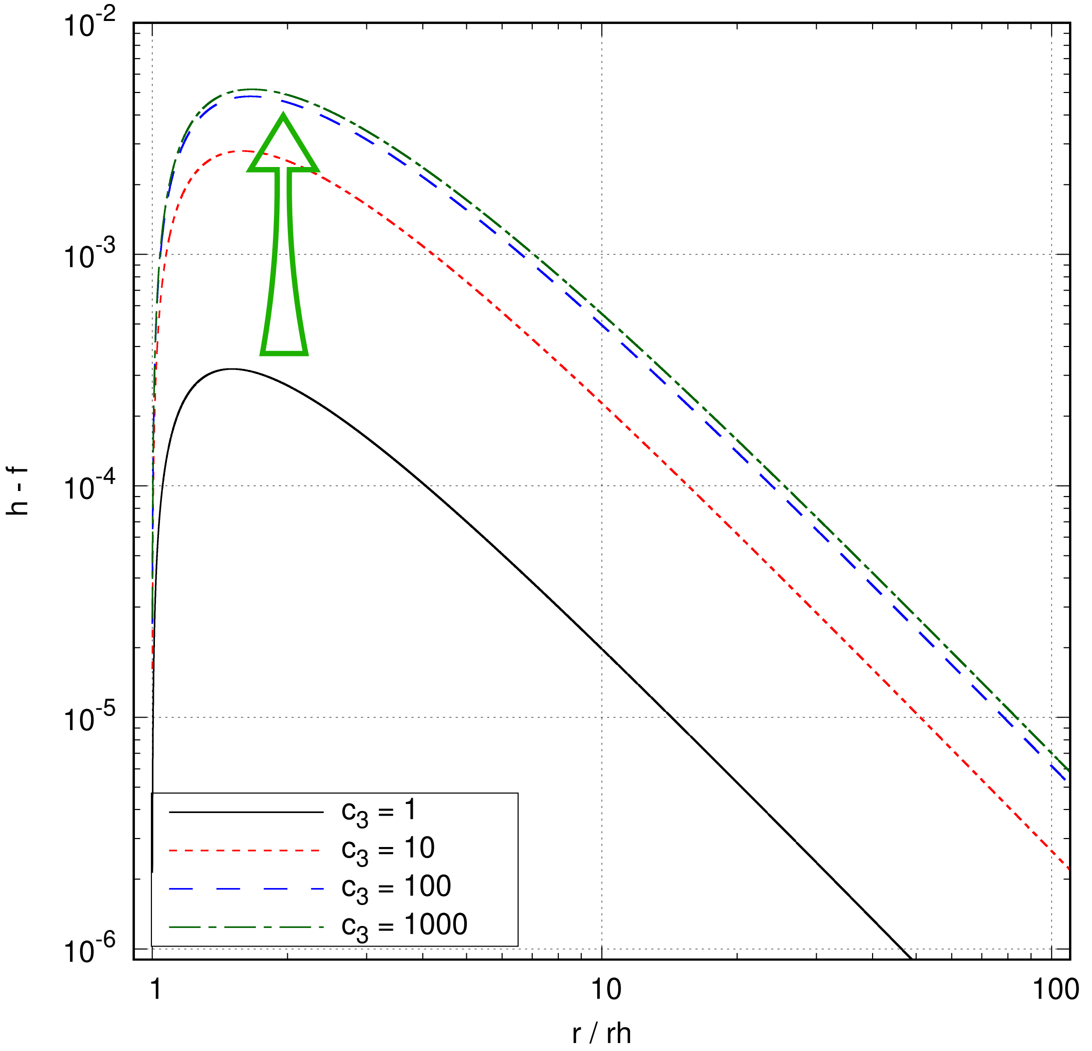
$$g_i(\phi, X) = c_i\phi/M_{\rm pl}$$

Analytic solution :

$$\frac{|h-f|}{(r-r_h)^2} \simeq \left| -\frac{(2\mu M_{\rm pl}^2 r_h^2 - P^2)^2}{4(1-\mu)\phi_0^2 M_{\rm pl}^2 r_h^6} + \frac{(2\mu M_{\rm pl}^2 r_h^2 - P^2)^2}{2(1-\mu)c_3\phi_0^3 M_{\rm pl} r_h^6} \right| + \mathcal{O}\left(\frac{1}{c_3^2}\right)$$

Numerical solution :



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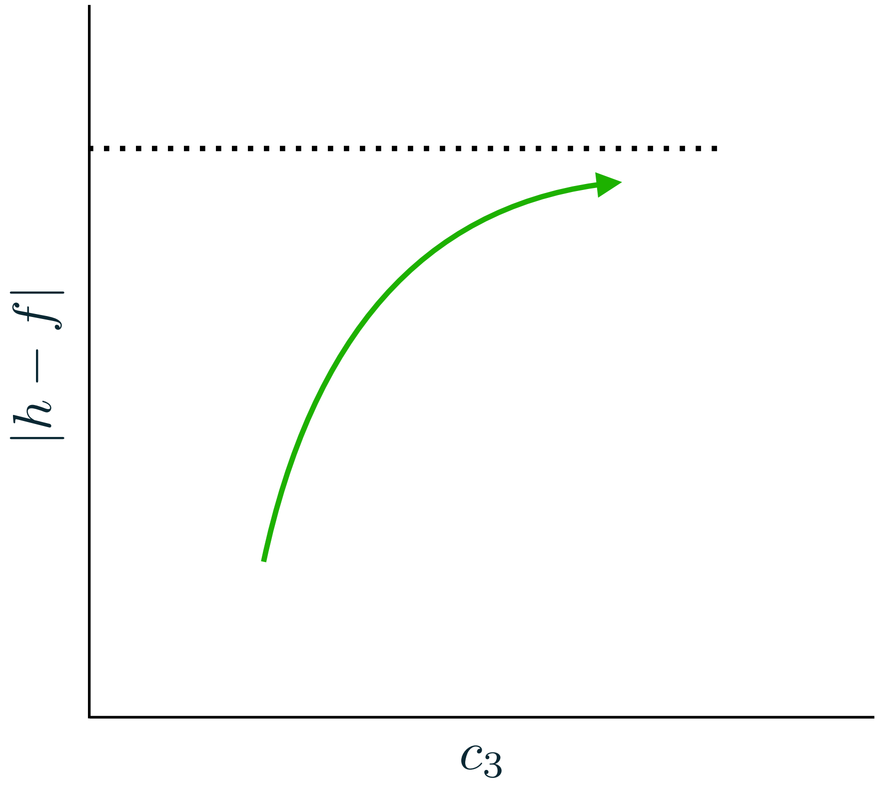


$|h-f|$ saturates
in the large limit of c_3

5. Conclusions

objectives & conclusions

$$Y = \nabla_\mu \phi \nabla^\nu \phi F^{\mu\alpha} F_{\nu\alpha},$$

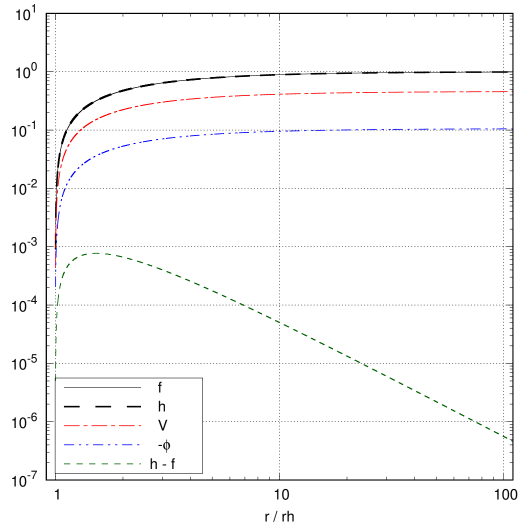
dilatonic/axionic coupling $g_1(\phi)F,\ g_2(\phi)\tilde{F}$ ⇨ ✓ hairy solutions

derivative coupling $\bar{g}_3(\phi, X)Y$ ⇨ ?

◆ **Can we find new hairy BH solutions arising from the derivative interactions?**

$\bar{g}_3(\phi, X)Y$ ⇨ **hairy solutions** \ new /

ϕ-dependence in g_i is essential for scalar hair

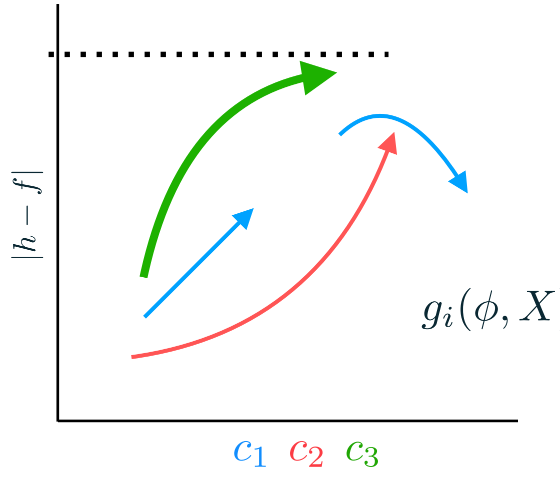


(confirmed analytically and numerically)

(X-dependence can modify the hairy solutions)

◆ **How to distinguish the scalar hair originated from different interactions?**

$|h - f|$ ⇨ different behavior at **large coupling limit**



$g_i(\phi, X) = c_i \phi / M_{\rm pl}$

Seventeenth Marcel Grossman Meeting

Monday, July 08, 2024

Hairy black holes in extended Einstein-Maxwell-scalar theories with magnetic charge and kinetic couplings

K. Taniguchi, S. Takagishi, R. Kase (Tokyo U. of Sci.)

Phys. Rev. D (in press), arXiv:2403.17484 [gr-qc]

Appendix A | equations of motion

$$
\begin{aligned}
&E_f \qquad 2M_{\rm pl}^2 r f h' = 2M_{\rm pl}^2 f(1-h) - r^2 h\left[f\phi'^2 + (1+g_1+g_3)V'^2\right] - \frac{P^2 f(1+g_1)}{r^2}\,,\\
&E_h \qquad 2M_{\rm pl}^2 r h f' = 2M_{\rm pl}^2 f(1-h) + r^2 h\left[f\phi'^2 - (1+g_1+g_3)V'^2\right] - \frac{P^2 f(1+g_1)}{r^2}\\
&\qquad\qquad + \left[\left(r^2 h(g_{1,X}+g_{3,X})V' + 2P\sqrt{fh}\,g_{2,X}\right)V' - \frac{P^2 f g_{1,X}}{r^2}\right]h\phi'^2\,,\\
&E_\phi \qquad J_\phi' = \mathcal{P}_\phi\,,\\
&E_V \qquad J_A' = 0\,,
\end{aligned}
$$

$$
\begin{aligned}
J_\phi &= -\left[\frac{r^2}{2}\sqrt{\frac{h}{f}}\,(g_{1,X}+g_{3,X})\,V'^2 + g_{2,X}P\,V' + \sqrt{\frac{f}{h}}\left(r^2 - \frac{g_{1,X}P^2}{2r^2}\right)\right]h\,\phi'\,,\\
\mathcal{P}_\phi &= \frac{r^2}{2}\sqrt{\frac{h}{f}}\,(g_{1,\phi}+g_{3,\phi})\,V'^2 + g_{2,\phi}\,P\,V' - \sqrt{\frac{f}{h}}\,\frac{g_{1,\phi}\,P^2}{2r^2}\,,\\
J_A &= r^2\sqrt{\frac{h}{f}}\,(1+g_1+g_3)\,V' + g_2\,P\,.
\end{aligned}
$$

Appendix B ｜ possible parameter space

(1) a condition for the couplings to work as corrections to the RN solution

$$\left|\frac{\Phi_{1,\phi}^2}{8(\Phi_{1,X}-2r_h^4)M_{\rm pl}^2}\right| \ll |(1-\mu)(2\mu-1)|\ ,$$

$$f_2 = \frac{2\mu-1}{r_h^2} + \left(\frac{\Phi_{1,\phi}}{8M_{\rm pl}^2 r_h^3}\right)\phi_1\,,$$

$$\phi_1 = -\frac{r_h}{1-\mu}\left(\frac{\Phi_{1,\phi}}{\Phi_{1,X}-2r_h^4}\right)$$

(2) a condition for avoiding discontinuity

$$ad - bc < 0$$

$$E_\phi : a\phi'' + bV'' = \cdots$$
$$E_V : c\phi'' + dV'' = \cdots$$

⇨

$$\phi'' = \cdots/(ad-bc)$$
$$V'' = \cdots/(ad-bc)$$

$$a = \frac{J_\phi}{\phi'} - h\phi'\,\frac{\partial J_\phi}{\partial X}\,, \qquad b = -h\phi'\left[r^2\sqrt{\frac{h}{f}}\,(g_{1,X}+g_{3,X})\,V' + g_{2,X}P\right]$$

$$c = -h\phi'\,\frac{\partial J_A}{\partial X}\,, \qquad d = r^2\sqrt{\frac{h}{f}}\,(1+g_1+g_3)\,.$$

In the absence of any coupling, $ad-bc=-r^4h<0$

(3) a condition for the existence of horizon

$$\gamma_{g_1g_3}\left[2\mu M_{\rm pl}^2 r_h^2 - (1+g_1)P^2\right] \geq 0$$

Appendix C ｜ effects of X-dependence

◆ Effects of X-dependence in g_i (analytic)

$\left(\mathcal{L}_{\rm int} = g_1(\phi, X)F + g_2(\phi, X)\tilde{F} + \frac{g_3(\phi, X)}{4X}Y\right)$

Deviation from the RN solution (around the horizon) :

$$|h-f| = \left|\frac{\Phi_{1,\phi}^2}{2M_{\rm pl}^2 r_h^2\,(1-\mu)(2r_h^4 - {\color{red}\Phi_{1,X}})}\right|(r-r_h)^2 + \cdots$$

⇨ deviation should be suppressed

$g_i(\phi, X) = \frac{{\color{red}c_i}\phi}{M_{\rm pl}}\left(1 + \frac{{\color{red}d_i}X}{M_{\rm pl}^4}\right)$ To clarify the difference among $d_1,\ d_2,\ d_3$, take large coupling limit ${\color{red}|d_i| \gg 1}$

$$\frac{|h-f|}{(r-r_h)^2} \simeq \begin{cases} \left|-\frac{\left[\mu M_{\rm pl}^3 r_h^2 - P^2(c_1\phi_0 + M_{\rm pl})\right] c_1 M_{\rm pl}}{(1-\mu)(c_1\phi_0 + M_{\rm pl})\,{\color{red}d_1}\phi_0 r_h^2} + \frac{M_{\rm pl}^6 r_h^2}{(1-\mu)\,{\color{red}d_1^2}\phi_0^2}\right| + \mathcal{O}\left(\frac{1}{d_1^3}\right) & (|d_1| \gg \{|d_2|,\ |d_3|\}) \\ \left|-\frac{P c_2 M_{\rm pl}\sqrt{2\mu M_{\rm pl}^2 r_h^2 - P^2}}{(1-\mu)\,{\color{red}d_2}\phi_0 r_h^2} + \frac{M_{\rm pl}^6 r_h^2}{(1-\mu)\,{\color{red}d_2^2}\phi_0^2}\right| + \mathcal{O}\left(\frac{1}{d_2^3}\right) & (|d_2| \gg \{|d_3|,\ |d_1|\}) \\ \left|-\frac{(2\mu M_{\rm pl}^2 r_h^2)c_3 M_{\rm pl}^2}{2(1-\mu)(c_3\phi_0 + M_{\rm pl})\,{\color{red}d_3}\phi_0 r_h^2} + \frac{M_{\rm pl}^6 r_h^2}{(1-\mu)\,{\color{red}d_3^2}\phi_0^2}\right| + \mathcal{O}\left(\frac{1}{d_3^3}\right) & (|d_3| \gg \{|d_1|,\ |d_2|\}) \end{cases}$$

⇨ The qualitative behaviors for three cases are the same

Appendix C | effects of X-dependence

◆ Effects of X-dependence in g_i (numerical)

$$\left(\mathcal{L}_{\rm int} = g_1(\phi, X)F + g_2(\phi, X)\tilde{F} + \frac{g_3(\phi, X)}{4X}Y\right)$$

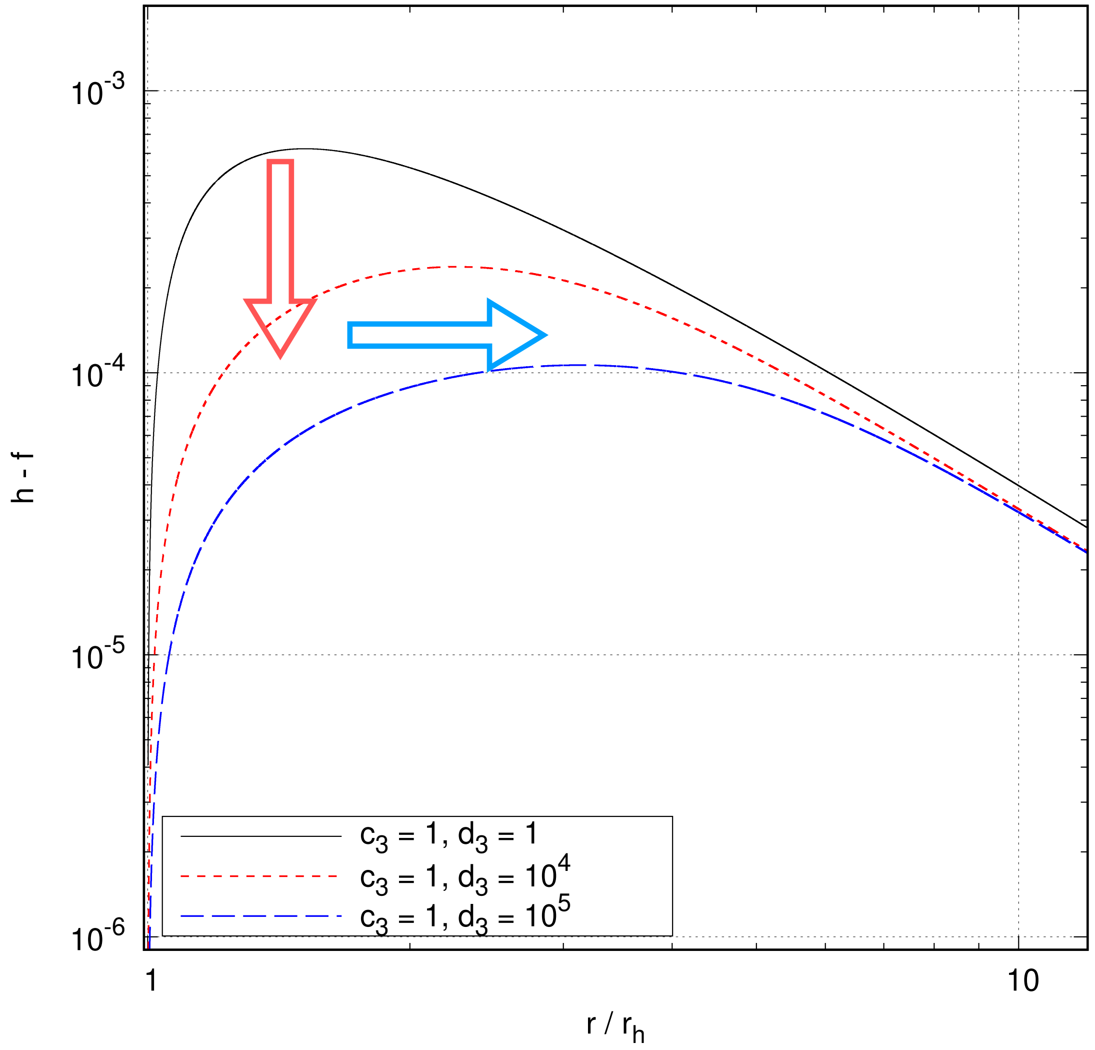
$$g_i(\phi, X) = \frac{c_i\phi}{M_{\rm pl}}\left(1 + \frac{d_i X}{M_{\rm pl}^4}\right)$$

Analytic solution :

$$\frac{|h-f|}{(r-r_h)^2} \simeq \left|-\frac{(2\mu M_{\rm pl}^2 r_h^2)c_3 M_{\rm pl}^2}{2(1-\mu)(c_3\phi_0 + M_{\rm pl})\, d_3\phi_0 r_h^2} + \frac{M_{\rm pl}^6 r_h^2}{(1-\mu)\, d_3^2\phi_0^2}\right| + \mathcal{O}\left(\frac{1}{d_3^3}\right)$$

$$X = -\frac{h\,\phi'^2}{2}$$

Numerical solution :



For the initial condition, we used the asymptotic solutions with at

⇩ : The deviation is suppressed when d_i is significantly large

⇨ : The maximum point shifts to the direction of large r

since X starts to grow
in the region away from the horizon