

Measuring a Gravitomagnetic Effect with the Triple Pulsar PSR J0337+1715

Lorenzo Iorio 

Ministero dell' Istruzione e del Merito, Viale Unità di Italia 68, I-70125 Bari, Italy; lorenzo.iorio@libero.it

Abstract: To the first post-Newtonian order, the orbital angular momentum of the fast-revolving inner binary of the triple system PSR J0337+1715, made of a millisecond pulsar and a white dwarf, induces an annular gravitomagnetic field which displaces the line of apsides of the slower orbit of the other, distant white dwarf by -1.2 milliarcseconds per year. The current accuracy in determining the periastron of the outer orbit is 63.9 milliarcseconds after 1.38 years of data collection. By hypothesizing a constant rate of measurement of the pulsar's times of arrivals over the next 10 years, assumed equal to the present one, it can be argued that the periastron will be finally known to a $\simeq 0.15$ milliarcseconds level, while its cumulative gravitomagnetic retrograde shift will be as large as -12 milliarcseconds. The competing post-Newtonian gravitoelectric periastron advance due to the inner binary's masses, nominally amounting to 74.3 milliarcseconds per year, can be presently modelled to an accuracy level as good as $\simeq 0.04$ milliarcseconds per year. The mismodeling in the much larger Newtonian periastron rate due to the quadrupolar term of the multipolar expansion of the gravitational potential of a massive ring representing the inner binary, whose nominal size for PSR J0337+1715 is 0.17 degrees per year, might be reduced down to the $\simeq 0.5$ milliarcseconds per year level over the next 10 years. Thus, a first measurement of such a novel form of gravitomagnetism, although undoubtedly challenging, might be, perhaps, feasible in a not too distant future.

Keywords: classical general relativity; experimental studies of gravity; experimental tests of gravitational theories; binary and multiple stars; pulsars



Citation: Iorio, L. . Measuring a Gravitomagnetic Effect with the Triple Pulsar PSR J0337+1715. *Universe* **2024**, *10*, 206. <https://doi.org/10.3390/universe10050206>

Academic Editor: Nicolas Chamel

Received: 15 April 2024

Revised: 28 April 2024

Accepted: 2 May 2024

Published: 6 May 2024



Copyright: © 2024 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

PSR J0337+1715 [1] is a peculiar hierarchical triple system made of three astrophysical compact objects: an inner, tight binary \mathcal{S} composed of a 2.7 ms millisecond pulsar A and a white dwarf B revolving one around each other in a relative circular orbit with orbital period $P_b \simeq 1.6$ d and another white dwarf C moving about \mathcal{S} along a wider circular path with orbital period $P \simeq 327$ d and coplanar with that of \mathcal{S} itself, both inclined by $I_b = I = 39.2^\circ$ to the plane of the sky, assumed as a reference $\{x, y\}$ plane. Their masses are $M_A = 1.44M_\odot$, $M_B = 0.2M_\odot$, and $M_C = 0.4M_\odot$, respectively, where M_\odot is the mass of the Sun.

So far, PSR J0337+1715 is the sole relatively tight member discovered in the class of hierarchical triple systems hosting stellar corpses, apart from B162-26, whose pulsar has a white dwarf as an inner companion and a roughly Jupiter-mass at 35 au as the outer orbiter [2]. According to Ransom et al. [1], $\lesssim 1\%$ of the millisecond pulsars population resides in stellar triples, and $\lesssim 100$ such systems exist in the Galaxy. In general, it is expected that, in triple systems hosting a millisecond pulsar, dubbed as triple pulsars in the following, the orbit of the distant companion is more eccentric due to dynamical interactions between the stars during stellar evolution [3]. The coplanarity and circularity of the orbits of PSR J0337+1715, along with the fact that $P/P_b \simeq 200$, imply that the current configuration is stable on long timescales, which favored its discovery. It is likely that the angular momentum vectors of the inner and outer orbits were torqued into near alignment in a

phase lasting about 1 Gyr, during which the outermost member evolved and transferred mass onto the inner binary, perhaps within a common envelope [4].

On one hand, PSR J0337+1715 proved unsuitable, at least until now, to perform the usual tests of the General Theory of Relativity (GTR) performed with some tight binary pulsars [5–7] like, e.g., the Hulse–Taylor pulsar PSR B1913+16 [8] and the double pulsar PSR J0737-3039A/B [9,10] because of its orbital configuration. Indeed, the argument of periastron ω is badly defined because of the almost vanishing eccentricity e of the orbit of the outer white dwarf. Furthermore, the mass of the inner one is far too small for the gravitational redshift of the pulsar signal to be measurable. Finally, the Shapiro delay is negligible; the radio waves traveling along the line of sight toward the Earth pass very distantly from its companion because the orbital plane is far from being seen edge-on. On the other hand, PSR J0337+1715 turned out to be a valuable laboratory to use the tightest constraints so far on the (absence of the) Nordtvedt effect [11,12]. The latter is an orbital consequence of a possible violation of the strong equivalence principle, which would take place should bodies with different amounts of gravitational self-energy, just like a neutron star and a white dwarf, fall with different accelerations in an external gravitational field. As a consequence, if the falling objects orbit one around each other while moving altogether about a third body, their barycentric orbits should experience a differential elongation toward the source of the external field. Actually, no Nordtvedt effect was found in PSR J0337+1715 to a relative accuracy of two parts per million at a 95% confidence level [13,14].

Does PSR J0337+1715 offer the possibility of testing some other predictions of GTR in a not too far future? The answer may become positive within a reasonable timeframe, as soon as a sufficiently large number of times of arrivals (TOAs) has been collected.

This paper is organized as follows. Section 2 is devoted to the orbital precessions of a test particle induced by the gravitomagnetic field of a distant, inner rotating massive ring. In Section 3, they are calculated for PSR J0337+1715 (Section 3.1), and an evaluation of the experimental accuracy obtainable in a not too remote future is given (Section 3.2). The competing effects due to the Schwarzschild-like gravitoelectric field of \mathcal{S} (Section 3.3) and to the expansion of the Newtonian gravitational field of a massive ring to the quadrupolar order (Section 3.4) are worked out as well. Section 4 summarizes our findings and offers conclusions.

2. The Gravitomagnetic Orbital Precessions Due to the Orbital Angular Momentum of the Inner Binary

The effect in question, to the first post-Newtonian (1pN) order, would not only be the usual gravitoelectric periastron advance due solely to the masses of the system but, in principle, also a particular form of gravitomagnetic orbital shift.

At first, this might sound strange, given that the generalizations of the well-known Lense–Thirring (LT) orbital precessions [15] to a binary system are [16]

$$\dot{i}_b^{LT} = \frac{2GJ_b(\hat{\mathbf{j}}_b \cdot \hat{\mathbf{i}}_b)}{c^2 a_b^3 (1 - e_b^2)^{3/2}} \tag{1}$$

$$\dot{\Omega}_b^{LT} = \frac{2GJ_b(\hat{\mathbf{j}}_b \cdot \hat{\mathbf{m}}_b)}{c^2 \sin I_b a_b^3 (1 - e_b^2)^{3/2}} \tag{2}$$

$$\dot{\omega}_b^{LT} = -\frac{2GJ_b \left[2(\hat{\mathbf{j}}_b \cdot \hat{\mathbf{h}}_b) + \cot I_b (\hat{\mathbf{j}}_b \cdot \hat{\mathbf{m}}_b) \right]}{c^2 a_b^3 (1 - e_b^2)^{3/2}} \tag{3}$$

induced by the spin angular momenta J_A and J_B of both bodies through

$$J_b := \left(1 + \frac{3}{4} \frac{M_B}{M_A}\right) J_A + \left(1 + \frac{3}{4} \frac{M_A}{M_B}\right) J_B \tag{4}$$

which are notoriously orders of magnitude smaller than the 1pN gravitoelectric periastron rate

$$\dot{\omega}_b^{GE} = \frac{3n_b\mu_b}{c^2 a_b (1 - e_b^2)} \tag{5}$$

In Equations (1)–(5), G is the Newtonian constant of gravitation, $\mu_b := G(M_A + M_B)$ is the standard gravitational parameter of the binary, c is the speed of light in vacuum, and a_b, e_b, ω_b , and Ω_b are the semimajor axis, the eccentricity, the argument of pericentre, and the longitude of the ascending node of the binary’s relative orbit, respectively, while $n_b := \sqrt{\mu_b/a_b^3}$ is the Keplerian mean motion related to the orbital period by $P_b = 2\pi/n_b$. Furthermore

$$\hat{l}_b := \{\cos \Omega_b, \sin \Omega_b, 0\} \tag{6}$$

$$\hat{m}_b := \{-\cos I_b \sin \Omega_b, \cos I_b \cos \Omega_b, \sin I_b\} \tag{7}$$

$$\hat{h}_b := \{\sin I_b \sin \Omega_b, -\sin I_b \cos \Omega_b, \cos I_b\} \tag{8}$$

are three mutually orthogonal unit vectors directed along the line of nodes in the reference $\{x, y\}$ plane, the orbital plane perpendicular to the latter, and the orbital angular momentum, respectively, in such a way that $\hat{l}_b \times \hat{m}_b = \hat{h}_b$. Suffice to say, for, e.g., the double pulsar PSR J0737–3039A/B [9,10], the gravitoelectric periastron precession of Equation (5) is as large as [5]

$$\dot{\omega}_b^{GE} = 16.89 \text{ deg yr}^{-1} \tag{9}$$

while the gravitomagnetic one, mainly due to the angular momentum of PSR J0737–3039A, is expected to be as small as [17,18]

$$\dot{\omega}_b^{LT} \simeq -0.00060 \text{ deg yr}^{-1} = -2.16 \text{ arcsec yr}^{-1} \tag{10}$$

Intense efforts to measure it, not yet crowned with success, are currently underway [17,19,20]. The figure quoted in Equation (10) is inferred from²

$$J_A \simeq 4.5 \times 10^{40} \text{ kg m}^2 \text{ s}^{-1} \tag{11}$$

for the angular momentum of PSR J0737–3039A; it can be calculated by assuming for the pulsar’s moment of inertia (MOI) the value [21]

$$\mathcal{I}_A \simeq 1.6 \times 10^{38} \text{ kg m}^2 \tag{12}$$

and recalling that the spin period of the A component of the double pulsar is 22.7 ms [5].

As pointed out in Iorio [22], a hierarchical triple system offers, in principle, the possibility of measuring the orbital precessions due to the gravitomagnetic field generated by the mass current generated by the fast orbital motion of the inner binary, seen as a rotating matter ring. In other words, one has to replace³ J_b in Equations (1)–(3) with the orbital angular momentum L_b of the inner binary. Its magnitude is

$$L_b = M_{\text{red}} \sqrt{\mu_b a_b (1 - e_b^2)} \tag{13}$$

where

$$M_{\text{red}} := \frac{M_A M_B}{M_b} \tag{14}$$

is the reduced mass, and

$$M_b := M_A + M_B \tag{15}$$

is the total mass of the inner binary. Usually, L_b can be orders of magnitude larger than the spin angular momentum of any of the inner binary’s components; in the case of PSR J0337+1715, it is

$$L_b = 3.5 \times 10^{44} \text{ kg m}^2 \text{ s}^{-1} \tag{16}$$

while the pulsar’s spin angular momentum J should be about three orders of magnitude smaller. From Equations (1)–(3), it straightforwardly follows that, for a coplanar triple system, i.e., if $\hat{L}_b \cdot \hat{h} = 1$, $\hat{L}_b \cdot \hat{l} = \hat{L}_b \cdot \hat{m} = 0$, the inclination and the node stay constant, while the periastron precesses at a rate

$$\dot{\omega}_{\text{LT}} = -\frac{4GL_b}{c^2 a^3 (1 - e^2)^{3/2}} \tag{17}$$

3. The Case of PSR J0337+1715

3.1. The Effect of the 1pN Gravitomagnetic Annular Field

In the particular case of PSR J0337+1715, Equation (17) predicts for it a gravitomagnetic precession of the order of

$$\dot{\omega}_{\text{LT}} = -1.2 \text{ mas yr}^{-1} \tag{18}$$

where mas stands for milliarcseconds, while the current level of uncertainty in measuring its periastron over 1.38 yr during which

$$N_0 = 26280 \tag{19}$$

TOAs were collected can be calculated from Table 1 of Ransom et al. [1] to be

$$\sigma_{\omega}^0 \simeq 63.9 \text{ mas} \tag{20}$$

3.2. Prospects for Future Accuracy Improvements

By provisionally assuming that about same number of TOAs as given by Equation (19) will be collected in 1.38 yr over, say, the next 10 years, the resulting accuracy would be improved down to⁴

$$\sigma_{\omega} \simeq 0.15 \text{ mas} \tag{21}$$

with respect to the figure quoted in Equation (20), while the total gravitomagnetic shift would amount to

$$\Delta\omega_{\text{LT}} \simeq -12 \text{ mas} \tag{22}$$

3.3. The 1pN Gravitoelectric Periastron Precession

The 1pN gravitoelectric apsidal rate of the orbit of the outer companion around the inner binary, whose nominal value is

$$\dot{\omega}_{\text{GE}} = 74.3 \text{ mas yr}^{-1} \tag{23}$$

could be safely subtracted from the measured total periastron precession since its uncertainty, calculated by propagating in Equation (5) the errors of P , μ , and e from Table 1 of Ransom et al. [1], is currently as little as

$$\sigma_{\dot{\omega}_{\text{GE}}} = 0.04 \text{ mas yr}^{-1} \tag{24}$$

3.4. The Newtonian Periastron Precession Due to the Quadrupolar Term of the Gravitational Potential of a Matter Ring

By expanding to the quadrupolar order the Newtonian gravitational potential of a circular massive ring, calculated in points far from the latter and lying in its plane [24], an extra-potential falling as $1/r^3$ arises; in the case of the inner binary of PSR J0337+1715, it can be cast into the form [25]

$$U_{qp} = -\frac{\mu_b a_b^2}{4r^3} \tag{25}$$

where the ring’s mass and radius are assumed to be equal to the total mass M_b and to the semimajor axis a_b of S , respectively.

It can be straightforwardly calculated that the periastron of the distant companion is shifted by Equation (25) according to [26,27]

$$\dot{\omega}^{qp} = \frac{3\mu_b a_b^2}{4\sqrt{\mu a^7 (1 - e^2)^2}} \tag{26}$$

where

$$\mu := G(M_A + M_B + M_C) \tag{27}$$

is the total standard gravitational parameter of PSR J0337+1715. The nominal precession of Equation (26) amounts to

$$\dot{\omega}^{qp} = 636775 \text{ mas yr}^{-1} = 0.17 \text{ deg yr}^{-1} \tag{28}$$

It turns out that the main sources of uncertainty in Equation (26) are the masses of the pulsar A and of the outer white dwarf C. By assuming that their errors will reduce by a factor of $1/\sqrt{N}$ in the next 10 yr, where N is calculated as in Equation (20), the mismodeling in Equation (28) should finally be

$$\sigma_{\dot{\omega}^{qp}} \simeq 0.5 \text{ mas yr}^{-1} \tag{29}$$

4. Summary and Conclusions

By assuming a rate of collection of TOAs over the next 10 years equal to the present one, it can be guessed that the accuracy in determining the periastron precession of PSR J0337+1715 may reach the $\simeq 0.15$ mas level, allowing, in principle, for a measurement of its cumulative gravitomagnetic shift of -12 mas due to the orbital angular momentum of the inner binary. The competing gravitoelectric advance should be subtracted from the total periastron variation without problems, since its uncertainty is, as of now, as little as 0.04 mas yr^{-1} . On the other hand, relying upon the aforementioned assumptions, it can be argued that the accuracy in modelling the nominally much larger Newtonian periastron precession due to the quadrupolar term of the multipolar expansion of the gravitational potential of a massive ring should reach the $\simeq 0.5 \text{ mas yr}^{-1}$ level.

The situation may be more favorable for a hypothetical triple system, yet to be discovered, whose outer companion’s orbit had a smaller size and was more eccentric, a scenario that should not be deemed as unrealistic. Furthermore, if \hat{L}_b and \hat{h} were misaligned, the inclination and the node precessions would also come into play, as per Equation (1) and Equation (2).

Funding: This research received no external funding.

Data Availability Statement: No new data were generated or analysed in support of this research.

Conflicts of Interest: The author declares no conflicts of interest.

Notes

- ¹ The orbital parameters of the outer companion are written without any label, while those of *S* are dubbed with “b”.
- ² The angular momentum of the B component of the double pulsar, which rotates more slowly, is smaller by about two orders of magnitude.
- ³ The gravitomagnetic field induced by a rotating matter ring is, far from the latter, identical to the usual one for a rotating body whose angular momentum *J* is replaced with the ring’s one [23].
- ⁴ Such an estimate is obtained by dividing Equation (20) by \sqrt{N} , where $N = (10/1.38) \times N_0 = 190435$.

References

1. Ransom, S.M.; Stairs, I.H.; Archibald, A.M.; Hessels, J.W.T.; Kaplan, D.L.; Van Kerkwijk, M.H.; Boyles, J.; Deller, A.T.; Chatterjee, S.; Schechtman-Rook, A.; et al. A millisecond pulsar in a stellar triple system. *Nature* **2014**, *505*, 520–524. [[CrossRef](#)] [[PubMed](#)]
2. Thorsett, S.E.; Arzoumanian, Z.; Camilo, F.; Lyne, A.G. The Triple Pulsar System PSR B1620–26 in M4. *Astrophys. J.* **1999**, *523*, 763–770. [[CrossRef](#)]
3. Portegies Zwart, S.; van den Heuvel, E.P.J.; van Leeuwen, J.; Nelemans, G. The Formation of the Eccentric-orbit Millisecond Pulsar J1903+0327 and the Origin of Single Millisecond Pulsars. *Astrophys. J.* **2011**, *734*, 55. [[CrossRef](#)]
4. Larwood, J.D.; Papaloizou, J.C.B. The hydrodynamical response of a tilted circumbinary disc: Linear theory and non-linear numerical simulations. *Mon. Not. Roy. Astron. Soc.* **1997**, *285*, 288–302. [[CrossRef](#)]
5. Kramer, M.; Stairs, I.H.; Manchester, R.N.; McLaughlin, M.A.; Lyne, A.G.; Ferdman, R.D.; Burgay, M.; Lorimer, D.R.; Possenti, A.; D’Amico, N.; et al. Tests of General Relativity from Timing the Double Pulsar. *Science* **2006**, *314*, 97–102. [[CrossRef](#)]
6. Wex, N.; Kramer, M. Gravity Tests with Radio Pulsars. *Universe* **2020**, *6*, 156. [[CrossRef](#)]
7. Kramer, M.; Stairs, I.H.; Manchester, R.N.; Wex, N.; Deller, A.T.; Coles, W.A.; Ali, M.; Burgay, M.A.R.T.A.; Camilo, F.; Cognard, I.; et al. Strong-Field Gravity Tests with the Double Pulsar. *Phys. Rev. X* **2021**, *11*, 041050. [[CrossRef](#)]
8. Hulse, R.A.; Taylor, J.H. Discovery of a pulsar in a binary system. *Astrophys. J. Lett.* **1975**, *195*, L51–L53. [[CrossRef](#)]
9. Burgay, M.; D’Amico, N.; Possenti, A.; Manchester, R.N.; Lyne, A.G.; Joshi, B.C.; McLaughlin, M.A.; Kramer, M.; Sarkissian, J.M.; Camilo, F.; et al. An increased estimate of the merger rate of double neutron stars from observations of a highly relativistic system. *Nature* **2003**, *426*, 531–533. [[CrossRef](#)]
10. Lyne, A.G.; Burgay, M.; Kramer, M.; Possenti, A.; Manchester, R.N.; Camilo, F.; McLaughlin, M.A.; Lorimer, D.R.; D’Amico, N.; Joshi, B.C.; et al. A Double-Pulsar System: A Rare Laboratory for Relativistic Gravity and Plasma Physics. *Science* **2004**, *303*, 1153–1157. [[CrossRef](#)]
11. Nordtvedt, K.L. Testing Relativity with Laser Ranging to the Moon. *Phys. Rev.* **1968**, *170*, 1186–1187. [[CrossRef](#)]
12. Nordtvedt, K.L. Equivalence Principle for Massive Bodies. I. Phenomenology. *Phys. Rev.* **1968**, *169*, 1014–1016. [[CrossRef](#)]
13. Archibald, A.M.; Gusinskaia, N.V.; Hessels, J.W.T.; Deller, A.T.; Kaplan, D.L.; Lorimer, D.R.; Lynch, R.S.; Ransom, S.M.; Stairs, I.H. Universality of free fall from the orbital motion of a pulsar in a stellar triple system. *Nature* **2018**, *559*, 73–76. [[CrossRef](#)] [[PubMed](#)]
14. Voisin, G.; Cognard, I.; Freire, P.C.C.; Wex, N.; Guillemot, L.; Desvignes, G.; Kramer, M.; Theureau, G. An improved test of the strong equivalence principle with the pulsar in a triple star system. *Astron. Astrophys.* **2020**, *638*, A24. [[CrossRef](#)]
15. Lense, J.; Thirring, H. Über den Einfluß der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Gravitationstheorie. *Phys. Z* **1918**, *19*, 156–163.
16. Soffel, M.H. *Relativity in Astrometry, Celestial Mechanics and Geodesy*; Springer: Berlin/Heidelberg, Germany, 1989. [[CrossRef](#)]
17. Hu, H.; Kramer, M.; Wex, N.; Champion, D.J.; Kehl, M.S. Constraining the dense matter equation-of-state with radio pulsars. *Mon. Not. Roy. Astron. Soc.* **2020**, *497*, 3118–3130. [[CrossRef](#)]
18. Iorio, L. The impact of the spin-orbit misalignment and of the spin of B on the Lense–Thirring orbital precessions of the double pulsar PSR J0737-3039A/B. *Mon. Not. Roy. Astron. Soc.* **2021**, *507*, 421–430. [[CrossRef](#)]
19. Kehl, M.S.; Wex, N.; Kramer, M.; Liu, K. Future measurements of the Lense–Thirring effect in the Double Pulsar. In *Proceedings of the Fourteenth Marcel Grossmann Meeting. Proceedings of the MG14 Meeting on General Relativity, Rome, Italy, 12–18 July 2015*; Bianchi, M., Jantzen, R., Ruffini, R., Eds.; World Scientific: Singapore, 2017; pp. 1860–1865. [[CrossRef](#)]
20. Hu, H.; Freire, P.C.C. Measuring the Lense–Thirring Precession and the Neutron Star Moment of Inertia with Pulsars. *Universe* **2024**, *10*, 160. [[CrossRef](#)]
21. Silva, H.O.; Holgado, A.M.; Cárdenas-Avendaño, A.; Yunes, N. Astrophysical and Theoretical Physics Implications from Multimessenger Neutron Star Observations. *Phys. Rev. Lett.* **2021**, *126*, 181101. [[CrossRef](#)]
22. Iorio, L. Frame-Dragging in Extrasolar Circumbinary Planetary Systems. *Universe* **2022**, *8*, 546. [[CrossRef](#)]
23. Ruggiero, M.L. Gravitomagnetic field of rotating rings. *Astrophys. Space Sci.* **2016**, *361*, 140. [[CrossRef](#)]
24. Ciftja, O.; Babineaux, A.; Hafeez, N. The electrostatic potential of a uniformly charged ring. *Eur. J. Phys.* **2009**, *30*, 623–627. [[CrossRef](#)]
25. Demetrian, M. Motion of a testing particle in gravitational field of a ring. *arXiv* **2006**, arXiv:physics/0602034. [[CrossRef](#)]

-
26. Adkins, G.S.; McDonnell, J. Orbital precession due to central-force perturbations. *Phys. Rev. D* **2007**, *75*, 082001. [[CrossRef](#)]
27. Iorio, L. Model-independent constraints on r^{-3} extra-interactions from orbital motions. *Ann. Phys.* **2012**, *524*, 371–377. [[CrossRef](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.