An Exact One Body Approach to the Binary Problem In General Relativity

Dingfang Zeng

The 17th Marcel Grossmann Meeting

Pescara Italy, July 12, 2024

ref: EPJC84(2024), NPB990(2023),NPB977(2022), NPB954(2020), NPB941(2019),NPB930,NPB917(2017)

Three methods to the binary merger dynamics of GR

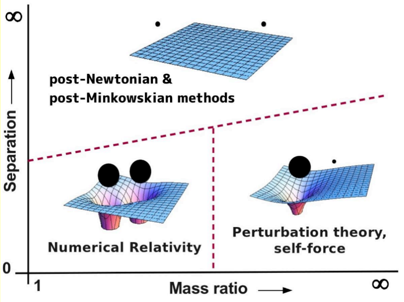


fig credit
L. Barack

NR is the only one which applies to the whole three stages of binary merger problem of GR. However, it requires huge amount of computational resources and is almost impractical for EMRI systems

EOB is a refined PN/PM approximation which works only for the inspiral phase of the merger process

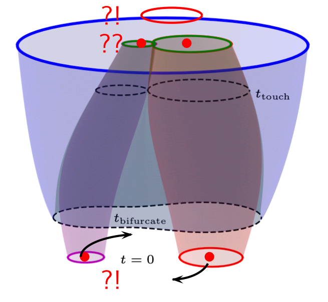
Self force method is designed for EMRI system which relies on analytic and numerics equally well, works only for the inspiral phase of the merger process

the problem of NR method

◇ It hides important physics about the inner structure of BHs in its code and boundary data setting

♣ Is there singularity inside BHs? If yes, how two Schwz's point like singularity becomes a circular line like singularity of Kerr during the merger process?

▷ PRL 123(2019)171102, "Interior of a Binary Black Hole Merger"



ric with respect to this isometry. We solve the Einstein equations only for the "upper" sheet, i.e., only for the space exterior to the throats, with boundary conditions given by

$$N|_{S_1}=0 \quad \text{and} \quad N|_{S_2}=0 \tag{11}$$

$$\vec{\beta}|_{S_1}=0 \quad \text{and} \quad \vec{\beta}|_{S_2}=0 \tag{12}$$

$$\left(\frac{\partial\Psi}{\partial r_1}+\frac{\Psi}{2r_1}\right)\bigg|_{S_1}=0 \quad \text{and} \quad \left(\frac{\partial\Psi}{\partial r_2}+\frac{\Psi}{2r_2}\right)\bigg|_{S_2}=0, \tag{13}$$

where r_1 and r_2 are the radial coordinates associated with spheres S_1 and S_2. Equations (11) reflect the antisymmetry of the lapse function N. The boundary conditions for the shift

◁ both the excision method PRD65(2002)044021 and moving puncture PRD70(2004)064011 cannot avoid BC setting on the horizon.

♠ the general idea of Green-func theorem and "bulk/boundary correspondence", any BC ∼ some special inner structure

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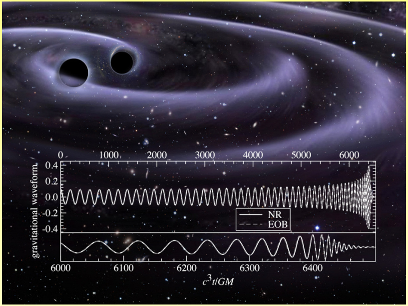
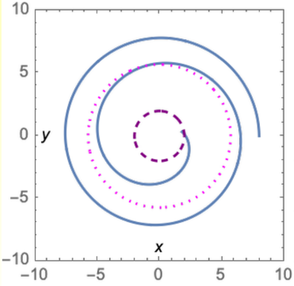
the problem of EOB method

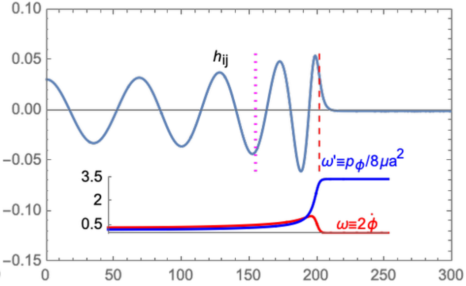
♠ EOB must be combined with NR&BHPT to yield full GW forms

▽ the WF of EOB has no QNM feature, the variation trends of ω & ω' contradicts with each other

▽ the test body falls into the horizon $r_h \sim 2GM_{1+2}$ in finite t-time $\sim$ wrong



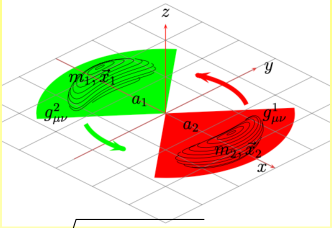




the new idea of XOB method

♠ all problems of EOB arise from its static eqv-bgrnd and artificial adding of dissipation

♡ XOB resolves these problems with a static force field whose controlling region rotates synchronously with the inspiral objects



$$L(x_A, x_B) = -M_A\sqrt{-g^A_{\mu\nu}\dot{x}^\mu_A\dot{x}^\nu_A} - M_B\sqrt{-g^B_{\mu\nu}\dot{x}^\mu_B\dot{x}^\nu_B} + L_{\rm diss} \quad (1)$$

$$g^A_{\mu\nu}\dot{x}^\mu_A\dot{x}^\nu_A = -h_B + h_B^{-1}\dot{a}_A^2 + \dot{\phi}_A^2 a_A^2 \quad (2)$$

$g^B_{\mu\nu}\dot{x}^\mu_B\dot{x}^\nu_B$ similar

$$h_B = 1 - \frac{2GE_B}{a_A} + \cdots,\ h_A = 1 - \frac{2GE_A}{a_B} + \cdots \quad (3)$$

$g^A \cup g^B$ is not the full spacetime sourced by the two inspiral objects, but a static patch-worked geometry designed to account for the force between them. Only its form on the equatorial plane and the region hosts the two objects is needed

The ellipsised terms originate from the non-linear feature of GR. This feature $\sim$ both A and B cannot be considered test particles of their inspiral partner's Schwz field

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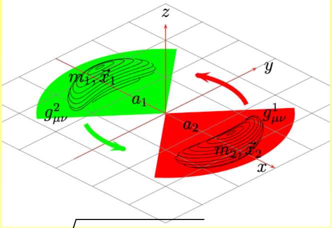

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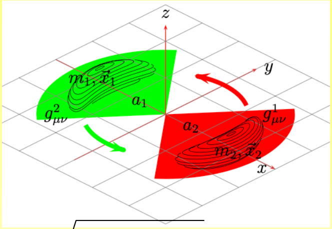

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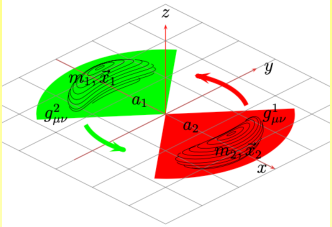
XOB method, equation of motion & hamiltonian



♠ the frequency of the force field's rotation will be determined by the flux balance condition at infinite after the radiation back-reaction is considered.

♡ central fixing and synchronicity the two particle's inspiral motion ⇒

$$a_A = \frac{M_B a}{M}, a_B = \frac{M_A a}{M}, E_A = \frac{M_B^3}{M^2}, E_B = \frac{M_A^3}{M^2}, M = M_A + M_B \quad (4)$$

this will translate the conservative part of GR binary hamiltonian into that of an exact one body form

$$\omega^2 \equiv \dot{\phi}^2 = \frac{GM}{a^3}, \ \frac{da}{dt} = -(\frac{dH}{da})^{-1} F_{\rm diss} \quad (5)$$

$$H = \frac{M_B(1 - \frac{2GM_A^2}{Ma})}{\sqrt{1 - \frac{2GM_A^2}{Ma} - \frac{M_A^2 \omega^2 a^2}{M^2}}} + (A \leftrightarrow B) + \int F_{\rm diss} dt \quad (6)$$

the quadrupole formula for GW radiation will be used for $F_{\rm diss}$

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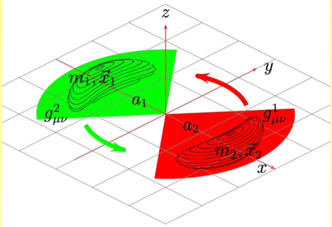

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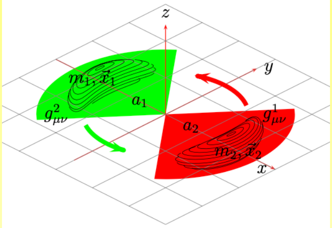

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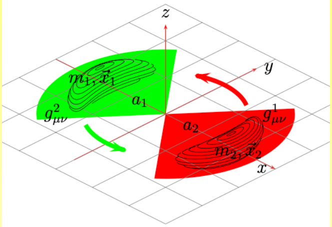

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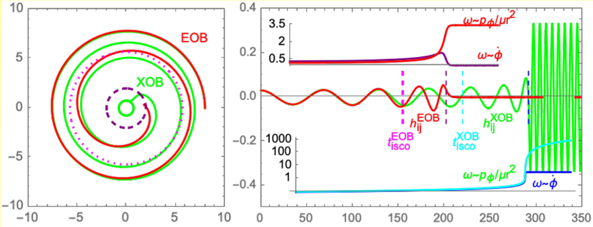
Applications of XOB method I: Schwz BHB case

when two merging participants are objects with exact horizon and singular mass central

$$\frac{dH^{\mathrm{part}}_{\mathrm{consv}}}{dt}=-\frac{32}{5}G\mu^2a^4\omega^6,\, h_{ij}\propto G\mu a^2\omega^2\cos(\int 2\omega\, dt) \quad (7)$$



♠ EOB fails when the two bodies get closer than $2GM_{\mathrm{tot}}$, but XOB method works well there

♣ the late time WF of EOB decays monotonically while that of XOB oscillate without decaying

◇ the late time trends of ω' & ω in EOB are opposite while those of XOB are the same

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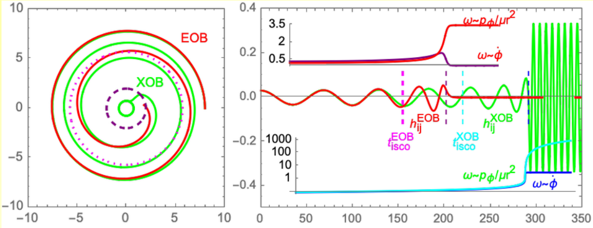

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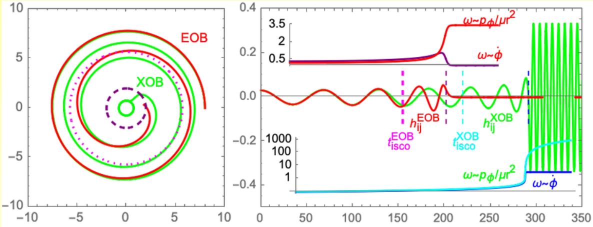

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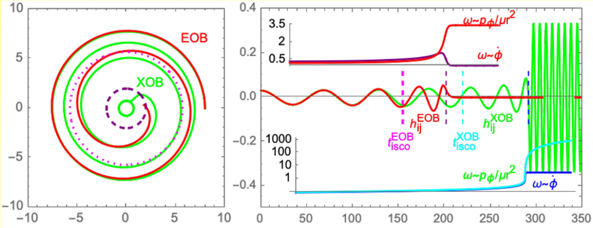

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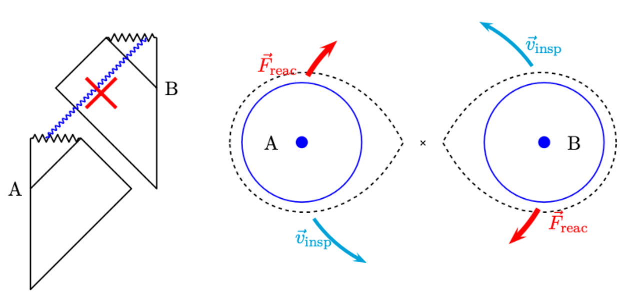

the binSchwz in GR is very like binPoinParticle in Newt mechanics, both of them have infinite descendable potential energy to support an equal amplitude GW-radiation

$$H^{\bf GR}_{binShwz} \propto \frac{-GM_A M_B^2/(Ma)}{\sqrt{1-3GM_B^2/(Ma)}} + (M_A \leftrightarrow M_B) \xrightarrow{a\to a_f} -\infty \quad (8)$$

$$H^{\bf Nwt}_{binPPtcl} \propto \frac{GM_A M_B}{a} \xrightarrow{a\to 0} -\infty \quad (9)$$

where $a_f^{GR} = \max\{\frac{3GM_A^2}{M}, \frac{3GM_B^2}{M}\}$

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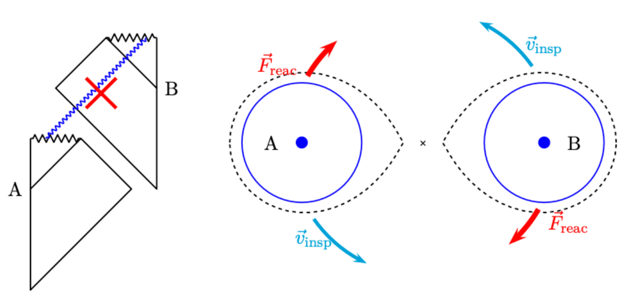

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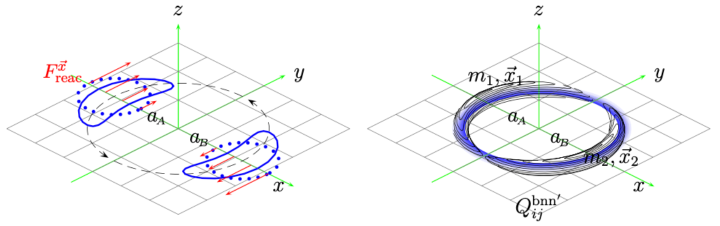
when the merger participants has extended inner mass distribution and carries no exact horizon, radiation back-reaction will cause banana deformation



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ζ - radiation-activity factor, z - banana-shape deformation, $z = z(t)$, parameterises the shape deformation progression of the physical black holes, e.g. frozen star with only asymptotically implementable horizon

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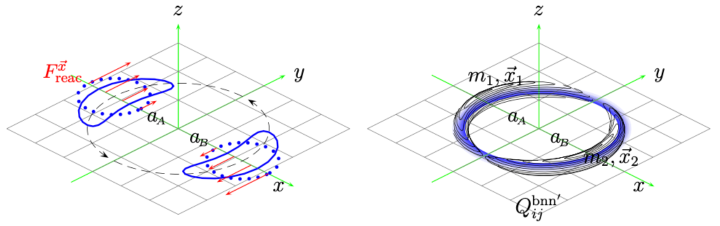

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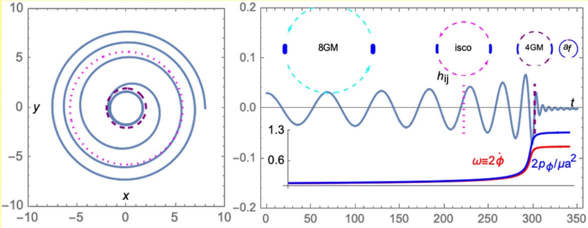

the late time WF of XOB will exhibit decaying oscillatory feature due to the rotational symmetry enhancing. Its full stage GW forms for BHB is comparable with EOB+NR+BHPT

setting $z(t)=z_f[\frac{4GMz_f}{\pi a(t)}]^n$, the match between XOB and EOBNRv2 is 70% when $z_f=2, n=1.2$

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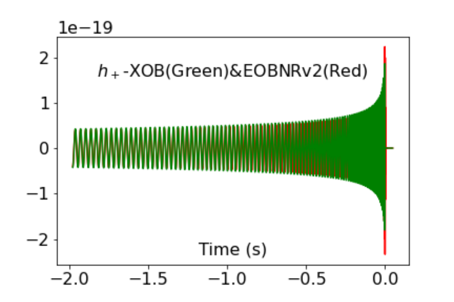

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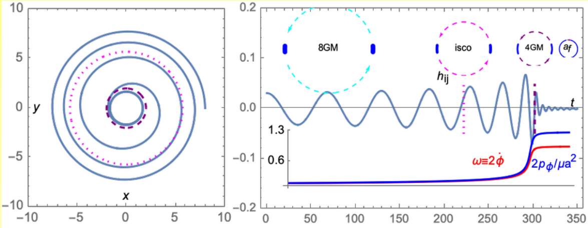

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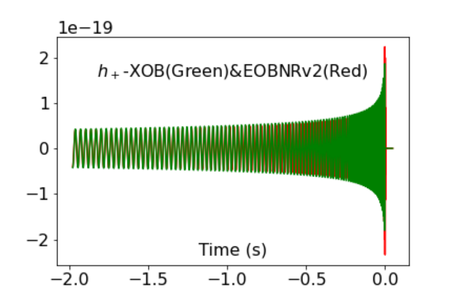

XOB method, prediction

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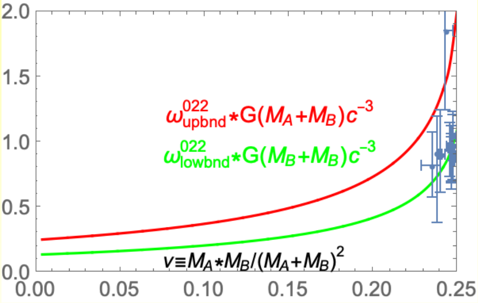
Setting this Hamiltonian $M_A + M_B$ will give an upper bound for the angular frequency of the merger product, i.e. the real part of the final black hole's quasi-normal frequency ω^{re}_{022}; while setting it to the minimal allowed value will yield the lower bound on ω^{re}_{022}

XOB prediction for the quasi-normal frequency of BHB final products. observational data from GWTC1,2,3 are compiled. this prediction is obtained from XOB method with the assumption that physical black holes have extended inner-mass distribution and no exact horizon.



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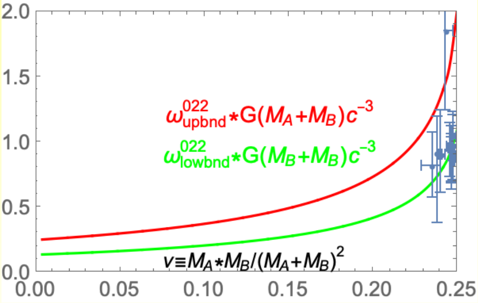
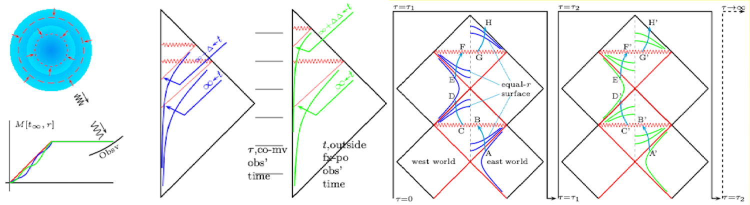


Are Black Holes Inner-structured?



- ♠ By the time definition of OFO, infinitely long time is needed for the horizon's formation, statistical description in the Boltzmann sense
- ♡ By the time of ICO, collapse across the horizon and singularity in finite duration. Hw, EP∼reaching to the singularity is not the terminal of physical evolution. OCO and ergodic evolution happens, Lagrangian description
- ♣ The domain of OFO's time is covered by that of ICO infinitely many times, OFO's statistic ensemble and ICO's ergodic OCO are equally right and complete
- ♢ 'tHooft-Suss: complementarity; Zeng, Boltzman=Lagrange and no hybridising is allowable, BH's area-law formula provable

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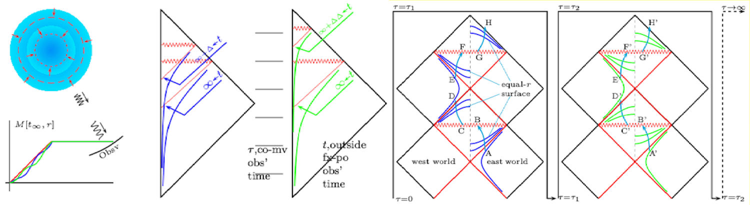

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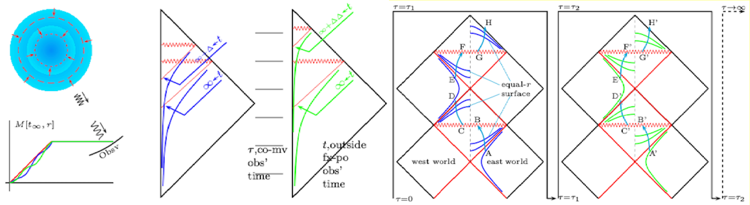

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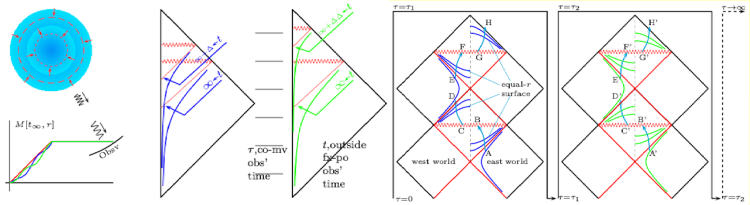

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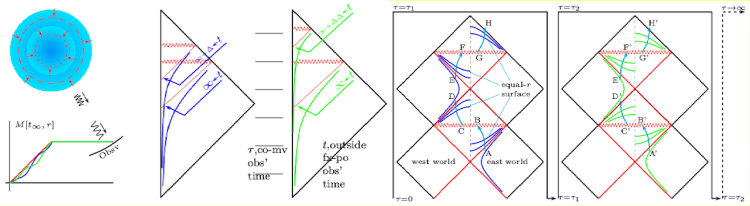

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conclusion

♠ The GW of BHB merger event is an idea lab for BH internal physics. NR encodes such physics on its horizon boundary data. EOB method contains inconsistency so cannot tell us such information

♣ XOB method gives a one-line formula for the full three stages' GWF of binary merger events, and relates their feature to the inner structure of BHs definitely and intuitively

$$H = \frac{M_B\left(1 - \frac{2GM_A^2}{Ma}\right)}{\sqrt{1 - \frac{2GM_A^2}{Ma} - \frac{M_A^2 \omega^2 a^2}{M^2}}} + (A \leftrightarrow B) + \int F_{\mathrm{diss}} dt \qquad (13)$$

♡ The GWF of XOB implies that physical BHs are inner structured, understandable in standard GR. Applicable to other binary systems such as binNS.

♡ the equivalence of XOB with EOB at the early stage needs be established further, generalisation to spinning initials and eccentric orbit are possible future directions

Thank you for your attention!

for those interested with details:
EPJC84(2024)370,2311.11764
NPB990(2023)116171,2207.05158
NPB977(2022)115722,2112.12531
NPB954(2020)115001, 1812.06777
NPB941(2019)665,1804.06726
NPB930(2018)533,1802.00675
NPB917(2017)178,1606.06178

XOB method, exact in what sense?

working flow of EOB

$$\sum \epsilon^n \ddot{x}_i^{(n)} + \Gamma[\eta + \sum \epsilon^n h^{(n)}](\epsilon^p \dot{x}_i^{(p)})(\epsilon^q \dot{x}_i^{(q)}) = 0 \quad (14)$$

$$G[\eta + \sum \epsilon^n h^{(n)}] = 8\pi G_N \sum_{i=}^{A,B} m_i(\epsilon^n \dot{x}_i^{(n)})(\epsilon^q \dot{x}_i^{(q)}) \quad (15)$$

$$(14) \xrightarrow{\epsilon=0} x_i^{(0)}, (15) \xrightarrow{x_i^{(0)}} h^{(1)}, (14) \xrightarrow{x_i^{(0)}, h^{(1)}} x_i^{(1)}, (15) \xrightarrow{\cdots} h^{(2)}, \cdots$$

writing $x_r(\tau) = x_A - x_B$ as a hamiltonian dynamics of a test particle in $ds^2_{\mathrm{eff}} = -hdt^2 + f^{-1}dr^2 + r^2 d\Omega^2$ with dissipation, writing the GW form with quadrupole formula

XOB replaces EOB's perturbative ds_{eff} with a static n-perturbative $\mathrm{g}^A \cup \mathrm{g}^B$ whose controlling region rotates synchronously with $A \& B$.

Before considering dissipation, $g^A \& g^B$'s form is almost determined by the asymptotic Keppler's 3rd law and the synchronicity and central fixing of the two body's motion

given $\mathrm{g}^A \cup \mathrm{g}^B$, the two inspiral particle's $\mathcal{L} = -\sum m_i \sqrt{-g^i_{\mu\nu} \dot{x}_i^\mu \dot{x}_i^\nu}$ is exact