Universitat de les Illes Balears

IAC3 Institute of Applied Computing & Community Code.

*MG17-Pescara, July 11, 2024*

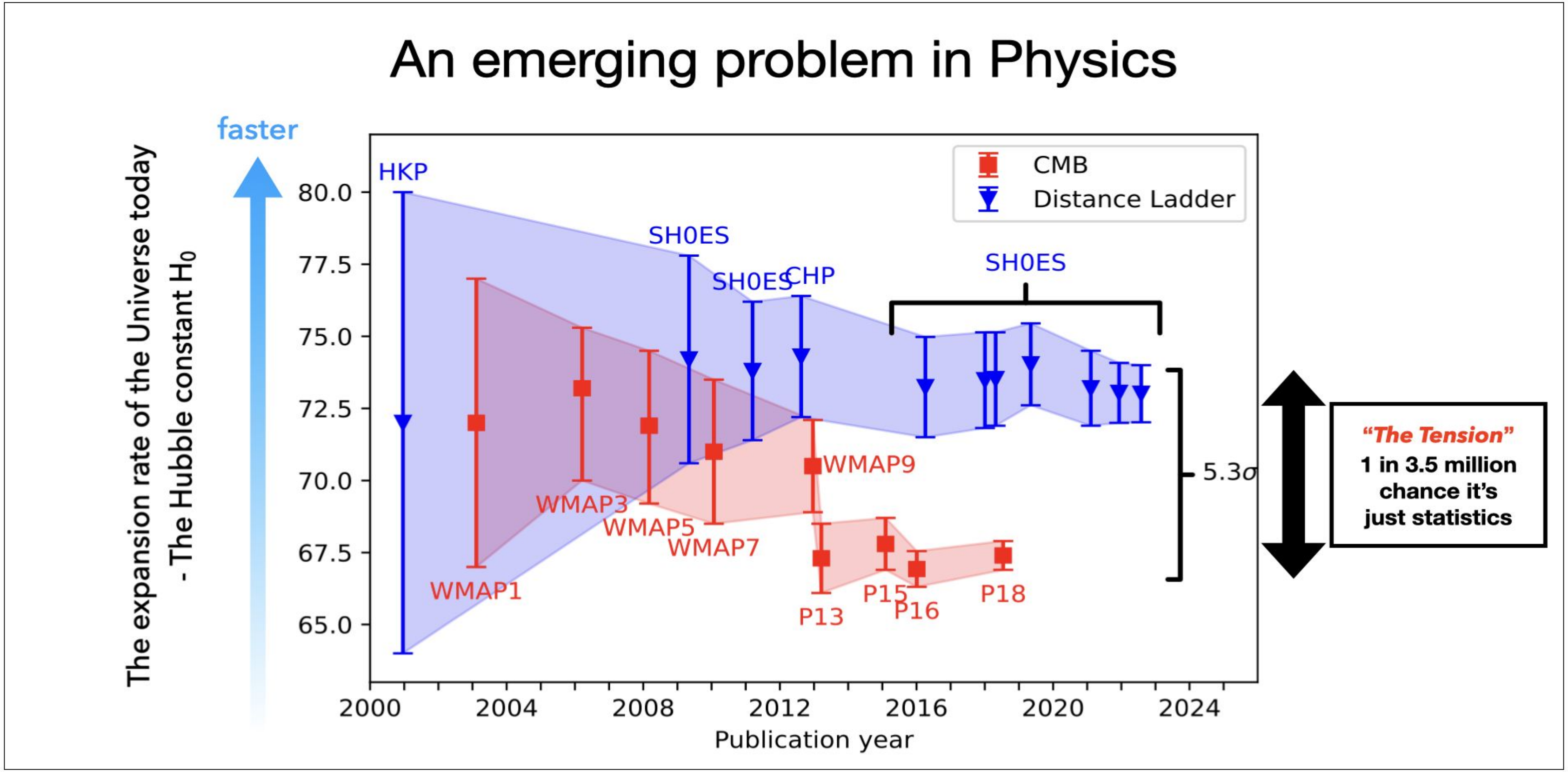
# Gravitational-Wave Cosmology with Large-Scale Structure Correlations

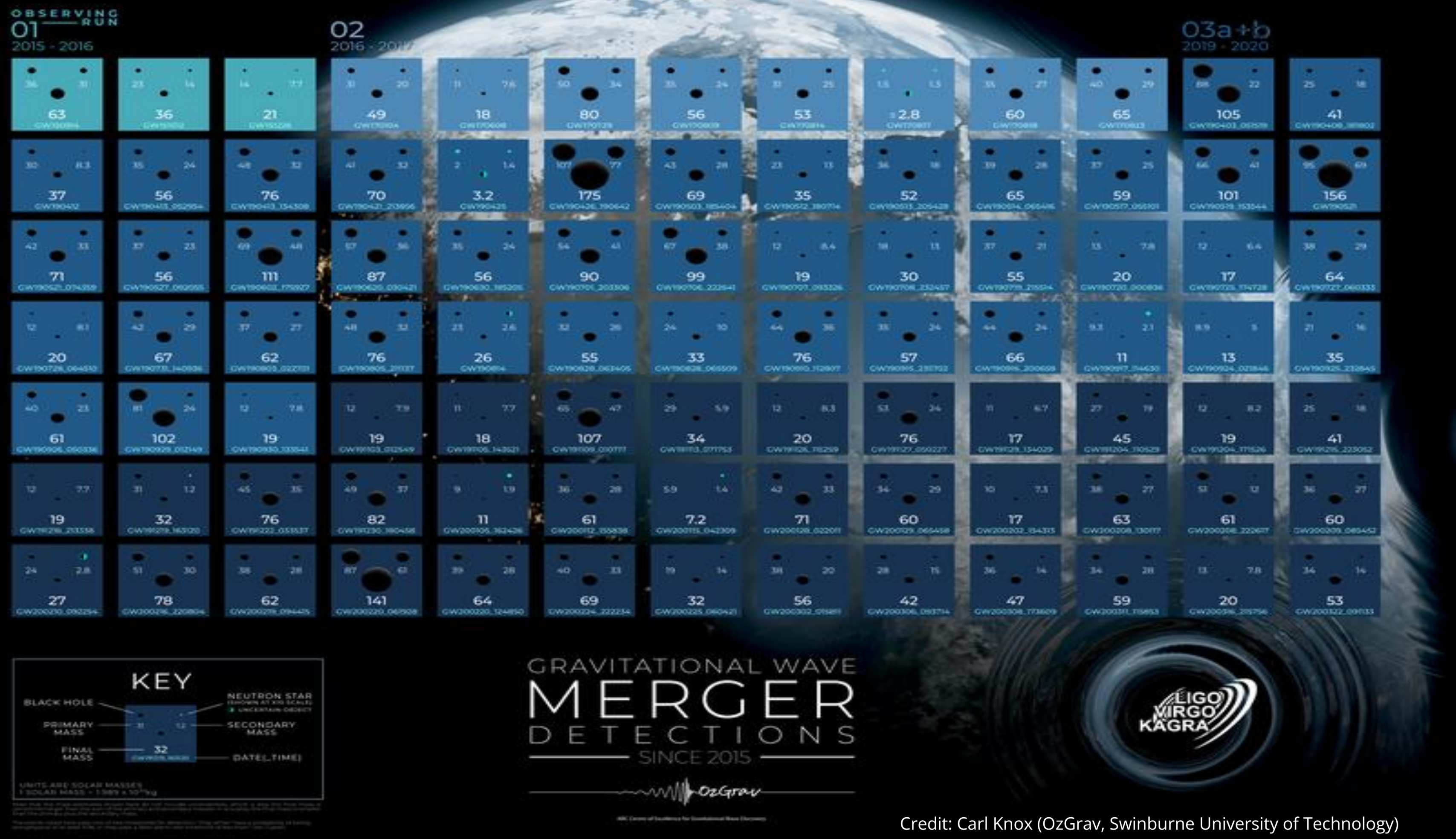
SAYANTANI BERA

With Tathagata Ghosh, Divya Rana, Surhud More and Sukanta Bose

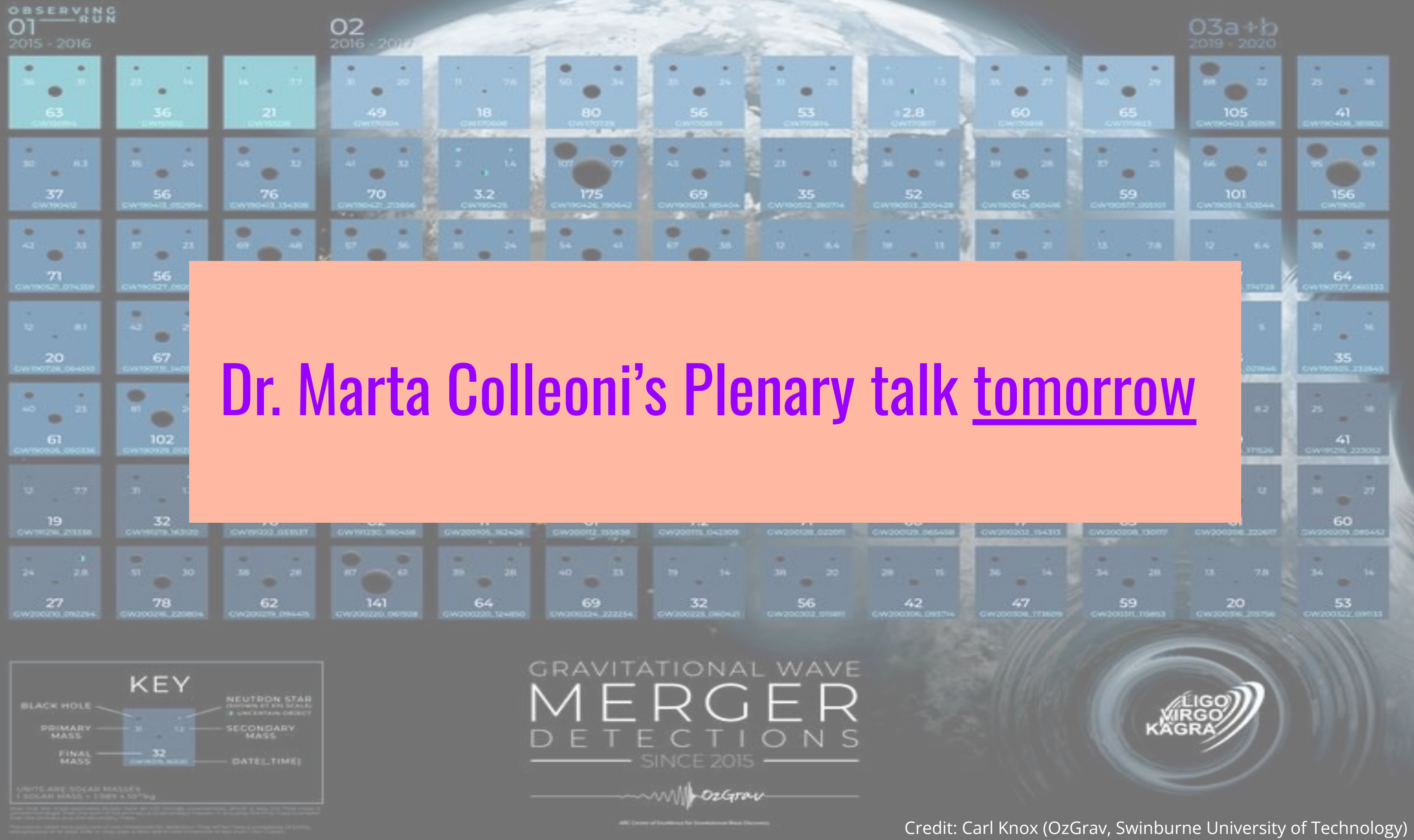
# Motivation

Image: *D'arcy Kenworthy*





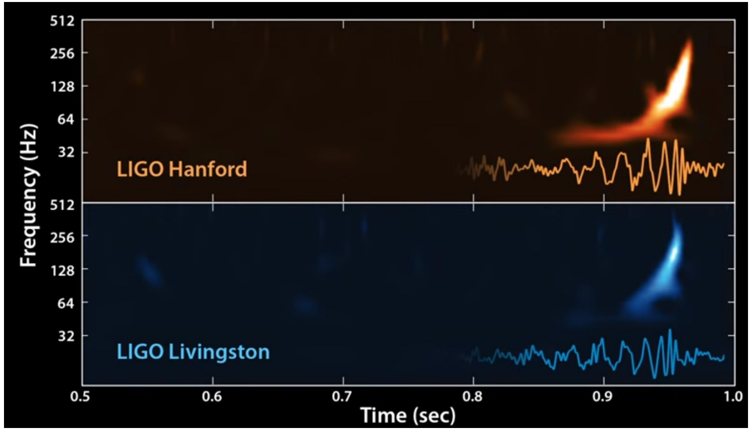

Credit: Carl Knox (OzGrav, Swinburne University of Technology)




# Dr. Marta Colleoni's Plenary talk tomorrow

Credit: Carl Knox (OzGrav, Swinburne University of Technology)

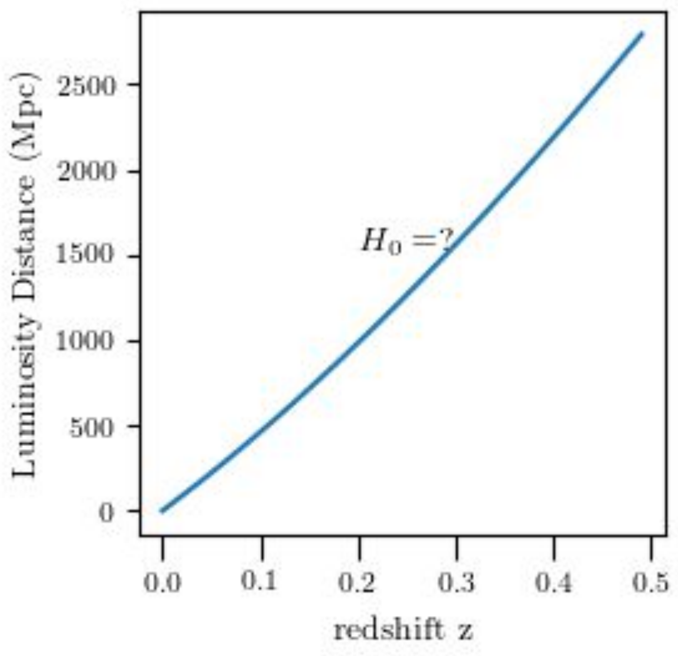
# Distance measurement from Gravitational Waves




$$h(t) = \frac{M_z^{5/3} f(t)^{2/3}}{d_L} F(\iota, \theta) \cos(\Phi(t))$$

$M_z$: Redshifted chirp mass
$\iota$: inclination angle
$\Phi(t)$: Accumulated phase

# Measuring $H_0$ with "standard sirens"



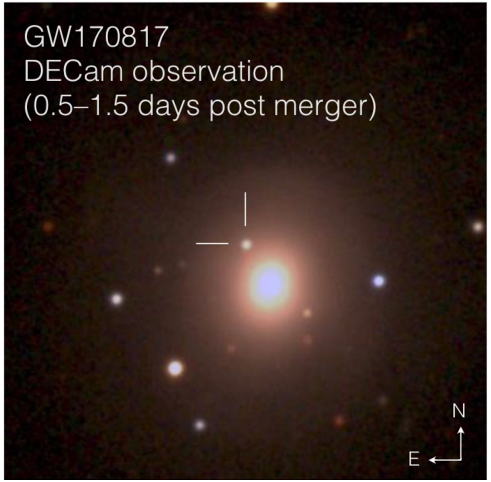

- Luminosity distance - redshift curve depends on the value of the Hubble parameter $H_0$

  $d_L \sim cz/H_0$ low redshift

- Luminosity distance - GW observation
- Redshift - Electromagnetic counterpart

**Thus an independent estimate of $H_0$ is possible**

# GW170817



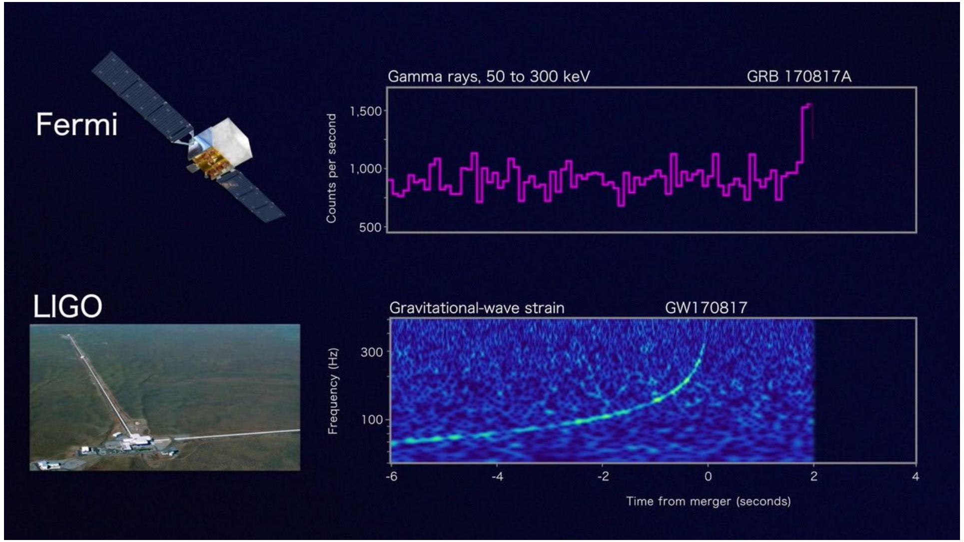
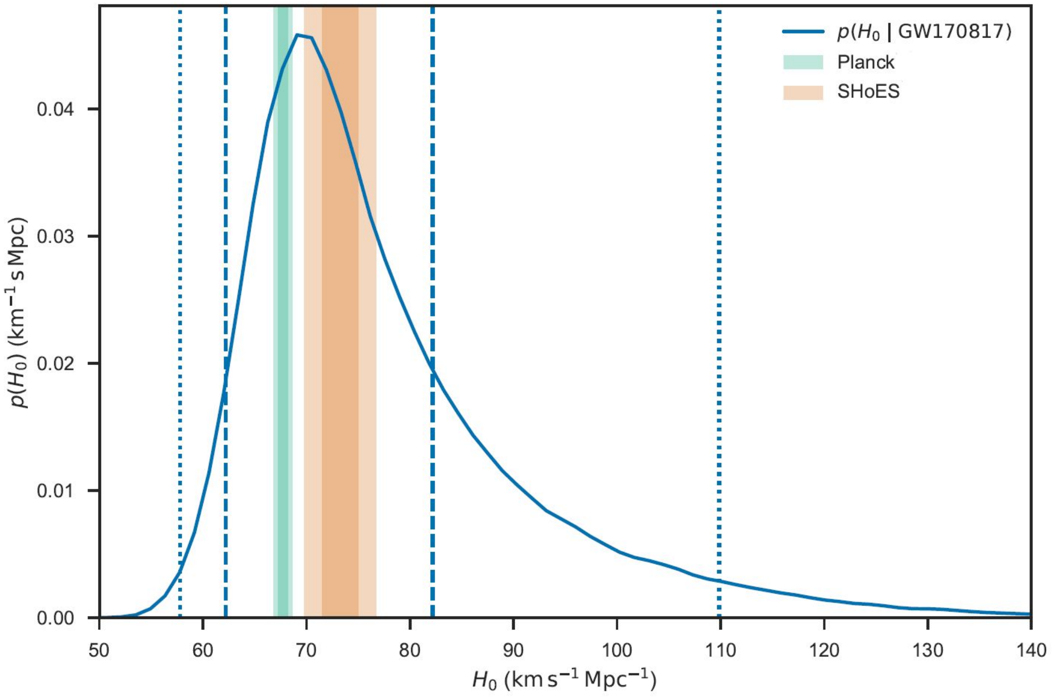




Image: https://www.ligo.org

- The only GW event detected along with a GRB: GRB 170817A
- Luminosity distance ~ 40 Mpc
- Host identification : NGC 4993

**For most of the detected events, the host identification is not possible**

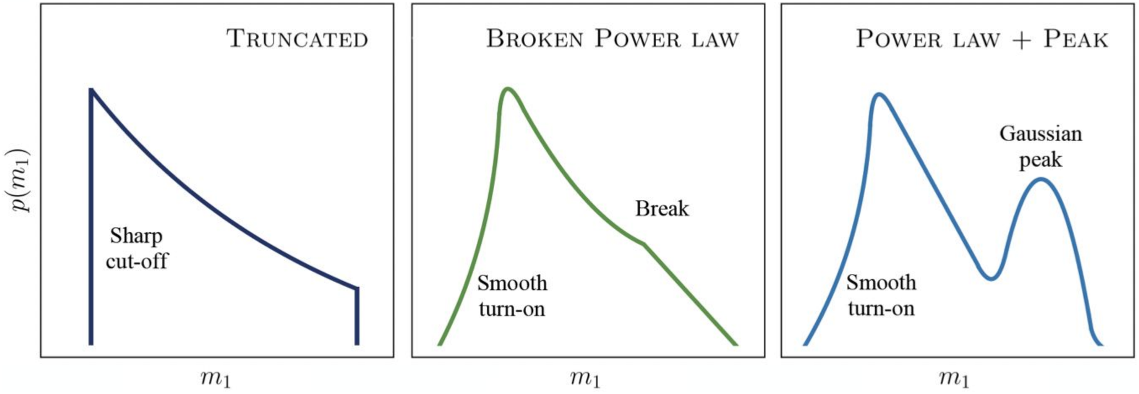
# Constraints on Hubble constant from GW170817



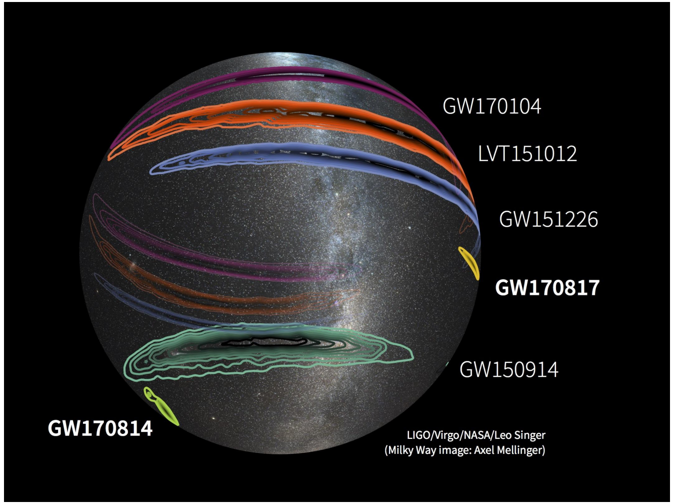

Abbott et. al. (2017)

**Measurement of $H_0$ with ~ 15% accuracy at 68.3% confidence**

# Inferring $H_0$ using population statistics



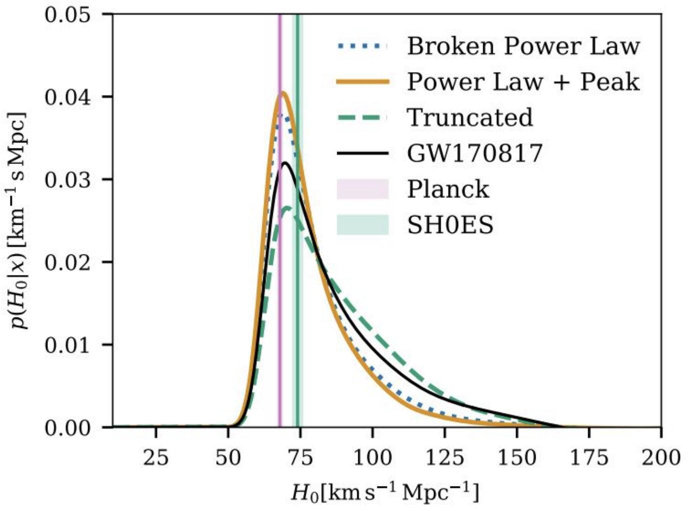

Abbott et al. (2021)



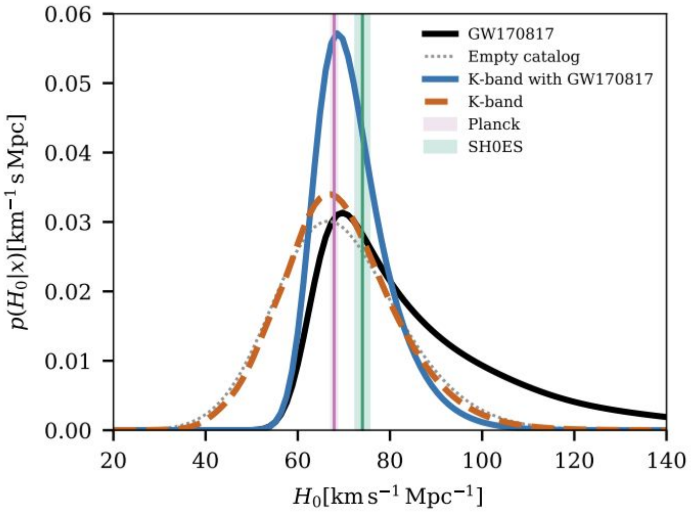

Credit : Leo Singer

- Map an astrophysically motivated source mass distribution to the detector frame - thus extract the redshift distribution (icarogw) Abbott et al. (2023)
- Consider galaxies (with known redshifts) in the localization region as potential hosts and compute $H_0$ distribution for each potential host (gwcosmo) Schutz(1986)

# Constraints from GWTC-3




Method 1: Astrophysical Population technique (icarogw)




Method 2 : Galaxy Catalogue technique (gwcosmo)

Abbott et al. (2023)

# An alternative approach: *The Large Scale Structures*

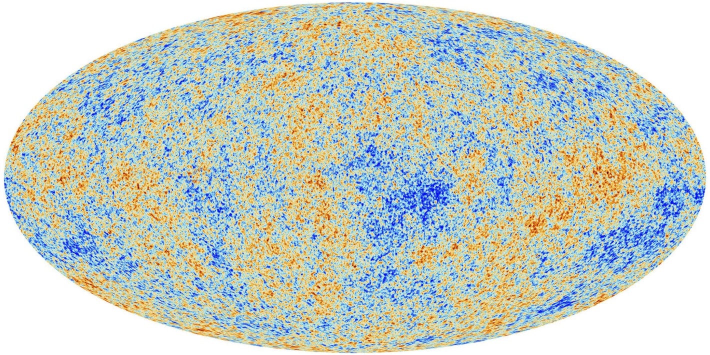


Image: ESA

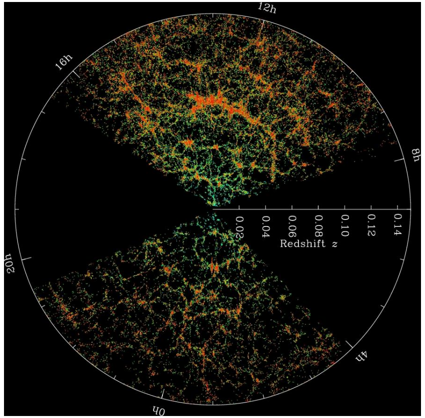



Image: SDSS

The Millennium simulation (z=0)

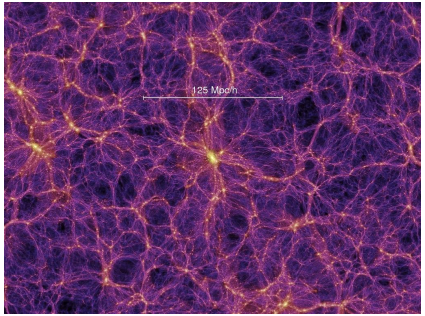
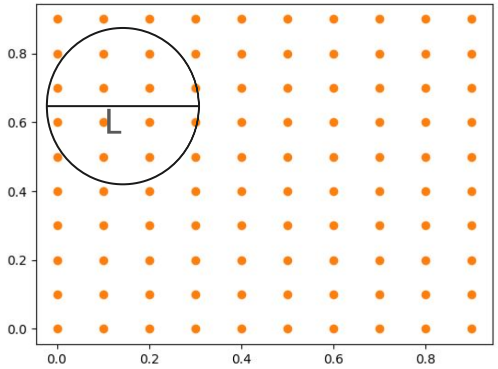


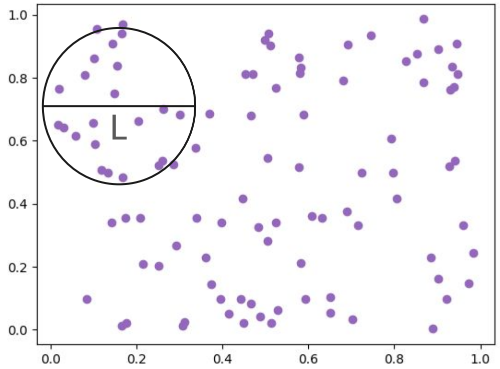

Image: https://wwwmpa.mpa-garching.mpg.de

# Measures of clustering: Density Contrast and cross-correlation

$$\delta(\mathbf{x}) \sim \frac{\rho(\mathbf{x})}{\bar{\rho}} - 1$$

$$\xi(\mathbf{x}, \mathbf{x}') \sim \langle \delta(\mathbf{x}) \delta(\mathbf{x}') \rangle$$




Angular cross-correlation

$$w(\theta, \theta') \sim \langle \delta(\theta) \delta(\theta') \rangle$$

Clustering ~ N/𝒩 -1
L : Clustering length

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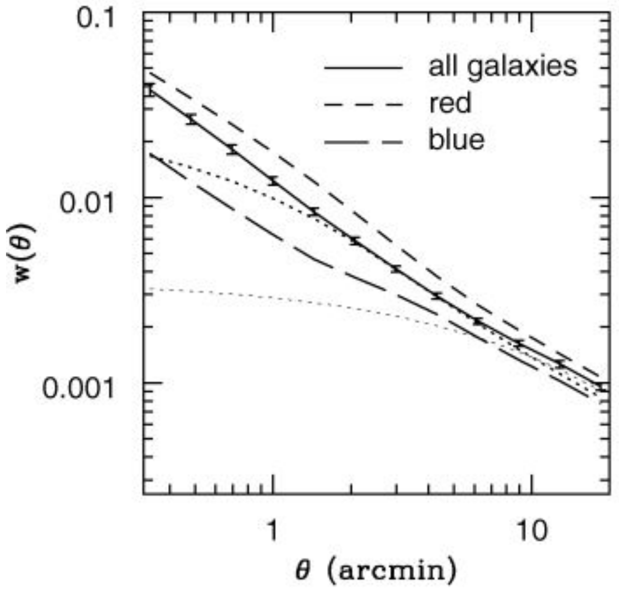



Jain, Scranton, Sheth (2003)

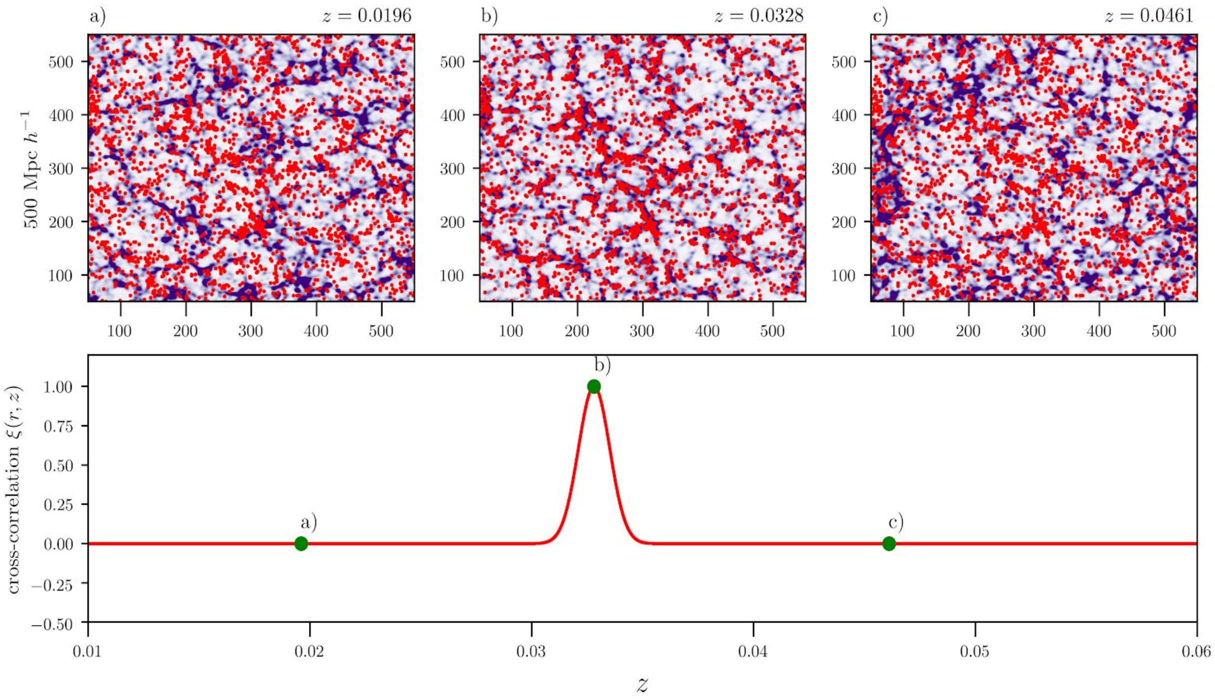
# Inferring redshift from cross-correlations

Red : BBH sources at a fixed unknown redshift

Blue: Galaxy distribution at different redshift slices

The BBH distribution is a part of the same large scale structure as the galaxies.

Cross-correlation of the two distributions provide a redshift estimate for the unknown BBH population




Newman (2008), Oguri (2016)

# A realistic Simulation of the catalogs

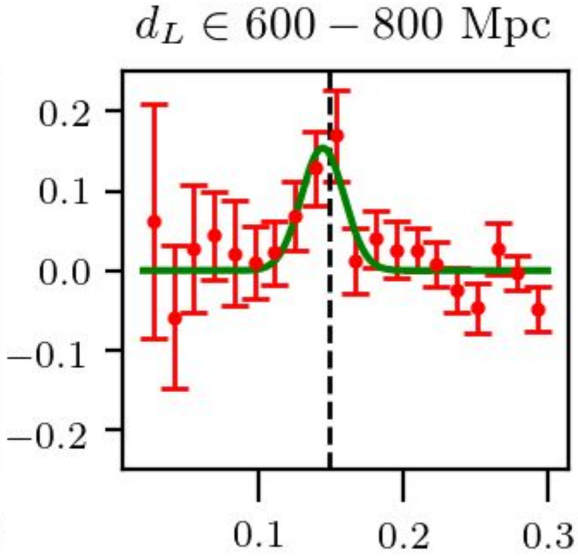
- ❏ The true locations of the GW events are sampled from the dark matter distribution of a cosmological N-body simulation (Big-MultiDark Planck)

- ❏ Massive dark matter halos act as galaxy markers in our simulation.

- ❏ Realistic simulation of the GW events and parameter estimations run using BILBY: A free Bayesian Inference library for GW (Ashton et al. 2019)

- ❏ 3 detector network (Advanced Ligo L +H + Advanced Virgo): combined SNR threshold of 8

# Modelling the cross-correlation

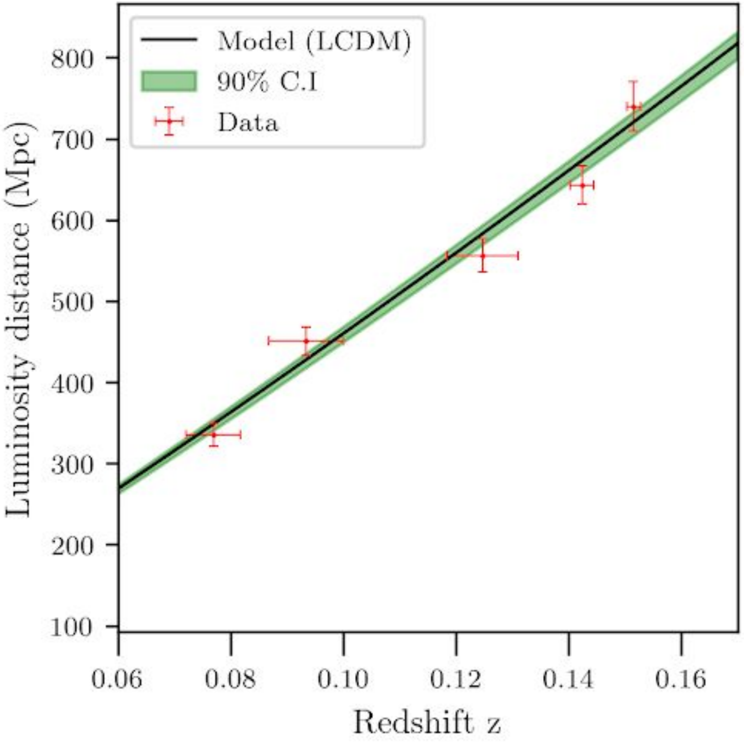


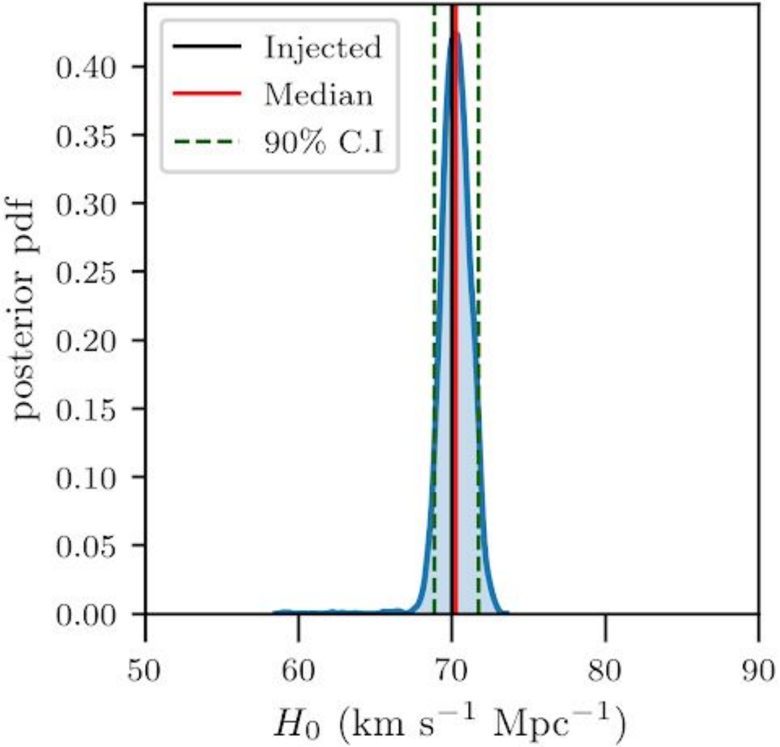

Assume power law three-dimensional cross-correlation function:

$$\xi_{\mathrm{gw,g}}(r) = \left[\frac{r}{r_0}\right]^{-\gamma}$$

$$w(\leq \theta_{\max}, z, z') \propto \exp\left[-\frac{(z-z')^2}{2\sigma_z^2}\right]$$

# Hubble-Lemaitre diagram : 500 events




**SB,** Rana, More, Bose (2020)

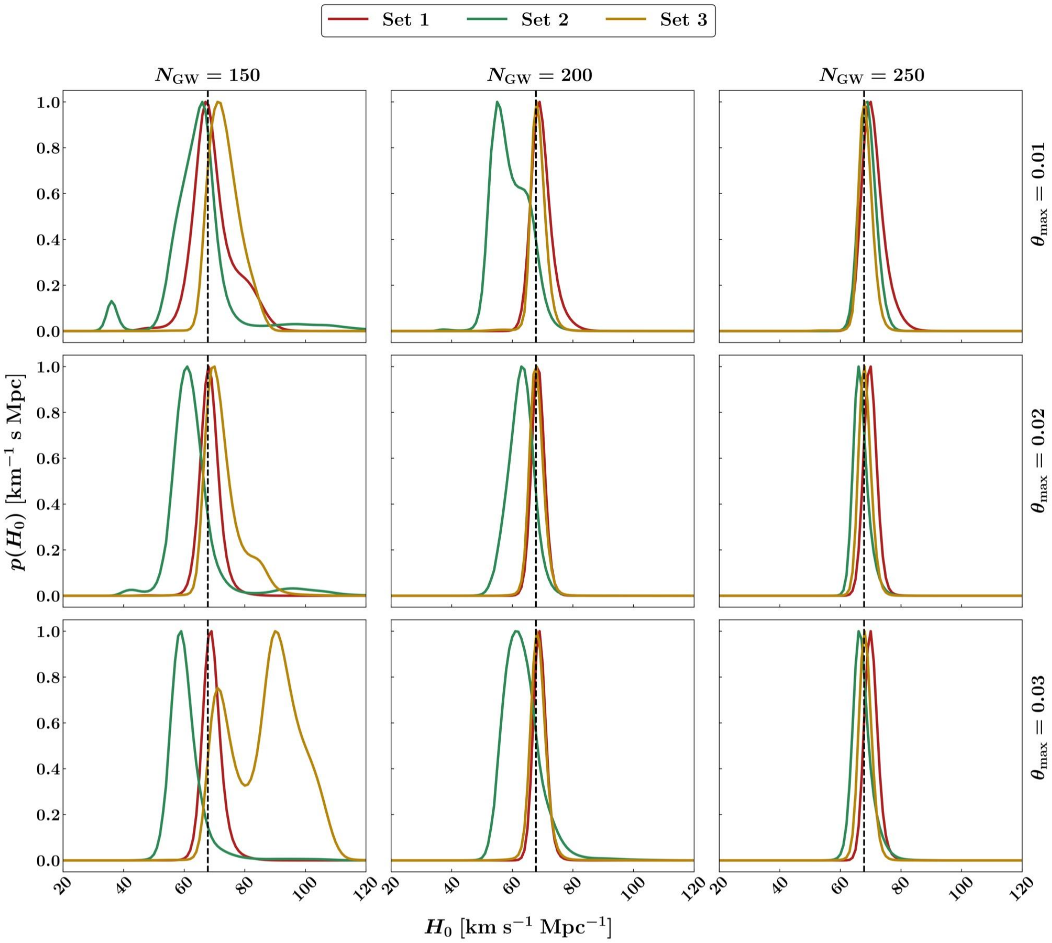
# An event-by-event analysis

$$p(H_0 \mid \boldsymbol{d}_{\text{strain}}, \boldsymbol{d}_g^{\text{obs}}) = \int p(H_0, \boldsymbol{d}_{\text{gw}} \mid \boldsymbol{d}_{\text{strain}}, \boldsymbol{d}_g^{\text{obs}}) d\boldsymbol{d}_{\text{gw}}$$
$$\propto \int \mathcal{L}(\boldsymbol{d}_{\text{strain}}, \boldsymbol{d}_{\text{g}}^{\text{obs}} | H_0, \boldsymbol{d}_{\text{gw}}) P(H_0, \boldsymbol{d}_{\text{gw}}) d\boldsymbol{d}_{\text{gw}}$$

→ For each GW event, the posterior is obtained by marginalizing over localization uncertainties $\boldsymbol{d}_{\text{gw}}$

Assuming independent probability distributions,
the single-event posteriors can be combined as :

$$P(H_0 \mid \{\boldsymbol{d}_{\text{strain}}\}, \{\boldsymbol{d}_g^{\text{obs}}\}) \propto P(H_0) \prod_i \mathcal{L}(\boldsymbol{d}_{\text{strain}_i}, \boldsymbol{d}_{g_i}^{\text{obs}} | H_0)$$
$$\propto P(H_0) \prod_i \int \mathcal{L}(\boldsymbol{d}_{\text{strain}_i}, \boldsymbol{d}_{g_i}^{\text{obs}} | H_0, \boldsymbol{d}_{\text{gw}}) P(\boldsymbol{d}_{\text{gw}}) d\boldsymbol{d}_{\text{gw}}$$




Set 1,2,3 : Different realizations of randomly generated events upto 1000 Mpc, SNR>12

Dependence on **sample size** and **correlation scale**

Injected value of $H_0$ = 70 km/s/Mpc

Ghosh, More, **SB**, Bose (arXiv: 2312.16305)

# Takeaway

Incorporation of the information from large-scale structure correlations is crucial to a more robust inference of the background cosmology

# Takeaway

**Incorporation of the information from large-scale structure correlations is crucial to a more robust inference of the background cosmology**

*Caveats:*

- Need ~**250 or more** well-localised GW sources for a meaningful estimate. Expected to be achieved in the 3G era of GW detectors.

- Effects due to weak lensing.

# ACKNOWLEDGMENTS

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# Extra Slides

# Waveform simulation: inputs

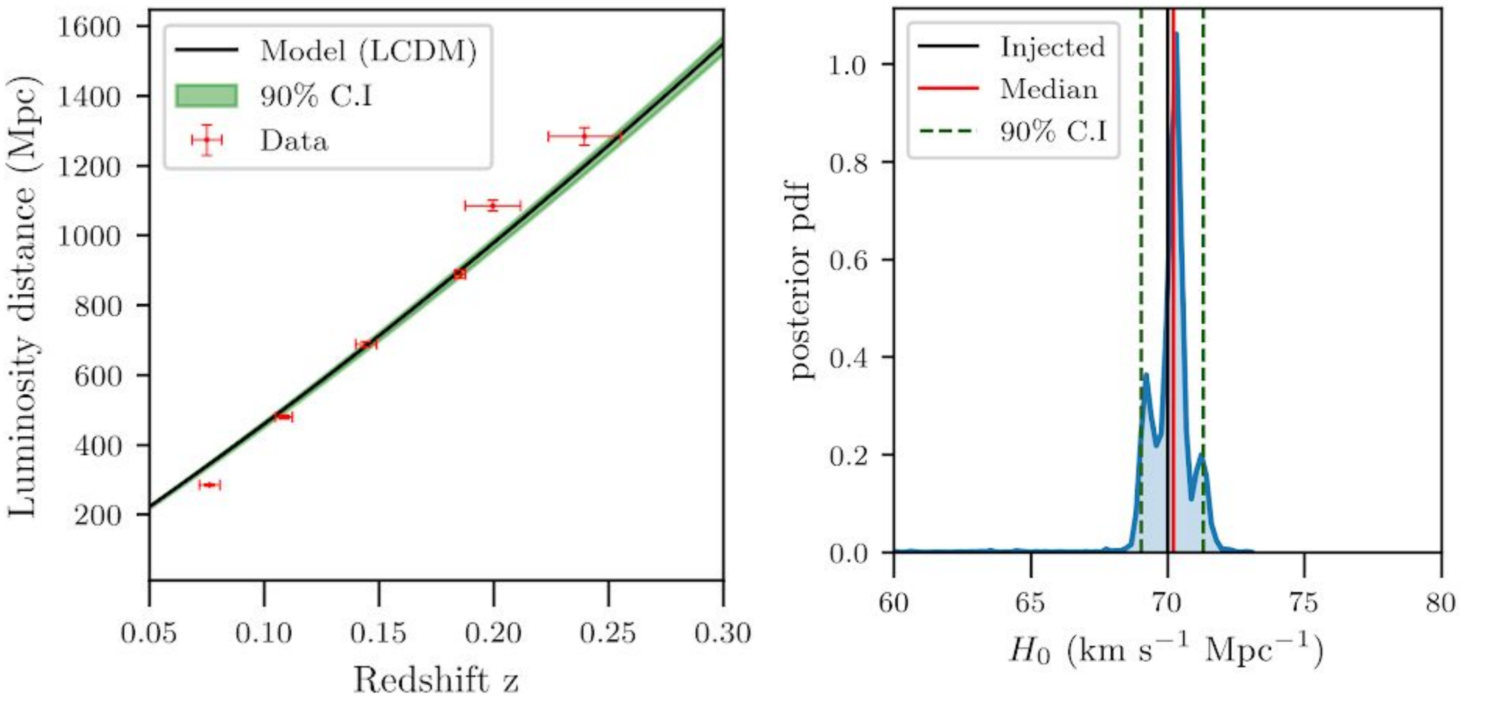
| Detectors | Sensitivity |
|---|---|
| Livingston | Advanced LIGO |
| Hanford | Advanced LIGO |
| Virgo | Advanced Virgo |

| Injection Parameters | | |
|---|---|---|
| Parameters | Distribution | Limits |
| $m_{1,2}$ | uniform | $[10, 35]\ M_\odot$ |
| $\chi_{1,2}$ | uniform | $[0, 0.8]$ |
| $\phi_{12}$ , $\phi_{jl}$ | uniform | $[0, 2\pi)$ |
| $\cos\theta_{1,2}$ , $\cos\iota$ | uniform | $[-1, 1)$ |
| $\psi$ , $\phi_c$ | Fixed | 0 |

**Detection criteria: At least two of the detectors SNR above a threshold value of 5 each, the third an SNR greater than 2.5, and network SNR of greater than 8.**

# Hubble-Lemaitre diagram : 5000 events

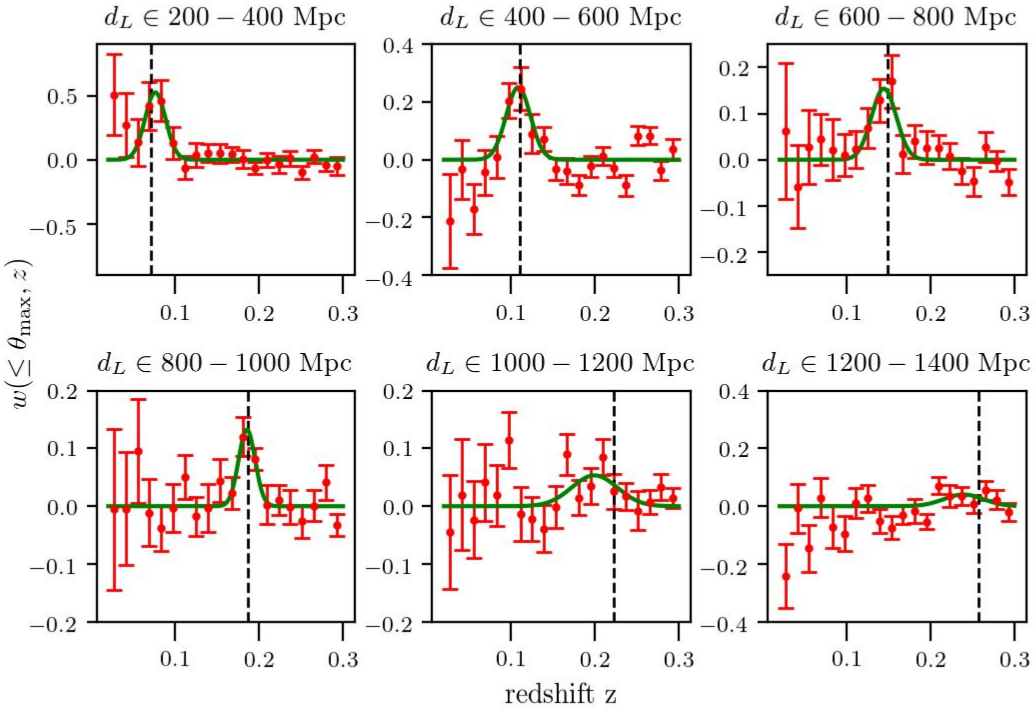
Red points: $d_L$ inferred from BBH merger waveforms, redshift from cross-correlations




Black solid line: the true value of $H_0$ in the simulation
Dashed lines: 90 percent credible interval

SB, Rana, More, Bose (2020)

# Redshift from angular cross-correlation

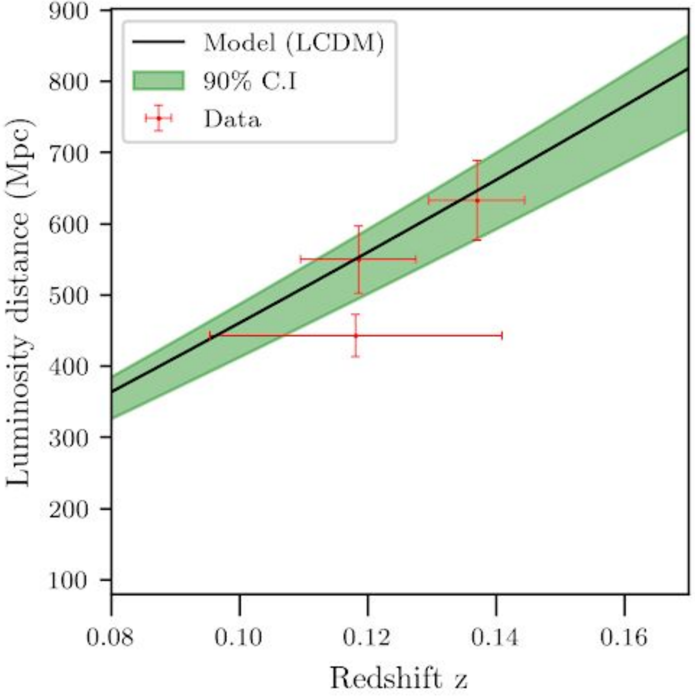


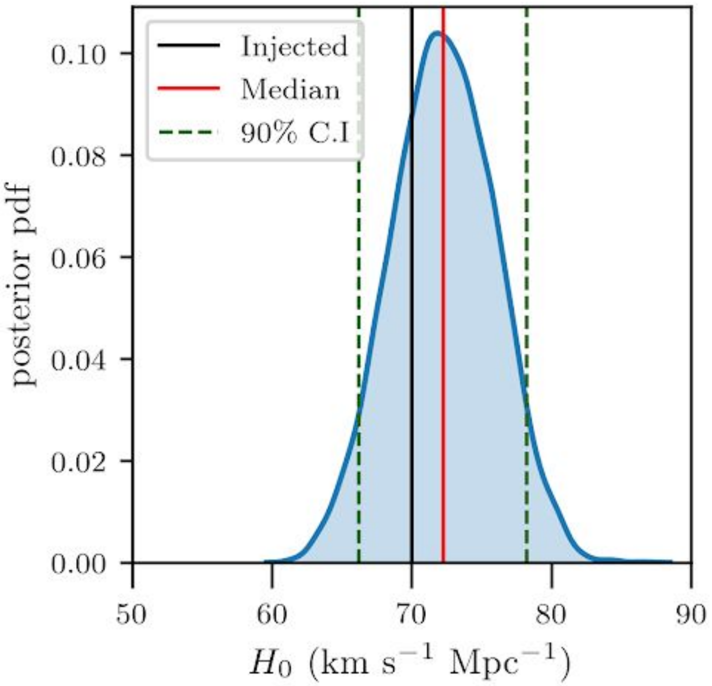

- ❑ 5000 BBH mergers divided into 6 bins in the inferred luminosity distances
- ❑ The mock galaxies are divided into 20 redshift bins

- ❑ Red points are the measured cross-correlations with error bars, peaking at the correct redshift
- ❑ The injected value of $H_0$ = 70 km/s/Mpc gives an average redshift of the GW sources in each bin (black vertical line)

**SB, Rana, More, Bose (2020)**

# Hubble-Lemaitre diagram : 50 events



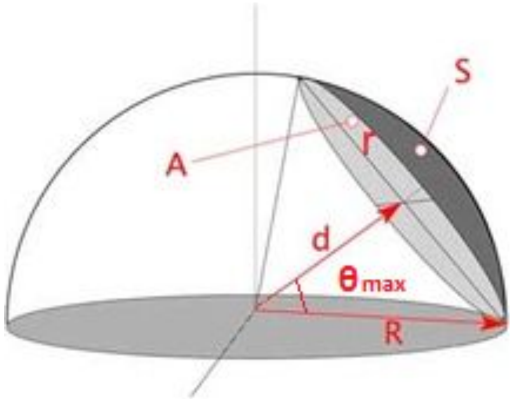

SB, Rana, More, Bose (2020)

# Constraints from the three samples

| Constraints on $H_0$ | | | |
|---|---|---|---|
| No. of GW events | Max $d_L$ (Mpc) | Injected $H_0$ (km $s^{-1}$ $Mpc^{-1}$) | Constraints on $H_0$ (km $s^{-1}$ $Mpc^{-1}$) |
| 5100 | 1400 | 70 | $70.22^{+1.09}_{-1.18}$ |
| 500 | 900 | 70 | $70.26^{+1.47}_{-1.40}$ |
| 50 | 900 | 70 | $72.24^{+5.98}_{-6.05}$ |

The error bars signify 90% credible interval around the the median of H0 posterior

# Angular Cross-correlation Estimator




We count the number of galaxy-BBH pairs which have an angular separation $\theta_{max}$ or less in the actual catalog and in a randomly distributed catalog.

**Angular cross-correlation estimator**

$$w(\leq \theta_{\rm max}) = \frac{n_{\rm D_1 D_2}(\leq \theta_{\rm max})}{n_{\rm R_1 R_2}(\leq \theta_{\rm max})} - 1$$

$D_1$, $D_2$ : Data catalogs
$R_1$, $R_2$ : Random catalogs