Linear Dynamics and Gravitational Waves in Gravitational Quantum Field Theory

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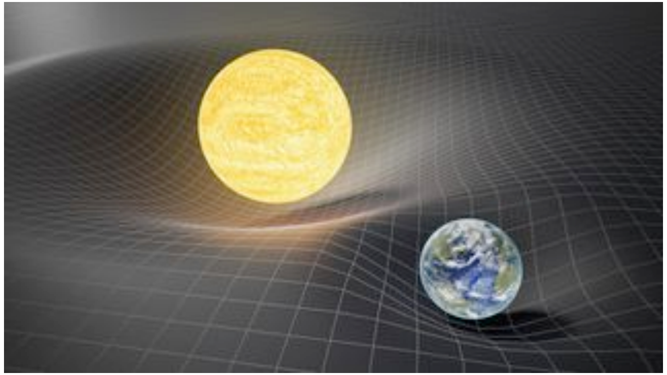
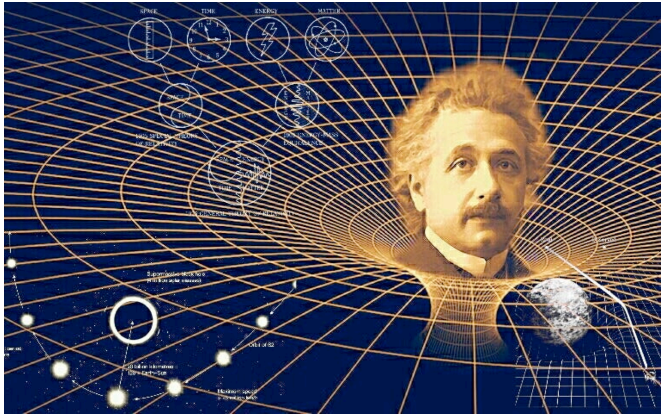
Content

Introduction

- Among four fundamental interactions, gravity is the weakest and most mysterious.
- It governs the evolutions of most astrophysical systems and even the whole Universe.
- Currently the standard theory of gravity is Einstein's General Relativity.

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

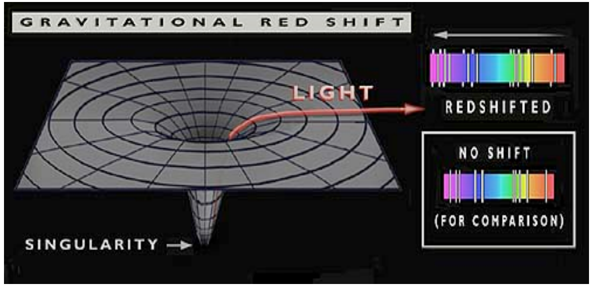




Introduction

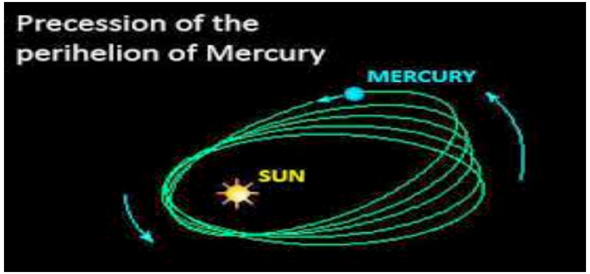
➢ GR has passed all astrophysical and cosmological tests:

Grav. Redshift



Light Deflection

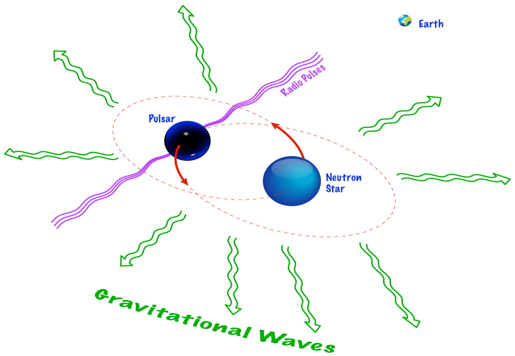
Mer. Perihelion

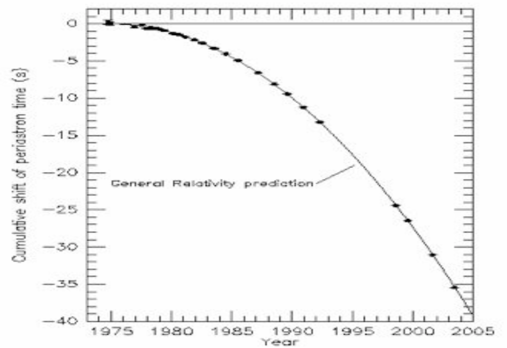


Time Delay

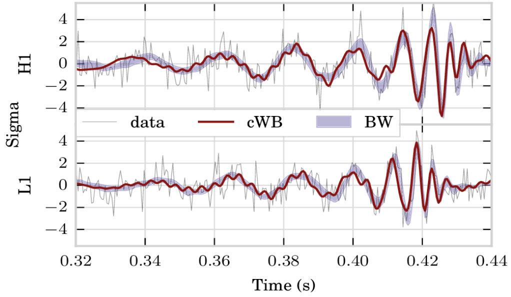
Introduction

- Hulse-Taylor binary pulsar B1913+16: Indirect Evidence of GWs



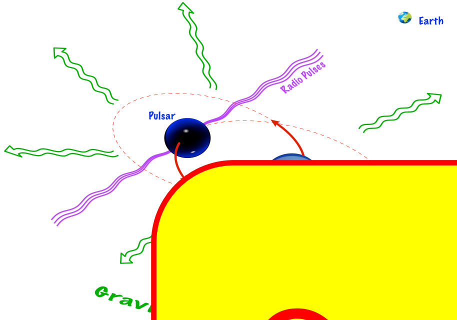


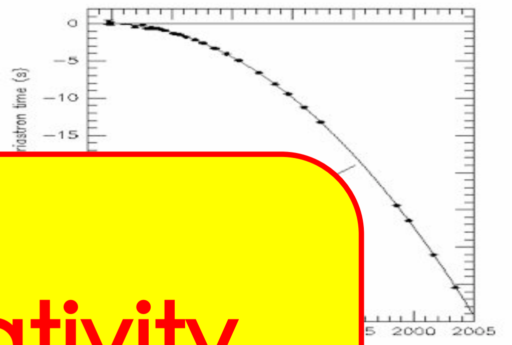
- GW150914 by LIGO: Direct Discovery of GWs



Introduction

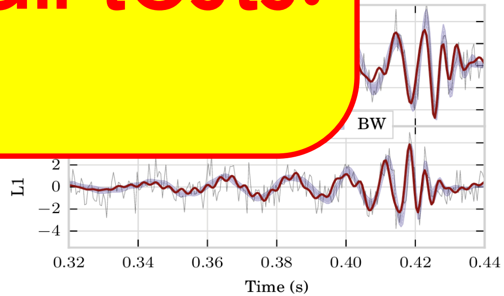
- Hulse-Taylor binary pulsar B1913+16: Indirect Evidence of GWs





General Relativity have passed all tests!

- GW150914



Motivation

- Despite its success, GR can be a low-energy effective field theory as indicated by its non-renormalizability.
- **Important Questions:**
  - ✓ What is the nature of gravity?
  - ✓ How to quantize gravity?
- **Hints:**
  - ✓ As inspired by electromagnetic, weak and strong interactions, we should formulate gravity with **gauge principle**.
  - ✓ Many attempts: Einstein-Cartan, Teleparallel Equivalent GR, ..

See e.g. F. W. Hehl et al. (1976) & L. Heisenberg (2023) for reviews

Brief Introduction to Gravitational Quantum Field Theory (GQFT)

- GQFT is a new gauge formulation of gravity, in which the gauge fields and transformations are represented wrt fermions.

- New formulation of 4-dimensional Dirac fermions

$$S_D = \int d^4x \, \frac{1}{2} \left(\bar{\psi}(x)\gamma^\mu i\partial_\mu \psi(x) + H.c. \right) - m \, \bar{\psi}(x)\psi(x),$$

$$S = \int d^4x \, \frac{1}{2} \{ \bar{\Psi}_-(x)\Gamma^a \Gamma_- \delta_a{}^\mu i D_\mu \Psi_-(x) - \bar{\Psi}_-(x)(m_5 \Gamma^5 \Gamma_- + m_6 \Gamma^6 \Gamma_-)\Psi_-(x) + H.c.\},$$

where

$$\Psi_-(x) = \Gamma_- \Psi(x) \equiv \begin{pmatrix} \psi_L(x) \\ \psi_R(x) \end{pmatrix}, \qquad \Gamma^5 = i\sigma_1 \otimes \gamma_5, \quad \Gamma^6 = i\sigma_2 \otimes \gamma_5,$$

$$\psi_{L,R}(x) = \gamma_\mp \psi(x), \qquad \Gamma^{\hat{a}} \equiv 2(\Sigma_-^{\hat{a}} + \Sigma_+^{\hat{a}}), \quad \Sigma_\mp^{\hat{a}} \equiv \frac{1}{2}\Gamma^{\hat{a}}\Gamma_\mp,$$

$$\Gamma_\mp = \frac{1}{2}(1 \mp \hat{\gamma}_7), \quad \gamma_\mp = \frac{1}{2}(1 \mp \gamma_5), \qquad \gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3 = \sigma_3 \otimes \sigma_0,$$

$$\Gamma^a = \sigma_0 \otimes \gamma^a, \quad \gamma^a = \delta^a_{\ \mu}\gamma^\mu, \qquad \hat{\gamma}_7 = -\Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^5\Gamma^6 = \sigma_3 \otimes \gamma_5,$$

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Brief Introduction to Gravitational Quantum Field Theory (GQFT)

- Gauge Transformation Generators

$$\Sigma^{ab} = \frac{i}{4}[\Gamma^a, \Gamma^b],$$
$$\Sigma_-^a = \frac{1}{2}\Gamma^a\Gamma_-, \quad \Gamma_- = \frac{1}{2}(1 - \hat{\gamma}_7),$$

- Commutators

$$[\Sigma^{ab}, \Sigma^{cd}] = i(\Sigma^{ad}\eta^{bc} - \Sigma^{bd}\eta^{ac} - \Sigma^{ac}\eta^{bd} + \Sigma^{bc}\eta^{ad}),$$
$$[\Sigma^{ab}, \Sigma_-^c] = i(\Sigma_-^a\eta^{bc} - \Sigma_-^b\eta^{ac}),$$
$$[\Sigma_-^a, \Sigma_-^b] = 0,$$

$\Rightarrow$ $WS(1,3) \equiv SP(1,3) \rtimes W^{1,3}$. Isometric to Poincare Group

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Brief Introduction to Gravitational Quantum Field Theory (GQFT)

➢ Inhomogeneous Spin Gauge Symmetry

✓ Covariant derivative: $i\partial_\mu \to i\hat{\mathcal{D}}_\mu \equiv i\partial_\mu + \hat{\mathcal{A}}_\mu(x)$

✓ Gauge Fields:

$$\hat{\mathcal{A}}_\mu(x) \equiv \mathcal{A}_\mu(x) + \check{\mathcal{A}}_\mu(x),$$
$$\mathcal{A}_\mu(x) \equiv \mathcal{A}_\mu^{ab}(x)\,\frac{1}{2}\Sigma_{ab}, \quad \check{\mathcal{A}}_\mu(x) \equiv \mathcal{A}_\mu^{a}(x)\frac{1}{2}\Sigma_{-a},$$

✓ Field Strengths

$$\hat{\mathcal{F}}_{\mu\nu} \equiv i[\hat{\mathcal{D}}_\mu, \hat{\mathcal{D}}_\nu] = \mathcal{F}_{\mu\nu} + \check{\mathcal{F}}_{\mu\nu} + F_{\mu\nu},$$
$$\mathcal{F}_{\mu\nu} \equiv \mathcal{F}_{\mu\nu}^{ab}(x)\frac{1}{2}\Sigma_{ab} = \partial_\mu\mathcal{A}_\nu(x) - \partial_\nu\mathcal{A}_\mu(x) - i[\mathcal{A}_\mu(x), \mathcal{A}_\nu(x)],$$
$$\check{\mathcal{F}}_{\mu\nu} \equiv \mathcal{F}_{\mu\nu}^{a}(x)\frac{1}{2}\Sigma_{-a} = \mathcal{D}_\mu\check{\mathcal{A}}_\nu(x) - \mathcal{D}_\nu\check{\mathcal{A}}_\mu(x)$$
$$\equiv \partial_\mu\check{\mathcal{A}}_\nu(x) - \partial_\nu\check{\mathcal{A}}_\mu(x) - i(\mathcal{A}_\mu(x)\check{\mathcal{A}}_\nu(x) - \mathcal{A}_\nu(x)\check{\mathcal{A}}_\mu(x)),$$

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Brief Introduction to Gravitational Quantum Field Theory (GQFT)

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➢ Identity:

$$\mathcal{F}^{ab}_{\mu\nu}(x) \equiv \mathbf{R}^{ab}_{\mu\nu}(x) + \mathbf{F}^{ab}_{\mu\nu}(x),$$

Riemann Tensor

Spin Gauge Field

➢ Lagrangian

$$\begin{aligned} \mathcal{S}_D \equiv & \int [d^4x]\chi(x)\{(\hat{\chi}^{\mu\nu}\bar{\Psi}_-\Sigma_-^a\chi_{\mu a} i\mathcal{D}_\nu\Psi_- - m\bar{\Psi}_-\Gamma^6\Psi_- + H.c.) \\ & - \frac{1}{4}\hat{\chi}^{\mu\mu'}\hat{\chi}^{\nu\nu'}F_{\mu\nu}F_{\mu'\nu'} - \frac{1}{4}\hat{\chi}^{\mu\mu'}\hat{\chi}^{\nu\nu'}\mathcal{F}_{\mu\nu}^{\ ab}\mathcal{F}_{\mu'\nu' ab} \\ & + \frac{1}{4}m_G^2\bar{\chi}_{aa'}^{\mu\nu\mu'\nu'}\mathbf{F}_{\mu\nu}^{\ a}\mathbf{F}_{\mu'\nu'}^{a'} + \frac{1}{4}g_G^{-2}M_\kappa^2\tilde{\chi}_{aa'}^{\mu\nu\mu'\nu'}F_{\mu\nu}^a F_{\mu'\nu'}^{a'}\}, \end{aligned}$$

➢ Note

$$\frac{1}{4}\chi\,\tilde{\chi}_{aa'}^{\mu\nu\mu'\nu'}F_{\mu\nu}^a F_{\mu'\nu'}^{a'} = \chi\,R + 2\partial_\mu(\chi\hat{\chi}^{\mu\rho}\hat{\chi}_a^{\ \sigma}F_{\sigma\rho}^a),$$

Unify Geometry and Gauge Field

➢ Gravitational Equations

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$$\partial_\nu \tilde{F}_a^{\mu\nu} = J_a^{\ \mu},$$

$$\partial_\mu J_a^{\ \mu} = 0.$$

$$\tilde{F}_a^{\mu\nu} \equiv \chi \tilde{\chi}_{aa'}^{[\mu\nu]\mu'\nu'} F_{\mu'\nu'}^{a'},$$

$$\bar{\chi}_{aa'}^{[\mu\nu]\mu'\nu'} \equiv \hat{\chi}_c^{\ \mu}\hat{\chi}_d^{\ \nu}\hat{\chi}_{c'}^{\ \mu'}\hat{\chi}_{d'}^{\ \nu'}\bar{\eta}_{aa'}^{[cd]c'd'},$$

$$\tilde{\chi}_{aa'}^{[\mu\nu]\mu'\nu'} \equiv \hat{\chi}_c^{\ \mu}\hat{\chi}_d^{\ \nu}\hat{\chi}_{c'}^{\ \mu'}\hat{\chi}_{d'}^{\ \nu'}\tilde{\eta}_{aa'}^{[cd]c'd'},$$

$$\bar{\eta}_{aa'}^{[cd]c'd'} \equiv \frac{1}{2}(\bar{\eta}_{aa'}^{cdc'd'} - \bar{\eta}_{aa'}^{dcc'd'}),$$

$$\tilde{\eta}_{aa'}^{[cd]c'd'} \equiv \frac{1}{2}(\tilde{\eta}_{aa'}^{cdc'd'} - \tilde{\eta}_{aa'}^{dcc'd'}).$$

$$J_a^{\ \mu} \equiv 16\pi G_N \hat{J}_a^{\ \mu} + \tilde{J}_a^{\ \mu}, \quad \hat{J}_a^{\ \mu} \equiv \mathrm{J}_a^{\ \mu} + \widetilde{J}_a^{\ \mu},$$

$$\tilde{J}_a^{\ \mu} = \hat{\chi}_a^{\ \rho} F_{\rho\nu}^c \tilde{F}_c^{\mu\nu} - \frac{1}{4}\hat{\chi}_a^{\ \mu} F_{\rho\nu}^c \tilde{F}_c^{\rho\nu},$$

$$\begin{aligned}\mathrm{J}_a^{\ \mu} = \ & \chi\{(\hat{\chi}_a^{\ \rho}\hat{\chi}_c^{\ \mu} - \hat{\chi}_a^{\ \mu}\hat{\chi}_c^{\ \rho})\frac{1}{2}(\bar{\psi}\gamma^c i\mathcal{D}_\rho\psi + H.c.) \\ & + \hat{\chi}_a^{\ \mu} m\bar{\psi}\psi - (\hat{\chi}^{\mu\mu'}\hat{\chi}_a^{\ \rho} - \frac{1}{4}\hat{\chi}^{\rho\mu'}\hat{\chi}_a^{\ \mu})\hat{\chi}^{\nu\nu'} F_{\rho\nu}F_{\mu'\nu'}\},\end{aligned}$$

$$\begin{aligned}\widetilde{J}_a^{\ \mu} = \ & \chi\{-(\hat{\chi}^{\mu\mu'}\hat{\chi}_a^{\ \rho} - \frac{1}{4}\hat{\chi}^{\rho\mu'}\hat{\chi}_a^{\ \mu})\hat{\chi}^{\nu\nu'} \mathcal{F}_{\rho\nu}^{bc}\mathcal{F}_{\mu'\nu' bc} \\ & + m_G^2(\bar{\chi}_{ba'}^{[\mu\nu]\mu'\nu'}\hat{\chi}_a^{\ \rho} - \frac{1}{4}\bar{\chi}_{ba'}^{[\rho\nu]\mu'\nu'}\hat{\chi}_a^{\ \mu})\mathbf{F}_{\rho\nu}^b \mathbf{F}_{\mu'\nu'}^{a'}\} \\ & - m_G^2 \mathcal{D}_\nu(\chi\,\bar{\chi}_{aa'}^{[\mu\nu]\mu'\nu'}\mathbf{F}_{\mu'\nu'}^{a'}),\end{aligned}$$

Linearized Gravitational Equations in GQFT

- Write $$\chi^a_\mu \equiv \eta^a_\mu + \frac{1}{2} h^a_\mu \,,$$

- Linearized Equations with h as perturbations

$$[\Box h^\rho_a - \partial^\rho \partial_\nu h^\nu_a - \partial_\nu \partial_a h^{\nu\rho} + \partial_a \partial^\rho h + \delta^\rho_a (\partial_\nu \partial_\sigma h^{\nu\sigma} - \Box h)] + \gamma_W (\Box h^\rho_a - \partial^\rho \partial_\nu h^\nu_a) = -16\pi G_\kappa \mathrm{J}^\rho_a \,,$$

- Note that indices a and μ does not have any symmetries, so we can depose this equation into its symm. and anti-symm. parts

$$\widetilde{G}_{\mu\nu} \equiv \frac{1}{2} [\Box h_{\mu\nu} - 2\partial^\sigma \partial_{(\mu} h_{\nu)\sigma} + \partial_\mu \partial_\nu h + \eta_{\mu\nu} (\partial^\rho \partial^\sigma h_{\rho\sigma} - \Box h)] + \frac{\gamma_W}{2} [\Box h_{\mu\nu} - \partial^\sigma \partial_{(\mu} h_{\nu)\sigma}] = -8\pi G_\kappa T_{(\mu\nu)} \,,$$

$$\widetilde{G}_{[\mu\nu]} \equiv -\frac{\gamma_W}{2} \partial^\sigma \partial_{[\mu} h_{\nu]\sigma} = -8\pi G_\kappa T_{[\mu\nu]} \,,$$

where
$$T_{(\mu\nu)} \equiv (\eta_{\mu\rho} \eta^a_\nu + \eta_{\nu\rho} \eta^a_\mu) \mathrm{J}^\rho_a / 2 \,,$$
$$T_{[\mu\nu]} \equiv (\eta_{\mu\rho} \eta^a_\nu - \eta_{\nu\rho} \eta^a_\mu) \mathrm{J}^\rho_a / 2 \,,$$

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Free Field Equations and Physical GW Degrees

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- Polarization Decomposition

- Spin-2 Tensor Modes $\hat{h}_{ij}$

$$\hat{h}_{it} = \hat{h}_{tt} = 0\,,\ \hat{h}^i_i = 0\,,\ \partial^i \hat{h}_{ij} = 0\,,$$

- Spin-1 Vector Modes S_i and F_i

$$h_{tt} = 0\,,\ h_{it} = S_i\,,\ h_{ij} = 2\partial_{(i}F_{j)}\,,\ \partial^i S_i = \partial^i F_i = 0\,,$$

- Spin-0 Scalar Modes ϕ, B, ψ and E

$$h_{tt} = -2\phi\,,\ h_{it} = -\partial_i B,\ h_{ij} = -2\psi\delta_{ij} + 2\partial_i\partial_j E.$$

- Line Element:

$$ds^2 = (1+2\phi)dt^2 - 2(S_i - \partial_i B)dx^i dt - [\hat{h}_{ij} - (1-2\psi)\eta_{ij} + 2\partial_{(i}F_{j)} + 2\partial_i\partial_j E]dx^i dx^j$$

Free Field Equations and Physical GW Degrees

- Scalar-Type Gauge Symmetry: $\delta h_{\mu\nu} = \partial_\mu \partial_\nu \zeta$

$$\phi \to \phi + \partial_t^2 \zeta/2\,, \quad B \to B + \partial_t \zeta\,, \quad E \to E - \zeta/2$$

- Gauge-Invariant Variables

$$\Phi = \phi - \partial_t B/2 \quad A = B + 2\partial_t E\,.$$

- Field Equations

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- (t,t) $$2\partial^k \partial_k \psi + \gamma_W \partial^k \partial_k \Phi = 0\,.$$
- Trace $$3\Box\psi = \partial^i \partial_i \left(\psi + \Phi - \partial_t A/2\right)$$
- (t,i) $$\left[\partial^k \partial_k (S_i - \partial_t F_i) + 4\partial_t \partial_i \psi\right] + (\gamma_W/2)\left[\Box S_i - \Box \partial_i A + \partial^k \partial_k (S_i - \partial_t F_i) + 2\partial_i \partial_t (\Phi - \psi + \partial_t A/2)\right] = 0\,. \quad (16)$$
- (i,j) $$(1+\gamma_W)\Box \hat{h}_{ij} + \gamma_W \Box \partial_{(i} F_{j)} - (\gamma_W + 2)\partial_t \partial_{(i}[S - \partial_t F]_{j)} + 2\eta_{ij}(\gamma_W + 1)\Box\psi + 2\partial_i \partial_j \left[(1-\gamma_W)\psi - \Phi + (\gamma_W + 1)\partial_t A/2\right] = 0\,,$$

Free Field Equations and Physical GW Degrees

- Scalar Sector:

$$\Box\Phi = 0\,,$$

$$\psi = -\gamma_W \Phi/2\,, \qquad \partial_t A = -(\gamma_W - 2)\Phi\,.$$

- Vector Sector:

$$\Box F_i = 0\,, \qquad S_i = \partial_t F_i\,.$$

- Tensor Sector

$$\Box\hat{h}_{ij} = 0\,,$$

Five GW Degrees:
Two Tensor + Two Vector + One Scalar

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Newtonian Limit

- Newtonian Limit: static weak gravitational field + static dust matter
- $T_{(00)} = \rho(\mathbf{x})$ is the only nonzero EM tensor component

$$\begin{aligned}
\widetilde{G}_{00} &= \partial_i\partial^i(2\psi + \beta\Phi) = -8\pi G_\kappa \rho(\mathbf{x})\,, \\
\widetilde{G}_{0i} &= -\partial_k\partial^k\left[(1+\gamma_W)S_i - \gamma_W\partial_i A/2\right]/2 = 0\,, \qquad (30)\\
\widetilde{G}_{ij} &= -(1/2)\{(1+\gamma_W)\partial_k\partial^k\hat{h}_{ij} + \gamma_W\partial_k\partial_k\partial_{(i}F_{j)} \\
&-2\partial_i\partial_j[(\gamma_W - 1)\psi + \Phi] - 2\eta_{ij}\partial_k\partial^k[(1-\gamma_W)\psi - \Phi]\} = 0\,.
\end{aligned}$$

- Solution: Poisson Equation $\quad \Delta\Phi \equiv -\partial_k\partial^k\Phi = 4\pi G_N\rho(\mathbf{x})\,,$

  + Constraint: $\quad \psi = \Phi/(1-\gamma_W)\,,$

- Newton Constant: $\quad G_N \equiv \dfrac{1-\gamma_W}{(1-\gamma_W/2)(1+\gamma_W)}G_\kappa$

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Newtonian Limit

- Probing Newtonian Limit with a Non-Relativistic Test Body
- At low-energies, all objects move along the geodesics

$$\frac{d^2x^\rho}{d\tau^2} + \Gamma^\rho_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0\,,$$

- The probing object moves very slowly, i.e., $dx^i/d\tau \ll dt/d\tau$,

$$\frac{d^2x^i}{dt^2} = \partial^i\Phi(\mathbf{x}) = -\partial_i\Phi(\mathbf{x})\,.$$

Universal Acceleration!

⇒ The GQFT obeys the Weak Equivalence Principle

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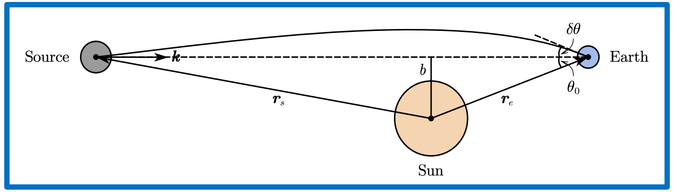
Newtonian Limit

- Detecting the Newtonian Limit with Photons, which would follow the light-like geodesics in the following metric Y.-K. Gao, **DH**, et al.(2022)

$$ds^2 = (1+2\Phi)dt^2 - (1-2\psi)\delta_{ij}dx^i dx^j \,,$$

- In the Solar system, these potentials can be written as

$$\Phi = -G_N M_\odot / r \,, \quad \psi = \Phi/(1-\gamma_W) \,,$$



- Following C.M. Will (2018), we can derive

- Deflection angle: $\delta\theta \approx (2+\gamma_W)\,(G_N M_\odot/b)\,(1+\cos\theta_0)$
- Shapiro Time Delay: $\delta t_{\mathrm{Shapiro}} \approx 2(2+\gamma_W) G_N M_\odot \ln\left(4 r_e r_s / b^2\right) \,,$

- Experimental Constraints:

- VLBI obs. of quasars: $\gamma_W = (-0.8 \pm 1.2) \times 10^{-4}$
- Cassini spacecraft (radar): $\gamma_W = (2.1 \pm 2.3) \times 10^{-5}$.

Conclusions

- Motivated by the difficulty of quantizing gravity, the **GQFT** has been proposed based on the gauge principle.
- We have studied linearized gravitational dynamics in the GQFT;
- We found five propagating GW dofs, with 2 tensors, 2 vectors and 1 scalar;
- We also studied the Newtonian limit of this theory, found the dynamics modified by a single parameter γ_W.
- We can probe Newtonian theory with non-relativistic objects, which are found to obey the Weak Equivalence Principle;
- For photons, we have derived GQFT corrected formulas for the light deflection angle and Shapiro time delay. When compared with experimental data, γ_W has been well constrained.

THANK YOU !

Newtonian Limit

- Another test of GQFT is provided by the gravitational redshift of photons;

- Ratio of Photon frequencies at different locations

$$\frac{\omega(\mathbf{x}_2)}{\omega(\mathbf{x}_1)} = \left(\frac{1+2\Phi(\mathbf{x}_1)}{1+2\Phi(\mathbf{x}_2)}\right)^{1/2} \approx 1 + \Phi(\mathbf{x}_1) - \Phi(\mathbf{x}_2)$$

- This formula is the same as in GR, so that gravitational redshift CANNOT be used to distinguish GQFT from GR.