Measuring a novel form of gravitomagnetism with hierarchical triple systems

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universe *time and space*

Outline

The Lense-Thirring Effect (I)

- In a two–body system, the Lense–Thirring (LT) orbital precessions are induced by the individual spin angular momenta $\boldsymbol{J}_{\mathrm{A}}$ and $\boldsymbol{J}_{\mathrm{B}}$ of both bodies A and B of masses M_{A} and M_{B} according to

$$\dot{I}_{\mathrm{b}}^{\mathrm{LT}} = \frac{2GJ_{\mathrm{b}}\left(\hat{\boldsymbol{J}}_{\mathrm{b}}\cdot\hat{\boldsymbol{\ell}}_{\mathrm{b}}\right)}{c^2 a_{\mathrm{b}}^3\left(1-e_{\mathrm{b}}^2\right)^{3/2}}, \tag{1}$$

$$\dot{\Omega}_{\mathrm{b}}^{\mathrm{LT}} = \frac{2GJ_{\mathrm{b}}\left(\hat{\boldsymbol{J}}_{\mathrm{b}}\cdot\hat{\boldsymbol{m}}_{\mathrm{b}}\right)}{c^2 \sin I_{\mathrm{b}}\, a_{\mathrm{b}}^3\left(1-e_{\mathrm{b}}^2\right)^{3/2}}, \tag{2}$$

$$\dot{\omega}_{\mathrm{b}}^{\mathrm{LT}} = -\frac{2GJ_{\mathrm{b}}\left[2\left(\hat{\boldsymbol{J}}_{\mathrm{b}}\cdot\hat{\boldsymbol{h}}_{\mathrm{b}}\right)+\cot I_{\mathrm{b}}\left(\hat{\boldsymbol{J}}_{\mathrm{b}}\cdot\hat{\boldsymbol{m}}_{\mathrm{b}}\right)\right]}{c^2 a_{\mathrm{b}}^3\left(1-e_{\mathrm{b}}^2\right)^{3/2}}. \tag{3}$$

The Lense-Thirring effect (II)

- Eqs. (1)–(3), in which

$$\boldsymbol{J}_\mathrm{b} := \left(1 + \frac{3}{4}\frac{M_\mathrm{B}}{M_\mathrm{A}}\right)\boldsymbol{J}_\mathrm{A} + \left(1 + \frac{3}{4}\frac{M_\mathrm{A}}{M_\mathrm{B}}\right)\boldsymbol{J}_\mathrm{B}, \tag{4}$$

  are orders of magnitude *smaller* (5 for Mercury, 2 for the Earth's LAGEOS satellite) than the Schwarzschild–like pericentre rate

$$\dot\omega_\mathrm{b}^\mathrm{Sch} = \frac{3 n_\mathrm{b}\mu_\mathrm{b}}{c^2 a_\mathrm{b}\left(1 - e_\mathrm{b}^2\right)}. \tag{5}$$

- In eqs. (1)–(5), G is the Newton's constant of gravitation, c is the speed of light, $a_\mathrm{b}, e_\mathrm{b}, I_\mathrm{b}, \omega_\mathrm{b}$ are the semimajor axis, eccentricity, inclination and pericentre of the binary's relative orbit, $\hat{\boldsymbol{\ell}}_\mathrm{b}, \hat{\boldsymbol{m}}_\mathrm{b}, \hat{\boldsymbol{h}}_\mathrm{b}$ are three orthonormal vectors depending on I_b and the node Ω_b, $\mu_\mathrm{b} := GM_\mathrm{b} = G\left(M_\mathrm{A} + M_\mathrm{B}\right)$ is the binary's standard gravitational parameter, and $n_\mathrm{b} := \sqrt{\mu_\mathrm{b}/a_\mathrm{b}^3}$ is its Keplerian mean motion.

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Hierarchical triple systems

- Hierarchical triple systems like circumbinary exoplanets and the triple pulsar PSR J0337 + 1715 are made of an inner tight binary $\mathcal{B}$ orbited by a distant companion C
- The *orbital* angular momentum $\boldsymbol{h}_\mathrm{b}$ of the inner binary $\mathcal{B}$

$$h_\mathrm{b} = m_\mathrm{red}\sqrt{\mu_\mathrm{b} a_\mathrm{b}\left(1 - e_\mathrm{b}^2\right)}, \tag{6}$$

  where $m_\mathrm{red} := M_\mathrm{A} M_\mathrm{B}/M_\mathrm{b}$ is the binary's reduced mass, is usually much *larger* than the individual spin angular momenta of A and B
- $\boldsymbol{h}_\mathrm{b}$ generates an additional gravitomagnetic field which affects the orbit of the distant companion C from which the inner binary $\mathcal{B}$ is seen as a *rotating matter ring* around $\hat{\boldsymbol{h}}_\mathrm{b}$. Circumbinary exoplanets and triple pulsars may be used to test it [Iorio 2022, Iorio 2024].

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LT versus Schwarzschild pericenter precessions (I)

- For a circumbinary planet P orbiting a pair of stars A and B, by imposing

$$\Delta\omega_{\mathrm{P}}^{\mathrm{LT}} = k\Delta\omega_{\mathrm{P}}^{\mathrm{Sch}},\ k > 0, \tag{7}$$

one has

$$a_{\mathrm{P}} = \frac{16 a_{\mathrm{b}}\left(1-e_{\mathrm{b}}^2\right) M_{\mathrm{A}}^2 M_{\mathrm{B}}^2}{9k^2 M_{\mathrm{b}}\left(M_{\mathrm{b}}+M_{\mathrm{P}}\right)^3\left(1-e_{\mathrm{P}}^2\right)}. \tag{8}$$

- If, say, $M_{\mathrm{A}} = M_{\mathrm{B}} = M_{\odot}$ and $M_{\mathrm{P}} = M_{♃}$, where $M_{\odot}$ and $M_{♃}$ are the masses of the Sun and Jupiter, respectively, and both the orbits are circular and coplanar, for

$$a_{\mathrm{P}} = 11.5\ a_{\mathrm{b}}, \tag{9}$$

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$$k \simeq 0.1; \tag{10}$$

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- From eq. (8) one has that
$$k = 1 \tag{11}$$
would imply
$$a_{\mathrm{P}} = 0.1 a_{\mathrm{b}}; \tag{12}$$
the LT shift cannot be as large as the Schwarzschild one.
- In order to have
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The expected gravitomagnetic effect

- PSR J0337 + 1715 is made of an inner binary $\mathcal{B}$ composed by a 2.7 ms millisecond pulsar A and a white dwarf B revolving one around each other in a circular orbit with orbital period $P_{\mathrm{b}} \simeq 1.6\,\mathrm{d}$, and another white dwarf C moving about them along a wider circular path with orbital period $P_{\mathrm{C}} \simeq 327\,\mathrm{d}$ and coplanar with that of $\mathcal{B}$ itself, both inclined by $I_{\mathrm{b}} = I_{\mathrm{C}} = 39.2^{\circ}$ to the plane of the sky. The masses are $M_{\mathrm{A}} = 1.44 M_{\odot}$, $M_{\mathrm{B}} = 0.2 M_{\odot}$ and $M_{\mathrm{C}} = 0.4 M_{\odot}$, respectively.
- Due to the peculiar orbital geometry of this system, the inclination and the node of the orbit of C about $\mathcal{B}$ are *constant*, while only the periastron is shifted at a rate

$$\dot{\omega}_{\mathrm{C}}^{\mathrm{LT}} = -\frac{4 G h_{\mathrm{b}}}{c^2 a_{\mathrm{C}}^3 \left(1 - e_{\mathrm{C}}^2\right)^{3/2}} = -1.2\,\mathrm{milliarcseconds\ per\ year\ (mas/yr)} \tag{15}$$

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The present and future experimental accuracy

- The *current uncertainty* in determining the periastron after N_0=26280 times of arrival (TOAs) collected during 1.38 yr is

$$\sigma^{0}_{\omega_{\mathrm{C}}} \simeq 63.9\,\mathrm{mas}. \tag{16}$$

- By assuming the *same* rate of data collection over the next 10 yr, the resulting *uncertainty* in determining the periastron should be

$$\sigma_{\omega_{\mathrm{C}}} \simeq 0.15\,\mathrm{mas}. \tag{17}$$

- The total gravitomagnetic periastron shift over the same time span would amount to

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Systematic errors

- To the *Newtonian* level, a matter ring induces an extra–precession of periastron whose quadrupolar term turns out to be

$$\dot{\omega}_{\rm C}^{\rm qp} = \frac{3\mu_{\rm b}a_{\rm b}}{4\sqrt{\mu a_{\rm C}^7}\left(1 - e_{\rm C}^2\right)^2}, \tag{19}$$

where $\mu := G\left(M_{\rm A} + M_{\rm B} + M_{\rm C}\right)$.

- Its *nominal* value is

$$\dot{\omega}_{\rm C}^{\rm qp} = 0.17^\circ/{\rm yr}. \tag{20}$$

- Its *uncertainty* is dominated by the errors in the masses of A and C. By assuming that they will improve by $1/\sqrt{N}$, where N is the number of TOAs which are expected to be collected in the next 10 yr at the *same* rate of those already recorded, the *uncertainty* in the quadrupole–induced periastron rate should be reduced to

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Concluding remarks

- The *orbital* angular momentum $\boldsymbol{h}_{\rm b}$ of a two–body system can act on the orbit of a distant companion C revolving about the former through its own gravitomagnetic field which is usually *larger* than that due to the individual spin angular momentum $\boldsymbol{J}$ of any of the inner binary's members A or B
- Circumbinary exoplanets and triple pulsars may be used to test this novel gravitomagnetic effect
- Over the next 10 yr, the experimental *uncertainty* in determining the periastron of the outer orbit of the triple pulsar PSR J0337 + 1715 may be $\sigma_{\omega_{\rm C}} \simeq 0.15\,{\rm mas}$, while the total LT periastron *shift* would be as large $\Delta\omega_{\rm C}^{\rm LT} = -12\,{\rm mas}$. The systematic bias due to the *Newtonian* quadrupolar precession, whose nominal value is currently too large, should be reduced to the $\sigma_{\dot\omega_{\rm C}^{\rm qp}} \simeq 0.5\,{\rm mas/yr}$ level. A measurement might, thus, be feasible in the future.

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